MODULE - VI

MODULE VI

PRESTRESSED CONCRETE

Prestressed concrete is basically concrete in which internal stresses of suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counter acted to a desired degree.

In reinforced concrete members pre stress is commonly introduced by tensioning the steel reinforcement. In RCC concrete and steel are combined such that concrete resist compression, and steel resist tension.

In pre stressed concrete high strength steel and high strength concrete are combined such that full section is effective in compression and tension.

Comparison: RCC beams v/s pre stressed concrete beams.

R.C.C beams	Pre-stressed concrete beams
1. Concrete in compression side of N.A above is effective. The concrete in tension side of N.A is ineffective.	In pre stressed concrete beam entire section is effective.
2. RCC beams are heavy. need shear reinforcement besides the longitudinal reinforcement for flexure.	Pre stressed concrete beams are lighter. for shear resistance provide curved tendons and pre compression.

3. High strength steel and high strength concrete are not needed.	High strength concrete and high strength steel are required. (high strength concrete is to resist stress at anchorage and steel is needed to transfer large prestressing force).
4. RCC beams being massive and heavy are more suitable in situations where weight is more desired than strength.	Suitable for heavy loads and long spans.
5. No way of testing the steel and concrete.	Testing of steel and concrete can be made by prestressing.
6. Does not involve any auxiliary unit	It involves many auxiliary units like prestressing equipments, anchoring etc.

Applications

- long span bridge constructions.
- construction of folded plate roof.
- marine structures such as floating docks, off shore oil drilling platform etc.
- construction of large capacity liquid retaining structure such as prestress concrete tanks
- construction of tall towers.
  - → air craft hangers.

→ pavements

→ rail road sleepers

→ poles

→ piles

→ television tower

→ flat slab floor construction.

Need for high strength steel and concrete

minimum grade of M_{35}

steel : HYSD bars.

- high strength concrete offers high resistance in tension, shear and bond and bearing.
- minimized shrinkage cracks.
- capacity to resist bearing stresses at zones of anchorage.
- youngs modulus = $5700\sqrt{f_{ck}}$ (IS 1343-1980). high strength concrete has higher modulus of elasticity.
- smaller ultimate creep strain resulting in smaller loss of prestress
- CSA can be reduced, self wt is reduced
- longer span will be economical.

High strength steel is also known as tendons. Tendon consist of no. of strands. Normal loss of stress in steel is generally about 100-240 N/mm² and if this loss is to be a small portion of initial stress. Stress in steel in initial stage must be very high about 1200-2000 N/mm². these high stress ranges are possible only with help of high strength steel.

Advantages of pre stressed concrete

fully prestressed members are free from tensile stresses under working loads.

Crossection is more effectively utilized.

No cracks under working load.

Effective saving in use of material.

Improved resistance to shearing force

use of high strength concrete and steel in pre-stress members result in lighter and slender members.

highly improved durability of structure.

high resistance to impact load.

high resistance to repeated working loads.

Assumptions for analysis of pre stress members.

1) concrete is homogeneous and elastic material.
2) plain section before bending is assumed to be plain after bending.
3) within permissible stresses both concrete and steel behave elastically.
4) Material will not subjected to any amount of creep under sustained loading.

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SYSTEMS OF PRESTRESSING

Classification based on method of design construction and Application of prestress

I: External prestressing
Internal prestressing

II: Linear prestressing
Circular prestressing

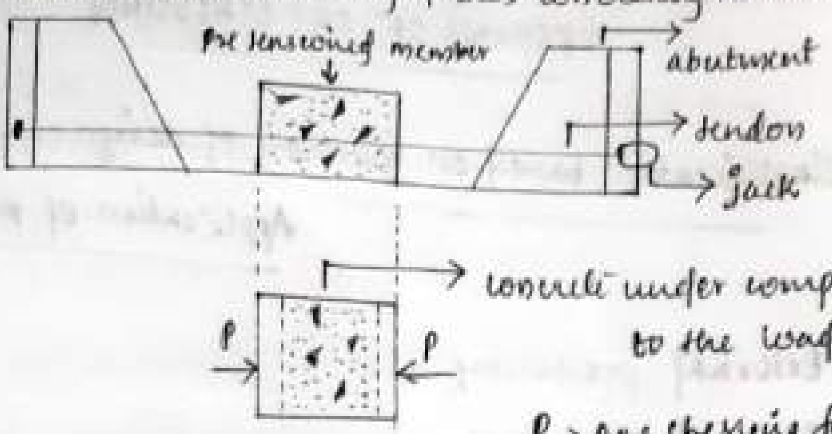
III: Pre tensioning
Post tensioning

Pre-tension

In pre tension members, the tendons are tensioned even before casting the concrete. One end of tendon is secured to an abutment while other end of tendon is pulled by using a jack, and this end is then fixed to other abutment.

The concrete is now poured, after the concrete is cured and hardened the ends of tendons are released from abutment. The tendon which tend to resume its original length and it will compress the concrete surrounding it by bond action.

(a) Reinforcement has been tensioned & then concreting has been done.



P → prestressing force.

Post tension

Post tension members is one in which the reinforcement is tensioned after concrete is fully hardened. The beam is first cast leaving a duct for placing the tendon. The duct are made in no: of ways;

a) while giving corrugated steel tube in concrete.

b) by providing steel spirals.

c) providing steel metal tubing.

d) rubber hose.

When concrete has hardened and developed its strength, tendon is passed through duct.

One end is provided with an anchor the other end of tendon is pulled by a jack which is budding against end of member.

The jack simultaneously pulls tendon and compress the concrete after the tendon is subjected to a desired stress. This end of tendon is also properly anchored to concrete.

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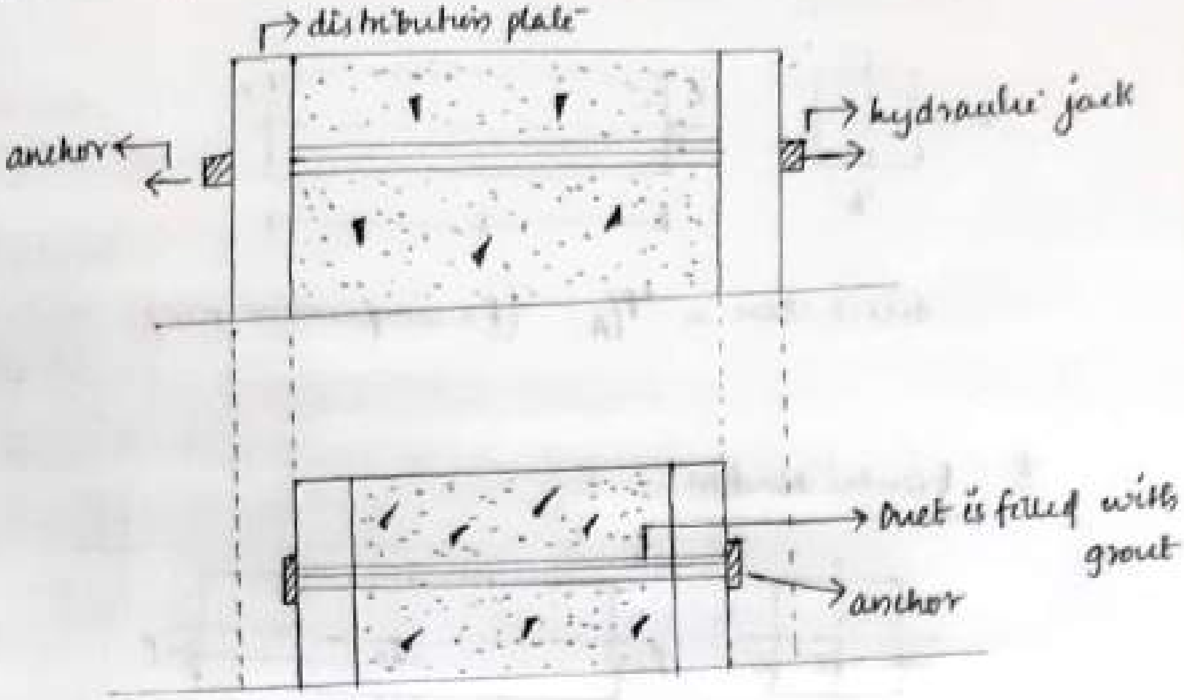
To avoid crushing of concrete due to excessive bearing stress a distribution plate is provided at each end.



Analysis of pre-stressed beam

There are 3 methods of analysis ;

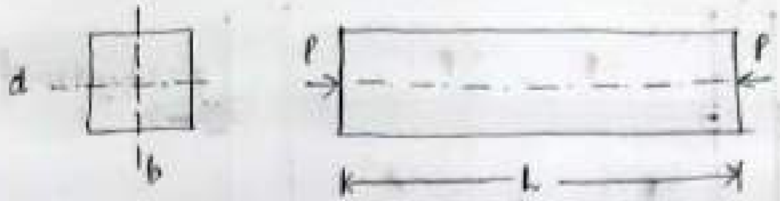
1) based on stress concept
2) " " force concept / strength concept
3) Load balancing.

Analysis by stress concept

In pre stress problems : tension → (-)ve
compression → (+)ve

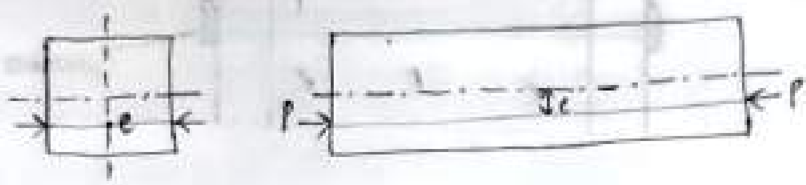
I : Concentric tendon

Center of gravity of beam and C.G of cable are same.



direct stress $= {}^{+}P/A$ (P = compressive force)

II : Eccentric tendon



In this case 2 stresses are developed;

1) direct stress due to pre stressing.

2) Bending stress due to eccentricity.

and also stresses at top and bottom fibres are different.

direct stress $= {}^{+}P/A$ (Stress for top & bottom fibres).

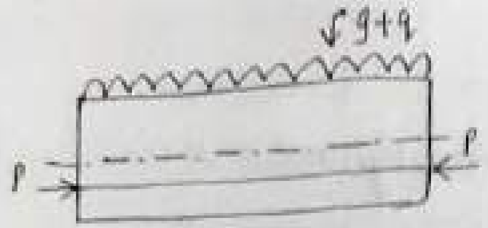
Bending stress

General bending eqn $= \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

$$\frac{f}{y} = \frac{M}{I}$$

$$f = \frac{My}{I}$$

Resultant stress at any s/c of beam subjected to live load and dead load

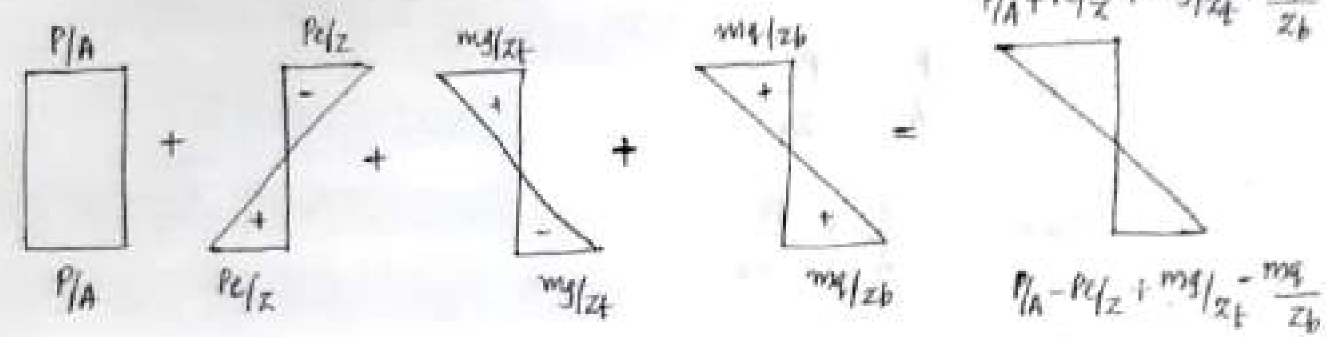


dead load = g

live load = q

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z_t} + \frac{m_g}{Z_t} + \frac{m_q}{Z_t}$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z_b} - \frac{m_g}{Z_b} - \frac{m_q}{Z_b}$$



Problem

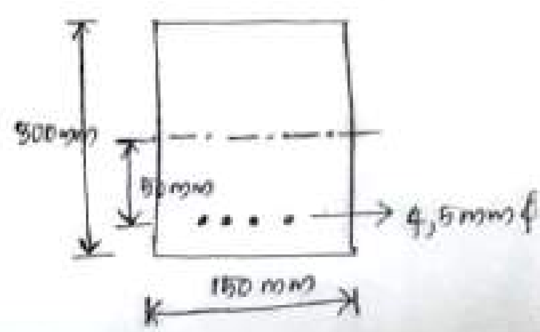
1) A concrete beam of rect. s/c 150 mm x 300 mm is pre stressed by 4 high tensile wires of 5mm dia stresses to 1200 N/mm². The wires are located at an eccentricity of 50 mm. Calculate stress developed at the soffit of the beam due to pre stress.

b = 150 mm.

d = 300 mm.

Stress = 1200 N/mm².

e = 50 mm.



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$$\therefore f = \frac{Pe}{I/y}$$

$$\boxed{f = \frac{Pe}{Z}}$$

here due to eccentricity top fibre is subjected to tensile force and bottom fibre is subjected to comp. force.

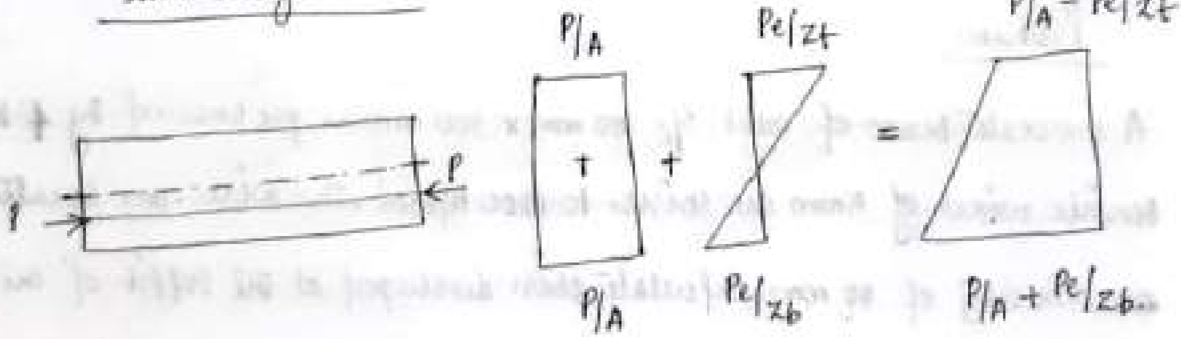
@ top fibre ; bending stress $= \frac{-Pe}{Z_t}$

@ bottom fibre ; bending stress $= \frac{+Pe}{Z_b}$

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z_t}$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z_b}$$

Stress diagram.



Stress = load/area

load = stress x area

$= 78.53 \times 1200$

$= 94.24 \times 10^3$ N

$Z = I/y$

$= \dfrac{bd^3/12}{150}$

$= \dfrac{150 \times 300^3/12}{150}$

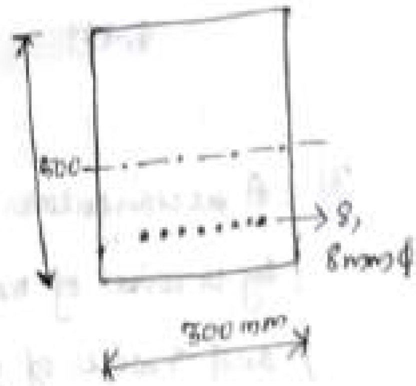
$= 2.25 \times 10^6 \text{ mm}^4$

$f_{bottom} = \dfrac{P}{A} + \dfrac{Pe}{Z}$

$= \dfrac{94.24 \times 10^3}{300 \times 150} + \dfrac{94.24 \times 10^3 \times 60}{2.25 \times 10^6}$

$= 4.188 \text{ N/mm}^2$

2 A concrete beam rectangular S/c 300 x 500 mm is prestressed by 8 high tensile wires of 8mmΦ is stress to 1600 N/mm². The wires are located at an eccentricity of 75 mm. Calculate stresses developed at top and bottom fibres.



$b = 300$ mm

$d = 500$ mm

$e = 75$ mm

Stress $= 1600 \text{ N/mm}^2$

P = stress x area

$= 1600 \times 8 \times \pi/4 \times 8^2 = 643.39 \times 10^3$ N

$f_{bottom} = P/A + P.e/Z_b$

$Z_b = I/y = \dfrac{300 \times 500^3/12}{500/2} = 12.5 \times 10^6 \text{ mm}^3$

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$$f_b = \frac{643.39\times10^3}{300\times500} + \frac{643.398\times10^3\times75}{12.5\times10^6}$$

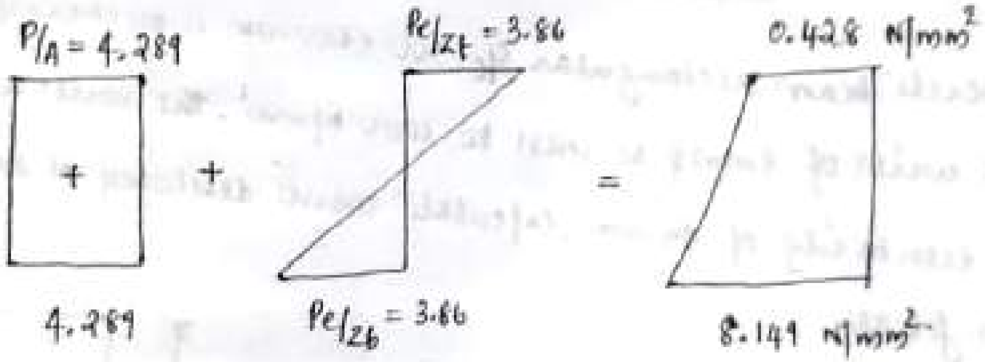
$$= \underline{\underline{8.149\ N/mm^2}}$$

$$f_{top} = P/A - Pe/Z_t$$

$$= \frac{643.398\times10^3}{300\times500} - \frac{643.398\times10^3\times75}{12.5\times10^6}$$

$$= \underline{\underline{0.428\ N/mm^2}}$$

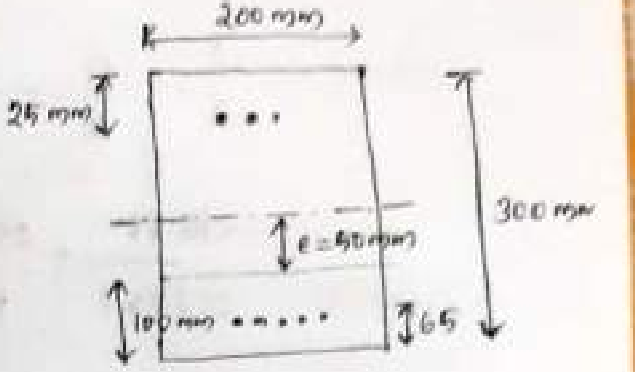
Stress diagram



3) A rectangular concrete beam C/S of 30 cm deep & 20 cm wide is prestress by 15 wires of 5mmϕ located at a distance of 6.5 cm from bottom of beam and 3 wires of 5mmϕ is located at a distance of 2.5 cm from top. Assume prestress in steel is 840 N/mm^2. Calculate stress @ extreme fibre of midspan S/C when beam is supporting its own weight over a span of 6m if UDL of 6kN/m is imposed. Calculate max stress in concrete. Assume density of concrete = 24 kN/m^3.

$d = 300$ mm

$b = 200$ mm



Distance of centroid of prestressed force;

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{15 \times \pi/4 \times 5^2 \times 65 + 3 \times \pi/4 \times 5^2 \times 275}{15 \times \pi/4 \times 5^2 + 3 \times \pi/4 \times 5^2}$$

$$= \underline{\underline{100 \text{ mm}}}$$

$e = 150 - 100 = \underline{\underline{50 \text{ mm}}}$

P = stress × area

$$= 840 \times 18 \times \pi/4 \times 5^2 = \underline{\underline{296.88 \times 10^3 \text{ N}}}$$

$$Z = \frac{I}{y} = \frac{200 \times 300^3/12}{300/2} = \underline{\underline{3 \times 10^6 \text{ mm}^3}}$$

Self wt; $g = bd \times 24 = 0.2 \times 0.3 \times 24 = \underline{\underline{1.44 \text{ kN/m}}}$

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{m_g}{Z} + \frac{m_q}{Z} \quad (q = \text{Live load})$$

$$= \frac{296.88 \times 10^3 \times 10^{-3}}{0.2 \times 0.3} - \frac{296.88 \times 50 \times 10^{-3}}{3 \times 10^6 \times (10^{-3})^3} + \frac{\frac{1.44 \times 6^2}{8}}{3 \times 10^6 \times (10^{-3})^3} + \frac{\frac{6 \times 6^2}{8}}{3 \times 10^6 \times (10^{-3})^3}$$

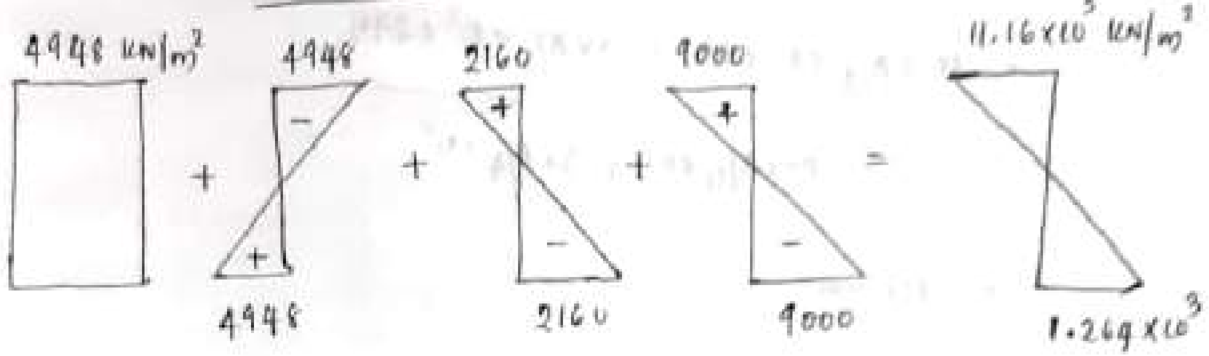
$$= 11.16 \times 10^3 \text{ kN/m}^2 = \underline{11.16 \text{ N/mm}^2}$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{m_g}{Z} - \frac{m_q}{Z}$$

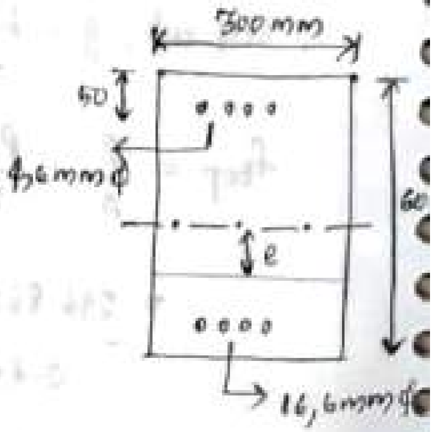
$$= \frac{296.88}{0.2\times0.3} + \frac{296.88\times50\times10^{-3}}{3\times10^{6}\times(10^{-3})^{3}} - \frac{\frac{1.44\times6^{2}}{8}}{3\times10^{6}\times(10^{-3})^{3}} - \frac{\frac{6\times6^{2}}{8}}{3\times10^{6}\times(10^{-3})^{3}}$$

$$= -1.264\times10^{3}\ kN/m^{2}$$

$$= \underline{\underline{-1.26\ N/mm^{2}}}$$



4) A rectangular beam of c/s 60 cm deep, 30 cm wide is prestressed by means of 16 wires of 6mm ϕ located @ 9 cm from bottom of beam & 4 wires of 6mm ϕ is located at 5 cm from top. Assuming the prestress in steel of 1200 N/mm². Calculate stress at extreme fibre of midspan s/c when beam is supported on its own weight over span of 8m. If a UDL of 8 kN/m is imposed calculate maximum stress in concrete. concrete density = 24 kN/m³



$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{16\times\pi/4\times6^{2}\times90 + 4\times\pi/4\times6^{2}\times550}{16\times\pi/4\times6^{2} + 4\times\pi/4\times6^{2}}$$

$$= \underline{\underline{182\ mm}}$$

$$e = 300 - 182 = \underline{118\ mm}$$

$$P = \text{stress} \times \text{area} = 1200 \times (16+4) \times \pi/_4 \times 6^2 = \underline{678.58 \times 10^3\ N}$$

$$Z = \frac{300 \times 600^3/12}{600/2} = \underline{18 \times 10^6\ mm^3}$$

$$g = b \times d \times 24 = 0.3 \times 0.6 \times 24 = 4.32\ kN/m$$

$$q = 8\ kN/m$$

$$P/_A = \frac{678.58 \times 10^3 \times 10^{-3}}{0.3 \times 0.6} = 3.769 \times 10^3\ kN/m^2 = \underline{3.769\ N/mm^2}$$

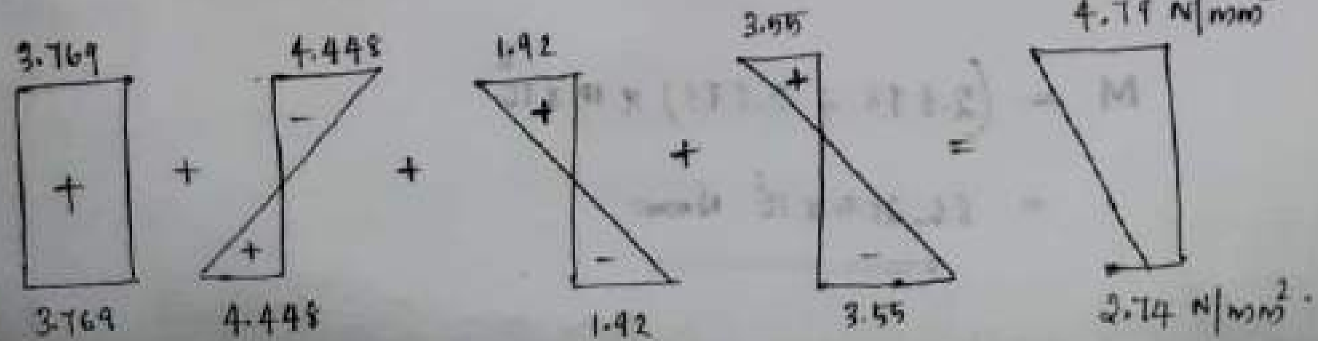
$$Pe/_Z = \frac{678.58 \times 10^3 \times 118}{18 \times 10^6} = 4.448\ N/mm^2$$

$$\frac{M_g}{Z} = \frac{\frac{4.32 \times 8^2}{8}}{18 \times 10^6 \times (10^{-3})^3} = 1920\ kN/m^2 = 1.92\ N/mm^2$$

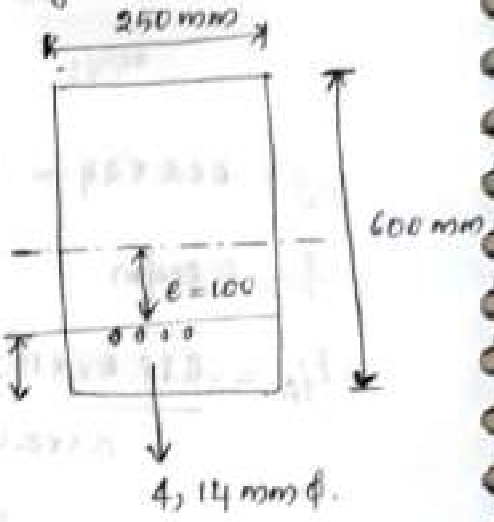
$$\frac{M_q}{Z} = \frac{\frac{8 \times 8^2}{8}}{18 \times 10^6 \times (10^{-3})^3} = 3555.55\ kN/m^2 = \underline{3.55\ N/mm^2}$$

$$f_{top} = 3.769 - 4.448 + 1.92 + 3.55 = 4.791\ N/mm^2$$

$$f_b = 3.769 + 4.448 - 1.92 - 3.55 = 2.747\ N/mm^2$$



5) A rectangular concrete beam 250 mm (wide) x 600 mm (deep) is prestressed by means of 4 nos of 14 mm ϕ wires located 200 mm from soffit of beam. If the effective stress in wire is 700 N/mm². What is max. bending moment that can be applied to s/c without causing tension at the soffit of beam.



$P = \text{stress} \times \text{area}$

$= 700 \times 4 \times \pi/4 \times 14^2 = \underline{431.02 \times 10^3 \text{ N}}$

$e = 300 - 200 = 100 \text{ mm}.$

$$Z = \frac{250 \times 600^3/12}{600/2} = 15 \times 10^6 \text{ mm}^3$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

$$0 = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

$$\frac{P}{A} = \frac{431.02 \times 10^3}{250 \times 600} = 2.873 \text{ N/mm}^2$$

$$\frac{Pe}{Z} = \frac{431.02 \times 10^3 \times 100}{15 \times 10^6} = 2.873 \text{ N/mm}^2.$$

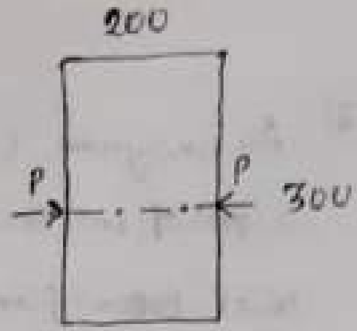
$$\frac{M}{Z} = \frac{P}{A} + \frac{Pe}{Z}$$

$$M = (2.873 + 2.873) \times 15 \times 10^6$$

$$= 86.205 \times 10^6 \text{ Nmm}$$

6) A prestressed concrete beam of S/c 200 mm x 300 mm is used over an effective span of 6m to support an imposed load of 4 kN/m. Density of concrete = 24 kN/m³ at the centre of span of the S/c of beam find magnitude of (a) the concentric prestressing force necessary for zero fibre stress at soffit when beam is fully loaded

(b) the eccentric prestressing force located 100mm from the bottom of beam which would nullify the bottom fibre stress due to loading.



(a) $f_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{m_g}{Z} - \frac{m_q}{Z}$

$\frac{Pe}{Z} = 0$ (since it is concentric)

$0 = \frac{P}{A} - \frac{m_g}{Z} - \frac{m_q}{Z}$

$g = b \times d \times 24 = .2 \times .3 \times 24 = 1.44$ kN/m

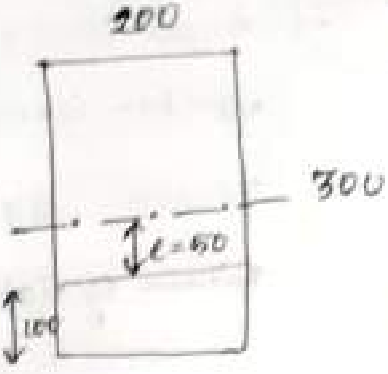
$Z = \frac{200 \times 300^3 / 12}{300/2} = 3 \times 10^6 \ mm^3$

$\frac{m_g}{Z} = \frac{\frac{1.44 \times 6^2}{8}}{3 \times 10^6 \times (10^{-3})^3} = 2160 \ kN/m^2 = 2.16 \ N/mm^2$

$\frac{m_q}{Z} = \frac{\frac{4 \times 6^2}{8}}{3 \times 10^6 \times (10^{-3})^3} = 6000 \ kN/m^2 = \underline{\underline{6 \ N/mm^2}}$

$P = (2.16 + 6) \times 200 \times 300 = \underline{\underline{489.6 \times 10^3 \ N}}$

(b) $f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_g}{Z} - \frac{M_q}{Z}$



$$\frac{P}{A} + \frac{Pe}{Z} = \frac{M_g}{Z} + \frac{M_q}{Z}$$

$$\frac{P}{300\times200} + \frac{P\times50}{3\times10^6} = 2.16 + 6$$

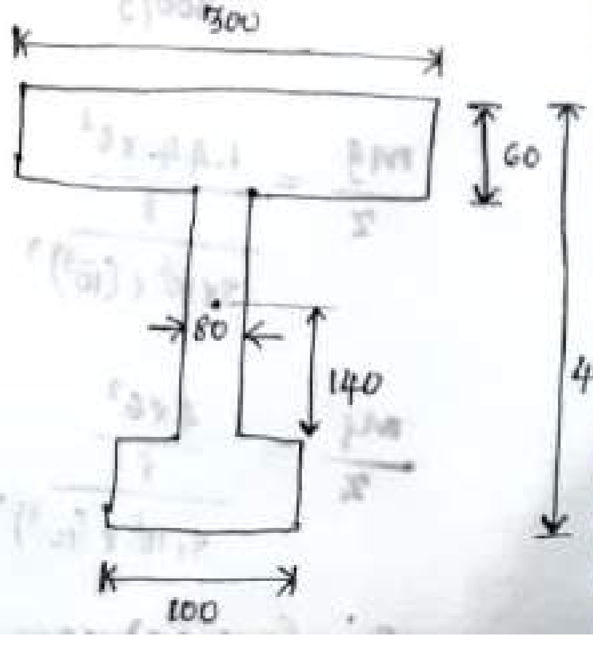
$$3\times10^6 P + 300\times200\times50\, P = 1.4688\times10^{12}$$

$$\underline{\underline{P = 244.8\times10^3 N}}$$

7) An unsymm. I/c beam is used to support an imposed load of 2 kN/m over a span of 8m. The sectional details are top flange 300 mm wide & 60 mm thick. Bottom flange 100 mm wide and 60mm thick. thickness of web is 80 mm. Over all depth of beam is 400mm. At the centre of span, the effective prestressing force 100 KN is located at 50 mm from the soffit of the beam. Estimate stress at centre of span of beam for following conditions-

(a) prestress + self wt.

(b) prestress + self wt + live load.



$$I = \frac{bd^3}{12} + Ah^2$$

$$\bar{y} = \frac{60\times100\times30 + 80\times280\times200 + 600\times300\times(30+280+60)}{60\times100 + 80\times280 + 60\times300}$$

$$= \underline{244\ mm}$$

$\bar{y}$ (from top) $= 400 - 244 = 156$ mm

$$I_1 = \frac{bd^3}{12} + Ah^2 = \frac{100 \times 600^3}{12} + 100 \times 60 \times (244-30)^2$$

$$= 276.57 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{80 \times 280^3}{12} + 80 \times 280 \times (244 - 140 - 60)^2 = 189.71 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{300 \times 60^3}{12} + 300 \times 60 \times (156 - 30)^2 = 291.168 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 757.45 \times 10^6 \text{ mm}^4$$

$$Z_t = \frac{757.45 \times 10^6}{156} = 4.855 \times 10^6 \text{ mm}^3$$

$$Z_b = \frac{757.45 \times 10^6}{244} = 3.104 \times 10^6 \text{ mm}^3$$

$P = 100$ KN

$$\frac{P}{A} = \frac{100 \times 10^3}{6000 + 22400 + 18000} = 2.155 \text{ N/mm}^2$$

$$\frac{Pe}{Z_t} = \frac{100 \times 10^3 \times (244 - 50)}{4.855 \times 10^6} = 3.995 \text{ N/mm}^2$$

$$\frac{Pe}{Z_b} = \frac{100 \times 10^3 \times (244 - 50)}{3.104 \times 10^6} = 6.25 \text{ N/mm}^2$$

$g = b\times d \times 24 = (6000 + 22400 + 16000) \times (10^{-3})^2 \times 24$

$= 1.1136 \ KN/m.$

$q = 2 \ KN/m.$

$$\frac{m_g}{Z_t} = \frac{1.1136 \times 8^2/8}{4.855 \times 10^6 \times (10^{-3})^3} = 1.834 \times 10^3 \ KN/m^2 = 1.834 \ N/mm^2$$

$$\frac{m_g}{Z_b} = \frac{1.1136 \times 8^2/8}{3.104 \times 10^6 \times (10^{-3})^3} = 2.870 \times 10^3 \ KN/m^2 = 2.87 \ N/mm^2$$

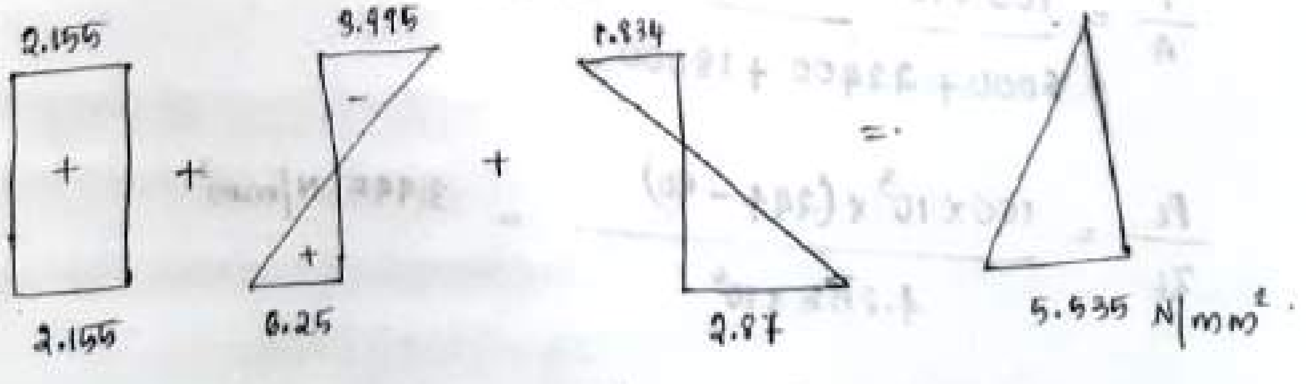
$$\frac{m_q}{Z_t} = \frac{2\times 8^2/8}{4.855 \times 10^6 \times (10^{-3})^3} = 3.295 \times 10^3 \ KN/m^2 = 3.295 \ N/mm^2.$$

$$\frac{m_q}{Z_b} = \frac{2\times \frac{8^2}{8}}{3.104 \times 10^6 \times (10^{-3})^3} = 5.155 \times 10^3 \ KN/m^2 = 5.155 \ N/mm^2.$$

A)

(a) $f_{top} = 2.155 - 3.995 + 1.834 = -.006 \approx \underline{0}$

(b) $f_b = 2.155 + 6.25 - 2.87 = \underline{\underline{5.535 \ N/mm^2}}$



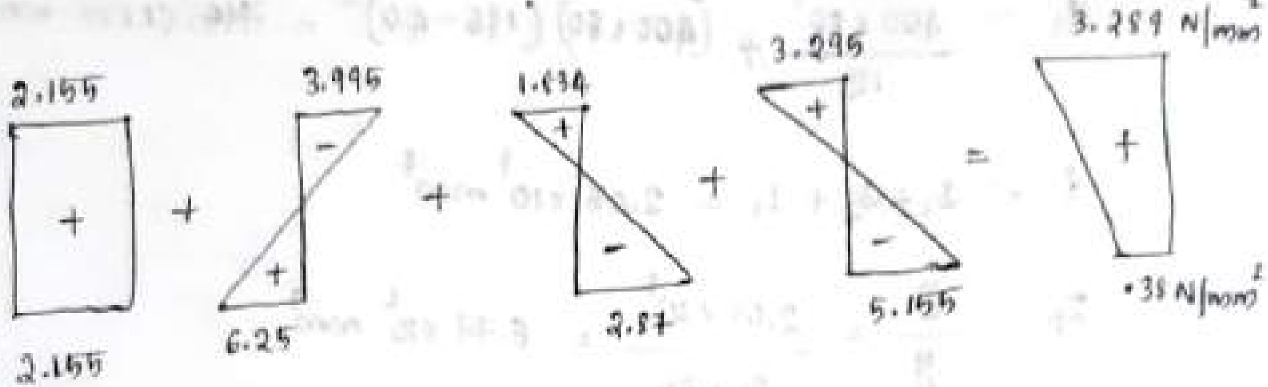
B)

$$f_{top} = P/A - Pe/z_t + mg/z_t + mq/z_t$$

$$= 2.155 - 3.995 + 1.834 + 3.295 = 3.289 \text{ N/mm}^2$$

$$f_b = \frac{P}{A} + \frac{Pe}{z_b} - \frac{mg}{z_b} - \frac{mq}{z_b}$$

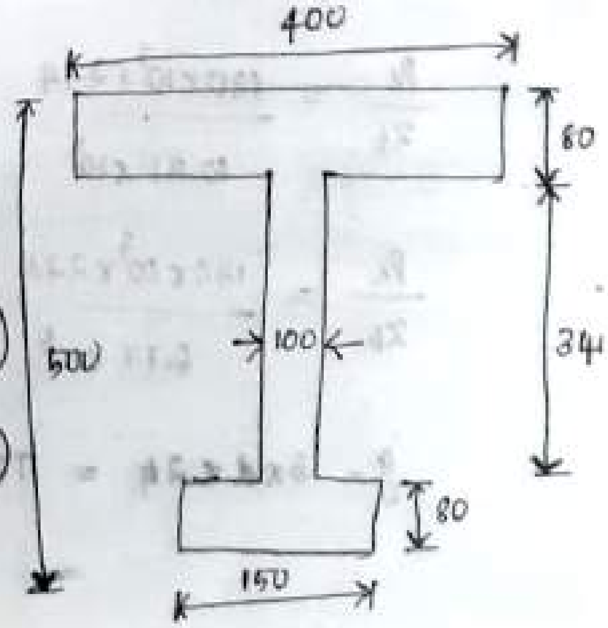
$$= 2.155 + 6.25 - 2.87 - 5.155 = 0.38 \text{ N/mm}^2$$



8) An unsym. I S/c beam is used to support an imposed load of 4 kN/m over a span of 12m. The S/c details are top flange 400 mm wide & 80 mm thick, thickness of web is 100 mm bottom flange 150 mm wide and 80 mm thick, overall depth of beam is 500 mm. At the centre of span effectively prestress force is 150 kN is located at 80 mm from the soffit of beam. Estimate stress at centre of the span of beam for following conditions

a) prestress + self wt

b) prestress + self wt + live load.



$$\bar{y} = \frac{(150\times80\times40)+(100\times340\times(170+80)) + (400\times80\times(340+80+40))}{(150\times80)+(100\times340)+(400\times80)}$$

$$= 303.84 \approx \underline{304 \text{ mm}}$$

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Load Balancing Concept

* Conditions;

(i) moment due to prestress = moment due to applied load.

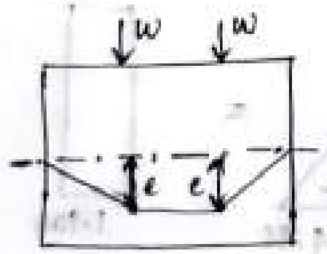
(ii) vertical reaction of prestress = externally applied load.

Suitable cable profile can be selected such that transverse component of cable force balances the given type of external load. The various types of reaction of cable depends upon shape of cable profile.

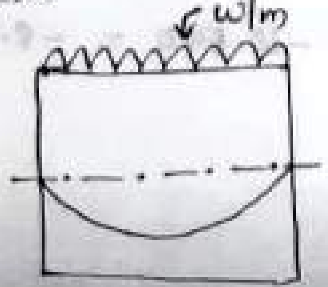
The concept of load balancing is useful in selecting the tendon profile which can supply most desirable system of force in concrete.

The requirement will be satisfied if cable profile in pre stress member corresponds to shape of BMD results to the external load.

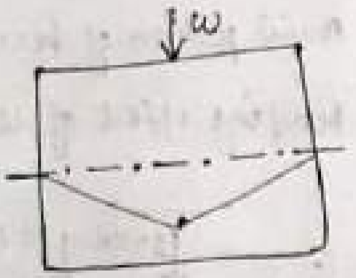
- if the beam supports 2 con. load, cable should follow a trapezoidal profile



- if beam supports UDL, corresponding tendon should follow parabolic profile



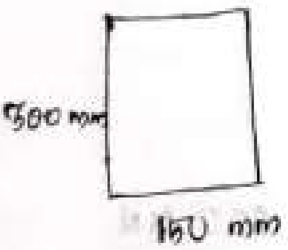
• if beam is subjected to single con. load corresponding tendon should form triangular shape

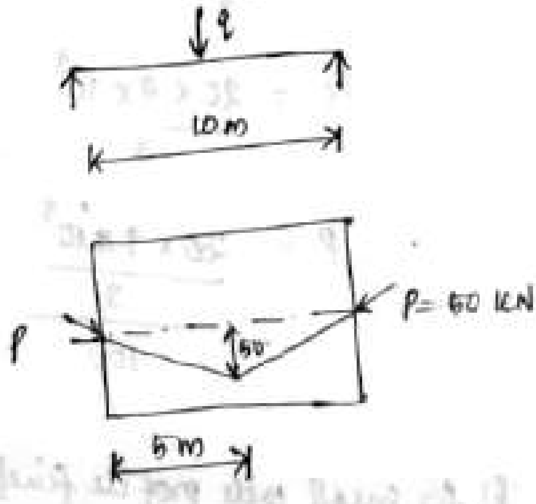


$$P_e = \frac{wl}{4}$$

Problem

1) A rectangular prestressed beam 150 mm x 300 mm is used over an effective span 10 m. cable with zero eccentricity at support and linearly varying to 50 mm @ centre. and carries an effective prestressing force of 50 kN, find magnitude of con. load 'q' located @ centre of span for the following conditions of centre of span i.e. if load counter acts the bending effect of prestressing force (neglect self wt of beam).



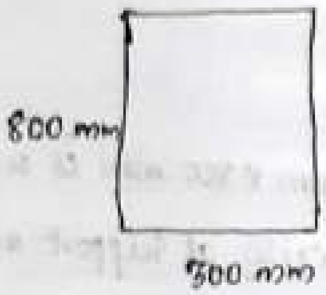


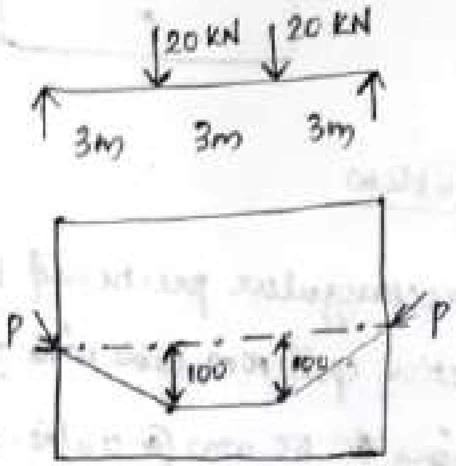
moment due to prestress = moment due to applied load.

$$P_e = ql/4$$

$$q = \frac{4Pe}{l} = \frac{4 \times 50 \times 10^3 \times 50}{10 \times 10^3} = \underline{1000\ N}$$

2) A rect. beam 300 x 800 mm supports 2 con. loads of 20 kN each @ 3rd point of span of 9 m. Suggest suitable cable profile if eccentricity of cable profile is 100 mm @ middle third portion of beam calculate prestressing force reqd. to balance bending effect of con. load (neglect self wt of concrete).





Suitable cable profile is trapezoidal in shape (BMD is in shape of trapezoid).

$$Pe = \frac{Wl}{3}$$

$$Pe = \frac{20 \times 9 \times 10^3}{3}$$

$$P = \frac{\frac{20 \times 9 \times 10^3}{3}}{100} = \underline{600 \text{ kN}}$$

(b) for small cable profile find ~~diff~~ effective force in cable if resulting stress due to self wt. impose an prestress force zero of bottom fibre of mid span s/c. Density of concrete = 24 kN/m³.

$$f_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{mg}{Z} - \frac{mq}{Z} = 0$$

$$\Rightarrow \frac{P}{A} + \frac{Pe}{Z} - \frac{mg}{Z} - \frac{mq}{Z} = 0$$

$g = b \times d \times 24 = .8 \times .3 \times 24 = 5.76$ kN/m.

$q = 20$ kN

$$Z = \frac{\frac{300 \times 800^3}{12}}{400} = 32 \times 10^6 \ mm^4$$

$$\frac{mg}{Z_b} = 1.822.$$

$$mq = \frac{20 \times 10^3 \times 4 \times 10^3}{3} = 60 \times 10^6$$

$$\frac{mq}{Z_b} = \frac{60 \times 10^6}{32 \times 10^6} = 1.875$$

$$\frac{P}{240 \times 10^3} + \frac{P \times 100}{32 \times 10^6} - 1.822 - 1.875 = 0.$$

$$P\left[4.166 \times 10^{-6} + 3.125 \times 10^{-6}\right] = 3.697$$

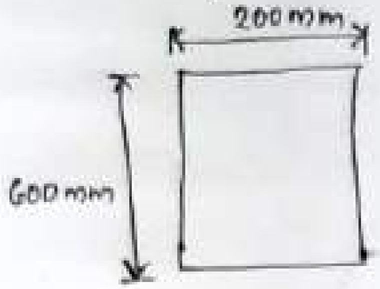
$$\underline{\underline{P = 507.06 \text{ kN}}}$$

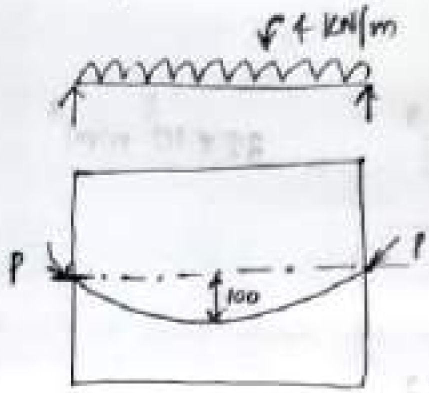
3) A prestress concrete beam supports an imposed load of 4 kN/m over an effective span 10m. beam has a rectangular c/s with width of 200 mm and depth of 600 mm. find effective prestress force in cable. if it is parabolic with an eccentricity of 100mm at centre and zero @ ends for following condition.

(a) if bending effect of pre stress force is nullify by imposed load for mid span S/c. (neglect self wt. of beam).

(b) if resulting stress due to imposed load and prestressing force is zero at soffit of beam for mid span S/c.

Density of concrete is 24 kN/m³.





(a) $Pe = \frac{wl^2}{8}$

$P \times 100 \times 10^{-3} = \frac{4 \times 10^2}{8}$

$P = 500 \text{ kN}$

(b) $f_b = 0;$

$\frac{P}{A} + \frac{Pe}{Z} - \frac{Mg}{Z} - \frac{Mq}{Z} = 0.$

$Z = \frac{200 \times 600^3 / 12}{300} = 12 \times 10^6 \text{ mm}^3$

$\frac{Mg}{Z} = \frac{\frac{2.88 \times 10^2 \times 10^6}{8}}{12 \times 10^6} = \underline{3}$

$$q = \frac{wL^2}{8} = \frac{4 \times 10^2}{8} = 50$$

$$\frac{Mq}{z} = \frac{50}{12 \times 10^6 \times 10^6} = 4.166 \text{ N/mm}^2$$

$$\frac{P}{A} + \frac{Pe}{z} = \frac{Mg}{z} + \frac{Mq}{z}$$

$$P\left[\frac{1}{200 \times 600} + \frac{100}{12 \times 10^6}\right] = 3 + 4.166$$

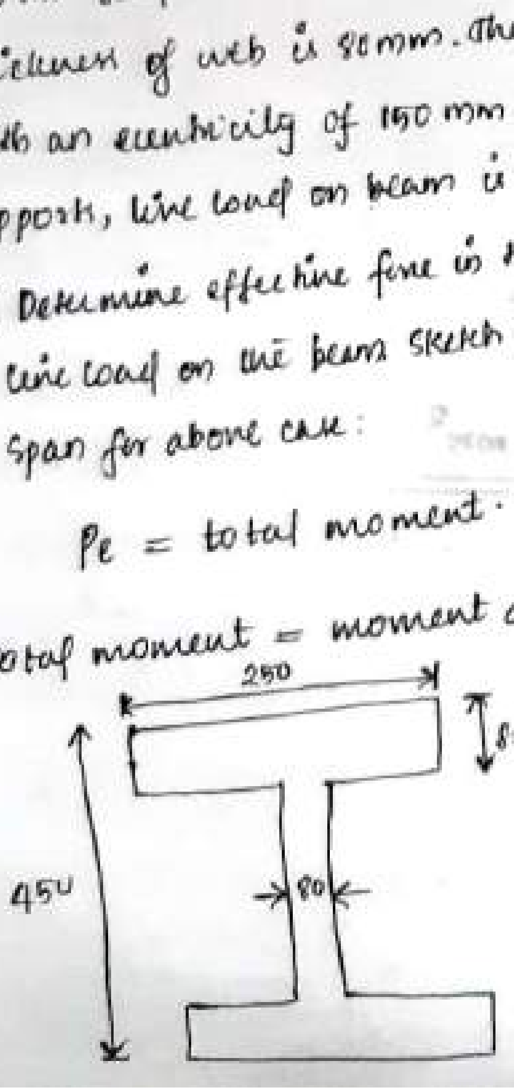
$$P = \underline{430 \text{ kN}}$$

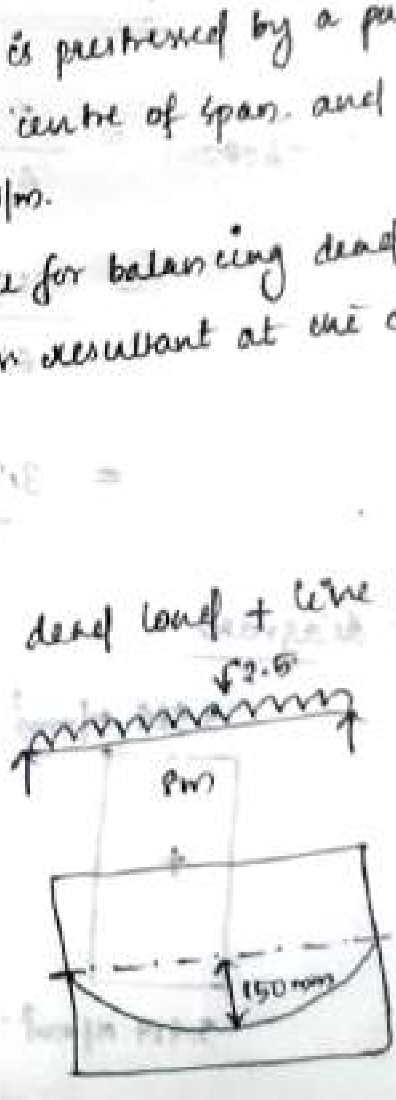
4) A beam of symmetrical ⊕ I/c with span 8m has a flange width of 250 mm and thickness 80 mm resp. The overall depth of beam is 450 mm. Thickness of web is 80 mm. The beam is prestressed by a parabolic cable with an eccentricity of 150 mm at the centre of span and zero at supports, live load on beam is 2.5 kN/m.

(a) Determine effective fine in the cable for balancing dead load and live load on the beam sketch the shen resultant at the centre of span for above case:

Pe = total moment.

Total moment = moment due to dead load + live load





(a) $P_e = M$

$g = b \times d \times 24$

$= (250 \times 80 + 290 \times 80 + 250 \times 80) \times 24 \times 10^{-6}$

$= \underline{\underline{1.51 \text{ kN/m}}}$

$$P_e = \frac{Mg l^2}{8} + \frac{Mq L^2}{8}$$

$$= \frac{1.5 \times 8^2 \times 10^6}{8} + \frac{2.5 \times 8^2 \times 10^6}{8}$$

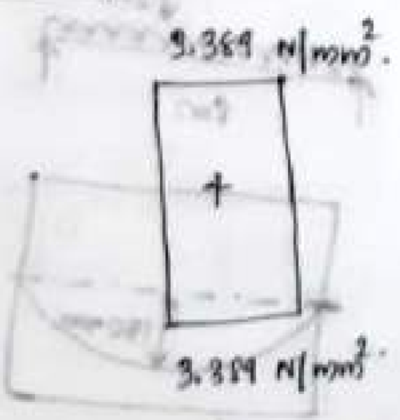
$$= \underline{\underline{214.3 \times 10^3 \text{ kN}}}$$

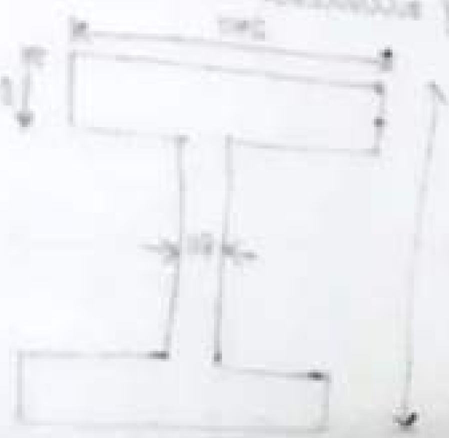
$$f_{top} = f_{bottom} = \frac{P}{A}$$

$$= \frac{214 \times 10^3}{63.2 \times 10^3}$$

$$= \underline{\underline{3.389 \text{ N/mm}^2}}$$

Stress diagram





5) A rect concrete beam 150 mm x 300 mm is prestressed by straight cable carrying an effective prestress force of 225 kN @ an eccentricity of 50 mm. Beam supports UDL of 7.2 kN/m inclusive of self wt of beam. Span of beam is 5 m. modulus of rupture of concrete 5 N/mm^2. calculate load factor against cracking.

$$\text{load factor} = \frac{\text{ultimate load}}{\text{working load.}}$$

modulus of rupture = max tensile stress which can withstand by concrete just before cracking.

f_{bottom} = modulus of rupture = -5 N/mm^2 (tensile stress).

$$f_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

so; ultimate load = live load + dead load

$$-5 = \frac{225}{150 \times 300} + \frac{225 \times 50}{\frac{150 \times 300^3}{12 \times \frac{300}{2}}} - \frac{Mu}{\frac{150 \times 300^3}{12 \times 150}}$$

$$Mu = 33.75 \text{ kNm}$$

$$Mu = \frac{wu l^2}{8}$$

$$wu = \frac{Mu \times 8}{l^2} = \frac{33.75 \times 8}{5^2} = \underline{\underline{10.8 \text{ kN/m}}}$$

$$\text{load factor} = \frac{\text{ultimate load}}{\text{working load}}$$

$$= \frac{10.8}{7.2}$$

$$= \underline{1.5}$$

LOSS OF PRESTRESS

Classification of losses

1. loss of pre stress during tensioning process.
2. loss of pre stress at anchoring stage.
3. loss of prestress after application of load.

Pre tensioning	Post tensioning
• Elastic deformation of concrete.	• No loss due to elastic deformation [if all wires are simultaneously tensioned there is no loss. if wires are successively tensioned there will be loss of tension due to elastic deformation of concrete].
• Relaxation of stresses in steel.	• Relaxation of stresses in steel.

• Shrinkage of concrete	• Shrinkage of concrete.
• Creep of concrete.	• Creep of concrete.
	• loss due to friction
	• Anchorage slip.
• loss of prestress may occur due to sudden change in temperature.	• loss of prestress may occur due to sudden change in temperature.

I: loss due to creep of concrete (IS 1343-1980)

$$\boxed{\text{Loss} = E_s \times \text{creep strain}}$$

$$\phi = \text{Creep coefficient} = \frac{\text{creep strain}}{\text{Elastic strain}} = \frac{\varepsilon c}{\varepsilon e}$$

$$\phi = \frac{\varepsilon c}{\varepsilon e} \Rightarrow \varepsilon c = \phi \varepsilon e.$$

$$\therefore \text{loss} = E_s \times \phi \varepsilon e.$$

f_c = stress of concrete @ level of steel

E_c = youngs modulus of concrete.

E_s = youngs modulus of steel.

modular ratio; $m = \frac{E_s}{E_c}$

$$\text{loss} = E_s \times \phi \times \text{elastic strain}$$

$$\text{loss} = E_s \cdot \phi \, \varepsilon_c$$

$$\text{loss} = E_s \phi \frac{f_c}{E_c}$$

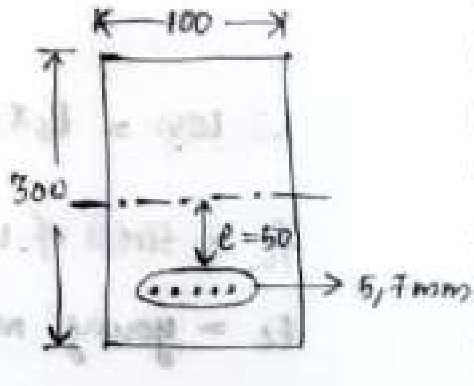
$$\therefore \boxed{\text{loss} = m \phi f_c}$$

from IS 1343 – 1980; pg: 17 cl. 5.2.5.1

loss of prestress due to creep of concrete is obtained as the product of modulus of elasticity of prestressing steel and ultimate creep strain of fibre integrated along line of center of gravity of prestressing steel over its entire length.

Problems

1) A concrete beam of rectangular s/c 100 x 300 mm is prestressed by 5 wires of 7mm dia located at an eccentricity of 50 mm initial stressing wire is 1200 N/mm². Estimate loss of prestress due to creep of concrete given $E_s = 210$ KN/mm², $E_c = 35$ KN/mm².



Assume age of concrete = 28 days;

SO 1343-1980 pg: 17, $\phi = 1.6$

$b = 100$ mm.

$d = 300$ mm

5, 7 mmϕ.

$e = 50$ mm.

$E_s = 210$ N/mm²

$E_c = 35$ N/mm²

$loss = m \phi f_c$

$m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

$A = \pi/4 \times 7^2 \times 5$

$= 192.42 \, mm^2$

$f_b = \frac{P}{A} + \frac{Pe}{Z}$

$P = 1200 \times 5 \times \pi/4 \times 7^2$

$= 230 \times 10^3 \, N/mm$

$= \frac{230 \times 10^3}{30 \times 10^3} + \frac{230 \times 10^3 \times 50}{1.5 \times 10^6}$

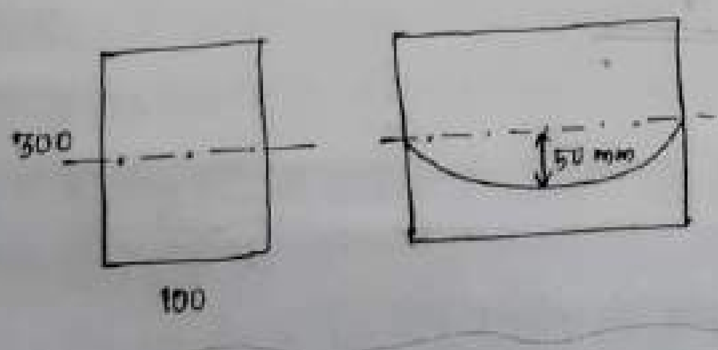
$Z = \frac{\frac{100 \times 300^3}{12}}{150} = 15 \times 10^6 \, mm$

$= 15.33$

$loss = 6 \times 15.33 \times 1.6$

$= 147.16 \, N/mm^2$

2) A pretensioned concrete beam 100x300 mm is prestressed by a parabolic cable with zero at cable and e = 50 mm @ center area of cable is 200 mm² & initial stress in cable is 1200 N/mm². Calculate loss due to creep of concrete given $E_s = 210 \, N/mm^2$, $E_c = 35 \, N/mm^2$.



$\phi = 1.6$ from IS 1343-1980, 28 days curing

$m = \frac{210}{35} = 6$

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$f_b = \frac{P}{A} + \frac{Pe}{Z}$ (@ mid span)

$P = 1200 \times 200 = 240 \times 10^3$

$Z = I/y = 1.6 \times 10^6 \ mm^3$

$= \frac{240 \times 10^3}{30 \times 10^3} + \frac{240 \times 10^3 \times 50}{1.6 \times 10^6}$

$= \underline{16}$

$f_{support} = \frac{P}{A}$

$= \frac{240 \times 10^3}{30 \times 10^3} = \underline{8}$

$loss = m \phi f_c$

$= 6 \times 1.6 \times \left[\frac{f_{supp} + f_{mid\ span}}{2} \right]$

$= 6 \times 1.6 \times \left(\frac{8 + 16}{2} \right)$

$= \underline{\underline{115.2 \ N/mm^2}}$

B: Loss due to shrinkage of concrete

Influenced by;

- type of cement and aggregate
- method of curing used.

shrinkage of concrete used in prestress members result in shortening of tensioning wires and hence contribute loss of stress.

$\boxed{\text{loss} = E_s \cdot \text{shrinkage strain}}$ pg:16, cl. 5.2.4.1

value of shrinkage strain;

for pretensioning $= 0.0003$

for post tensioning $= \dfrac{0.0002}{\log_{10}(t+2)}$

$t \rightarrow$ age of concrete @ transfer in days.

Problem

1. prestressed concrete beam is prestress by cable carrying an initial force of 300 KN, CSA of wire in cable is 300 mm². calculate % loss of stress in cable only due to shrinkage of concrete by assuming beam to be

(a) pre tensioned.

(b) post tensioned.

Ans: (a) Pre tensioned

$\text{loss} = E_s \times \text{shrinkage strain} = 210 \times 0.0003$

$= .063\ KN/mm^2 = \underline{\underline{63\ N/mm^2}}$

(b) Post tensioned

loss = E_s x shrinkage strain

$$= 210 \times \frac{.0002}{\log_{10}(28+2)}$$

$= .0284\ kN/mm^2$

$= 28.4\ N/mm^2$

% loss

initial stress $= \frac{P}{A} = \frac{300\ kN}{300\ mm^2} = 1\ kN/mm^2 = 1000\ N/mm^2$.

Pre tensioned

$$\% = \left(\frac{63}{1000} \times 100\right) = 6.3\%$$

Post tensioned

$$\%\ \text{loss} = \left(\frac{28.4}{1000} \times 100\right)$$

$= 2.84\%$

III : loss due to elastic deformation of concrete

$$\boxed{\text{loss} = m \times f_c}$$

where; $m \rightarrow$ modular ratio $= E_s/E_c$

$f_c \rightarrow$ stress in concrete @ level of steel.

Problem

1. A pretensioned concrete beam 100 X 300 mm is prestressed by straight wires carry a force of 150 KN @ e = 50 mm, $E_s = 210$ KN/mm^2, $E_c = 35$ KN/m. Estimate % loss of stress in steel due to elastic deformation of concrete if area of steel is 188 mm^2.

loss $= m \times f_c$.

$$m = \frac{210}{35} = 6$$

$$Z = \frac{\frac{100 \times 300^3}{12}}{150} = 1.5 \times 10^6 \text{ mm}^3$$

$$f_c = f_{bottom} = \frac{P}{A} + \frac{Pe}{Z}$$

$$= \frac{150 \times 10^3}{30 \times 10^3} + \frac{150 \times 50 \times 10^3}{1.5 \times 10^6}$$

$$= 10 \text{ N/mm}^2$$

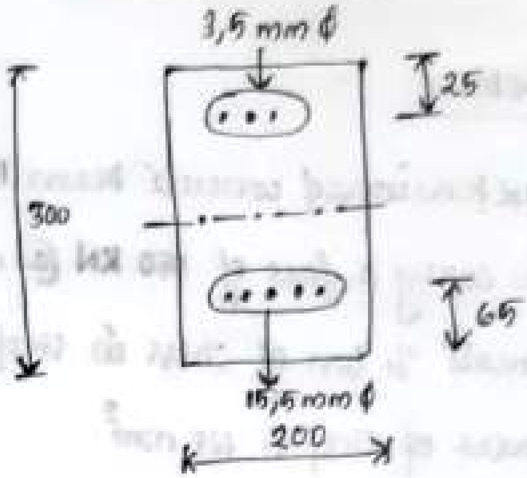
$$\text{loss} = m \times f_c = 60 \text{ N/mm}^2.$$

$$\text{initial stress} = \frac{150}{188} = 0.7978 = 797.8 \approx 800 \text{ N/mm}^2$$

$$\% \text{ loss} = \frac{60}{800} \times 100 = 7.50\ \%$$

2. A rectangular concrete beam 200×300 mm prestress by means of 15 no. of 5 mm ϕ wires located @ 65 mm from bottom of beam & 3 no. of 5 mm ϕ located at distance of 25 mm from top of beam if wire are initially tensioned to stress of 840 N/mm². calculate % loss of stress in steel immediately after transfer allowing loss of stress due to elastic deformation of concrete only.

$$loss = m \times f_c$$



$$\bar{y} = \frac{ast_1 \cdot y_1 + ast_2 \cdot y_2}{ast_1 + ast_2}$$

$$= \frac{19144.08 + 58.90 \times 275}{58.90 + 294.54}$$

$$= 100.07 \approx \underline{100\ mm}$$

$$Z = \frac{\frac{200 \times 300^3}{12}}{(150-65)}$$

$$= \underline{5.29 \times 10^6\ mm^3}$$

$$P = 840 \times 18 \times \pi/4 \times 5^2$$

$$= 296.88 \times 10^3\ N$$

$$e = 150 - 100 = 50\ mm.$$

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z}$$

$$= \frac{296.88 \times 10^3}{60 \times 10^3} - \frac{296.88 \times 10^3 \times 50}{\frac{200 \times 300^3/12}{150-25}}$$

$= \underline{\underline{0.825 \ N/mm^2}}$

$$f_b = \frac{P}{A} + \frac{Pe}{Z}$$

$$= \frac{296.88 \times 10^3}{200 \times 300} + \frac{296.88 \times 10^3 \times 50}{\frac{\frac{200 \times 300^3}{12}}{150-65}}$$

$= \underline{\underline{7.751 \ N/mm^2}}$

Top loss $= m \times f_{top}$

$= 6 \times 0.825 = 4.948 \ N/mm^2$

$\% \text{ loss} = \frac{4.948}{840} \times 100 = \underline{\underline{0.589 \%}}$

Bottom loss $= m \times f_{bottom}$

$= 6 \times 7.751 = 46.506 \ N/mm^2$

$\% \text{ loss} = \frac{46.506}{840} \times 100 = \underline{\underline{5.536 \%}}$

Note:

Here we take the value of y = eccentricity of steel

So, for bottom $y = 150 - 65 = 86$ mm.

becoz we calculate stress at level of steel.

(Contd)

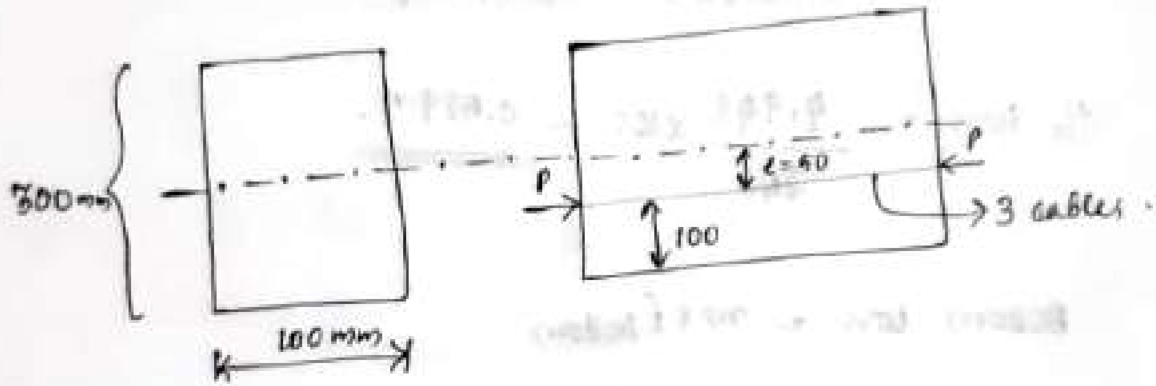
3) A post tension concrete beam 100 x 300 mm deep is prestressed by 3 cables each with CSA of 50 mm² and with an initial stress of 1200 N/mm² all the 3 cables are straight and located @ 100 mm from soffit of beam if modular ratio is 6. calculate loss of stress in 3 case due to elastic deformation of concrete for following cases.

(a) Simultaneous tensioning, anchoring of all 3 cables.

(b) successive tensioning of 3 cables one at a time.

soln :- (a) There is no elastic deformation ; loss = 0.

(b) when cable is successively tensioned;



when cable 1 is tensioned and anchored;

No loss due to elastic deformation (equalent to normal post tension beam).

cable 2 is tensioned and anchored.

loss of prestress in cable 1 = mf_c

$m = 6.$

$P = 1200 \times 50$

$= \underline{60 \times 10^3 \text{ N}}$

$$f_c = \frac{P}{A} + \frac{Pe}{Z}$$

$$= \frac{P}{A} + \frac{Pe.y}{I}$$

$$= \frac{60\times10^3}{30\times10^3} + \frac{60\times10^3\times50\times50}{\frac{100\times300^3}{12}}$$

$$= 2.67\ N/mm^2.$$

loss $= mfc = 6\times2.67 = \underline{\underline{16.02\ N/mm^2}}$

cable 3 is tensioned & anchored

loss of stress in both cables 1 and 2

loss of stress in cable 1 $= 16\ N/mm^2$ (same as above case).

loss of stress in cable 2 $= 16\ N/mm^2$ (same as above case since position of cable is not changed).

total loss of concrete due to elastic deformation in the cable.

Cable 1

loss $= 16 + 16 = 32\ N/mm^2$. (due to cable 2,3).

Cable 2

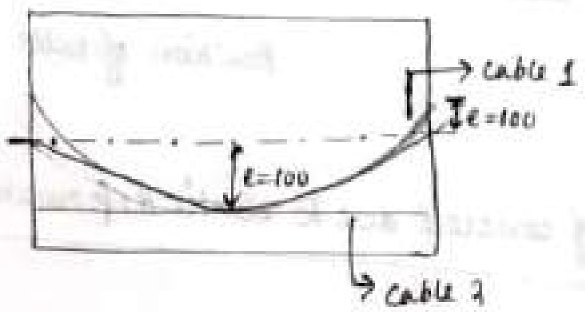
loss $= 16\ N/mm^2$ (due to cable 3 tension)

Cable 3

loss $= 0$.

IV : Loss of stress due to successive tensioning of curved cables

1) A simply supported concrete beam of uniform S/c is post tensioned by means of 2 cables both of which have an eccentricity 100 mm below centroid of S/c at mid span. The first cable is parabolic and anchored at an eccentricity 100 mm above centroid @ each end. The second cable is straight and parallel to line joining support. CSA of each cable is 100 mm². The concrete beam has sectional area 2×10^4 mm² and radius of gyration of 120 mm. Calculate loss of stress in first cable when second is tensioned due to a stress of 1200 N/mm². modular ratio = 6.



$A_c = 2\times10^4\ mm^2$

$A_{cable} = 100\ mm^2$

$P_{stress} = 1200\ N/mm^2$.

$i = 120\ mm$

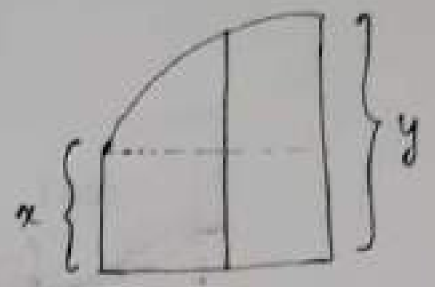
$i = \sqrt{I/A}$

$14400 = \dfrac{I}{2\times10^4}$

$\underline{I = 288\times10^6\ mm^4}$

cable 2 is tensioned there is loss in cable 1. Cable 1 is in shape of parabola the stress variation so we have to consider average stress value and draw fig:

Stress @ top fibre, $y = e$ (in loss)



$\text{avg} = x + \frac{2}{3}(y - x)$

$$f_{top} = \frac{P}{A} - \frac{Pe \cdot y}{I}$$

$$= \frac{120 \times 10^3}{2 \times 10^4} - \frac{120 \times 10^3 \times 100 \times 100}{288 \times 10^6}$$

$$= \underline{\underline{1.833 \ N/mm^2}}$$

$$f_b = \frac{120 \times 10^3}{2 \times 10^4} + \frac{120 \times 10^3 \times 100 \times 100}{288 \times 10^6}$$

$$= \underline{\underline{10.16 \ N/mm^2}}$$

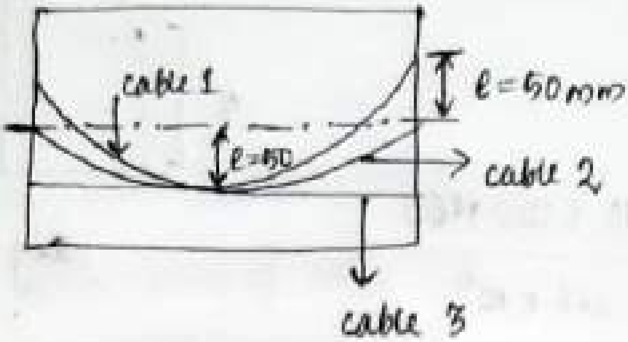
$$\text{Avg} = 1.833 + 2/3\,(10.16 - 1.833)$$

$$= 7.38 \ N/mm^2$$

$$\text{loss} = mf_c = 6 \times 7.38 = \underline{\underline{44.28 \ N/mm^2}}$$

2) A post tensioned concrete beam 100 x 300 mm spaning over 10 m is stressed by successive tensioning and anchoring of 3 cables 1, 2 and 3 resp. CSA of each cable is 200 mm^2 and initial stress in cable is 1200 N/mm^2 modular ratio is 6. first cable is parabolic with an eccentricity 50 mm below centroidal axis @ center of span and 50 mm above centroidal axis @ support s/c second cable is parabolic with zero eccentricity at support and eccentricity of 50 mm @ center of span 3rd cable is straight with uniform e of 50 mm below centroidal axis. and Estimate % loss of stress in each of cable if they are successively tensioned and

(nn)



$P = 1200 \times 200$

$P = 240 \times 10^3\, N$

$A = 30 \times 10^3\, mm^2$

$$I = \frac{bd^3}{12}$$

$$= \frac{100 \times 300^3}{12} = \underline{\underline{225 \times 10^6}}$$

<u>Cable 1 is tensioned</u>

There is no loss due to elastic deformation.

<u>Cable 2 is tensioned</u>

Elastic deformation in cable 1

loss = mf_c

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z}$$

$$= \frac{240 \times 10^3}{30 \times 10^3} - \frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}$$

$$= 5.33\ N/mm^2$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z}$$

$$= \frac{240 \times 10^3}{30 \times 10^3} + \frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}$$

$$= 10.66\ N/mm^2.$$

$$Avg = 5.33 + \frac{2}{3}(10.66 - 5.33)$$

$$= 8.9\ N/mm^2.$$

$$loss = mf_c = 6 \times 8.9 = \underline{53.3\ N/mm^2}.$$

<u>cable 3 is tensioned and anchored</u>

loss of stress due to elastic deformation of cable 1 = 53.3
(same as above case).

<u>loss of stress due to elastic deformation cable 2.</u>

At support $f = P/A$ $(e=0)$

At mid span $f = \frac{P}{A} + \frac{Pey}{I}$

$$P/A = 8$$

$$\frac{Pey}{I} = 2.66$$

$$Avg = 9.78\ N/mm^2.$$

$$loss = 6 \times 9.78 = 58.6$$

$$loss = 53.3 + 58.6 = 106.6$$

V : loss due to anchorage slip

loss due to anchorage slip $= \frac{E_s \Delta}{L}$

Δ = slip of anchorage in mm.

L = length of cable in mm.

E_s = modulus of elasticity of steel.

Note :-

this loss is higher for short members.

1. Concrete beam is post tensioned by cable and carrying an initial stress of 1000 N/mm². The slip at jacking end is 5mm. $E_s = 210$ KN/mm². Estimate % loss of stress due to anchorage slip. if length of cable

(a) 30 m (b) 3m.

loss of slip $= \frac{E_s \Delta}{L} = \frac{210 \times 10^3 \times 5}{30 \times 10^3} = \underline{35}$

$E_s = 210$; $\Delta = 5$; $L = 30 \times 10^3$.

loss of slip $= \frac{210 \times 10^3 \times 5}{3 \times 10^3} = \underline{350 \text{ N/mm}^2}$

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VI: Loss due to relaxational strain

Loss due to relaxational strain = 5% of initial stress.

VII: Loss due to friction

2 types;

- Loss due to curvature effect.
- Loss due to length effect / wave effect / wobble effect.

Loss due to curvature effect

In curved ducts loss of prestress depend upon radius of curvature 'R' of duct and coefficient of friction μ between duct surface and tendon.

$$\boxed{P_x = P_0 e^{-\frac{\mu x}{R}}}$$

$$x/R = \alpha$$

$$P_x = P_0 e^{-\alpha\mu}$$

P_x = prestressing force at any distance x.

μ = coeff. of friction

R = radius of curvature of tendon.

P_0 = Prestressing force at jacking end.

α = cumalative angle in radians through which tangent to cable profile has turn b/w any 2 pts under consideration.

$\alpha = 2\theta$.

loss due to length or wave or wobble effect

It depends upon the local deviation in alignment of cable,

$$P_x = P_0 \cdot e^{-kx}$$

k = wobble correction / friction coefficient for wave effect.

$$P_x = P_0 e^{\frac{-\mu x}{R}} + P_0 e^{-kx}$$

$$= P_0 e^{\frac{-\mu x}{R} - kx}$$

$$= P_0 e^{-\left(\frac{\mu x}{R} + kx\right)}$$

$$e^{-\left(\frac{\mu x}{R} + k\right)} = 1 - \left(kx + \frac{\mu x}{R}\right) + \cancel{\left(kx + \frac{\mu x}{R}\right)^2}$$

neglect

neglecting higher terms;

$$e^{-\left(kx + \frac{\mu x}{R}\right)} = 1 - \left(kx + \frac{\mu x}{R}\right)$$

$$\therefore \; P_x = P_0 - \left[1 - \left(kx + \frac{\mu x}{R}\right)\right]$$

In code; $\boxed{P_x = P_0 e^{(-\mu \alpha + kx)}}$

$$\text{Loss} = \text{initial stress} \times (\mu \alpha + kx)$$

$$= \frac{P_0}{A} (\mu \alpha + kx)$$

1)- A concrete beam of 10m span 100 X 300 mm deep is prestress by 3 cables each of area 300 mm², initial stress in cable is 1200 N/mm². Cable 1 is parabolic with eccentricity of 50 mm about centroid at support and 50 mm below center of span. Cable 2 is also parabolic with 0 eccentricity @ support and 50 mm below @ center of span. Cable 3 is straight with uniform 'e' of 50 mm below the centroid. If cables are tensioned from one end only, Estimate the % loss of stress in each cable due to friction. given that $\mu = 0.35$, $k = .0015/m$.

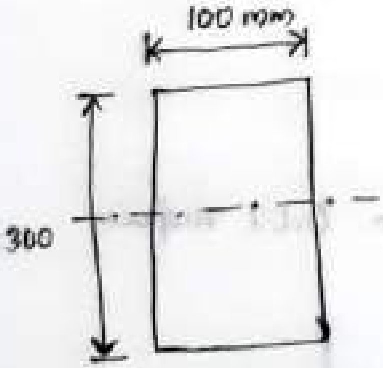
General eqn. of parabola;

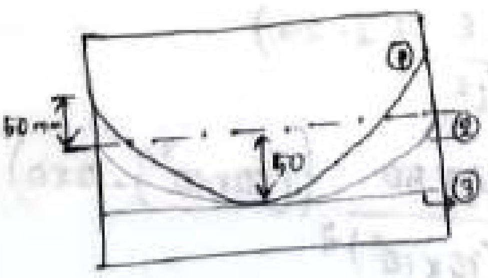
$$y = \frac{4e}{l^2} x(l-x)$$

e = central dip

$$\theta = \frac{dy}{dx} = \frac{4e}{l^2}(l-2x)$$

$$\boxed{\theta = \frac{4e}{l^2}(l-2x) = \text{slope.}}$$





P_0 = initial stress X area

$= 1200 \times 300 = 360 \times 10^3$ N

a) Consider cable ①:

@ support, slope $\theta = \frac{4e}{l^2}(l-2x)$

$= \frac{4\times(50+50)}{(10\times10^3)^2}\times((10\times10^3)-2\times0)$

$= 0.04$ radians

$\alpha = 2\theta = 2\times.04 = .08$ radians.

loss of prestress $= \frac{P_0}{A}(\mu\alpha + Kx)$

$x = l$ (because loss is maximum @ ends).

loss $= 1200\times(0.35\times0.08 + 0.0015\times10)$

$= 51.6$ N/mm^2.

% loss $= \frac{51.6}{1200}\times100 = \underline{\underline{4.3\%}}$

<u>Cable 2:</u>

$\theta = \frac{4e}{l^2}(l-2x)$

$= \frac{4\times50}{(10\times10^3)^2}((10\times10^3)-2\times0) = 0.02$ radians

$\alpha = 2\theta = 0.04$ radians

loss $= 1200\times(0.35\times0.04 + 0.0015\times10)$

$= 34.8$ N/mm^2.

Scanned by CamScanner

$$\% \text{ loss} = \frac{34.8}{1200} \times 100 = \underline{\underline{2.9\%}}$$

cable 3;

$\theta = 0.$

$\alpha = 0.$

$$\text{loss} = 1200 \times (0.35 \times 0 + 0.0015 \times 10)$$
$$= 18 \text{ N/mm}^2$$

$$\% \text{ loss} = \frac{18}{1200} \times 100 = \underline{\underline{1.5\%}}$$

2) A prestressed concrete beam 200 x 300 mm is prestressed with wires (area = 320 mm^2) located at constant eccentricity of 50 mm and carrying initial stress of 1000 N/mm^2 and span 10 m. calculate % loss of stress in wires.

a) if the beam is pre tensioned.

b) if the beam is post tensioned using following details.

$E_s = 210$ kN/mm^2.

$E_c = 35$ kN/mm^2

Relaxation of steel stress = 5% of initial stress.

shrinkage of concrete = 300×10^{-6} for pre tensioning

$= 200 \times 10^{-6}$ for post tensioning

creep coefficient = 1.6.

slip at anchorage = 1mm.

coefficient of friction for wave effect = 0.0015/m.

Soln

Given that initial stress = 1000 N/mm²

prestress = 1000 × 320 = 320×10^3 N

modular ratio = $\frac{210}{35} = 6$.

Stress at level of steel; $f_c = \frac{P}{A} + \frac{Pe \cdot y}{I}$

$$= \frac{320 \times 10^3}{200 \times 300} + \frac{320 \times 10^3 \times 50 \times 50}{\frac{200 \times 300^3}{12}}$$

$$= \underline{7.11 \text{ N/mm}^2}$$

Losses

I: loss due to creep of concrete = $m\phi f_c$

$\phi = 1.6$ for 28 days (IS1343 pg: 17)

∴ loss = 6 × 1.6 × 7.11 = $\underline{68.26 \text{ N/mm}^2}$

II: loss due to shrinkage of concrete = E_s × shrinkage strain

given the shrinkage strain for pretension = .0003.

post tension = $\frac{.0002}{\log_{10}(t+2)}$

∴ loss of shrinkage (Pretension) = 0.0003 × 210 × 10^3

$= \underline{63 \text{ N/mm}^2}$

loss of shrinkage (post tension) $= \frac{0.0002}{\log_{10}(t+2)} \times 210 \times 10^3$

$= \frac{0.0002}{\log(28+2)} \times 210 \times 10^3$

$= \underline{\underline{28.43 \text{ N/mm}^2}}$

III: loss due to elastic deformation $= mf_c$

$= 6 \times 7.11$

$= \underline{\underline{42.66 \text{ N/mm}^2}}$

IV: loss due to anchorage slip $= \frac{E_s \Delta}{L}$

$= \frac{210 \times 10^3 \times 1}{10 \times 10^3}$

$= \underline{\underline{21 \text{ N/mm}^2}}$

V: loss due to relaxation of strain $=$ 5% of initial stress

$= \frac{5}{100} \times 1000$

$= \underline{\underline{50 \text{ N/mm}^2}}$

VI: loss due to friction $=$ initial stress $\times (\mu\alpha + Kx)$

$= 1000 \times (0 + .0015 \times 10)$

$= \underline{\underline{15 \text{ N/mm}^2}}$

(962)

Types of losses	Pre tensioned	Post tensioned
creep of concrete	68.26	68.26
shrinkage of concrete	63	28.43
Elastic deformation	42.66	—
Relaxation strain	50	50
slip of anchorage	—	21
friction effect	—	15

7th hour 0-17

1st 2, 3, 10, 12, 14, 15, 20, 21, 24, 26, 27, 33, 34, 35, 36, 37, 38, 41, 42, 47, 49, 50, 51, 54, 55, 56, 58, 59, 60, 62, 63, 68, 73.

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39, 44, 35, ~~14~~, 27, 45, 1, 43, 42