PHYSICS FOR INFORMATION SCIENCE
(Group A)
(KTU - 2024 Scheme)

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PREFACE

Authors are very happy to introduce the first edition of book **Physics For Information Science (Group A)""** as per the revised syllabus and regulations 2024 of **APJ Abdul Kalam Technological University, Thiruvananthapuram, Kerala**.

Authors have a huge teaching experience in engineering colleges of high repute, which they have used to provide simplified version of complicated phenomena in Physics. Hope this book will cater to all the needs for organized studies of the subject.

This book covers the topic like Electrical Conductivity, Superconductivity, Quantum Concept, Semiconductor Physics, Semiconductor Devices and Optoelectronic Devices.

This textbook presents the fundamental principles of conducting, semiconducting and superconducting materials and their applications in simple language. Numerous solved problems and exercise problems have been given at the end of each module.

The salient features of this book is its complete syllabus coverage, simple and lucid writing style, solved problems and exercise questions.

We would like to extend our heartfelt thanks to the Management, Principal and colleagues of Ilahia College of Engineering and Technology, Mulavoor, Muvattupuzha, Ernakulam district, Kerala for providing us with a wonderful environment and encouraging us to bring out this book. We are also thankful to our Ilahia Publishers for the co-operation and support.

We hope that, this book will be well received by B.Tech students and faculties alike. We invite suggestions from the reader for making further improvement to the text.

Authors

SYLLABUS

MODULE 1: Electrical conductivity

Classical free electron theory, Electrical conductivity in metals, Fermi Dirac distribution, Variation of Fermi function with temperature, Fermi Energy, Energy bands, Classification of materials into conductors, semiconductors and insulators.

Superconductivity, Transition temperature, Critical field, Meissner effect, Type I and Type II superconductors, BCS Theory, Applications of superconductors.

MODULE 2: Quantum Mechanics

Introduction, Concept of uncertainty and conjugate observables (qualitative), Uncertainty principle (statement only), Application of uncertainty principle - Absence of electron inside nucleus - Natural line broadening. Wave function - properties - physical interpretation, Formulation of time dependent and time independent Schrodinger equations, Particle in one-dimensional box - Derivation of energy eigen values and normalized wave function, Quantum Mechanical Tunnelling (qualitative).

MODULE III: Semiconductor Physics

Intrinsic Semiconductor, Derivation of density of electrons in conduction band and density of holes in valence band, Intrinsic carrier concentration, Variation of Intrinsic carrier concentration with temperature, Extrinsic semiconductor (qualitative).

Formation of p-n junction, Fermi level in semiconductors - intrinsic and extrinsic, Energy band diagram of p-n junction - Qualitative description of charge flow across a p-n junction - Forward and reverse biased p-n junction, Diode equation (Derivation), I-V characteristics of p-n junction.

MODULE IV: Semiconductor Devices

Semiconductor devices - Rectifiers- Full wave and Half wave, Zener diode - VI characteristics, Tunnel diode - VI characteristics, Semiconductor Laser (Construction and working), Applications.

Photonic devices (Qualitative treatment only) - Photo detectors,(Junction and PIN photodiodes), Solar cells - IV characteristics, Efficiency, Stringing of Solar cells to solar panel, Light Emitting Diode, Applications.

Text Books

1. H.K. Malik and A.K. Singh "Engineering Physics", McGraw Hill Publishers, 2^{nd} Edition, 2017.

2. Arthur Beiser "Concepts of Modern Physics", Tata McGraw Hill Publishers, 6^{th} Edition, 2003.

3. MN Avadhanulu, PG Kshirsagar, TVS Arunmurthy "A Textbook of Engineering Physics", S. Chand Publishers, 11^{th} Edition, 2018.

Reference Books

1. Robert F Pierret, "Semiconductor Devices Fundamental", Pearson Education, 1995.

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3. Ben G Streetman and Sanjay Kumar Banerjee "Solid State Electronic Devices", Pearson Education 6/e, 2010.

4. S.O. Pillai, " Solid State Physics" New Age International Publishers, 10^{th} Edition, 2022.

5. Charles Kittel, "Introduction to Solid State Physics", Wiley India Edition,2019.

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Contents

Chapter 1

Electrical Conductivity

In the study of solid state materials, the metals and their alloys occupy a special position because of their variety of striking properties such as high electrical and thermal conductivities. The conducting materials play an important role in the field of engineering and technology. Materials having low electrical resistivity are known as conductors. Metals and their alloys belong to this group of materials. In metals, the valence electrons are loosely bound to their individual atoms. They become free and are responsible for the conduction of electricity and heat in metals.

The experimental measurements have shown that the metals and their alloys exhibit large electrical conductivity in the order of $10^8\Omega^{-1}m^{-1}$. Hence they are known as conductors. The low resistive materials are also called as conducting materials. The high conductivity of this material is due to the presence of free electrons.

1.1 Electrical Conduction

If a potential difference V is applied across a solid, it establishes an electric field E in the solid.

$$E = \frac{V}{L} \tag{1.1}$$

where L is the length of the solid along which charge carriers move. The electric field accelerates the charge carriers and causes a flow of electric current through the solid. The current I passing across an area A is defined as the net charge Q transported through the area per unit time.

Thus

$$I = \frac{Q}{t} \tag{1.2}$$

Any material can conduct electricity if it contains mobile charge carriers. Examples for charge carriers are free electron, mobile positive or negative ions, holes etc.

The magnitude of the electrical current I, passing through a solid at a constant temperature is directly proportional to the potential difference V applied across the solid. This is *Ohm's Law*.

$$I = \frac{V}{R} \tag{1.3}$$

When electrons travel through solids, they encounter opposition while moving. This opposition a material offers to the flow of electric current is called *electrical resistance*. It is the measure of how much a material resists the movement of electrons through it.

The electrical resistance offered by a solid is found to be dependent on the dimensions of the solid. If L is the length and A is the area of cross-section of the solid, then

$$R \propto \frac{L}{A} \tag{1.4}$$

$$\text{i.e.,} \quad R = \rho \frac{L}{A}$$

Here ρ is called the *electrical resistivity*. It is a material constant and does not depend on the dimensions of the solid.

$$\rho = \frac{RA}{L} \quad \text{Ohm-meter or } \Omega m \tag{1.5}$$

The reciprocal of electrical resistivity is called *electrical conductivity* (σ).

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} \tag{1.6}$$

Using the equation (1.3) into equation (1.6), we get,

$$\sigma = \frac{IL}{VA} \quad \text{mho/ meter } or\, \Omega^{-1}m^{-1} \tag{1.7}$$

Electrical conductivity σ, characterizes the ability of a material to conduct electricity.

Current flowing per unit area of cross section of a current carrying conductor is called *Current density*.

If I is the current and A is the area of cross-section, then current density is given by

$$J = \frac{I}{A}. \quad \text{Its unit is } A/m^2. \tag{1.8}$$

$$\text{From equation (1.7), we have} \quad \sigma = \frac{IL}{VA} \text{ S/m}$$

$$\text{Rearranging,} \quad \frac{I}{A} = \sigma \frac{V}{L}$$

$$\text{Therefore,} \quad J = \sigma E \tag{1.9}$$

The conducting materials are classified into three major categories based on conductivity.

- Metals and alloys exhibit large conductivity of order $10^8\,\Omega^{-1}m^{-1}$ and are therefore, called *conductors*.
- Materials such as metal oxides, glasses, plastics are found to possess very low conductivity of the order less than $10^{-12}\,\Omega^{-1}m^{-1}$. They are called *insulators*.
- Materials such as silicon and germanium have values of conductivity, of the order of 10^4 to $10^{-4}\,\Omega^{-1}m^{-1}$, intermediate to those of conductors and insulators. They are hence called *semiconductors*.

1.2 Free Electron Model of Solids

The free electron model of solids was proposed by Paul Drude. He assumed that the valence electrons become free in solids and move about randomly within the solids much the same way as molecules in a gas confined to a container. This the *free electron model*. This theory is applicable to all solids, both metals and non-metals and it explains electrical, thermal, optical and magnetic properties of solids.

The free electron theory underwent successive modifications in an attempt to explain the electrical behaviour and the distinction between the three types of solids.

1. **Classical Free Electron Theory:** This theory was proposed by Paul Drude in 1900 and later was extended by Lorentz. Hence this theory is also known as the Drude-Lorentz Theory.

In this theory, it was assumed that valence electrons become free in metals and move about randomly within the metal. Further, it was assumed that the free electrons move in a region of constant potential. Just as the velocities of molecules in a container, the velocities of electrons in a solid obey the classical Maxwell-Boltzmann distribution. This theory successfully explained the Ohm's law and the high electrical conductivity of metals, but failed to explain other features and the distinction between conductors, insulators and semiconductors.

2. **Quantum Free Electron Theory:** This theory was developed by Sommerfield in 1928. This theory uses quantum concepts and hence it is known as quantum free electron theory. An assembly of free electrons obey Fermi-Dirac statistics. Based on this, Sommerfield modified Drude's Classical free electron theory.

   In this theory also, it was assumed that the free electrons move in a region of constant potential. This theory acknowledges that electrons exhibit both wave-like and particle-like properties. Due to the wave nature, electrons in a metal occupy discrete energy levels rather than a continuous distribution. The distribution of electrons among energy levels follows Fermi-Dirac statistics, accounting for the wave nature and the Pauli's exclusion principle. Even though,it explain most of the physical properties of the metals like electrical conductivity, thermal conductivity, specific heat capacity of metals etc, it fails to explain other features and to state the difference between conductor, semiconductor and insulator.

3. **Band Theory of Solids:** This theory was formulated by Felix Bloch in 1928. This theory takes into account that electrons exhibit wave character as they move between atoms in a solid. It further assumed that the potential varies in a periodic manner in the solid. This theory successfully explained the classification of solids into three groups, namely conductors, insulators and semiconductors.

1.2.1 Classical Free Electron Theory (CFE)

The classical free electron theory of metals was proposed by Paul Drude in the year 1900 to explain the electrical conduction in metals. This theory was further extended by H.A. Lorentz in the year 1909.

According to this theory, metals consist of positive ion cores and valence electrons. The ion cores are immobile and consists of positive nucleus and the bound electrons. The valence electrons get detached from the parent atom during the process of formation of the metal and move randomly among these cores. Hence they are known as *free electrons*.

In Drude's model, the potential field of the ion cores is considered to be constant all over the metal and the mutual repulsion among the electrons is neglected. The free electrons moving within the metal are supposed to be similar to the freely moving atoms in the perfect gas. These free electrons are called *free electron gas*. Since the potential energy of a stationary electron within the metal is less than the potential energy of an identical electron outside the metal, there is a potential barrier which prevents these free electrons from escaping from the surface of the metal. As the free electrons are responsible for conduction of electricity in the metals, they are called *conduction electrons*.

Postulates of Classical Free Electron Theory

According to this theory, a metal consists of a very large number of free electrons. These free electrons move freely throughout the volume of the metal. The movement of the free electrons is mainly responsible for the electrical conduction in the metal.

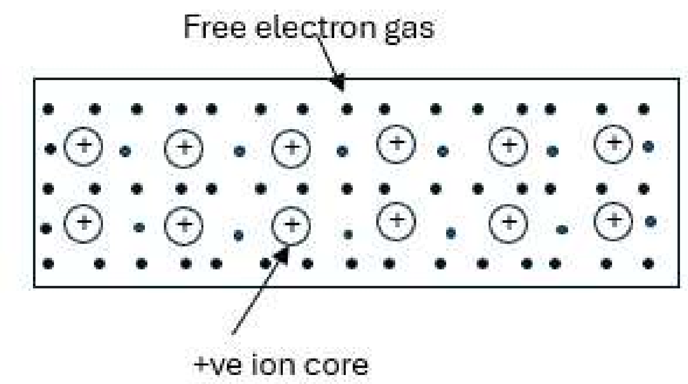

Figure 1.1: Metal consisting of positive ion cores with the valence electrons moving freely

- Drude assumed that the free electrons in the metal form an electron gas. They move randomly in all possible directions just like the gas molecules move in a container as shown in Figure 1.2.

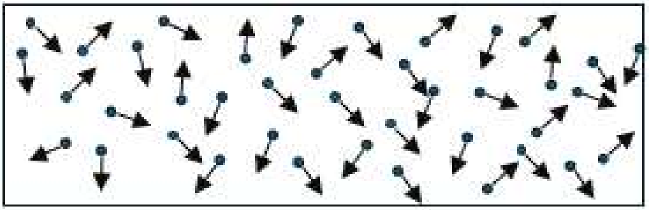

Figure 1.2: Random motion of free electrons in the absence of electric field

- In absence of an electrical field, the free electrons move in all directions in a random manner at an average speed of the order of 10^6 m/s. They collide with other free electrons and positive ion core during the motion and get deflected. This collision is known as *elastic collision*. As the motion is random, the resultant velocity in any particular direction is zero and hence this thermal motion of free electron does not cause flow of current through the metal.

- When the electric field is applied, the electrons get some amount of energy. These electrons moves in a direction opposite to that of electric field as shown in Figure 1.3. The directional motion of electrons due to the action of electric field is called drift.

  The drift velocity gained by an electron due to acceleration is lost completely whenever a collision occurs. After that, the electron gets accelerated once again and loses its velocity at the next collision. This process goes on repeating and the electron moves on an average with a *mean drift velocity (v_d)*. The drift velocity is of the order of 10^{-2}m/s. Thus the motion of free electrons in the presence of electric field, i.e., the drift motion is directional and causes current flow in a conductor called *drift current* or *conduction current*.

- The velocity and the energy distribution of free electrons are governed by classical Maxwell distribution function.

- Since the electrons are assumed to be a perfect gas, they obey the laws of kinetic theory of gases.

  Therefore, the free electrons are assigned with mean free path (λ), mean collision time (τ_c) and average velocity.

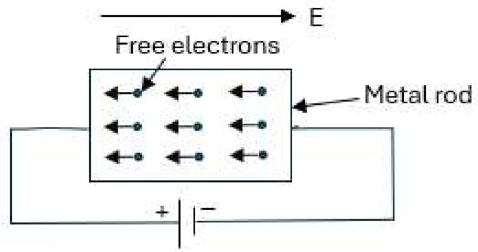

Figure 1.3: Movement of free electrons with applied electric field

- **Mean free path(λ):** The average distance travelled by a free electron between any two successive collision is known as mean free path.
- **Collision time (τ_c):** The average time taken by a free electron between any two successive collision is known as collision time of the electron.
- **Relaxation time (τ):** The average time taken by a free electron to reach its equilibrium state from its disturbed state due to the application of an external electric field is called relaxation time. Its value is of the order of $10^{-14}s$.

  For isotropic materials such as metals, collision time τ_c = τ

- **Drift velocity (v_d):** It is defined as the average velocity acquired by the free electrons of a metal in a particular direction by the application of an electric field.

  We know that the force experienced by the electron $F = eE$. This force accelerates the electron and hence it gains acceleration 'a'. The acceleration attained by electrons, a = $\frac{eE}{m}$. If τ is the relaxation time, the velocity attained by electrons,

$$v = 0 + \frac{eE}{m}\tau$$

.

$$\text{Therefore, Drift velocity} \quad v_d = \frac{eE}{m}\tau \tag{1.10}$$

Merits of Classical Free Electron Theory

The free electron model is highly successful in explaining many physical properties of metals.

- It is used to verify Ohm's Law.
- It is used to explain electrical and thermal conductivities of metals.
- It is used to derive Wiedemann-Franz law.
- It is used to explain the optical properties of metals.

Drawbacks of Classical Free Electron Theory

Classical Free Electron Theory failed in explaining certain properties of solids. Some of the important failures of this model are cited below.

- According to Classical Free Electron Theory, all the free electrons can absorb thermal energy which is not true according to quantum theory. By quantum theory, only a few electrons absorbs the supplied energy.
- The electrical conductivity of semiconductors and insulators cannot be explained by this theory.
- The photoelectric effect, Compton effect and black body radiation cannot be explained on the basis of Classical Free Electron Theory.
- This theory failed to explain heat capacity of metals.

1.3 Electrical Conductivity in Metals

Let 'n' be the free electron density (number of free electrons per unit volume of the conductor). The total number of electrons in the metal specimen is given by

N = free electron density $\times$ total volume

$$N = nAL \tag{1.11}$$

The total charge present in the conductor may be written as

$$Q = Ne = nALe \tag{1.12}$$

The current flowing in the conductor is given by

$$I = \frac{Q}{t} = \frac{nALe}{t} \tag{1.13}$$

The term $\frac{L}{t}$ represents velocity and gives the average drift velocity, v_d of electrons in the conductor.

$$I = neAv_d \tag{1.14}$$

The current density is defined $J = \frac{I}{A}$

$$J = nev_d \tag{1.15}$$

Substituting equation (1.10) in the above equation,

$$J = ne\left(\frac{eE}{m}\tau\right) = \frac{ne^2\tau}{m}E \tag{1.16}$$

From equation (1.7), we have electrical conductivity

$$\sigma = \frac{IL}{VA}$$

Since $E = \frac{V}{L}$ and current density $J = \frac{I}{A}$,
We have $\sigma = \frac{J}{E}$
Therefore, *Point form of Ohm's law* is

$$J = \sigma E \tag{1.17}$$

Equating RHS of equation (1.16) and equation (1.17), we obtain

$$\sigma = \frac{ne^2\tau}{m} \tag{1.18}$$

Similarly equating RHS of equation (1.15) and equation (1.17), we obtain

$$\sigma = \frac{nev_d}{E} \qquad \text{or} \qquad \sigma = ne\mu \tag{1.19}$$

where μ is called electron mobility.

Electron Mobility

Electron mobility is the drift velocity of electrons per unit electric field.

We have

$$v_d \propto E$$
$$v_d = \mu E$$
$$\text{or} \quad \mu = \frac{v_d}{E}$$

Mobility indicates the ease with which electrons move in a solid.

1.4 Fermi Dirac Distribution

Fermi-Dirac statistics deals with the particles having half integral spin like electrons, protons and neutrons. These particles are known as *Fermions*.

Fermi distribution function gives the distribution of electrons among the various energy levels as a function of temperature. It is a probability function $f(E)$ of an electron occupancy for a given energy level at absolute zero temperature. It is given by

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} \tag{1.20}$$

Here, E is the energy of the level whose occupancy is being considered. E_F is Fermi energy level; k is Boltzmann constant and T is the absolute temperature.

The probability value $f(E)$ lies between 0 and 1.

- If $f(E)$ = 1, the energy level is occupied by an electron.
- If $f(E)$ = 0, the energy level is vacant. i.e., it is not occupied by the electron.
- If $f(E)$ = 0.5, then there is a chance of 50$\%$ for the electron occupying in that energy level.

1.5 Variation of Fermi Function with Temperature

The Fermi-Dirac distribution is given as

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

Case (i): Probability of occupation of electrons at T = 0K

- When *T = 0K and E <* E_F, then the Fermi-Dirac distribution becomes

$$\begin{aligned} f(E) &= \frac{1}{1 + e^{(E-E_F)/0}} \\ &= \frac{1}{1 + e^{-\infty}} \quad = \frac{1}{1+0} \quad = 1 \end{aligned}$$

  Thus at T = 0K and E < E_F, there is 100% chance for the electrons to occupy the energy levels below Fermi energy levels.

- When *T = 0K and E >* E_F, then the Fermi-Dirac distribution becomes

$$\begin{aligned} f(E) &= \frac{1}{1 + e^{(E-E_F)/0}} \\ &= \frac{1}{1 + e^{\infty}} \quad = \frac{1}{1+\infty} = \frac{1}{\infty} \quad = 0 \end{aligned}$$

  In this case, there is 0% chance for the electrons to occupy the energy levels above Fermi energy levels. i.e., all the energy levels above Fermi energy level are empty.

- When *T = 0K and E =* E_F, then the Fermi-Dirac distribution becomes

$$\begin{aligned} f(E) &= \frac{1}{1 + e^{(E-E_F)/0}} \\ &= \frac{1}{1 + e^{0/0}} = \text{indeterminate} \end{aligned}$$

Case (ii): Probability of occupation of electrons at T > 0K

- On heating the conductor, as a result of thermal excitation, the probability of finding electrons in the levels immediately above E_F increases.

  At $T > 0K$, $E = E_F$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$
$$= \frac{1}{1 + e^0}$$
$$= \frac{1}{1+1} = \frac{1}{2} = 0.5$$

  Percentage of $f(E)$ = 50% Hence, there is 50% chance for the electrons to occupy the Fermi energy level. i.e., the value of $f(E)$ becomes $\frac{1}{2}$ at $E = E_F$. This result is used to define Fermi energy level.

- When $kT \gg E_F$, the electron lose their quantum mechanical character and Fermi distribution function reduces to classical Boltzmann distribution.

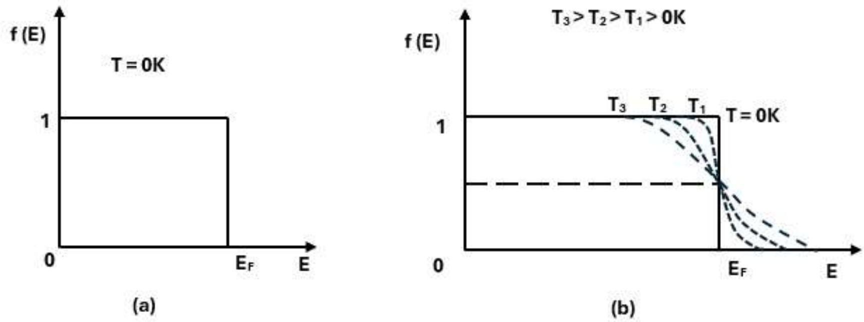

Figure 1.4: Variation of Fermi distribution function with E at different temperature

The variation of $f(E)$ for different values of energy at T = 0K become a step function as shown in Figure 1.4(a). Further for T > 0K and E > E_F, the probability value falls off rapidly to zero as in Figure 1.4(b).

1.6 Fermi Energy

Fermi energy represents the highest energy level occupied by an electron in the metal at 0 K. Fermi energy is the average energy possessed by electrons participating in conduction in metals at temperatures above 0K. Thus, the top most filled energy level at absolute zero temperature is known as Fermi level and the energy corresponding to this level is called as Fermi energy E_F. At any other temperature above 0K, Fermi Energy is defined as the energy corresponding to the level at which the probability of electron occupation is $\frac{1}{2}$ or 50%.

1.7 Energy Bands

In a single isolated atom, the electrons occupy discrete energy level. However, when atoms come together to form a solid, their electron orbitals overlap. This interaction causes the energy levels of the individual atoms to split and broaden into continuous bands of allowed energy levels. This is because the wave functions of electrons in adjacent atoms overlap, leading to the broadening of energy levels into a continuous range of energies as shown in Figure 1.5 . These range of energies possessed by an electron in a solid is known as *energy band*.

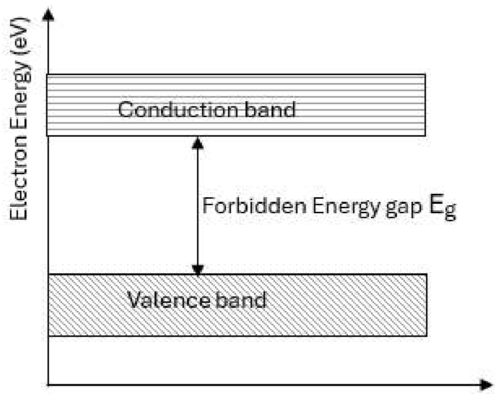

Figure 1.5: Energy band diagram

Valence Band

The electrons in the outermost orbit of an atom are known as valence electrons. The range of energies possessed by valence electrons is known as valence band. This is the highest energy band that is completely filled or partially filled with electrons at absolute zero temperature.

Conduction Band

This is the energy band immediately above the valence band. It is usually empty at absolute zero but can contain electrons at higher temperatures or when energy is supplied.

In certain cases, the valence electrons are loosely attached to the nucleus. Some of them can move through the solids like free electrons. These free electrons are responsible for the conduction of current in a conductor. So they are called conduction electrons. The range of energies possessed by such electrons is known as conduction band.

Forbidden Energy Gap

The gap between conduction band and valence band is called forbidden energy gap (E_g). This gap determines the electrical conductivity of the material. The greater the energy gap, more tightly the valence electrons are bond to the nucleus. An electron can be lifted from the valence band to the conduction band by applying an energy which is greater than forbidden energy gap.

1.8 Classification of Materials in Terms of Energy Bands

The concept of energy bands helps us in understanding the division of solids into three groups. The nature of energy bands determines whether the solid is an electrical conductor or insulator. According to the band theory, the electrical conductivity of a solid is characterised by the energy gap (E_g) separating the outermost energy bands namely the valence band and the conduction band.

Conductors

With no external energy, all the valence electrons will reside in the valence band. If the lowest level in the conduction band happens to be lower than the highest level of the valence band, the electrons from the valence band can easily move into the conduction band. Normally the conduction band is empty. But when it overlaps on the valence band electrons can move freely into it. Therefore, these solids exhibit good electrical conductivity and are called conductors. The energy band formation in conductors is as shown in Figure 1.6. Examples: Copper, silver etc.

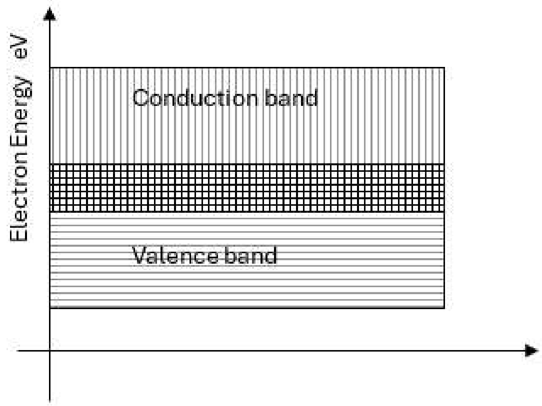

Figure 1.6: Energy band formation in Conductors

Insulators

Some solids have large energy gap (E_g) which is greater than 5eV. Even if a large electric field is applied, the electrons cannot jump from the valence band to the conduction band. So in such solids, valence band is partially or completely filled while conduction band is empty. Hence they do not allow the passage of electric current through it and thus they are poor conductors of electricity. Such solids are called insulators. Examples: Rubber, plastic, glass etc.

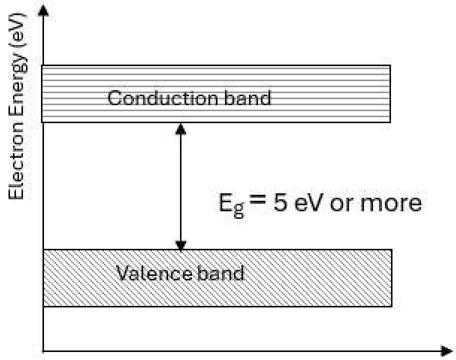

Figure 1.7: Energy band formation in Insulators

Semiconductors

In some solids, the band gap is narrow and is of the order of 2eV or less as shown in Figure 1.8.

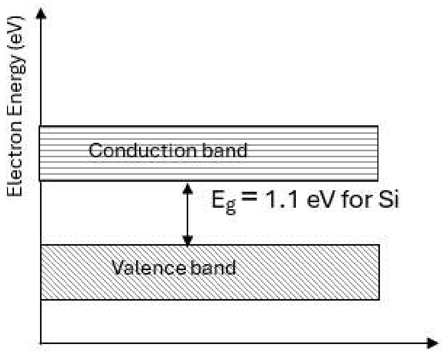

Figure 1.8: Energy band formation in Semiconductors

Valence band is completely filled and conduction band is empty. At low temperature (T = 0K), no electron is free to cross the small band gap energy into the conduction band. Those substances whose electrical conductivity lies in between that of conductors and insulators are called semiconductors. But when temperature increases, electron gain energy and cross the forbidden energy gap which enables conduction. Hence electrical conductivity of a semiconductor increases with rise in temperature. Examples of semiconductors are Si with (E_g) = 1.1eV and Ge with (E_g) = 0.72eV.

1.9 Solved Numerical Problems

Example 1.9.1 Find the drift velocity of free electrons in a copper wire of cross sectional area 10 mm^2 when the wire carries a current of 100 A. Assume that each copper atom contributes one electron to the free electron gas. Density of copper is 8969 kg/m^3 and its atomic weight is 63.54.

Solution:

Given that the density of the metal is 8969 kg/m^3; atomic weight of the copper is 63.54 g/mol = 63.54 $\times 10^{-3}$ kg/mol and area is 10 mm^2 = 10×10^{-6} m^2.

$$\text{Number of atoms per unit volume} = \frac{\text{Density} \times \text{Avogadro's number}}{\text{Atomic weight}}$$

Since each copper atom contributes one electron, the number of electrons per unit volume is given as

$$\begin{aligned}
n &= \frac{\text{Density} \times \text{Avogadro's number}}{\text{Atomic weight}} \times x \\
&= \frac{8969 \times 6.023 \times 10^{23}}{63.54} \times 1 \\
&= 8.49 \times 10^{28} \text{electrons}/m^3 \\
\text{Drift velocity}(V_d) &= \frac{I}{neA} \\
&= \frac{100}{8.49 \times 10^{28} \times 10 \times 10^{-6} \times 1.602 \times 10^{-19}} \\
&= 0.7352 \times 10^{-3} m/s
\end{aligned}$$

Example 1.9.2 Find mobility of electrons in copper if there are 9×10^{28} valence electrons/m^3 and the conductivity of copper is 6×10^7 mho/m.

Solution:

Given n = 9×10^{28} valence electrons/m^3 ; conductivity of copper is $\sigma = 6 \times 10^7$mho/m. Electronic charge e = 1.602×10^{-19}C

Conductivity σ = $ne\mu$ where μ is the mobility.

$$\begin{aligned}\text{Therefore,} \quad \text{Mobility} &= \frac{\sigma}{ne} \\ &= \frac{6 \times 10^7}{9 \times 10^{28} \times 1.602 \times 10^{-19}} \\ &= 4.16 \times 10^{-3} m^2/Vs\end{aligned}$$

Example 1.9.3 Find the relaxation time of conduction electrons in a metal, if its resistivity is $1.54 \times 10^{-8}\Omega m$ and it has 5.8×10^{28} conduction electrons /m^3.

Solution:

Given that the resistivity ρ = $1.54 \times 10^{-8}\Omega m$; number of conduction electrons n = $5.8 \times 10^{28}/m^3$.

We know that mass of electron is 9.11×10^{-31} kg and charge of electron is 1.6×10^{-19} C.

$$\begin{aligned}\text{Conductivity,} \quad \sigma &= \frac{ne^2\tau}{m} \\ \text{Relaxation time,} \quad \tau &= \frac{m\sigma}{ne^2} = \frac{m}{\rho ne^2} \qquad \left(\text{Since} \quad \sigma = \frac{1}{\rho}\right) \\ &= \frac{9.11 \times 10^{-31}}{1.54 \times 10^{-8} \times 5.8 \times 10^{28} \times (1.6 \times 10^{-19})^2} \\ &= 3.9 \times 10^{-14}\text{s}\end{aligned}$$

Example 1.9.4 Evaluate the Fermi Function for energy kT above the Fermi energy.

Solution:

For energy kT above the Fermi energy, we have E = kT + E_F

Then $E - E_F = (kT + E_F) - E_F = kT$

$$\text{Fermi function, } f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$
$$= \frac{1}{1 + e^{kT/kT}}$$
$$= \frac{1}{1 + e^{1}} = \frac{1}{1 + 2.78} = 0.269$$

Example 1.9.5 In a solid, consider the energy level lying 0.01eV above Fermi level. What is the probability of this level being occupied by an electron at 200K?

Solution:

The probability $f(E)$ that a quantum state at energy E is occupied by an electron is

$$f(E) = \frac{1}{e^{(E-E_F)/kT}}$$

For energy 0.01eV above the Fermi energy, we have E = 0.01 + E_F
Then $E - E_F = (0.01 + E_F) - E_F$ = 0.01 eV
We have Boltzmann constant k = 8.617×10^{-5} eV/K

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + e^{0.01/(8.617\times10^{-5}\times200)}}$$
$$= \frac{1}{1 + e^{(0.01/0.0172)}} = \frac{1}{1 + e^{0.58}}$$
$$= \frac{1}{1 + 1.786} = \frac{1}{2.786} = 0.359$$

Example 1.9.6 In a solid, consider the energy level lying 0.01eV below Fermi level. What is the probability of this level being occupied by an electron at 300K?

Solution:

The probability $f(E)$ that a quantum state at energy E is occupied by an electron is $f(E) = \dfrac{1}{e^{(E-E_F)/kT}}$

For energy 0.01eV below the Fermi energy, we have E = E_F - 0.01eV
Then $E - E_F = (E_F - 0.01) - E_F$ = - 0.01 eV
We have Boltzmann constant k = 8.617×10^{-5} eV/K

$$\begin{aligned} f(E) &= \frac{1}{1+e^{(E_F-E)/kT}} \quad = \frac{1}{1+e^{-0.01/(8.617\times10^{-5}\times300)}} \\ &= \frac{1}{1+e^{-0.01/0.02585}} = \frac{1}{1+e^{-0.387}} \\ &= \frac{1}{1+0.679} = \frac{1}{1.679} = 0.595 \end{aligned}$$

Example 1.9.7 In a solid, consider the energy level lying 0.01eV below Fermi level. What is the probability of this level not being occupied by an electron at 300K?

Solution:

The probability $f(E)$ that a quantum state at energy E is occupied by an electron is $f(E) = \dfrac{1}{e^{(E-E_F)/kT}}$
The probability of this level not being occupied is given by 1 - $f(E)$.
For energy 0.01eV below the Fermi energy, we have E = E_F - 0.01eV
Then $E - E_F = (E_F - 0.01) - E_F$ = - 0.01 eV
We have Boltzmann constant k = 8.617×10^{-5} eV/K

$$\begin{aligned} f(E) &= \frac{1}{1+e^{(E_F-E)/kT}} \quad = \frac{1}{1+e^{-0.01/(8.617\times10^{-5}\times300)}} \\ &= \frac{1}{1+e^{-0.01/0.02585}} = \frac{1}{1+e^{-0.387}} \\ &= \frac{1}{1+0.679} = \frac{1}{1.679} = 0.595 \end{aligned}$$

Therefore, 1 - $f(E)$ = 1 - 0.595 = 0.405

Example 1.9.8 The Fermi level for potassium is 2.1eV. Calculate the velocity of the electrons at the Fermi level.

Solution:

Given $E_F = 2.1eV$

$$\text{We have} \quad E_F = \frac{1}{2}m\mathrm{v}_F^2$$
$$\mathrm{v}_F^2 = \frac{2E_F}{m}$$
$$= \frac{2 \times 2.1 \times 1.602 \times 10^{-19}}{9.1 \times 10^{-31}}$$
$$= 0.74 \times 10^{12} m^2/s^2$$

Therefore, velocity of electrons at Fermi level v_F = 8.6×10^5m/s

1.10 Exercises

1. A copper wire whose diameter 0.16cm carries steady current of 10A. What is the current density of the wire? Calculate the drift velocity of the electrons in copper. Given density of electron in copper is 8.5×$10^{28}m^{-3}$.
Hint: J = 497.6×$10^4 A/m^2$; $v_d = 3.6 \times 10^{-4}$m/s

2. A uniform silver wire has a resistivity of 1.34×$10^{-8}\Omega m$ at room temperature for an electric field of 1volt/cm. Calculate (i) the drift velocity (ii) the mobility (iii) the relaxation time of electrons assuming that there are 5.8×10^{28} conduction electrons per m^3 of the material.
Hint: $v_d = 0.804m/s$; $\tau = 4.57{\times}10^{-14}s$; $\mu = 8.04{\times}10^{-3}m^2V^{-1}s^{-1}$

3. Use the Fermi distribution function to obtain the value of $f(E)$ for $E - E_F$ = 0.01eV at 300K
Hint: $f(E)$ = 0.4045

4. Using Fermi function, evaluate the temperature at which there is 1% probability that an electron in a metal will have an energy 0.5eV above E_F of 5eV.
Hint: T = 1263K

Chapter 2

Superconductivity

The phenomenon of sudden disappearance of electrical resistance in a material, when it is cooled below a certain temperature is known as superconductivity. This was discovered by Dutch physicist Heike Kammerlingh Onnes in 1911. During his investigations on the conductivity of metals at low temperature, he found that the resistance of a mercury sample dropped to a small value just at the boiling temperature of liquid helium.
T_c for Mercury is 4.2K and that for Aluminium is 1.175K

The variation of the electrical resistance with temperature for mercury is as shown in Figure 2.1. It is found that electrical resistance of pure mercury suddenly drops to zero when it is cooled below 4.2K.

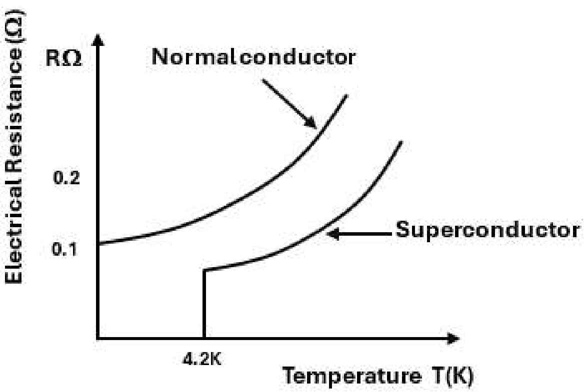

Figure 2.1: Resistance- Temperature graph for superconductors and non superconductors.

2.1 Physical Properties of superconductors

A material which exhibits superconductivity is called superconductor or superconducting material.

2.1.1 Effect of Temperature - Critical temperature (T_c)

The temperature at which material at normal conducting state changes into a superconducting state is known as *transition temperature* (T_c) *or critical temperature*.

Transition temperature depends on the property of the material. It is found that superconducting transition is reversible. That is, above critical temperature (T_c), the superconductor becomes a normal material. Every superconductor has its own transition temperature at which it changes into superconducting state.

2.1.2 Effect of Magnetic Field - Critical magnetic field (H_c)

Below transition temperature (T_c) of a superconducting material, its superconductivity can be destroyed by the application of a strong magnetic field. The minimum strength of magnetic field required to destroy the superconducting nature of a metal at transition temperature is called *critical field* (H_c).

The critical magnetic field (H_c) depends upon the temperature of the superconducting material. The relation between critical magnetic field and temperature is given by

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2}\right] \tag{2.1}$$

where H_c - critical field at TK; H_0 - critical field at 0K
T_c - transition temperature

It is noted that when the temperature of a material increases, the value of critical magnetic field decreases correspondingly. The critical magnetic field is zero at superconducting transition temperature. i.e., at T = T_c , H_c = 0. The variation of H_c with temperature T in a superconductor is shown in the Figure 2.2.

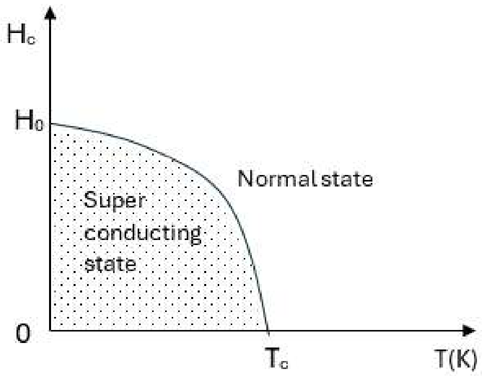

Figure 2.2: Variation of H_c with T

2.1.3 Effect of Electric Current: Silsbee's effect

A very high electrical current passing through a superconducting material destroys its superconducting property. The minimum current which can destroy the superconducting state of a superconductor is defined as *critical current* I_c.

Let 'I' be the current flowing through a superconducting wire. The flow of high current produces a magnetic field around the conductor which destroys the superconducting property. The critical current I_c required to destroy the superconducting property is given by

$$I_c = 2\pi r H_c \tag{2.2}$$

where H_c is critical magnetic field and r is radius of superconducting rod (wire).

The minimum current that can be passed through a superconductor per unit area of cross section, which destroys its superconducting property is called critical current density J_c.

$$J_c = \frac{I_c}{A} = \frac{2\pi r H_c}{\pi r^2} = \frac{2H_c}{r} \tag{2.3}$$

2.1.4 Isotope Effect

It was discovered by Maxwell and Reynold in 1950. The variation of transition temperature with isotopic mass is called isotope effect.
Transition temperature and isotopic mass are inversely proportional.

$$T_c \propto \frac{1}{M^\alpha} \quad \text{or} \quad M^\alpha T_c = \text{a constant} \tag{2.4}$$

where M is isotopic mass.

For most cases, $\alpha = \frac{1}{2}$, then

$$T_c \times \sqrt{M} = \text{a constant}$$

2.1.5 Meissner Effect

It was discovered by Meissner and Ochsenfeld in 1933. When a super conducting material in its normal conducting state is placed in a uniform magnetic field of flux density B, the magnetic lines of force penetrates through the material.

When a superconductor is cooled below the critical temperature (T_c) in an external magnetic field $(H < H_c)$, then the magnetic field lines are expelled out of the superconductor, so the magnetic field inside the superconductor is zero. This phenomenon is called Meissner effect. This is shown in the Figure 2.3.

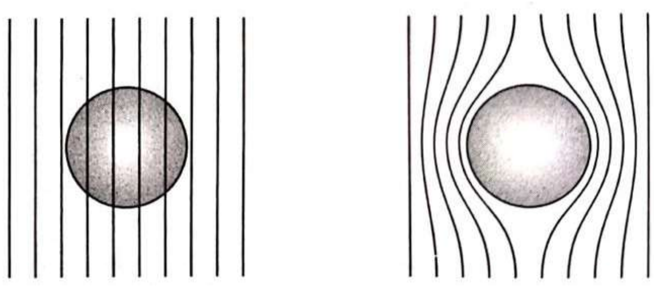

Figure 2.3: Magnetic field lines are expelled out of the superconductor below critical temperature and in a magnetic field $(H < H_c)$

If the superconducting specimen kept at a lower temperature than critical temperature T_c is slowly heated, the flux suddenly penetrates the specimen as it reaches T_c and the specimen becomes a normal conductor. Thus we can infer Meissner effect is reversible. Magnetic flux density,

$$B = \mu_0(M + H) \tag{2.5}$$

where μ_0 is permeability of free space; M is magnetization (that is the magnetic moment per unit volume); H is the intensity of the external magnetic field.

From Meissner effect, Magnetic field (B) inside the superconductor is zero. Hence,

$$\mu_0(M + H) = 0$$

$$\text{or} \quad M = -H$$

$$\text{Magnetic susceptibility } \chi = \frac{M}{H} = -1 \tag{2.6}$$

For diamagnets, magnetic susceptibility, $\chi = -1$ *This means that superconducting materials exhibit perfect diamagnetism.*

2.2 Types of Superconductors

Based on the magnetic behaviour of supercondutors in an external magnetic field, they are classified into two types.

1. Type I superconductors

2. Type II superconductors

2.2.1 Type I Superconductors

The superconductors which strictly follow Meissner effect are called Type I Superconductors. These superconductors exhibit perfect diamagnetism below a critical field H_c. As the applied field is increased beyond H_c, the field penetrates the material completely and the material loses its superconductivity abruptly.

The magnetisation curve for Type I Superconductor is shown in Figure 2.4. It is found that the transition from superconducting state to normal state in the presence of magnetic field occurs sharply at the critical magnetic field H_c.

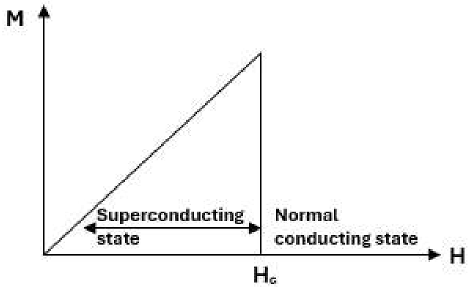

Figure 2.4: Magnetisation curve for Type I Super conductor

Characteristics of Type I Superconductors

- They exhibit complete Meissner effect. They are completely diamagnetic.
- They have only one critical magnetic field.The value of the critical magnetic field H_c is very low.
- The maximum known magnetic field for type I superconductor is of the order of 0.1 Tesla. So a small magnetic field is only required to destroy the superconducting nature of the material. Hence it is also known as *soft superconductors* . So these superconductors cannot be used for the coils of strong electromagnets.
- The magnetisation curve shows that the transition at H_c is reversible. This means that if the magnetic field is reduced below H_c, the material acquires superconducting property again and the magnetic field is expelled.
- Below H_c, the material behaves as a superconductor and above H_c it behaves as a normal conductor.
- When the field H increases, magnetization M also increases linearly up to H_c .
- Transition from superconducting state to normal state is a sudden process.
- Example: Lead, Tin, Aluminium, Mercury.

2.2.2 Type II Superconductors

Type II superconductor is one in which the material loses its magnetisation gradually rather than suddenly. The magnetisation curve for type II superconductor is given in Figure 2.5.

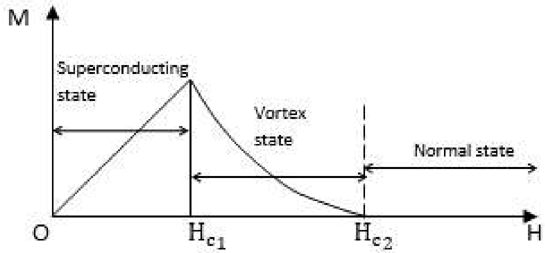

Figure 2.5: Magnetisation curve for Type II Super conductor

Characteristics of Type II Superconductors

- Type II superconductors are characterised by two critical fields H_{c1} and H_{c2}
- When external field H increases, magnetization M also increases linearly up to H_{c1} (lower critical field).
- Beyond H_{c1}, magnetic field lines slowly penetrate through the specimen, so magnetization M gradually decreases and is equal to zero at H_{c2} (upper critical field).
- Beyond H_{c2} , the material changes to normal conductor.
- The state in between H_{c1} and $H_{c2}$is called vortex state.
- In Type II superconductor, the transition from superconducting state to normal state is a gradual process.
- The value of H_{c2} is high . i.e., $H_{c2} \approx$ 10T to 20T. So high magnetic field is required to destroy superconducting nature. So type II superconductor is called *hard superconductor*.
- They do not show complete Meissner effect.
- They do not behave as perfect diamagnetic materials above H_{c1}.

- Magnetization curve is irreversible.
- Example: Niobium, Niobium - tin, Niobium – titanium, Germanium

Comparison of Type I And Type II Superconductors

SL.No	Type I Superconductor	Type II Superconductor
1	Type I superconductors are called soft superconductors.	Type II superconductors are called hard superconductors.
2	The critical field is very low which is about 0.1T to 0.2T	The critical field value is very high which is about 10T to 20T.
3	Only one critical field H_c exits for these superconductors	Two critical fields H_{c1} (lower critical field) and H_{c2} (higher critical field) exists for these superconductors.
4	Type I superconductors exhibit complete Meissner's effect	Type II superconductors do not exhibit complete Meissner's effect.
5	Type I superconductors undergo sudden transition from superconducting state to normal state at critical magnetic field.	Type II superconductors undergo gradual transition from superconducting state to normal state between two critical magnetic field.
6	Type I superconducting materials have limited technical applications because of their low critical field strength.	Type II superconducting materials have wider technological applications because of their high critical field strength.
7	Examples are Lead (Pb), Tin (Sn), Mercury (Hg), Zinc (Zn), Aluminium (Al) etc	Examples are Niobium (Nb), Niobium - tin (Nb_3Sn), Niobium – titanium (Nb_3Ti), Germanium (Ge), Vanadium (V) etc

Table 2.1: Comparison of Type I and Type II Superconductors

Applications of Type II Superconductor

Some of the applications of type II superconductor are listed below.

- Type II superconductors are used in power generators.
- The high magnetic fields produced by type II superconductors are used in particle accelerators, plasma production, fusion reaction etc.
- Superconducting magnets have been used for magnetic levitation.
- Because type II superconductors can carry very high current densities, they have great technological importance.

2.3 BCS Theory

In 1957, Bardeen, Cooper and Schriffer developed a new theory to explain superconductivity called BCS theory. It is based on the formation of Cooper pair of electrons which is purely a quantum mechanical concept.

During the flow of current in a superconductor, when an electron approaches a positive ion of the metal lattice, there is a coulomb attraction between the electron and the lattice ion. As a result, the positive ion will be displaced from the position due to this interaction. This interaction is called the electron-phonon interaction.

Now a second electron which approaches the distorted positive ion also experiences Coulomb attractive force. Thus an interaction occurs between these two electrons via the lattice. Because of this interaction, an apparent force of attraction develops between the electrons and they tend to move in pairs called *Cooper pair*.

Cooper pair is defined as the pair of electrons formed by the interaction between the electrons with opposite spin and momenta in the phonon field.

At normal temperature, the attractive force between the two electrons is too small and pairing of electrons does not take place. But, below the transition temperature Tc , the apparent force of attraction reaches a maximum value for any two electrons of equal and opposite spin. This force of attraction exceeds the Coulomb force of repulsion between two electrons and the electrons moves as pairs.

The dense cloud of cooper pairs form a collective state and they drift cooperatively through the material with identical velocity. The small velocity of cooper pairs combined with their precise ordering minimizes collision process. The extremely rare collisions of cooper pairs with the lattice leads to vanishing resistivity. At this stage, the cooper pairs of electrons smoothly move over the lattice point without any exchange of energy. As a result, the superconductors possesses infinite electrical conductivity.

The two electrons in a Cooper pair exchanging phonons through lattice ions is shown in Figure 2.6.

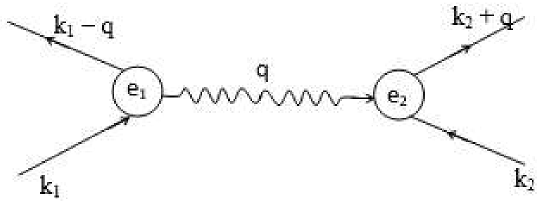

Figure 2.6: electron-phonon interaction

Here, an electron $\bar{e}_1$ with a wave vector k_1, emits a phonon and change its state to $(k_1 - q)$. A second electron $\bar{e}_2$ with a wave vector k_2 absorbs that phonon and change its state to $(k_2 + q)$.

Characteristics of Cooper Pairs

- Two electrons in a Cooper pair have opposite momenta and opposite spin.
- The mass of a Cooper pair is 2m where m is the effective mass of the electron. The charge of Cooper pair is -2e.
- As the spin of Cooper pair is zero, the Cooper pair behaves like a Boson, and it does not obey Pauli's exclusion principle.
- At a temperature less than Tc, almost all free electrons are paired as Cooper pair. Above Tc , this pairing is broken.
- The binding energy of Cooper pair is of the order of 10^{-3}eV to 10^{-4}eV. It is slightly less than twice the energy of free electron.

2.4 Applications of Superconductors

Superconductors is a basis of new generation of energy saving power systems. It has a wide range of application from large scale devices like generators, magnets etc to small scale devices like SQUID, cryotron etc.

1. Superconductors are used to produce a very strong and powerful magnetic field in the order of 20T. This high magnetic field is used in particle accelerators, cyclotrons, controlled nuclear fusion etc.
2. Medical application
   - They are used in MRI.
   - Superconducting magnetic field is used to remove tumour cell from the healthy cells.
   - Group of squids are used for the diagnosis of epilepsy.
3. Electronic and small devices:
   - Squid.
   - Frictionless bearing, magnetically controlled superconducting switches, superconductor fuses, breakers, superconducting transformers.
4. Electrical machines and measuring instruments.
   - Superconducting materials are used to manufacture small size electrical generators and transformers having high efficiency. They are used in the construction of very sensitive electrical measuring instruments such as galvanometer.
5. Computers:
   - High capacity and high speed computer chips can be developed with superconductors.
   - Used to perform logic and store functions in computers.
6. Low loss transmission lines can be made with superconductors.

2.5 Superconducting Quantum Interference Device

A Superconducting Quantum Interference Device (SQUID) is a very sensitive magnetometer used to measure extremely weak magnetic fields. It is based on the flux quantization in a superconducting ring. A small change in magnetic field produces variation in the quantum flux. It consists of superconducting ring with two Josephson junctions in parallel. They are capable of measuring magnetic fluctuations of the order of 10^{-18} T. Some of the applications of SQUID are mentioned below.

- It is used to detect the presence of ships, submarines, by detecting a small disturbances in the earth magnetic field.
- It is used to measure the weak magnetic pulse generated by heart, brain in their pathological analysis.
- Principle of SQUID is applied in MRI for the investigation and diagnosis of various diseases.
- It is used to explore the oil deposits and other mineral deposits in different parts of the world.
- It is useful in the study of earthquakes.

2.6 High Temperature Superconductors

To achieve superconductivity, we have to maintain very low temperature which is very difficult and expensive. This marks the need for superconductors with very high T_c. The discovery of superconductor with transition temperature 77K was a remarkable development. This was because we can use inexpensive liquid nitrogen as coolants to maintain low temperature.

The superconductors with high value of transition temperature T_c are called *High Temperature Superconductors*. Substance having $T_c > 24K$ are high temperature superconductors. All known high temperature superconductors are Type II superconductors. Some of the examples are
Yittrium barium copper oxide (Y-Ba-Cu-O) with T_c 93K;
Thallium barium calcium copper oxide (Tl-Ba-Ca-Cu-O) with T_c > 125K;
Mercury thallium barium calcium copper oxide (Hg-Tl-Ba-Ca-Cu-O) with T_c > 138K.

Characteristics of High - Temperature Superconductors

- They have high transition temperature.
- They have a modified perovskite crystal structure.
- They are oxides of copper in combination with other elements.
- They are reactive, brittle and cannot be easily modified.

2.7 Solved Numerical Problems

Example 2.7.1 For a certain metal, the critical magnetic field is $5 \times 10^3 A/m$ at 6K and $2 \times 10^4 A/m$ at 0K. Determine its transition temperature.
Solution:
Given $H_c = 5 \times 10^3 A/m$;$H_0 = 2 \times 10^4 A/m$;T = 6K

$$\text{We have} \quad H_c = H_0 \left[1 - \frac{T^2}{T_c^2}\right]$$

$$\text{Then} \quad T_c = \frac{T}{\left[1 - \frac{H_c}{H_0}\right]^{1/2}} \quad = \frac{6}{\left[1 - \frac{5 \times 10^3}{2 \times 10^4}\right]^{1/2}} = 6.93K$$

Example 2.7.2 Calculate the critical current which can flow through a long thin superconducting wire of diameter $10^{-3}m$.
Given $H_c = 7.9 \times 10^3$A/m.
Solution:
Given $H_c = 7.9 \times 10^3$A/m and $r = \frac{10^{-3}m}{2}$

$$\text{From Silsbee rule} \quad I_c = 2\pi r H_c$$

$$= \frac{2 \times 3.14 \times 10^{-3} \times 7.9 \times 10^3}{2} = 24.81A$$

Example 2.7.3 Superconducting tin has a critical temperature of 3.7K at zero magnetic field and a critical field of 0.0306 Tesla at 0K. Find the critical field at 2K.
Solution:
Given $T_c = 3.7K$; H_0 = 0.0306 Tesla; T = 2K

$$\text{We know that the critical field,} \quad H_c = H_0 \left[1 - \frac{T^2}{T_c^2}\right]$$

$$= 0.0306 \left[1 - \frac{(2)^2}{(3.7))^2}\right] = 0.02166T$$

Therefore, the critical field at 2K, H_c= 0.02166 Tesla

Example 2.7.4 The critical temperature for a metal with isotopic mass 199.5 is 4.185K. Calculate the isotopic mass if the critical temperature falls to 4.133K.

Solution:

Given $T_{c1} = 4.185K$; M_1 = 199.5; $T_{c2} = 4.133K$

$$\text{We know that} \quad T_{c1} \times (M_1)^{1/2} = \text{a constant}$$

$$\text{That is} \quad T_{c1}(M_1)^{1/2} = T_{c2}(M_2)^{1/2}$$

$$\left(\frac{T_{c2}}{T_{c1}}\right)^2 = \frac{M_1}{M_2}$$

$$M_2 = M_1 \times \left(\frac{T_{c1}}{T_{c2}}\right)^2$$

$$M_2 = \frac{199.5 \times (4.185)^2}{(4.133)^2} = 204.55amu$$

Example 2.7.5 The critical temperature of a superconductor at zero magnetic field is T_c. Determine the temperature at which the critical field becomes half of its value at 0K.

Solution:

We have $H_c = \dfrac{1}{2} H_0$

$$\text{We know that the critical field} \quad H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right]$$

$$\frac{1}{2}H_0 = H_0\left[1 - \frac{T^2}{T_c^2}\right]$$

$$\frac{1}{2} = 1 - \left(\frac{T}{T_c}\right)^2$$

$$\left(\frac{T}{T_c}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$