$$\frac{T}{T_c} = \frac{1}{\sqrt{2}}$$

$$\text{or} \qquad T = \frac{1}{\sqrt{2}} \times T_c = 0.707T_c$$

## 3.8 Exercises

1. Estimate the current through a thin wire of diameter 2mm at 2K, if $T_c$ =3.722K and critical field at 0K is 0.0305T
*Hint:* $I_c$ = 0.000136A

2. The lead material works as superconductor at a temperature of $T_c$ = 7.2K. If the constant characteristic of the lead material at 0K is $H_0$ = $\frac{8\times10^5}{4\pi} A/m$. What is the value of the magnetic field in the lead at 5K?
*Hint:* $3.18 \times 10^4$A/m

3. Calculate the critical current for a superconducting wire of lead having a diameter of 1mm at 4.2K. Critical temperature for lead is 7.18K and $H_c(0)$= $6.5 \times 10^4$ A/m.
*Hint:* $H_c = 42.758A/m$ and $I_c = 134.26A$

4. The critical temperature of Nb is 9.15K. At zero kelvin the critical field is 0.196T. Calculate the critical field at 6K.
*Hint:* $H_c = 0.112T$

5. For a specimen of a superconductor, the critical fields are respectively $1.4 \times 10^5$ A/m and $4.2 \times 10^5$ A/m for 14K and 13K. Calculate the transition temperature and critical field at 0K and 4.2K.
*Hint:* $T_c$ = 14.4K; $H_{c(0)}$ = $2.19 \times 10^5$A/m; $H_{c(4.2)}$ = $20.05 \times 10^5$A/m

# Chapter 4

# Dielectric Materials

*Dielectrics* or (insulators) as distinct from conductors have no free charges and they are non conducting materials. When they are placed in electric field, they undergo considerable changes which make them act as stores of electrical charges. A good dielectric is always an insulator. So any insulator is a dielectric.

*An insulator is a nonconducting material and its main function is to provide electrical insulation.*

*A dielectric material is an electrical insulator that can be polarized by an applied electric field and its main function is charge storage.*

A dielectric may be described in terms of the energy band structure.

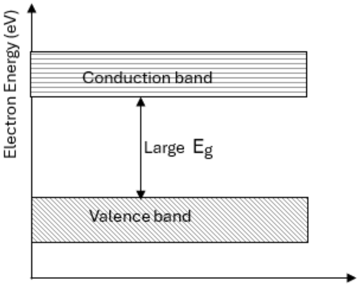


Figure 4.1: Energy band diagram of dielectric

The forbidden gap $E_g$ as shown in figure 4.1 is very large in dielectrics and excitation of electrons from the normally filled valence band to the empty conduction band is not possible under ordinary conditions. Therefore, conduction cannot occur in a dielectric. Even if a dielectric contains impurities, extrinsic conduction such as the one observed in doped semiconductors is not possible. The resistivity of an ideal dielectric should be therefore infinitely high.

## 4.1 Dielectric Constant

A dielectric is chiefly characterised by its dielectric constant. *Dielectric constant of a dielectric* is defined and measured as the ratio of capacitance of a capacitor containing the dielectric medium to the capacitance of the same capacitor with air as the medium.

$$\epsilon_r = \frac{C}{C_0} \tag{4.1}$$

where $C_0$ is the capacitance with air as the medium between the plates of the capacitor; C is the capacitance with dielectric as medium;

*Dielectric constant $\epsilon_r$ is also called as relative permittivity.*

Dielectric constant is a dimensionless quantity which is always greater than unity in case of dielectrics. It is independent of the size and shape of the dielectric.

*Dielectric constant or relative permittivity describes the ability of the dielectric material to store electric charges.* Also, dielectric constant is measured as the ratio of absolute permittivity of the medium ($\epsilon$) and permittivity of free space ($\epsilon_0$).

$$\text{Dielectric constant } \epsilon_r = \frac{\text{Permittivity of the medium}(\epsilon)}{\text{Permittivity of free space}(\epsilon_0)}$$

$$\text{Dielectric constant } \epsilon_r = \frac{\epsilon}{\epsilon_0} \tag{4.2}$$

Dielectric constant is a dimensionless quantity and it has no unit. The value of $\epsilon_r$ = 1 for air or vacuum.

*Permittivity of the medium* indicates easily polarizable nature of the material. Its unit is Farad/metre (F/m). $\epsilon = \epsilon_0 \epsilon_r$
Here $\epsilon_0 = 8.854 \times 10^{-12}$ F/m represents the permittivity of free space.

## 4.2 Dielectric Polarization

The electric behaviour of a dielectric material can be changed by the application of an external electric field.

When an external electric field is applied to a dielectric, it exerts a force on both positive and negative charged particles of each atom present in the dielectric material. The positive charges are pushed in the same direction as that of the electrical field while negative charges are pushed in the opposite direction as shown in figure 4.2.

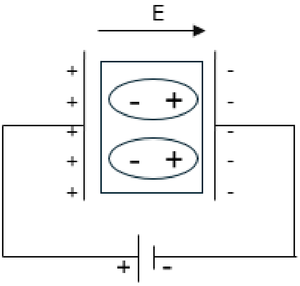


Figure 4.2: Electric Polarization

Consequently, the centres of positive and negative charges of each atom are displaced from their equilibrium positions in opposite directions.

Each atom or molecule then acts as an elementary dipole and acquires an electric dipole moment in the direction of the field. The cumulative effect of formation of such dipoles is that negative charge is induced by the electric field on the dielectric surface adjacent to the positive capacitor plate while a positive charge of equal magnitude is induced on the dielectric surface adjacent to negative capacitor plate.

Thus, the action of the electric field on dielectric is to induce charges on its surfaces. When charges of opposite polarity are induced on the surfaces of a dielectric, the dielectric is said to be polarized. The effect is known as dielectric polarization. The polarized dielectric is equivalent to a big dipole consisting of polarization charges separated by a distance d, which is the thickness of the slab.

*When a dielectric is placed in an electric field, electric charges slightly shift from their average equilibrium positions and induce charges on its surfaces and is known as dielectric polarization.*

*The intensity of polarization P is defined as the total dipole moment per unit volume of the material.*
The intensity of polarization is given by

$$P = \frac{\sum d\mu}{V} \tag{4.3}$$

where $\mu$ is the dipole moment (the product of the magnitude of the charge and the distance between the two charges) and V is the total volume of the dielectric.

Consider a polarized dielectric as big dipole consisting of induced charges separated by a distance d. Then,

$$\text{Dipole moment} \quad \mu = (A\sigma_p)d = \sigma_p V \tag{4.4}$$

where A is the area of the slab and $\sigma_p$ is the surface charge density due to polarization. Comparing the equation (4.3) and equation (4.4), we find that

$$P = \sigma_P \tag{4.5}$$

It follows that the polarization is equal to the surface density of the induced charges in a dielectric.

The effect of polarization is to reduce the magnitude of the external field $E_0$. The induced surface charges on the dielectric give rise to an induced electric field $E_i$ which opposes the external field $E_0$.

Therefore, the net electric field E in the dielectric has a magnitude given by

$$E = E_0 - E_i \tag{4.6}$$

## Polarizability $(\alpha)$

It is found that the average dipole moment $(\mu)$ of a system is proportional to the applied electric field E.
That is $\mu \propto E$ or $\mu = \alpha E$
where $\alpha$ is the proportionality constant and is known as the polarizability.

$$\alpha = \frac{\mu}{E}$$

*Polarizability* is defined as the ratio of average dipole moment to the electric field applied. Its unit is Farad metre$^2$.

### Polarization Vector

*Polarization vector* is defined as the average dipole moment per unit volume of a dielectric. If N is the number of atoms per unit volume and $\mu$ is average dipole moment per atom, then,

$$\text{Polarization vector} \quad P = N\mu \tag{4.7}$$

## 4.3 Dielectric Susceptibility

The magnitude of polarization is directly proportional to the intensity of the electric field.

$$P = \chi\epsilon_0 E \tag{4.8}$$

$\chi$(chi) is the proportionality constant and is called the dielectric susceptibility of the material. Dielectric susceptibility characterizes the ease with which a dielectric material can be influenced by an external electric field.

The *dielectric susceptibility* is a dimensionless proportionality constant that indicates the degree of polarization of a dielectric material in response to an applied electric field. It is a measure of polarization produced in the material per unit electric field.

## 4.4 The Three Field Vectors

The three field vectors electric field (E), electric displacement (D) and polarization (P) are related by

$$D = \epsilon_0 E + P \tag{4.9}$$

Electric displacement vector (D) is used for analysing electrostatic fields in the presence of dielectrics. This vector is related to the free charges that are stored on the capacitor plates. Polarization (P) is related to the bound charges only. The expression for the vector (E) shows that it is connected with both types of charges present.

## 4.5 Relation between $\epsilon_r$ and $\chi$

In order to describe the combined effects of the applied electric field E and electric polarization P, we have introduced displacement vector (D).

$$D = \epsilon_0 E + P$$

Substituting the expression for P from equation (4.8), we get

$$D = (1+\chi)\epsilon_0 E$$

We have $D = \epsilon E = \epsilon_0 \epsilon_r E$
Then $\epsilon_0 \epsilon_r E = (1+\chi)\epsilon_0 E$
Hence,

$$\epsilon_r = 1 + \chi \tag{4.10}$$

## 4.6 Relation between P and E

The electric displacement vector is used for analysing electrostatic fields in the presence of dielectrics. It is given by

$$D = \frac{q}{4\pi r^2} \tag{4.11}$$

We know that the electric field intensity, $E = \dfrac{q}{4\pi\epsilon r^2}$ (4.12)

Or

$$\epsilon E = \frac{q}{4\pi r^2}$$

From equations (4.11) and equation(4.12), we have

$$D = \epsilon E$$

$$D = \epsilon_0 \epsilon_r E \tag{4.13}$$

From equation (4.10), we have $\epsilon_r = 1 + \chi$, then

$$D = \epsilon_0 (1+\chi) E \tag{4.14}$$

where $\chi$ is electrical susceptibility.
i.e.,

$$D = \epsilon_0 E + \epsilon_0 \chi E$$

Since $P = \epsilon_0 \chi E$, we have

$$D = \epsilon_0 E + P \tag{4.15}$$

Equating equation (4.13) and equation (4.15), we have

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$P = \epsilon_0 (\epsilon_r - 1) E \tag{4.16}$$

This is the relation between electric field (E) and electric polarization (P).

## 4.7 Types of Polarization

Dielectric polarization is classified into four basic types.

(i) Electronic polarization

(ii) Ionic polarization

(iii) Orientation polarization

(iv) Space charge polarization

### 4.7.1 Electronic Polarization

This is the polarization that results from the displacement of the electron clouds of atoms, molecules and ions with respect to heavy fixed nuclei to a distance that is less than the dimensions of the atoms, molecules and ions. The electronic polarization sets in a short period of time, of the order of $10^{-14}$ to $10^{-15}$ seconds. It is independent of temperature.

Consider an atom placed in an electric field E. The electron cloud is acted on by a force and let it shift from the centre of the positive nucleus by *x*.

The induced dipole moment is *(Ze)x* where *Ze* is the nuclear charge. Assuming that the negative charge is uniformly distributed over a sphere of radius R, then the charge on the sphere of radius *x* is obtained as follows.

If $\rho$ is the charge density, then

$$\rho = -\frac{Ze}{\frac{4}{3}\pi R^3} = -\frac{3Ze}{4\pi R^3}$$

Now the charge in the sphere of radius *x* is q' and is given by

$$q' = \rho \times \frac{4}{3}\pi R^3 = -\frac{3}{4}\left[\frac{Ze}{\pi R^3}\right]\left[\frac{4}{3}\pi x^3\right] = -\frac{Zex^3}{R^3}$$

Now the coulomb force of attraction between the displaced electron cloud and positive nuclear charges is given by

$$F = \frac{1}{4\pi\epsilon_0} \times -\frac{(Ze)(Zex^3/R^3)}{x^2} = \frac{Z^2e^2x}{4\pi\epsilon_0 R^2} \tag{4.17}$$

In the equilibrium position, the restoring force ($F = -ZeE$) is equal and opposite to the Coulomb force. Then

$$-ZeE = \frac{Z^2e^2x}{4\pi\epsilon_0 R^2}$$

Therefore,

$$x = \frac{4\pi\epsilon_0 R^3 E}{Ze} \tag{4.18}$$

Here, we can conclude that the displacement of the electron cloud is proportional to the applied field E.
A dipole moment $\mu$ is induced in the atom due to the displacement and is given by

$$\begin{aligned}
\mu_{ind} &= (Ze)x \\
&= (Ze) \times \frac{4\pi\epsilon_0 R^3 E}{Ze} \\
\mu_{ind} &= 4\pi\epsilon_0 R^3 E \\
\mu_{ind} &= \alpha_e E
\end{aligned}$$

where $\alpha_e = 4\pi\epsilon_0 R^3$ is the electronic polarizability.

The dipole moment for unit volume is called electronic polarization. It is obtained as follows.

$$P_e = N\alpha_e E$$

where N is the number of atoms/$m^3$.

From equation (4.16), we know, $P = E\epsilon_0(\epsilon_r - 1) = N\alpha_e E$

$$(\epsilon_r - 1) = \frac{N\alpha_e}{\epsilon_0} \tag{4.19}$$

Equation (4.19) shows that electronic polarization induced by an electric field is independent of temperature. Thus we conclude that

- Electronic polarization is independent of temperature.
- Electronic polarization is proportional to the volume of atoms in the material ($\alpha_e \propto R^3$).
- Electronic polarization takes place in all dielectrics

### 4.7.2 Ionic Polarization

*Ionic polarization* occurs in ionic crystals. It occurs due to the displacement of positive and negative ions from their equilibrium positions.

Let us take the example of sodium chloride crystal. A sodium chloride molecule consists of $Na^+$ ions bound to $Cl^-$ ions through ionic bond. If the interatomic distance is *d*, the molecule exhibits an intrinsic dipole moment equal to *ed*. When a dc electric field is applied to the molecule, the sodium and chlorine ions are displaced in opposite directions as seen in the figure 4.3 until ionic bonding stop the process. The dipole moment of the molecule increases subsequently. When the field direction is reversed the ions move closer and again the dipole moment undergoes a change.

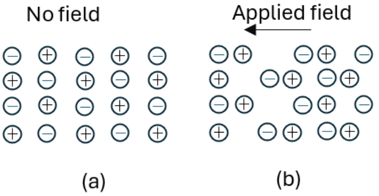


Figure 4.3: Ionic Polarization

Thus dipoles are induced. The induced dipole moment is proportional to the applied field and is expressed as $\mu_i = \alpha_i E$ where $\alpha_i$ is known as ionic polarizability.

## Expression For Ionic Polarization

Consider a NaCl crystal kept in an external electric field E. Due to the action of electric field, the positive ion ($Na^+$) ions with mass $M$ displace in the direction of electric field through a distance of say $x_1$ units and the negative ion ($Cl^-$) ions with mass $m$ displace in the direction opposite to that of the field through a distance of, say $x_2$ units.

The net displacement of the ions is $x = x_1 + x_2$
The force acting on $Na^+$ ions due to electric field is +eE and that acting on $Cl^-$ ions is -eE.
Also there will be a restoring force on both ions which is $-k_1x_1$ for $Na^+$ ions and $+k_2x_2$ for $Cl^-$ ions.

The shifting of ions occurs until the equilibrium is attained when the electric forces and restoring forces are equal and opposite to each other. That is when

$$eE = k_1x_1 \qquad \text{and} \qquad eE = k_2x_2$$

We have

$$k_1 = M\omega_0^2 \qquad \text{and} \qquad k_2 = m\omega_0^2$$

Using the above equations, we get,

$$x = \frac{eE}{M\omega_0^2} + \frac{eE}{m\omega_0^2}$$

$$x = \frac{eE}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] \tag{4.20}$$

The induced dipole moment $\mu = ex$

$$\mu = \frac{e^2E}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] \tag{4.21}$$

Since $\mu = \alpha_i E$, we have ionic polarizability

$$\alpha_i = \frac{e^2}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] \tag{4.22}$$

From this, we can conclude that

- Ionic polarizability ($\alpha_i$) is inversely proportional to the square of angular frequency of ionic molecule.
- It is directly proportional to the reduced mass of the molecule $\left[\frac{1}{M}+\frac{1}{m}\right]$.
- It does not depend on temperature.
- It occurs in ionic substance.

The most materials, the ionic polarizability is less than the electronic polarizability and is given as

$$\alpha_i = \frac{\alpha_e}{10}$$

The ionic polarization is given as

$$P_i = N\alpha_i E$$

$$P_i = \frac{Ne^2}{\omega_0^2}\left[\frac{1}{M}+\frac{1}{m}\right] \tag{4.23}$$

Ionic polarization takes $10^{-11}$ to $10^{-14}$ seconds to build up, and is not influenced by temperature.

## 4.8 Orientation Polarization

*Orientation polarization* is the polarization arising from the orientation of molecules which have permanent dipoles occurring from an asymmetric charge distribution. It is also known as dipole polarization.

In the absence of external electric field, the orientation of dipoles is random resulting in a complete cancellation of each other's effect as shown in figure 4.4.

In the presence of electric field, the molecular dipoles rotate about their axis of symmetry to align with the applied field. In the case of electronic and ionic polarizations, the force due to the external field is balanced by a restoring force due to coulomb attractions. But for orientation polarization, restoring forces do not exist. However, the dipole alignment is counteracted by thermal agitation. The higher the temperature, the greater is the thermal agitation.

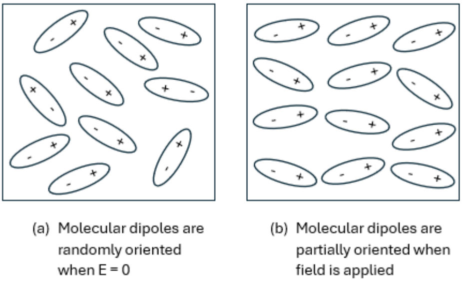


Figure 4.4: Orientation polarization

The orientation polarization occurs in gases, liquids and amorphous viscous substances. In case of solids, the molecules are fixed in their positions and their rotation is highly restricted by the lattice forces. Hence orientation polarization have no contribution to the total polarization in solids. Because of this reason, the dielectric constant of the water is about 80, but that of solid is 10.

The process of orientation polarization takes relatively longer time than the electronic and ionic polarization.

The total orientation polarization of the dielectric is

$$P_o = N\alpha_o E = \frac{N\mu^2 E}{3kT}$$

$\alpha_o$ is the orientation polarizability and is given by $\alpha_o = \dfrac{\mu^2}{3kT}$

Orientation polarizability occurs only in polar dielectrics such as $H_2O$, HCl and CO etc.

## 4.9 Space Charge Polarization

*Space charge Polarization* is the polarization of a dielectric which occurs when charge carriers are present which can migrate an appreciable distance through the dielectric but which become trapped or cannot discharge at an electrode.

Space charge polarization is very small when compared to other polarization mechanisms and it is not common in most of the dielectrics. This type of polarisation occures in ferrites and semiconductors.

### 4.9.1 Total Polarization

In a material, which can experience all forms of polarization, the total polarization is equal to the sum of the electronic, ionic, orientation and space charge polarization. In general, space charge polarization is very small and negligible. Therefore, total polarization in a material is

$$P_{total} = P_e + P_i + P_o$$

$$P_{total} = N\left[4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}\right]E$$

Therefore, the total polarizability is given by

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

$$\alpha = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}$$

Often one or more of the contributions to the polarization may be either absent or insignificant. For example, orientation polarization does not present in non polar dielectrics. In the same way, ionic polarization is not found in covalently bonded materials. Electronic polarization will be negligible compared to orientation polarization in polar dielectrics.

## 4.10 Internal Field in Solids And Liquids

In gaseous state, the atoms are separated by large distances and the interaction between the atoms can be neglected. When an external electric field E is applied, the intensity of the electric field experienced by an atom in gaseous state will be equal to the applied electric field E.

But in solids and liquids, the atoms are close to each other leading to strong interaction between them. In solids and liquids the intensity of the electric field at a given point of the material is not equal to the applied electric field but is equal to internal field which is the sum of applied electric field and field due to the neighbouring atoms present in the material. This resultant electric field acting on the atom or molecules of dielectric substance is called local field (or) *internal field.*

The local field is calculated by using the method suggested by Lorentz. According to this method consider a small spherical region of the dielectric with an atom at the center of the sphere for which the local field is to be

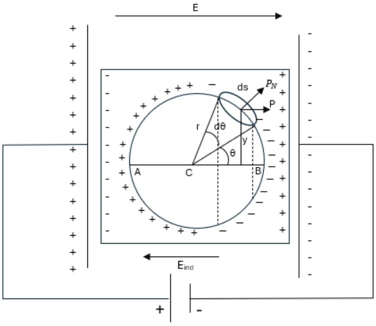


Figure 4.5: Calculation of internal field in solids

calculated.
The internal field $E_{int}$ at the atom site is considered to be the resultant of the following four components $E_1$, $E_2$, $E_3$, $E_4$.

$$E_{int} = E_1 + E_2 + E_3 + E_4 \tag{4.24}$$

$E_1 = \frac{\sigma}{\epsilon_0}$ is the field due to the charges on the surface of the capacitor plates with no dielectric medium where
$E_2$ is the field at A due to induced charge carriers on the surface of the dielectric. The value of $E_2$ is given as $E_2 = -\frac{\sigma_p}{\epsilon_0}$. Here $E_2$ is directed in the opposite direction of applied electric field $E_1$
$E_3$ is the field at A due to polarised charge carriers on the surface of the spherical cavity;
$E_4$ is the electric field at A due to electric dipoles of atoms inside the spherical cavity and is zero. Here $E_4$ = 0. Therefore,

$$E_{int} = \frac{\sigma}{\epsilon_0} + -\frac{\sigma_p}{\epsilon_0} + E_3 + 0$$

$\frac{\sigma}{\epsilon_0} + -\frac{\sigma_p}{\epsilon_0}$ represents the net electric field between the two plates and is considered as E. Then $E = E_1 + E_2$
Therefore

$$E_{int} = E + E_3 \tag{4.25}$$

*To find the value of $E_3$,*

Let us consider a small area ds on the surface of spherical cavity. It is confined within an angle $d\theta$ at an angle $\theta$ in the direction of electrical field E.

Polarisation (P) is parallel to E. $P_N$ is the component of polarization perpendicular to the area ds as shown in figure 4.5 .

$$P_N = Pcos\theta$$

$q'$ is the charge on the area ds.

Polarization is also defined as the surface charges per unit area.

$$P_N = Pcos\theta = \frac{q'}{ds}$$

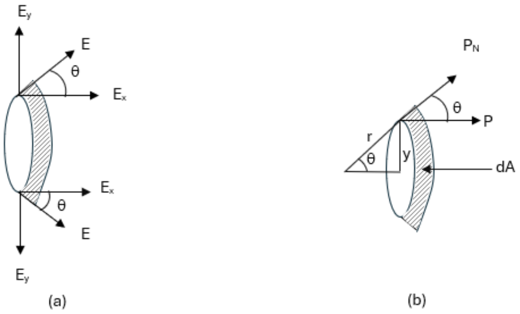
$$\text{Charge on ds,} \quad q' = Pcos\theta ds \tag{4.26}$$

Electric field intensity at C due to charge $q'$ is given by

$$E = \frac{q'}{4\pi\epsilon_0 r^2}$$

Substituting for $q'$ from equation (4.26) ,

$$\text{we have, ,} \quad E = \frac{Pcos\theta ds}{4\pi\epsilon_0 r^2} \tag{4.27}$$



(a) (b)

Figure 4.6: Computation of Lorentz field

This electric field intensity is along the radius r and it is resolved into two components ($E_x$ and $E_y$) as shown in figure 4.6 (a).
The component of intensity parallel to the electric field direction

$$E_x = Ecos\theta \tag{4.28}$$

Since the perpendicular component $E_y = Esin\theta$ are in opposite directions as in figure4.6 (a), they cancel out each other. Hence the parallel components alone are taken into consideration. Substituting E from equation (4.26) and equation (4.27), we have

$$E_x = \frac{Pcos\theta dscos\theta}{4\pi\epsilon_0 r^2}$$

$$E_x = \frac{Pcos^2\theta ds}{4\pi\epsilon_0 r^2} \tag{4.29}$$

Now consider a ring of area dA which is obtained by revolving ds about AB as shown in figure 4.6 (b).
Ring area dA = Circumference × Thickness

$$dA = 2\pi y \times rd\theta$$

$$dA = 2\pi rsin\theta \times rd\theta \tag{4.30}$$

The electric field intensity due to elemental ring dA is

$$dE = \frac{Pcos^2\theta dA}{4\pi\epsilon_0 r^2} \tag{4.31}$$

Substituting equation (4.30) in equation (4.31) ,

$$dE = \frac{Pcos^2\theta}{4\pi\epsilon_0 r^2} \times 2\pi r^2 sin\theta d\theta \tag{4.32}$$

Therefore, electrical field intensity due to charges present in the whole sphere is obtained by integrating the equation (4.32) within the limits 0 to

$\pi$. This electric field is taken as $E_3$.

$$E_3 = \int_0^\pi \frac{Pcos^2\theta sin\theta d\theta}{2\epsilon_0}$$

$$E_3 = \frac{P}{2\epsilon_0}\int_0^\pi cos^2\theta sin\theta d\theta$$

since, $\int_0^\pi cos^2\theta sin\theta d\theta = \frac{2}{3}$

$$E_3 = \frac{P}{2\epsilon_0} \times \frac{2}{3}$$

$$\text{Therefore,} E_3 = \frac{P}{3\epsilon_0} \tag{4.33}$$

Substituting equation (4.33) in equation (4.25), we get,

$$E_{int} = E + \frac{P}{3\epsilon_0} \tag{4.34}$$

This equation shows that $E_{int}$ is larger than the macroscopic field intensity E. Hence the molecules are more effectively polarised.

## 4.11 Clausius-Mosotti Equation

Let N be the number of molecules per unit volume and $\alpha$ be the molecular polarizability.

$$\text{Total polarization P } = N\alpha E_{int}$$

$$E_{int} = \frac{P}{N\alpha} \tag{4.35}$$

We know that

$$D = \epsilon E = \epsilon_0 E + P$$

$$\epsilon E - \epsilon_0 E = P$$

$$(\epsilon - \epsilon_0)E = P$$

$$E = \frac{P}{\epsilon - \epsilon_0} \tag{4.36}$$

Lorentz field is given by

$$E_{int} = E + \frac{P}{3\epsilon_0} \tag{4.37}$$

Substituting equation (4.36) in equation (4.37), we have

$$\begin{aligned} E_{int} &= \frac{P}{\epsilon - \epsilon_0} + \frac{P}{3\epsilon_0} \\ &= P\left[\frac{1}{\epsilon - \epsilon_0} + \frac{1}{3\epsilon_0}\right] \\ &= \frac{P}{N\alpha}\left(\frac{3\epsilon_0 + \epsilon - \epsilon_0}{3\epsilon_0(\epsilon - \epsilon_0)}\right) \\ &= \frac{P}{N\alpha}\left(\frac{\epsilon + 2\epsilon_0}{\epsilon - \epsilon_0}\right) \end{aligned} \tag{4.38}$$

Equating equations (4.35) and equation (4.38), we have

$$\begin{aligned} \frac{P}{N\alpha} &= \frac{P}{3\epsilon_0}\left(\frac{\epsilon + 2\epsilon_0}{\epsilon - \epsilon_0}\right) \\ \text{or} \quad \frac{N\alpha}{3\epsilon_0} &= \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \\ &= \frac{\left(\frac{\epsilon}{\epsilon_0}\right) - 1}{\left(\frac{\epsilon}{\epsilon_0}\right) + 2} \end{aligned}$$

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \text{where } \epsilon_r = \frac{\epsilon}{\epsilon_0} \tag{4.39}$$

$\epsilon_r$ is dielectric constant. This is *Clausius-Mosotti Equation*. This relates the dielectric constant of the material with polarizability.

## 4.12 Dielectric Loss

When a dielectric is applied with AC voltage, the electrical energy is absorbed by the dielectric and certain quantity of electrical energy is dissipated in the form of heat energy. This dissipation of electrical energy is known as *dielectric loss*.

Dielectric loss can occur in both direct and alternating voltages. It is less in direct voltage than that of alternating voltage.

When an AC voltage is applied to a perfect dielectric like vacuum or purified gas, it does not absorb electrical energy and there is no loss of electrical energy.

**The origin of dielectric loss:** An ac field changes its direction with time. With each direction reversals, the molecules are required to follow the field reversals in order to contribute to the polarization of the dielectric. When a capacitor is charged in one half cycle, the molecules of the dielectric medium are polarized. When the capacitor is discharged in the second half cycle, the molecules should revert to their initial condition. While reverting to its initial state, the molecules push each other and lose energy due to friction. The energy lost due to friction takes the form of heat. This energy loss will increase with increase in frequency.

Dielectric loss can be represented in terms of *loss tangent* $tan\delta$. It represents the electrical power lost which is often in the form of heat. Hence it is also called dissipation factor.

$$tan\delta = \frac{I_a}{I_d} = \frac{V/R}{\omega CV} = \frac{1}{\omega CR}$$

## 4.13 Dielectric Breakdown

When a dielectric is subjected to a very high electric fields, a considerable number of covalent bonds may be broken and electrons may get excited to energies within the conduction band. These electrons acquire a large kinetic energy and cause localized melting, burning and vapourization of material leading to irrreversible degradation and failure of the material. This results in high electrical conductivity and total loss of the charge storage property of the dielectric. This phenomenon is called *Dielectric Breakdown*.

*Dielectric strength:* It is a measure of the ability of that material to withstand high electric fields.

*Dielectric strength is defined as the maximum electric field that the dielectric can withstand without suffering electrical breakdown.* It is the breakdown voltage per unit thickness of the material.

Dielectric strength depends on the thickness of the insulating material and on the length of time for which the dielectric is subjected to electric field.

Dielectric strength, $$E_{max} = \frac{\text{Breakdown voltage}(V_{max})}{\text{Thickness of the dielectric}(d)}$$

**Types of Dielectric Breakdown**

Some important types of dielectric breakdown are

- Intrinsic breakdown and avalanche breakdown
- Thermal breakdown
- Chemical and electrochemical breakdown
- Discharge breakdown
- Defect breakdown

## 4.14 Solved Numerical Problems

**Example 4.14.1** When NaCl crystal is subjected to an electric field of 50V/cm, the resulting polarization is $2.215 \times 10^{-7} C/m^2$. Calculate relative permittivity of NaCl.

**Solution:**

Given $E = 50V/cm = 50V/10^{-2}m = 50 \times 10^2 V/m$;
$P = 2.215 \times 10^{-7} C/m^2$

$$P = \epsilon_0(\epsilon_r - 1)E$$

$$\epsilon_r = 1 + \frac{P}{\epsilon_0 E}$$

$$\epsilon_r = 1 + \frac{2.215 \times 10^{-7}}{8.85 \times 10^{-12} \times 50 \times 10^2} = 5.0056$$

**Example 4.14.2** Calculate the electronic polarizability of argon atom. Given $\epsilon_r$ = 1.0024 at NTP and N = $2.7 \times 10^{25}$ atoms/$m^3$.

**Solution:**

**Given** $N = 2.7 \times 10^{25}$ **and** $\epsilon_r$ **= 1.0024**

**We have** $P = \epsilon_0(\epsilon_r - 1)E$ **and** $P = N\alpha_e E$

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

$$\alpha_e = \frac{(8.85 \times 10^{-12})(1.0024 - 1)}{2.7 \times 10^{25}} = 7.9 \times 10^{-40} F.m^2$$

**Example 4.14.3** The atomic weight and density of sulphur are 32 units and $2.08 gm/cm^3$ respectively . The electronic polarizability of the atom is $3.28 \times 10^{-40} Fm^2$. If sulphur solid has cubic symmetry, what will be its relative permittivity?

**Solution:**

**We know that a volume of $\frac{M}{\rho}$ contains Avagadro number $(N_a)$ of atoms.**

**Then $1cm^3$ has $\frac{\rho N_A}{M}$ = N atoms.**

$$N = \frac{2 \times 6.02 \times 10^{23}}{32} = 0.37 \times 10^{23}/cm^3 = 0.37 \times 10^{29}/m^3$$

**From Clausius-Mosotti relation,**

$$\begin{aligned} \frac{N\alpha}{3\epsilon_0} &= \frac{\epsilon_r - 1}{\epsilon_r + 2} \\ &= \frac{0.37 \times 10^{29} \times 3.28 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} = 0.4 \\ \epsilon_r &= 3 \end{aligned}$$

**Example 4.14.4** An atom of oxygen on being polarized produces a dipole moment of $0.5 \times 10^{-22}$Cm. If the distance of the centre of negative charge cloud from the nucleus be $4 \times 10^{-17}$m. Calculate the polarizability of oxygen atom.

**Solution:**

**We have**

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

**For oxygen atom** $q = 8e = 8 \times 1.6 \times 10^{-19}$

$$E = \frac{8 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (4 \times 10^{-17})^2} = 2.6 \times 10^{24} V/m$$

**Dipole moment** $\mu$ **=** $\alpha E$ **or** $\alpha = \dfrac{\mu}{E}$

$$\alpha = \frac{0.5 \times 10^{-22}}{2.6 \times 10^{24}} = 0.19 \times 10^{-46} Fm^2$$

**Example 4.14.5** Find the electronic polarizability of Helium gas for a given dielectric constant is 1.0000684 and $N = 2.7 \times 10^{25} atoms/m^2$.
**Solution:**

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\alpha = \frac{3\epsilon_0}{N}\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)$$

$$\begin{aligned}\alpha &= \frac{3 \times 8.85 \times 10^{-12}}{2.7 \times 10^{25}}\left(\frac{1.0000684 - 1}{1.0000684 + 2}\right)\\ &= 2.24 \times 10^{-41} C^2m^2/N\end{aligned}$$

**Example 4.14.6** A solid contains $5 \times 10^{28}$ atoms/$m^3$ each with a polarizability of $2 \times 10^{-40} Fm^2$. Assuming that the internal field is given by Lorentz formula. Calculate the ratio of internal field to the external field. $\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$.
**Solution:**
**Given number of atoms** $N = 5 \times 10^{28}$ **atoms/**$m^3$ **; Polarizability** $\alpha = 2 \times 10^{-40} Fm^2$

**We know Lorentz field is given by**

$$E_i = E + \frac{P}{3\epsilon_0}$$

$$P = N\alpha_e E_i$$

$$E_i = E + \frac{N\alpha_e E_i}{3\epsilon_0}$$

$$E_i\left[1 - \frac{N\alpha_e}{3\epsilon_0}\right] = E$$

$$\frac{E_i}{E} = \frac{1}{\left[1 - \frac{N\alpha_e}{3\epsilon_0}\right]}$$

$$\frac{E_i}{E} = \frac{1}{\left[1 - \frac{5 \times 10^{28} \times 2 \times 10^{\breve{}40}}{3 \times 8.85 \times 10^{-12}}\right]}$$

$$\frac{E_i}{E} = 1.604$$

## 4.15 Exercises

1. The number of atoms in hydrogen gas is $9.8 \times 10^{20}$ atoms/cc. The radius of hydrogen atom is 0.053nm. Calculate its electronic polarizability and relative permittivity.
*Hint:* $1.67 \times 10^{-41} C^2 m^2/N$

2. A monoatomic gas contains $3 \times 10^{25}$ atoms/$m^3$ at a certain temperature at one atmosphere pressure. The radius of the atom is 0.19 nm. What is the polarizability of the atom
*Hint:* $7.63 \times 10^{-40} C^2 m^2/N$

3. Find the total polarizability of $CO_2$ if its susceptibility is $0.985 \times 10^{-3}$