INTERFERENCE

When two or more light waves of the same frequency and amplitude with a constant phase difference travel in the same direction, superimpose upon each other, the resultant intensity of light is not distributed uniformly in space. This non uniform distribution of the light intensity due to the superposition of light waves is called interference.

Principle of Superposition of Waves

When two or more light wave travels simultaneously in a medium, the resultant displacement at any point is the vector sum of the displacement due to each wave. This is the principle of superposition of waves and this forms the basis of interference.

Consider two waves travelling simultaneously in a medium. At any point, let y_1 be the displacement due to one wave at any instant and let y_2 be the displacement of the other wave at the same instant.

Then resultant displacement is $y = y_1 \pm y_2$. Here + ve sign is taken when both waves are in same direction and –ve sign is taken when both waves are in opposite direction.

Constructive and Destructive Interference

When two light waves superimpose at points where crest falls on crest and trough falls on trough, the amplitude adds up and intensity of light increases. [intensity $\propto (\text{Amplitude})^2$]. This is known as *constructive interference*. i.e., The superposition of waves resulting in maximum intensity is called constructive interference.

When two light waves superimpose at points where crests falls on trough or trough falls on crests, amplitude are reduced (i.e., resultant amplitude is zero). This is known as *destructive interference*. i.e., The superposition of waves resulting in minimum intensity is called destructive interference.

So when two or more light waves interfere, we get alternate dark and bright bands of equal width. These bands are called interference fringes.

Condition for constructive interference

When the path difference between two interfering beams is either zero or integral multiple of λ, constructive interference occurs.

Hence path difference $\Delta = n\lambda$ where $n = 0, 1,2, 3,\ldots\ldots$

Phase difference = $2n\pi$ where $n = 0, 1,2, 3,\ldots\ldots$

Condition for destructive interference

When the path difference between two interfering beams is odd multiple of $\frac{\lambda}{2}$, destructive interference occurs.

Hence path difference $\Delta = (2n+1)\frac{\lambda}{2}$ where $n = 0, 1,2, 3,\ldots\ldots$

Phase difference = $(2n+1)\,\pi$ where $n = 0, 1,2, 3,\ldots\ldots$

Condition for sustained interference

- The two light sources must be monochromatic, they must emit light of same wavelength or frequency.
- The two sources must be coherent.
- The two sources must be very close to each other.
- Light waves from the two sources should superimpose at the same point and at the same place.

Optical Path

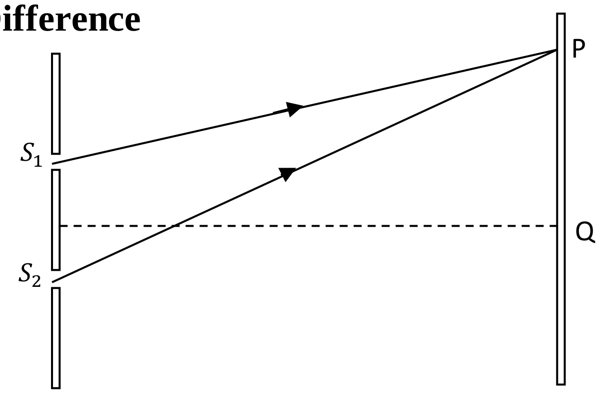
The distance travelled by light through a medium is optical path.

Optical path = geometrical path $\times \mu$ where μ is refractive index. For air, μ = 1.

Relation Between Path Difference and Phase Difference

The difference between optical path of two rays which are in constant phase difference with each other is known as path difference. In the figure, path difference between light waves emerging from two sources S_1 and S_2 is $\Delta = S_2P - S_1P$

Phase difference (ϕ) = $\frac{2\pi}{\lambda}$ × path difference (x)

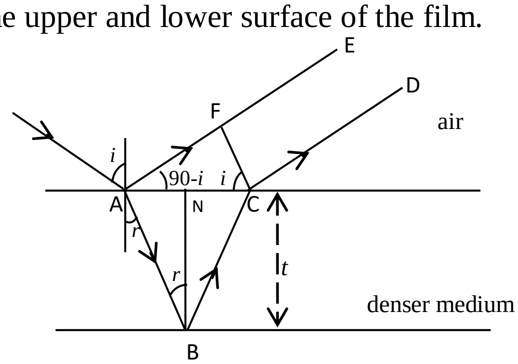


Interference in thin films – Reflected system

We see beautiful colours on oil films or soap bubbles. This is due to interference phenomenon involving multiple reflections. Here the method involved is interference by division of amplitude. The light reflected from the upper and lower surfaces of a thin film interferes and interference patterns are produced.

Interference due to reflected light

Consider a thin film of thickness 't' and refraction index μ. Let light from a monochromatic source be incident on the surface of the film. Reflections takes place from the upper and lower surface of the film.



Optical path difference between two beams is

(AB + BC) in film - AF in air

Since AB = BC,

Optical path difference = (2AB × μ) − AF → (1)

To find AB,

Consider Δ ABN, $\cos r = \frac{BN}{AB} = \frac{t}{AB}$

So, $AB = \frac{t}{\cos r}$ → (2)

To find AF, consider Δ AFC

$$AF = AC \sin i \rightarrow (3)$$

We have, AC = 2 AN

To find AN, consider Δ ABC

$\tan r = \frac{BN}{AN} = \frac{t}{AN}$

AN = $t \tan r$

Therefore, AC = $2t \tan r$

Hence eqn (3) becomes, AF = $2t \tan r \sin i$

By Snell's law, $\mu = \frac{\sin i}{\sin r}$; $\sin i = \mu \times \sin r$

Hence AF = $2t \tan r \times \mu \sin r = 2t \frac{\sin r}{\cos r} \times \mu \sin r$

$$AF = 2\mu t \times \frac{\sin^2 r}{\cos r} \rightarrow (4)$$

Substituting eqn (3) and (4) in eqn (1), we get

Optical path difference $= \frac{2\mu t}{\cos r} - 2\mu t \times \frac{\sin^2 r}{\cos r} = \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r}(\cos^2 r)$

Optical path difference is $\mathbf{2\mu t \cos r}$.

This is known as Cosine's law.

When light is reflected from the surface of an optically denser medium, like the air film interface, then the reflected rays undergoes a path change of $\frac{\lambda}{2}$. (This is in the case of reflection only.)

Then the actual path difference is $\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

Condition for constructive interference

$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$ where n = 1,2,3…….

$$\mathbf{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$$

Condition for destructive interference

$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$ where n = 1,2,3…….

$$\mathbf{2\mu t \cos r = n\lambda}$$

Colour of thin films

When a beam of white light is incident normally on a thin film, different colours are seen. This is due to interference involving multiple reflections. The condition for destructive interference is $2\mu t \cos r = n\lambda$. At any region of the film, the colour with the wavelength which satisfies this condition will be dark .

For example, if blue colour satisfies this condition, blue colour will be absent and a combination of other colours will be seen at that region.

⇒ If the thickness of the film varies continuously in the case of oil film, the colour of the film continuously changes.

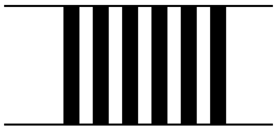
⇒ If we observe the film from different positions, the colour of the film will be varying as the angle of refraction *r* changes.

⇒ If the thickness of the film *t* is very small ($t = 0$), then destructive interference occur due to the additional path difference of $\frac{\lambda}{2}$ and film appears dark. If '*t*' is very large, almost all the colours will undergo constructive interference and white light is produced. Only a thin film of a few wavelengths thick, alone will show different colours.

⇒ The condition of brightness and darkness in the reflected system and transmitted system are opposite to each other.

Air Wedge

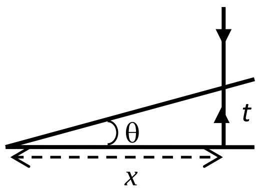
A wedge shaped film is constructed using two glass plates with one end placed in contact and the other end separated by a thin paper or wire of thickness '*t*'. A wedge shaped air film is formed between them. When the film is illuminated normally by monochromatic light, interference occurs between the rays reflected form the top and bottom surfaces of the film. As a result, a large number of equidistant parallel dark and bright bands are observed.



The spacing between two consecutive dark or bright fringes is called band width.

Interference produced by wedge shaped films

When the film is illuminated normally by monochromatic light, interference occurs between the rays reflected form the top and bottom surfaces of the film.



Let 't' be the thickness of film at a distance *x* from the edge.

We have path difference between two rays $\Delta = 2\mu t \cos r$

For a normal incidence, $r = 0$ (i.e., $\cos 0 = 1$) ,

Then path difference between the rays is $\Delta = 2\mu t$ → (1)

From figure, $t = x \tan\theta$ → (2)

Putting (2) in (1), we get $\Delta = 2\mu \times x \tan\theta$

The condition for destructive interference in thin film is $2\mu\, x \tan\theta = n\lambda$ where n = 0,1,2,3 ……

or $x = \dfrac{n\lambda}{2\mu \tan\theta}$ → (3)

If x_1 is the distance of the nth dark band from the edge and x_2 is the (n+m)th dark band,

then $x_1 = \dfrac{n\lambda}{2\mu \tan\theta}$ and $x_2 = \dfrac{(n+m)\lambda}{2\mu \tan\theta}$

Then band width $\beta = \dfrac{x_2 - x_1}{m} = \dfrac{\frac{(n+m)\lambda}{2\mu\tan\theta} - \frac{n\lambda}{2\mu\tan\theta}}{m} = \dfrac{m\lambda}{2\mu\, m\tan\theta}$

Band width $\beta = \dfrac{\lambda}{2\mu\tan\theta}$ $\rightarrow$ (4)

Since for air film, $\mu = 1$, $\boldsymbol{\beta = \dfrac{\lambda}{2\tan\theta}}$ $\rightarrow$ (5)

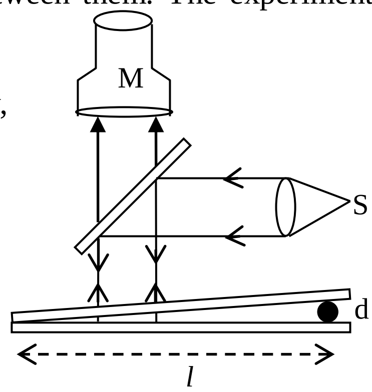
Since angle of wedge (θ) is too small, then $\tan\theta = \theta$, then $\boldsymbol{\beta = \dfrac{\lambda}{2\,\theta}}$ $\rightarrow$ (6)

The same relation holds good for bright bands also. Since the locus of all points having same thickness of the film being a straight line, we get straight line fringes.

Applications

Diameter of a thin wire

A thin wire is wound near one end of a plane glass plate. It is kept over another plane glass plate so that they touch along one edge and a wedge shaped air film is formed between them. The experimental arrangement for determining the diameter of the wire is as shown in figure.



When a parallel beam of monochromatic light is made to fall normally, we can observe a large number of straight parallel alternate bright and dark fringes using a microscope.

Let 'd' be the diameter of the wire and 'l' be the length of the wedge, then,

$\tan\theta = \dfrac{d}{l}$ $\rightarrow$ (1)

Band width $\beta = \dfrac{\lambda}{2\tan\theta} = \dfrac{\lambda}{2\, d/l} = \dfrac{l\,\lambda}{2\,d}$ $\rightarrow$ (2)

Diameter of thin wire, $\mathbf{d} = \boldsymbol{\dfrac{l\,\lambda}{2\,\beta}}$ $\rightarrow$ (3)

Testing of planeness of surfaces

Air wedge experiment can be used to test the optical planeness of surfaces. The surface to be tested is used as one of the plates in air wedge experiment. The other will be a standard plane surface. If the surface is optically plane, then straight fringes of equal thickness are observed. If the fringes are non uniform and distorted, the given surface is not optically plane.

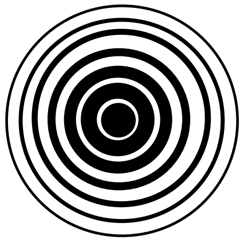
A surface is said to be optically plane, if it is flat up to $\frac{1}{10}^{\text{th}}$ of the wavelength of the light used. With the air wedge experiment, a flatness of a surface up to $\frac{1}{10}^{\text{th}}$ of the wavelength of the light used can be determined.

Newton's Rings

When a plano convex lens is placed on a plane glass plate, with its convex surface touching the plate, an air film of gradually increasing thickness is formed between the two. Light reflected from the top and bottom surfaces of the air film between lens and the glass plate interferes to produce interference pattern. The thickness of the film is zero at the point of contact at O and increases radially outwards.

The thickness of the air film will be constant over a circle and the pattern consists of concentric bright and dark rings. These rings are called **Newtons rings**.

Newton's rings consist of a set of alternate dark and bright rings with central spot dark.

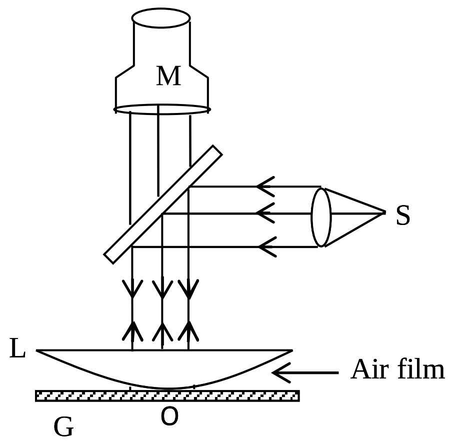


From the interference pattern, it can be seen that the centre spot is dark.

At the centre, i.e., at the point of contact, the thickness of the air film is zero. But the beam of light gets reflected from the upper surface of the glass plate G and undergoes an additional path difference of $\frac{\lambda}{2}$. This results in destructive interference. Hence the **central spot is dark**.

Experimental arrangement for Newton's rings:

Light from a monochromatic source S is allowed to fall on a glass plate kept at 45° to the incident beam. This beam is reflected normally on to the planoconvex lens L placed on a glass plate G.

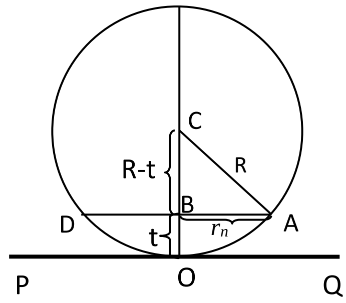


Light rays reflected from top and bottom surface of the air film interferes. Circular bright and dark fringes can be observed by looking through a travelling microscope focused on to the system.

The locus of points having the same thickness of the air film falls on a circle. Therefore fringes take the form of concentric rings.

Determination of radius of n^th ring

Let DOA is the lens placed on the glass plate PQ. DOA is a part of the spherical surface with centre C. Let R be the radius of curvature of the plano convex lens and r_n be the radius of the n^th ring. Let λ be the wavelength of the light used.



The condition for minimum intensity in thin interference film is

$2\mu t\cos r = n\lambda \quad \rightarrow (1)$

Since light is falling normally $r = 0$, and $\cos r = 1$ and for air film $\mu = 1$

So equation (1) becomes $2t = n\lambda \quad \rightarrow (2)$

In figure, from ΔABC,

$$R^2 = r_n^2 + (R - t)^2 = r_n^2 + R^2 - 2Rt + t^2$$

Since t is very small compared to R, $R^2 = r_n^2 + R^2 - 2Rt$

Or $r_n^2 = 2Rt$ i.e., $t = \frac{r_n^2}{2R} \quad \rightarrow (3)$

Substituting (3) in (2), $2 \times \frac{r_n^2}{2R} = n\lambda$

Or $\boldsymbol{r_n = \sqrt{nR\lambda}}$ where n = 0,1,2,3….. for dark ring.

Similarly, for bright ring, $r_n = \sqrt{\frac{(2n\pm1)R\lambda}{2}}$ where n is the order of the ring.

Determination of wavelength of monochromatic light used

Let D_n and D_{n+m} are diameters of n^{th} and $(n+m)^{th}$ dark rings respectively.

We have $r_n = \sqrt{nR\lambda}$ where R is the radius of curvature of the plano convex lens

Here, $D_n = 2r_n$

Hence diameter of n^{th} dark ring is given by $(D_n)^2 = 4(r_n)^2 = 4nR\lambda$

Similarly, the diameter of $(n+m)^{th}$ dark ring is given by $\left(D_{(n+m)}\right)^2 = 4(n+m)R\lambda$

Then $\left(D_{(n+m)}\right)^2 - (D_n)^2 = 4mR\lambda$

Wavelength of the monochromatic light used $\lambda = \frac{\left(D_{(n+m)}\right)^2 - (D_n)^2}{4mR}$

Determination of refractive index of a liquid

If a liquid of refractive index μ is introduced between the lens and the plate, path difference between the rays is $\Delta = 2\mu t$.

The condition for minimum intensity in thin interference film is $2\mu t = n\lambda$ and $t = \frac{r_n'^2}{2R}$ where r_n' is the radius of the ring when liquid is introduced.

So, $2\mu\frac{r_n'^2}{2R} = n\lambda$ or $r_n'^2 = \frac{nR\lambda}{\mu}$

Hence, diameter of n^{th} dark ring when liquid is introduced is given by $(d_n)^2 = \frac{4nR\lambda}{\mu}$ and diameter of $(n+m)^{th}$ dark ring when liquid is introduced is given by $\left(d_{(n+m)}\right)^2 = \frac{4(n+m)R\lambda}{\mu}$

Then $\left(d_{(n+m)}\right)^2 - (d_n)^2 = \frac{4mR\lambda}{\mu}$

Hence, $\frac{\left(D_{(n+m)}\right)^2 - (D_n)^2}{\left(d_{(n+m)}\right)^2 - (d_n)^2} = \frac{4mR\lambda}{\frac{4mR\lambda}{\mu}}$ or $\boxed{\mu = \frac{\left(D_{(n+m)}\right)^2 - (D_n)^2}{\left(d_{(n+m)}\right)^2 - (d_n)^2}}$

Since, $r_n'^2 = \frac{nR\lambda}{\mu}$ where r_n' is the radius of the ring when liquid is introduced and $r_n^2 = nR\lambda$ is the radius of the dark ring, then $\mu = \frac{r_n^2}{r_n'^2}$.

Also, $\mu = \frac{(D_n)^2}{(d_n)^2}$. Since μ for liquid is greater than 1, $r_n' < r_n$. Hence we conclude that the rings contract with introduction of a liquid.

Newton's rings in white light

⇒ With white light, few colour rings appear near the point of contact. The radius of the rings depends on wavelength. So bright rings of the same order for several colours will be seen with different radii.

A few coloured fringes are observed around the point of contact. Due to the overlapping of the higher order of fringes, the pattern disappears in the outer portion.

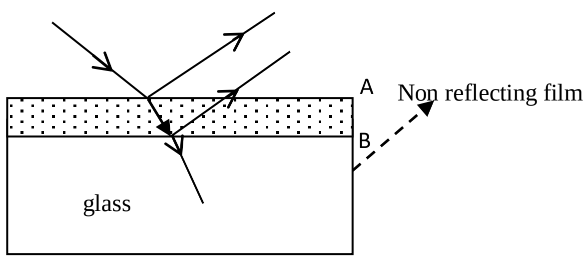
⇒ In transmitted light, there is a central spot which is bright and rings are less distinct.

Antireflection coating or nonreflecting films

⇒ Antireflection coating or non reflecting films are a type of coating applied to the surface of lenses, and other optical instruments to reduce reflection.

⇒ It is a transparent dielectric material whose refractive index lies between refractive index of air and that of glass. It should be equal to square root of the refractive index of the glass used.

Magnesium fluoride or cryolite is the most widely used material for antireflection coating. They are usually deposited by vacuum evaporation technique.



Anti reflection coating

a⇒ The incident light is reflected from the upper and lower surface of thin film with a phase change of π. If the thickness of the film is such that both the reflected rays are in opposite phase , they cancels each other and the intensity of transmitted light is increased.