MODULE 2

FIRST LAW OF THERMODYNAMICS FOR A CLOSED SYSTEM UNDERGOING A CYCLE

- The first law of thermodynamics states that, for a closed system undergoing a cycle, the cyclic integral of the heat is proportional to the cyclic integral of work.

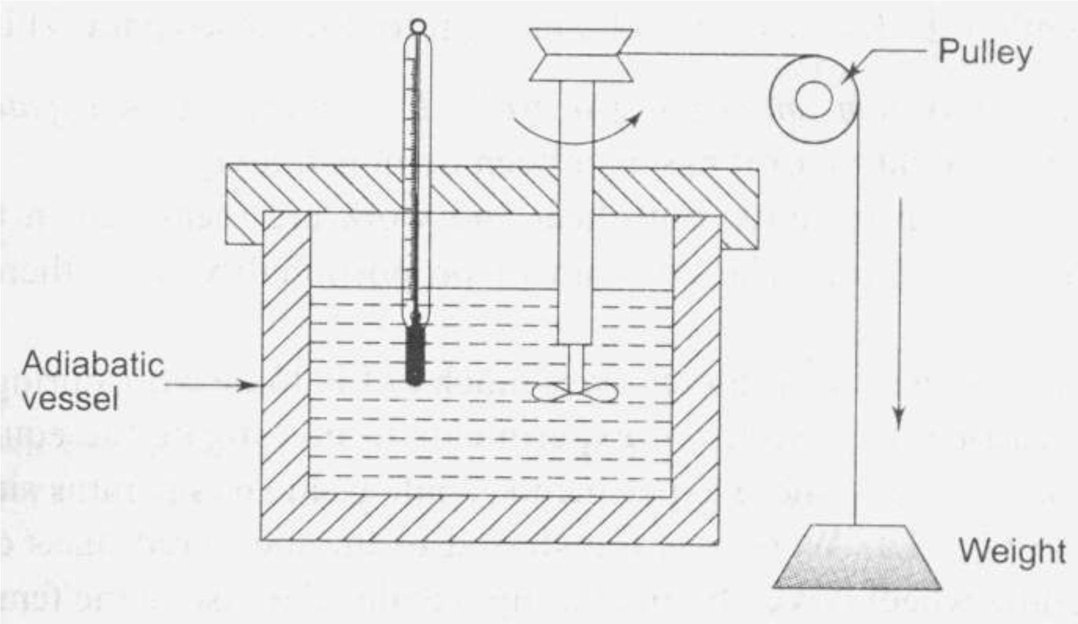
$$\oint dQ \propto \oint dw$$

- The transfer of heat & the performance of work may both cause the same effect in a system.

- Energy which enters a system as heat may leave the system as work, or energy which enters the system as work may leave as heat.

- Consider the gas in a container as shown. Let this system go through a cycle that is made up of two processes.

- In the first process work is done on the system by the paddle that turns as the weight is lowered. Let the system then return to its initial state by transferring heat from the system until the cycle has been completed.



- When the amounts of work & heat are compared, it is found that they are always proportional. This is known as the first law of thermodynamics, which is in the equation form, is:

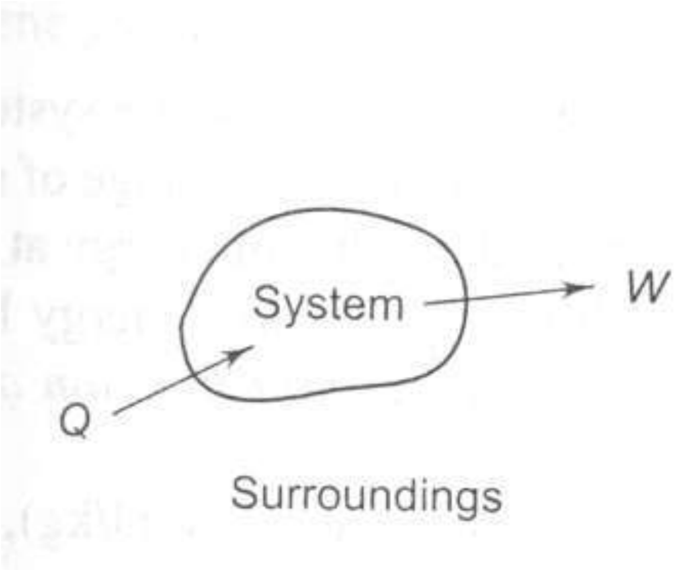
- $J\phi\, dQ = \phi\, dw$

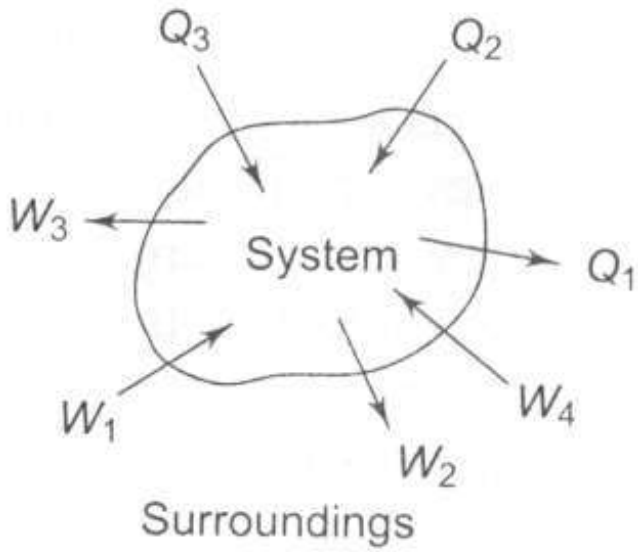
FIRST LAW FOR A CLOSED SYSTEM UNDERGOING A CHANGE OF STATE

- The first law of thermodynamic for a closed system undergoing a change of state states that the net energy transfer will be stored or accumulated within the system. If Q is the amount of heat transferred to the system & W is the amount of work transferred from the system during the process, the net energy transfer.

- (Q – W) will be stored in the system. Energy storage is neither heat nor work.
- ie. $Q - W = \Delta E$

  Where Q, W & ΔE are expressed in Joules.





- If there are more energy transfer quantities involved in the process as shown;

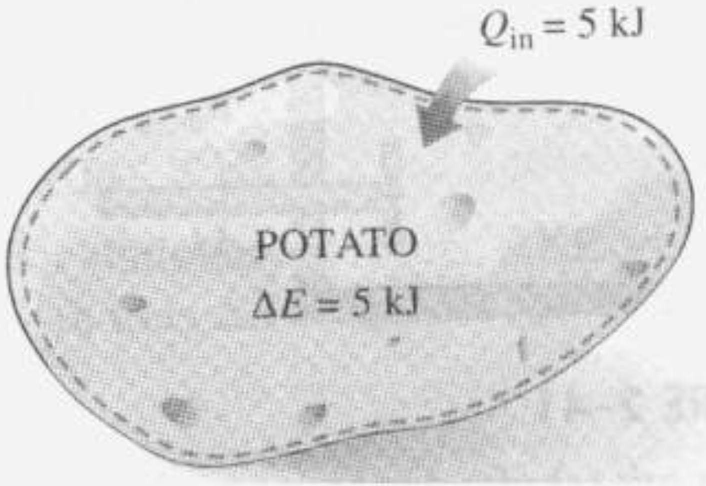
  The first law gives:
- $(Q_2 + Q_3 – Q_1) = \Delta E + (W_2+W_3-W_1 –W_4)$

  Energy is thus conserved in the operation.

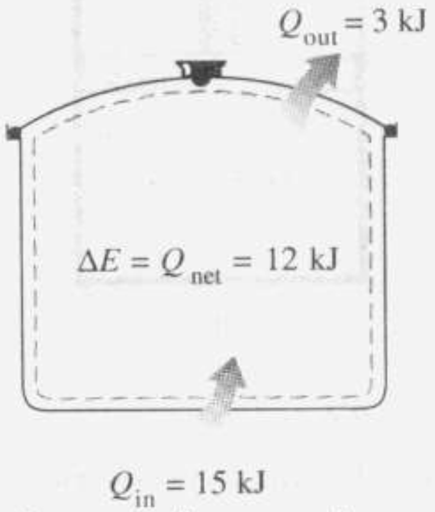
- eg. 1.Consider a process that involves heat transfer but no work transfer as in the case of baking a potato. That is if 5 kJ of heat is transferred to the potato the energy increase of the potato will also be 5 kJ.

- Consider a process that involves work transfer but no heat transfer as in the case of an electric heater in an adiabatic room. The electrical WD on the system is equal to the increase in energy of the system.

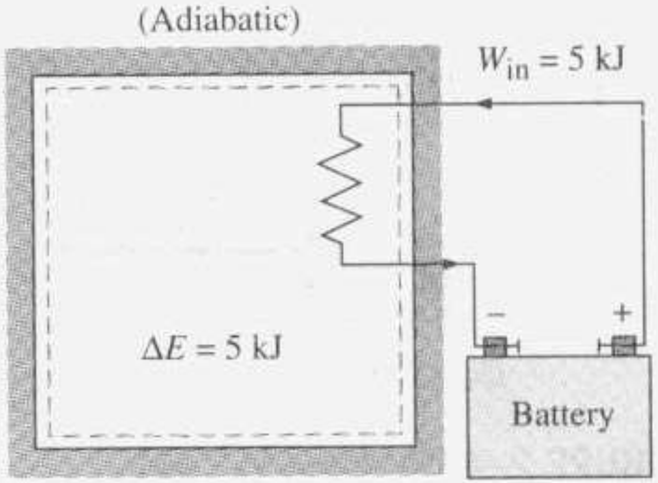
- Consider a system that involve both heat & work interaction. If a system gains 12 kJ of heat during a process while 6 kJ of work is done on it the increase in energy of the system during that process is 18 kJ.



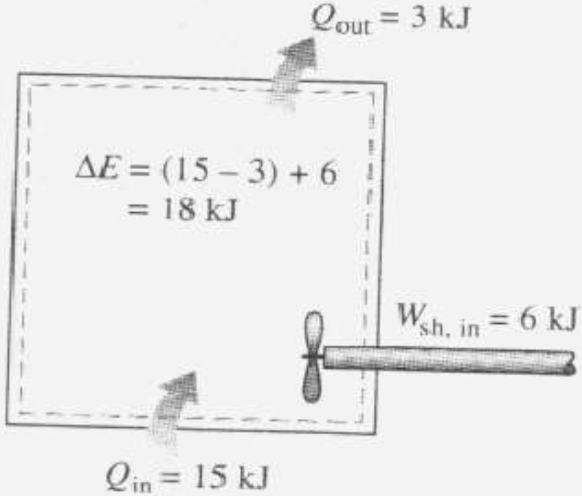
The increase in the energy of a potato in an oven is equal to the amount of heat transferred to it.



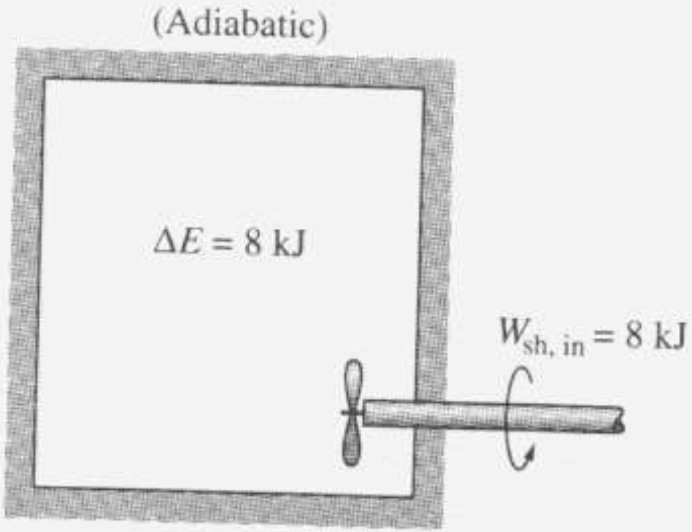
In the absence of any work interactions, the energy change of a



The work (electrical) done on an adiabatic system is equal to the increase in the energy of the system.



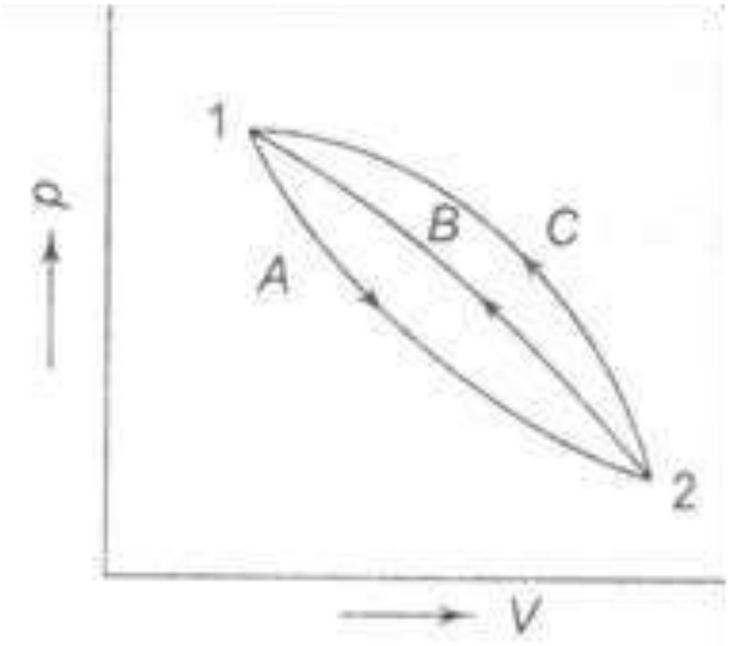
The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.



The work (shaft) done on an adiabatic system is equal to the increase in the energy of the system.

CONCEPT OF ENERGY

- Consider a system which changes its state from. State 1 to state 2 by following the path A, & returns from state 2 to state 1 by following the path B & C. So the system undergoes a cycle.



Applying first law for path A & B separately,

$Q_A = \Delta E_A + W_A$ ………………… (1)

$Q_B = \Delta E_B + W_B$ ………………… (2)

- The process A & B together constitutes a cycle, then;
- ϕ dQ = ϕdw
- $Q_A + Q_B = W_A + W_B$
- ie, $Q_A - W_B = W_B – Q_B$
- From (1) & (2) => ie $\Delta E_A = - \Delta E_B$ ………………. (3)
- Similarly considering the process A & C constituting a cycle we get.
- $\Delta E_A = - \Delta E_C$ ……………… (4)
- From (3) & (4), we get
- $\Delta E_B = - \Delta E_C = \Delta E_A$

- Therefore, it is seen that the change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state.

- If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follow.

- Therefore, energy has a definite value for every state of the system. Hence it is a point function & thus a property of the system.

ENTHALPY

Consider a system undergoing a quasi-equilibrium constant pressure process. Applying first law

${}_1Q_2 = U2 – U_1 + {}_1W_2$

The work can be calculated from ${}_1W_2 = {}^2\int_1 PdV$

Since pressure is constant,

${}_1W_2 = {}^2\int_1 dV = P\,(V_2 – V_1)$

$\therefore\ {}_1Q_2 = U_2 – U_1 + P_2V_2 – P_1V_1$

ie, ${}_1Q_2 = (U2 + P_2V_2) – (U_1 + P_1V_1)$

ie, heat transfer for constant pressure is can be shown as the change in quantity U + PV between initial & find states.

Since U, P & V are thermodynamic properties; their combination of (U +PV) has the same characteristic. ie (U + PV) is also a thermodynamic property & is termed as enthalpy.

SPECIFIC HEATS

1) SPECIFIC HEAT AT CONSTANT VOLUME (C_V)

- It is the amount of heat added or removed per degree change in temperature when the system is kept at constant volume. It is denoted as $C_{V.}$

$$\text{ie } C_v = \left(\frac{\partial Q}{\partial T} \right)_V$$

- By first law, ${}_1Q_2$ - ${}_1W_2 = \Delta U$
- For constant volume process, dV = 0
- $\therefore$ WD = PdV= 0
- ie, ${}_1Q_2 = \Delta U$

$$\text{ie } C_v = \left(\frac{\partial U}{\partial T} \right)_V$$

- Or, $\Delta U = mC_V \Delta T$

2) SPECIFIC HEAT AT CONSTANT PRESSURE (Cp)

- It is the amount of heat added or removed per degree change in temperature when the system is kept under constant pressure. It is denoted by Cp.

$$\text{ie } C_p = \left(\frac{\partial Q}{\partial T}\right)_p$$

- For a constant pressure process; ${}_1Q_2 = \Delta H$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

RELATIONSHIP FOR Cp, Cv, R

Enthalpy, $H = U + PV$

For an ideal gas, $PV = mRT$

$\therefore H = U + mRT$

$\therefore \Delta H = \Delta U + mR\,\Delta T$ …………. (1)

But $\Delta H = mCp\,\Delta T$ and $\Delta U = mC_V \Delta T$

Substituting in (1) we get,

$mCp\,\Delta T = mC_V \Delta T + mR\,\Delta T$

ie, $Cp = Cv + R$

$Cp - Cv = R$ ……………. (2)

Dividing by C_V,

$\{Cp/ C_V\}- 1 = R/C_V$

$Cp/ C_V = \gamma$

$\therefore\ \gamma – 1 = R/C_V$

$\therefore\ C_V = R/ (\gamma – 1)$

Substituting for C_V in equation (2),

$$Cp - \frac{R}{\gamma - 1} = R$$

$$Cp = R + \frac{R}{\gamma - 1}$$

$$Cp = \frac{(\gamma - 1)R + R}{\gamma - 1} = \frac{R(\gamma - 1 + 1)}{\gamma - 1}$$

$$\text{ie, } Cp = \frac{R\gamma}{\gamma - 1}$$

- Q. In an internal combustine engine, during the compression stroke the heat rejected to the cooling water is 50kJ/kg and the work input is 100 kJ/kg. Calculate the change in internal energy of the working fluid stating whether it is a gain or loss.
- Ans. $Q = -50$ kJ/kg, $W = -100$ kJ/kg, $\Delta u = ?$

$$Q = \Delta u + W$$

$$-50 = \Delta u - 100$$

$$\Delta U = -50 + 100 = 50 \text{ kJ/kg}$$

- Q. In an air motor cylinder the compressed air has an internal energy of 450 kJ/kg at the beginning of the expansion and an internal energy of 220 kJ/kg after expansion. If the work done by the air during the expansion is 120 kJ/kg, calculate the heat flow to and from the cylinder.
- Ans.

$U1 = 450$ kJ/kg, $U2 = 220$ kJ/kg, $W = 120$ kJ/kg

$Q = (220 - 450) + 120 = -110$ kJ/kg

ie, heat is rejected from the air.

- Q. 0.3 Kg of nitrogen gas at 100 kPa & 40^0C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa & temperature becomes 160^0C. The work done during the process is 30 kJ. Calculate the heat transferred from nitrogen to the surroundings. C_V for N_2 = 0.75 kJ/kg K.
- Ans.

m = 0.3 kg, P_1 = 100 kPa, P_2 = 1 MPa, T_1 = 40^0C = 313 K, T_2 = 600C = 433K

W = -30 kJ (compression), Q =?

$$Q = \Delta U + W$$

$$\Delta U = \Delta m\ C_V T = 0.3 \text{ x } 0.75(433 - 313) = 27 \text{ kJ}$$

$$\Delta Q = \Delta U + W = 27 – 30 = -3 \text{ kJ}$$

ie, heat is rejected during the process.

- Q. When a stationary mass of gas was compressed without friction at constant pressure its initial state of 0.4 m^3 and 0.105 MPa was found to change to final state of 0.20 m^3 and 0.105 MPa. There was a transfer of 42.5 kJ of heat from the gas during the process. How much did the internal energy of the gas change?
- Ans.

$V_1 = 0.4m^3$, $P_1 = 0.105$ MPa, $V_2 = 0.2m^3$, $P_2 = 0.105$ MPa, $Q = -42.5$ kJ

$\Delta U = ?$

- $Q = W + \Delta U$
- $W = PdV$
- ie, $W = 0.105 \times 10^3 (0.2 - 0.4) = -21$ kJ
- $\Delta U = Q - W = -42.5 - (-21) = -21.5$ kJ
- ie, there is a decrease in internal energy.

Q. A cylinder containing the air comprises the system. Cycle is completed as follows:

i) 82,000 Nm of work is done by the piston on the air during compression stroke & 45 kJ of heat are rejected to the surroundings.

ii) During expansion stroke 100000 Nm of work is done by the air on the piston.

Calculate the quantity of heat added to the system

- Ans.
- Compression stroke
- Work done = -82000 Nm = -82 kJ
- Q = -45 kJ
- $\Delta U = Q - W = -45 - (-82) = 37$ kJ
- Expansion stroke
- W = 1000000 Nm = 100 kJ
- $\Delta U = -37$ kJ
- $\therefore Q = U + W = -37 + 100 = 63$ kJ
- The total heat added to the system.

- Q. A rigid tank contains a hot fluid that is cooled while being stirred by a paddle wheel. Initially the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat, & the paddle wheel does 100 kJ of work on the fluid. Determine the final energy of the fluid.
- Ans.
- $U1 = 800$ kJ, $Q = -500$ kJ, $W = -100$ kJ
- $U2 =?$

- $Q = \Delta U + W$
- $-500 = (U2 - 800) + -100$
- $U2 – 800 = -500 - (-100) = - 400$
- $U2 = -400 + 800 = 400$ kJ

- Q. The properties of a certain fluid are related as follows. u = 196+0.718t, Pv = 0.287 (t + 273), where u is the specific internal energy (kJ/kg), t in ^{0}C. P is pressure (kN/m^2) and v is the specific volume in m^3/kg. Find C_P & C_V for the fluid.

- Ans.
- u = 196 + 0.718 t,   Pv = 0.287 (t + 273)

- du/dt = 0.718
- du = $C_V$dt
- Therefore, C_V = du/dt = 0.718 kJ/kg
- h = u + Pv
- u = 196 + 0.718 t ………………….. (1)
- Pv = 78.351 + 0.287 t   ………….. (2)
- (1) + (2), h = 274.351 + 1.005 t
- Therefore, dh/dt = 1.005
- dh = Cp dT
- Therefore, C_P = dh/dt = 1.005 kJ/kg

- Q. A fluid is contained in a cylinder with a piston so that the pressure in the fluid is a linear function of the volume P = a + bv. The internal energy of the fluid is given by U = 42+ 3.6 PV, where, U is the kJ, P in kPa, & V in m^3. If the fluid changes from an initial state of 190 kPa, $0.035m^3$ to a final state of 420 kPa, $0.07m^3$, with no work other than that done on the piston, find the direction & magnitude of work & heat transfer.

$P = a + bv, \quad P_1 = 190 \text{ kPa}, \quad P_2 = 420 \text{ kPa}, \quad V_{1} = 0.035m^3, \quad V_2 = 0.07m^3$

$U = 42+3.6\ PV$

$U = U2 – U_1 = (42+3.6\ P_2V_2) - (42+3.6\ P_1V_1)$

$= 36\ (P_2V_{2\ -}\ P_1V_1)$

$= 3.6 \times 10^3\ (4.2 \times 0.07 – 1.9 \times 0.035)\ kJ = 81.9\ kJ$

$P = a + bv$

$190 = \quad a + b \times 0.035 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$

$420 = \quad a + b \times 0.07 \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$

$(1)\ - (2) \Rightarrow 230 = 0.035\ b$

Therefore, $b = 230/0.035 = 6571\ kN/m^5$

$a = -40\ kN/m^2$

$W = \int pdV = \int (a+bV)\, dV$

$W = a\,(V_2 - V_1) + b\,\{(V_2^2 - V_1^2)/2\}$

ie, $W = (V_2 - V_1)\,[a + b/2\,(V_1 + V_2)]$

ie, $W = (0.074 - 0.035)\,[-40\ kN/m^2 + 6571\ kN/m^2\,(0.035 + 0.07)] = 10.67\ kJ$

$Q = \Delta U + W$

ie, $Q = 81.9 + 10.67 = 92.57\ kJ$

- Q. Gas from a bottle of compressed helium is used to inflate an inelastic flexible balloon, originally folded completely flat to a volume of $0.5m^3$. If the barometer reads 760 mm kg, what is the amount of work done upon the atmosphere by the balloon?

Ans.

V_1 of balloon = 0, V_2 of balloon = $0.5m^3$

$P = 760mm\ of\ Hg = 1.01325 \times 10^5\ N/m^2 = 101.325\ kN/m^2$

- $W_d = \int_{balloon} pdv + \int_{bottle} pdv$

  $= P (V_2 – V_1) + (P \times 0) = 101.325\ kN/m^2 \times 0.5\ m^3 = 50.66\ kJ$

  Work is +ve as work is done by the balloon.

- Q. When the valve of an evacuated bottle is opened, atmospheric air rushes into it. If the atmospheric pressure is 101.325 kPa, and $0.6m^3$ of air (measured at atmospheric condition) enter into the bottle. Calculate the work done by air.

  P = 101.325 kPa, V_1 of air in bottle = 0, V_2 of air in bottle = $0.6m^2$

  $W_d = \int_{Bottle} pdv + \int_{Free\ air} pdv$

- $W_d = 0 + 101.325\ (0.6 - 0) = 60.8\ kJ$

- Since the free air boundary is contracting, the work done by the system is negative,

  ie, $W_d = - 60.8$ kJ
- If the temperature and pressure becomes atmospheric after filling, determine the amount of heat transfer.

$$W = - 60.8 \text{ kJ}$$

$$\therefore \text{Heat transfer, } Q = 60.8 \text{ kJ}$$

- Q. A piston and cylinder machine containing a fluid system has a stirring device in the cylinder. The piston is frictionless and it is held down against the fluid due to atmospheric pressure of 101.325 kPa. The stirring device is turned 10,000 revolutions with an average torque against the fluid of 1.275 MN. Meanwhile the piston of 0.6m diameter moves out 0.8m. Find the net work transfer for the system.

Ans.

$P = 101.325$ kPa, $N = 10{,}000$, $d = 0.6$m, $T = 1.275$ MN, $L = 0.8$m

Work done by the stirring device upon the system = $W_1 = 2\,\pi$ TN

$W_1 = 2\pi \times 1.275 \times 10^3, 000$ Nm = 80 kJ

$W_1 = -80$ kJ as work done is on the system.

Work done by the piston on the surrounding, = $W_2 = (PA)\,L$

$W_2 = 101.325 \times \pi/4\; 0.6^2 \times 0.8 = 2.9$ kJ

$\therefore W = W_1 + W_2 = -80 + 22.9 = -57.1$ kJ

- Q. A piston and cylinder machine contains a fluid system which passes through a complete cycle of 4 processes. During a cycle, the sum of all heat transfers is –170kJ. The system completes 100 cycles per min. Complete the following table showing the method for each item, and compute the net rate if work output in kW.

Process	*Q kJ/min.*	*W (kJ/min)*	*ΔE (kJ/min)*
a-b	0	2170	-
b-c	21,000	0	-
c-d	-2,100	-	-36,600
d-a	-	-	-

- Ans.

a-b

$$Q = \Delta E + W$$

$$0 = \Delta E + 2170$$

$$\Delta E = -2170 \text{ kJ/min}$$

b- c

$$Q = \Delta E + W$$

$$21{,}000 = \Delta E + 0$$

$$\Delta E = 21{,}000 \text{ kJ/min}$$

c-d

$$Q = \Delta E + W$$

$$-2100 = -36{,}600 + W$$

$$W = 34{,}500 \text{ kJ/min}$$

- ε Q = -170 kJ

  The system completes 100 cycles / min.

  Qab + Qbc + Qcd + Qda = -17,000 kJ/min.

  0 + 21,000 – 2100 + Qda = -17,000

  Qda = -35,900 kJ/min.

  Now ϕ dE = 0, since cyclic integral of any property is zero.

  $\Delta E_{ab} + \Delta E_{bc} + \Delta E_{cd} + \Delta E_{da} = 0$

  $-21,70 + 21,000 - 36,000 + \Delta E_{da} = 0$

  $\Delta E_{da} = 17,770$ kJ/min

  $W_{da} = Q_{da} - \Delta E_{da} = -35,900 - 17,770 = -53,670$ kJ/min.

Since $\sum Q = \sum W$

Rate of work output = -17,000 kJ/min = -283.3 kW

PMM1

- The first law states the general principle of the conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another.

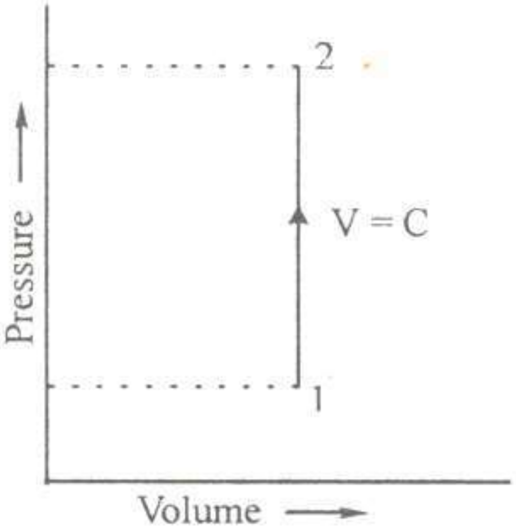
- There can be no machine which would continuously supply mechanical work without some other form of energy disappearing simultaneously.

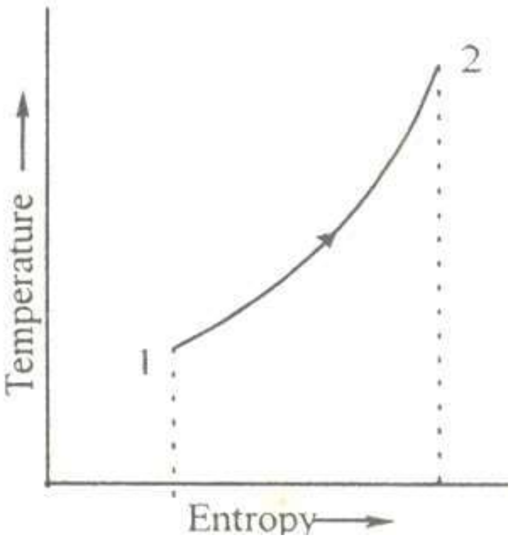
- Such a fictitious machine is called a perpetual motion machine of the first kind, or in brief, PMMI. A PMMI is thus impossible.

- The converse of the above statement is also true, i.e. there can be no machine which would continuously consume work without some other form of energy appearing simultaneously.

- **FIRST LAW APPLIED TO THERMODYNAMIC PROCESES**

- **A) CONSTANT VOLUME (ISOCHORIC PROCESS)**





- 1) P-V-T Relation

$$P_1V_1/ T_1 = P_2V_2/ T_2$$

$$V_1 = V_2$$

$$\therefore \mathbf{P_1/ T_1 = P2/T_2}$$

- 2) Work Done

$${}_1W_2 = \int PdV$$

$$dV = 0$$

$$\therefore \mathbf{{}_1W_2 = 0}$$

- 3) Change in internal energy

$$\mathbf{\Delta u = mC_V (T_2 - T_1)}$$

- 4) Heat supplied

$${}_1Q_2 = \Delta U + {}_1W_2$$

But ${}_1W_2 = 0$

$$\mathbf{\therefore {}_1Q_2 = \Delta U = mC_V (T_2 - T_1)}$$

Q) 1 kg of air has a pressure of 3 bar and a temperature of 125^0C. After it has received 500 kJ of heat at constant volume, find the final temperature and change in pressure.

Ans:

m = 1 kg, Q = 500 kJ, P_1 = 3 bar = 3x 105 N/m^2, T_1 = 125^0C = 398K

T_2, P_2 =?

$_1Q_2 = m\,c_v\,(T_2 - T_1)$

ie, $500 = 1 \times 0.718\,(T_2 - 398)$

$T_2 = 1094.38\ K = 821.38°C$

$P_1/T_1 = P_2/T_2$

$P_2 = P_1T_2/T_1 = (3 \times 1094.38)/\ 398 = 8.25$ bar

- Q) An insulated cylinder of capacity 2.8 m^3 contains 15 kg of nitrogen. Paddle work is done on the gas still the pressure inside the cylinder increases from 5 bar to l0 bar. Determine

(i) Change in internal energy

(ii) Work done

(iii) Heat transfer

Assume $c_p = 1.04$ kJ/kg K and $c_v = 0.7432$ kJ/kg K

Ans:

$V_1 = 2.8\ m^3$, m = 15 kg, $p_1 = 5$ bar = 5 x 105 N/m^2,

$p_2 = 10$ bar = 10 x 105 N/m^2, $C_p = 1.04$ kJ/kg and $C_v = 0.7432$ kJ/kgK

ΔU, ${}_1W_2$, ${}_1Q_2$ =?

$P_1V_1 = mRT_1$

$T_1 = P_1V_1/mR = \{5 \text{ x } 105 \text{ x } 2.8\}/ \{15 \text{ x } (1.04 – 0.7432) \text{ x } 1000\} = 314.47$ K

For constant volume process,

$P_1/ T_1 = P_2/T_2$

$T_2 = P_2/P_1 \text{ x } T_1 = 10/5 \text{ x } 314.47 = 628.94$K

1) $\Delta U = m\, C_v\, (T_2 - T_1)$

$= 15 \times 0.7432(628.94 - 314.47) = 3505.7$ kJ

2) ${}_1Q_2 - {}_1W_2 = \Delta U$

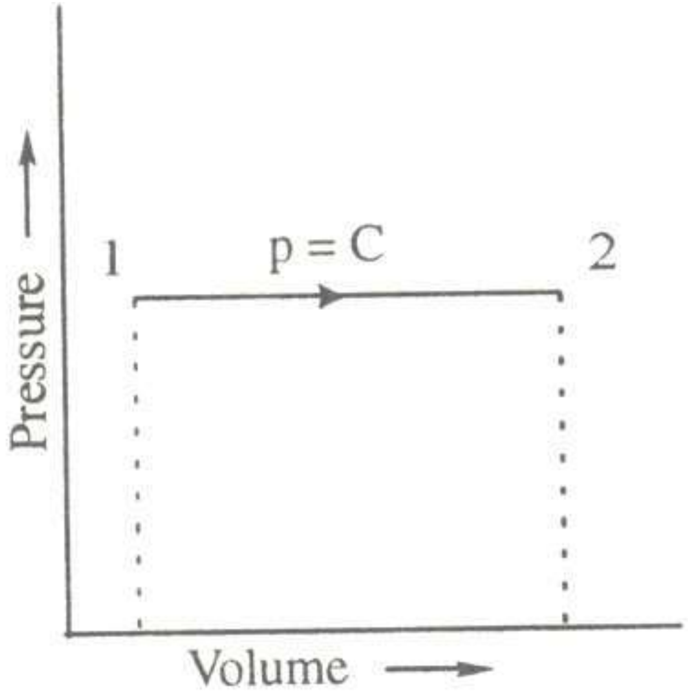
${}_1W_2 = {}_1Q_2 - \Delta U$

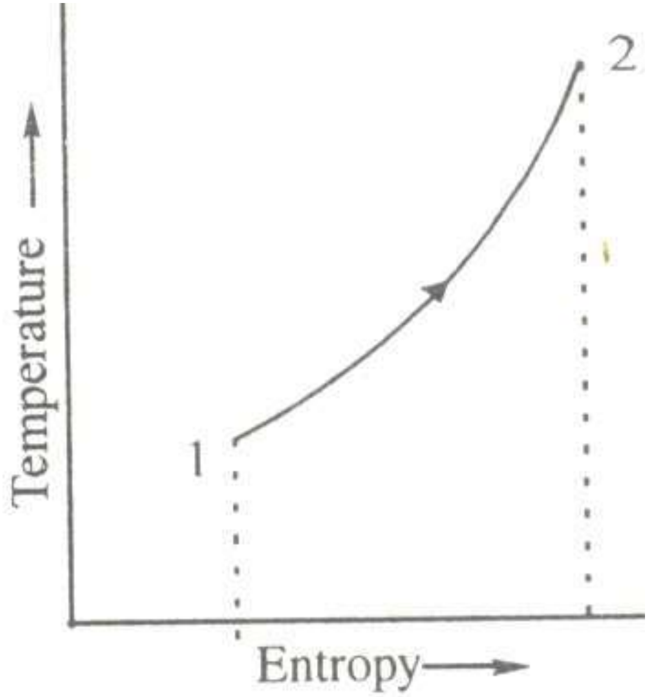
As the cylinder is insulated ${}_1Q_2 = 0$.

${}_1W_2 = -\Delta U = -3505.7$ kJ (Paddle work is not zero)

3) ${}_1Q_2 = 0$.

B) CONSTANT PRESSURE (ISOBARIC) PROCESS





1) P-V-T Relation

$P_1V_1/ T_1 = P_2V_2/ T_2$

$P_1 = P_2$

$V_1/T_1 = V_2 / T_2$

- 2) Work Done

$${}_1W_2 = 2\int 1\ PdV = P\,(V)v_2v_1$$
$${}_1W_2 = P\,(V_2 - V_1)$$

- 3) Change in internal energy

$$\Delta u = mC_V\,(T_2 - T_1)$$

- 4) Heat supplied

$${}_1Q_2 = \Delta u + {}_1W_2$$
$${}_1Q_2 = mC_V\,(T_2 - T_1) + P\,(V_2 - V_1)$$
$$P = P_1 = P_2$$
$$\Delta_1Q_2 = mC_V\,(T_2 - T_1) + (P_2\,V_2 - P_1V_1)$$

$PV = mRT$

$\Delta_1Q_2 = mC_V (T_2 - T_1) + (mRT_2 - mRT_1)$

$= mC_V (T_2 - T_1) + mR (T_2 - T_1)$

$= m (T_2 - T_1) (C_V + R)$

$Cp - C_v = R$ » $Cv + R = C_p$

$\Delta_1Q_2 = mC_p (T_2 - T_1)$

- Q) A stationary mass of gas is compressed at constant pressure from an initial state of 2.5 m^3 and 2 bar to a final volume of 1.5m^3. There is a transfer of 400 kJ of heat from the gas during compression. Find the change in internal energy of the gas.

Solution:

Given: $P_1 = P_2 = 2bar = 2 \times 10^5 N/m^2$

$V_I = 2.5m^3$, $V_2 = 1.5 m^3$, ${}_IQ_2 = - 400$ kJ

$\Delta U=?$

- For a constant pressure process,

$_1W_2 = P (V_2 - V_1)$

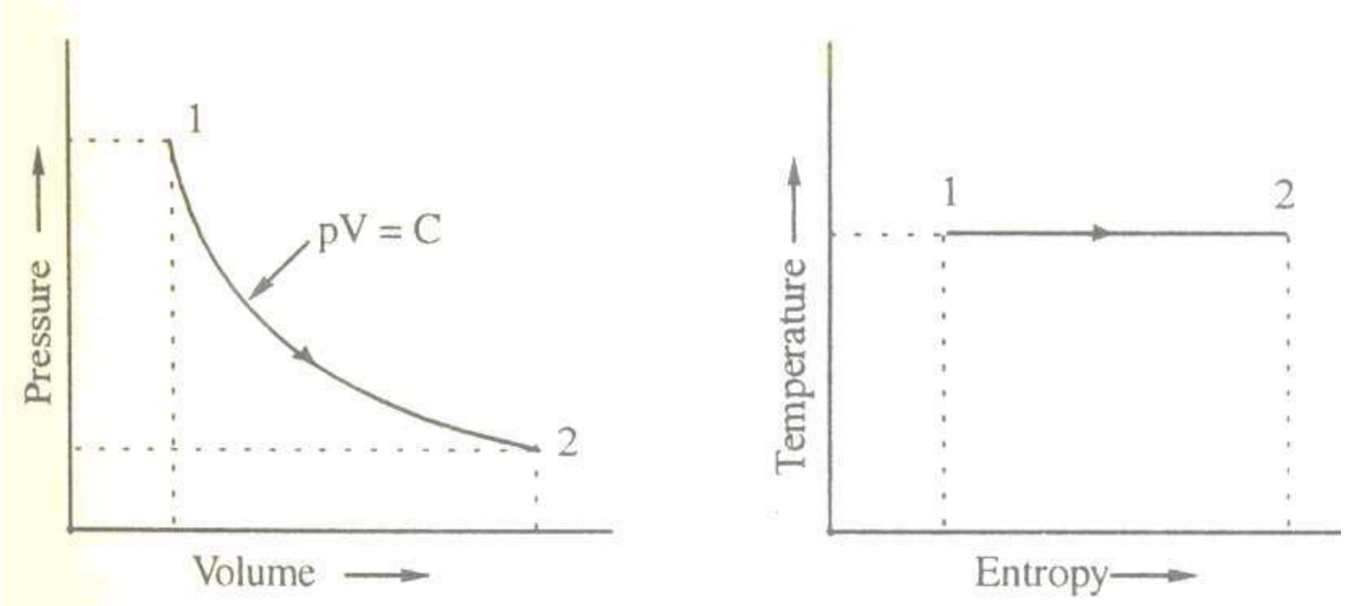
$= 2 \times 10^5 (1.5 - 2.5) = -2 \times 10^5$ J

$_1W_2 = -200$ kJ

$\Delta U = {_1Q_2} - {_1W_2}$

$= (-400) - (-200) = -200$ kJ

C) CONSTANT TEMPERATURE (ISOTHERMAL) PROCESS



1) P-V-T Relation

$P_1V_1/ T_1 = P_2V_2/ T_2$

$T_1 = T_2$

$\therefore \mathbf{P_1V_1 = P_2\ V_2}$

2) Work Done

$${}_1W_2 = \int_{v_1}^{v2} PdV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

PV = a const or $PV = P_1V_1 = P_2V_2$

$P = P_1V_1/V$

Substituting in (1),

$${}_1W_2 = \int_{v_1}^{v2} P_1V_1\, /V\, dV$$

$$= P_1V_1 \int_{v_1}^{v2} dV/V$$

$$= P_1V_1 \,[\ln V]^{v2}_{v1}$$

$$= P_1V_1\,(\ln V_2 - \ln V_1)$$

$${}_1W_2 = P_1V_1 \ln (V_2 /V_1)$$

$P_1V_1 = P_2 V_2$ or $V_2 /V_1 = P_1/ P_2$

$$\mathbf{{}_1W_2 = P_1V_1 \ln (P_1/ P_2)}$$

- 3) Change in internal energy

  $T_1 = T_2$

  $\Delta U = 0$

- 4) Heat supplied

  ${}_1Q_2 = \Delta U + {}_1W_2$

  $\Delta U = 0$

$$\mathbf{{}_1Q_2 = {}_1W_2 = P_1V_1 \ln V_2/V_1 = P_1V_1 \ln (P_1/P_2)}$$

- Q) Determine the volume of 2 kg of air at 30^{D}C and under a pressure of 2 bar, what would be its volume after isothermal compression to a pressure of 4 bar, calculate the work done.
- Solution:
- Given: $m = 2$ kg, $T_1 = 30^0C = 303K$, $P_1 = 2 \text{ bar} = 2 \times 10^5 \ N/m^2$,

$P_2 = 4 \text{ bar} = 4 \times 10^5 \ N/m^2$,

$V_1, V_2, {}_1W_2 = ?$

$$P_1V_1 = mRT_1$$

$$V_1 = mRT_1/P_1 = (2 \times 287 \times 303)/(2 \times 10^5)$$

$$V_1 = 0.8696 \ m^3$$

For iso thermal process,

$$P_1V_1 = P_2V_2$$

$$V_2 = P_1V_1/P_2 = (2 \times 10^5 \times 0.8696)/ (4 \times 10^5)$$

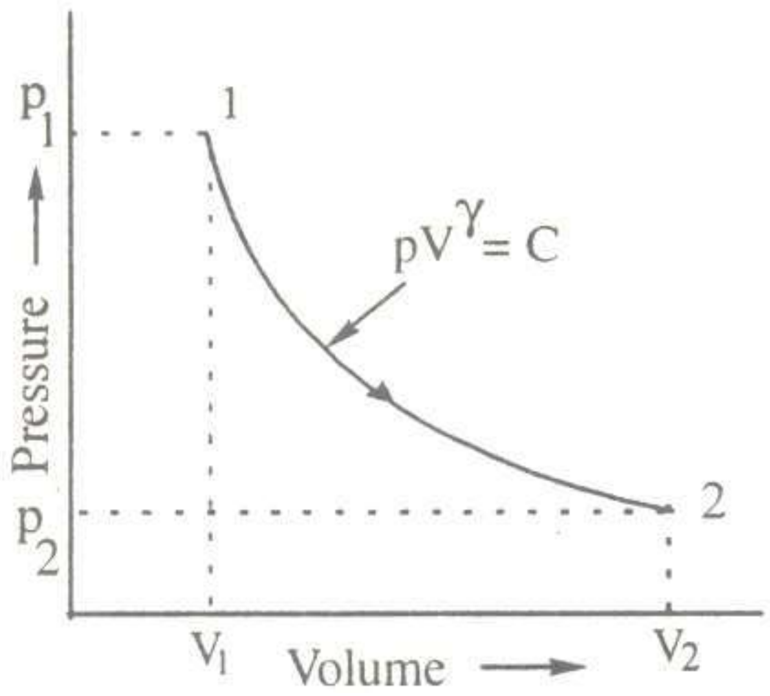
$$V_2 = 0.4348 \text{ m}^3$$

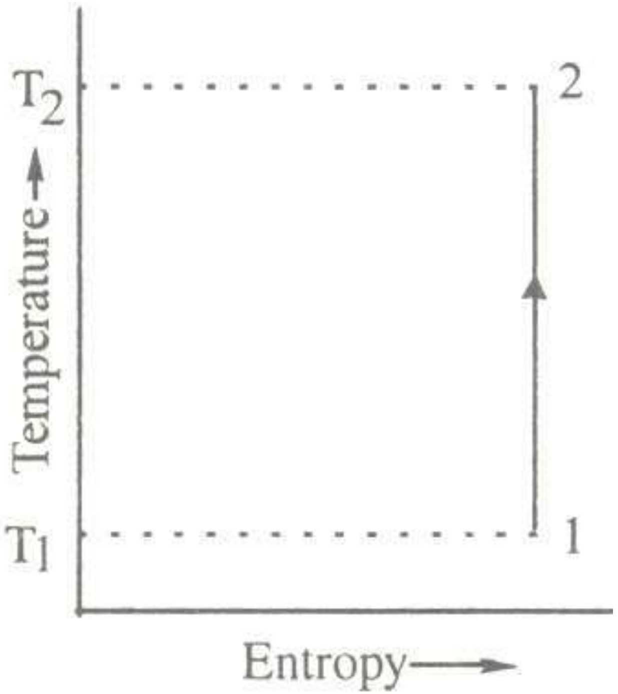
$${}_1W_2 = P_1V_1 \ln (V_2/V_1)$$

$$= 2 \times 10^5 \times 0.8696 \times \ln (0.4348/0.8696)$$

$${}_1W_2 = -120.55 \text{ kJ}$$

- **D) ADIABATIC PROCESS**
- In an adiabatic process, the gas neither receives nor rejects heat. In this process, the heat exchange Q = 0.Work is done by the gas at the expense of internal energy.





$_1W_2 = \int_{V_1}^{V_2} PdV$

$_1Q_2 = \Delta U + {_1W_2}$

$_1Q_2 = 0$

$\therefore {_1W_2} = -\Delta U = -mC_V (T_2 - T_1)$

$_1W_2 = \int_{V_1}^{V_2} PdV = -mC_V (T_2 - T_1)$

ie; $PdV = -mC_V \, dT$ ………..(1)

$PV = mRT$

Differentiating, $PdV + Vdp = mRdT$

ie; $mdT = \{PdV + Vdp\}/R$

Substituting for mdT in (1), we get;

$$PdV = -C_V\left\{\frac{PdV + Vdp}{R}\right\}$$

ie; $R\,PdV = -C_V\,(PdV + Vdp\,)$

$R = Cp - C_V$

$\therefore (Cp - C_V)\,PdV = -C_V\,PdV - Vdp$

ie; $Cp\,PdV = -\,C_V\,Vdp$

$$\frac{Cp}{C_V}\,\frac{dV}{V} = \frac{-dP}{P}$$

$Cp/C_V = \gamma$

$\gamma\,(dV/V) + dP/P = 0$

Integerating the above equation,

$\gamma \ln V + \ln P = C_1$

where C_1 is the constant of integration.

ie; $\ln PV^{\gamma} = C_1$

or $PV^{\gamma} = C$ where C is another constant.

ie. $P_1V_1^{\gamma} = P_2 V_2^{\gamma}$ = constant.

1) P-V-T Relation

$P_1V_1^{\gamma} = P_2 V_2^{\gamma}$

$\therefore$ **$P_1/P_2 = (V_2/V_1)^{\gamma}$ or $V_2/V_1 = (P_1/P_2)^{1/\gamma}$ ………… (1)**

$P_1V_1/T_1 = P_2V_2/T_2$

$V_2 /V_1 = (T_2/T_1) \times (P_1/P_2)$ …………… (2)

From (1) & (2);

$(P_1/ P_2)^{1/\gamma} = (T_2/ T_1) \times (P_1/P_2)$

$T_2 /T_1 = (P_1/ P_2)^{1/\gamma -1}$.

$T_2/T_1 = (P_1/ P_2)^{1-\gamma / \gamma}$

or $P_1/P_2 = (T_2/T_1)^{\gamma/1-\gamma} = (T_1/T_2)^{\gamma/\gamma-1}$

ie. $\mathbf{P_1/P_2 = (V_2/V_1)^{\gamma} = (T_1/T_2)^{\gamma/\gamma-1}}$

$\mathbf{(V_2/V_1) = (T_1/T_2)^{1/\gamma-1}}$

2) Work Done

$_1W_2 = \int_{v1}^{v2} PdV$

$P_1V_1^{\gamma} = P_2V_2^{\gamma} = PV^{\gamma} = C$

$\mathbf{P = C/V^{\gamma}}$

$_1W_2 = \int_{v1}^{v2} C\, dV/V^{\gamma}$

$= C \int_{v1}^{v2} V^{-\gamma}\, dV$

$$= C \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$= C/(1-\gamma)\ \{V_2^{-r+1} - V_1^{-r+1}\}$

$C = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

${}_1W_2 = 1/(1-\gamma)\ \{P_2V_2^{\gamma}\ V_2^{-\gamma+1} - P_1V_1^{\gamma}\ V_1^{-\gamma+1}\}$

$= 1/(1-\gamma)\ \{P_2V_2^{-\gamma+\gamma+1} - P_1V_1^{\gamma-\gamma+1}\}$

$= 1/(1-\gamma)\ \{P_2V_2 - P_1V_1\}$

$\mathbf{{}_1W_2 = (P_2V_2 - P_1\,V_1)/\,1 - \gamma}$

$\mathbf{{}_1W_2 = (P_1V_1 - P_2\,V_2)/\,\gamma - 1 = mR\,(T_1 - T_2)\,/\,\gamma - 1}$

$\mathbf{{}_1W_2 = \Delta U = -mC_V\,\Delta T}$

3) Change in internal energy

$\mathbf{\Delta U = -mC_V\,\Delta T = -{}_1W_2}$

4) Heat Exchanged

$\mathbf{Q = 0}$

- Q) 1 kg of gas expands adiabatically and its temperature is observed to fall from 240°C to 115^0C while the volume is doubled. The gas does 90 kJ of work in the process. Determine the value of C_p and C_v.
- Solution;
- m = 1kg, $T_1 = 240^0C = 513$ K, $T_2 = 115^0C = 388K$, ${}_1W_2 = 90kJ$, $V_2/V_1 = 2$

C_p and C_v =?

${}_1Q_2 = \Delta U + {}_1W_2$

${}_1Q_2 = 0$

$\therefore \Delta U = -{}_1W_2$

$mC_V \Delta T = -{}_1W_2$

$C_V = -{}_1W_2 / m\,(T_2 - T_1) = -90/\ 1 \text{ x } (388 - 513)$

$C_V = 0.72$ kJ/kgK

$(T_1/T_2)^{\gamma / \gamma\text{-}1 =} (V_2/V_1)^{\gamma}$

$513/388 = 2^{\gamma-1}$

$1.322 = 2^{\gamma-1}$

Taking logarithm,

$\ln 1.322 = (\gamma-1)\ln 2$

$\gamma - 1 = 0.4$

$\gamma = 1.4$

$C_p = \gamma\, C_V = 1.4 \times 0.72 = 1.008$ kJ/kg K

- Q) Find the work done in compressing 0.28 m3 of air at a pressure of 1.4 bar to a volume of 0.028m3 when the compression is adiabatic and isothermal.
- Solution;
- $V_1 = 0.28\ m^3$, $P_1 = 1.4$ bar $= 1.4 \times 10^5\ N/m^2$
- $V_2 = 0.028\ m^3$
- W adiabatic, W isothermal =?

- For adiabatic process,

$P_2/P_1 = (V_2/V_1)^{\gamma}$

$P_2 = P_1 (V_2/V_1)^{\gamma} = 1.4 \times 10^5 (0.28/0.028)^{1.4}$

$= 35.17 \times 10^5 \text{ N/m}^2$

${}_1W_2 = (P_2V_2 - P_1 V_1) / 1 – \gamma$

$= \{1.4 \times 105 \times 0.28 – 35.17 \times 105 \times 0.028\}/ 1.4 – 1 = -148190$ J

- For isothermal process,
- ${}_1W_2 = P_1V_1 \ln (V_2/ V_1)$

$= 1.4 \times 105 \times 0.28 \times \ln (0.028/0.28) = -90261.33$J

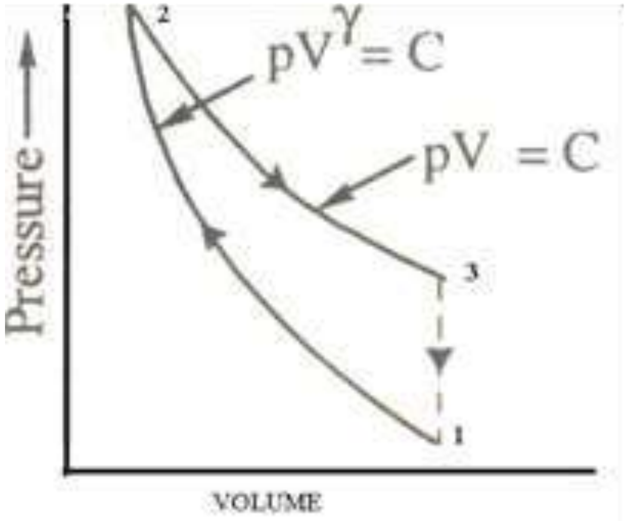
- The difference in work done = -148190 – (-90261.33) = 57928.67 J

- Q) 3.5 m^3 of hydrogen gas at a pressure of 100 kPa and 20°C are compressed adiabatically to 4.5 times its original pressure. It is then expanded isothermally to its original volume. Determine the final pressure of the gas and the heat transfer. Also determine the quantity of heat that is to be exchanged to reduce the gas to its original pressure and volume. Take Cp for hydrogen as 14.3 kJ/kg K and γ a s 1.4.
- Solution;
- $V_1 = 3.5\ m^3$, $P_1 = 100$ kPa $= 100 \times 10^3\ N/m^2$, $T_1 = 20^0C = 293$ K

  $P_2 = 4.5P_1 = 4.5 \times 100 \times 10^3 = 450 \times 10^3\ N/m^2$, $Cp = 14.3$ kJ/kg K,

  $V_3 = V_1$, $\gamma = 1.4$

  P_3, ${}_3Q_1$, ${}_2Q_3$ =?



- $P_1V_1^{\gamma} = P_2 V_2^{\gamma}$

- $100 \times 10^3 \times 3.5^{1.4} = 450 \times 10^3 \times V_2^{1.4}$

  $V_2 = 1.2 \text{ m}^3$

- $P_1V_1/T_1 = P_2V_2/T_2$

  $T_2 = \{P_2V_2/P_1V_1\} \times T_1 = \{450 \times 10^3 \times 1.2/100 \times 10^3 \times 3.5\} \times 293 = 452 \text{ K}$

For the process 2-3,

$P_2V_2 = P_3V_3$

$450 \times 10^3 \times 1.2 = P_3 \times 3.5$

$P_3 = 1.54 \times 10^5 \text{ N/m}^2$

- Heat transferred,

$_2Q_3 = P_2V_2 \ln (V_3/V_2)$

$= 450 \text{ x } 103 \text{ x } 1.2 \text{ x } \ln (3.5/1.2)$

$_2Q_3 = 578 \text{ kJ}$

$_3Q_1 – {_3W_1} = \Delta U$

$_3Q_1 = \Delta U$ since $_3W_1 = 0$

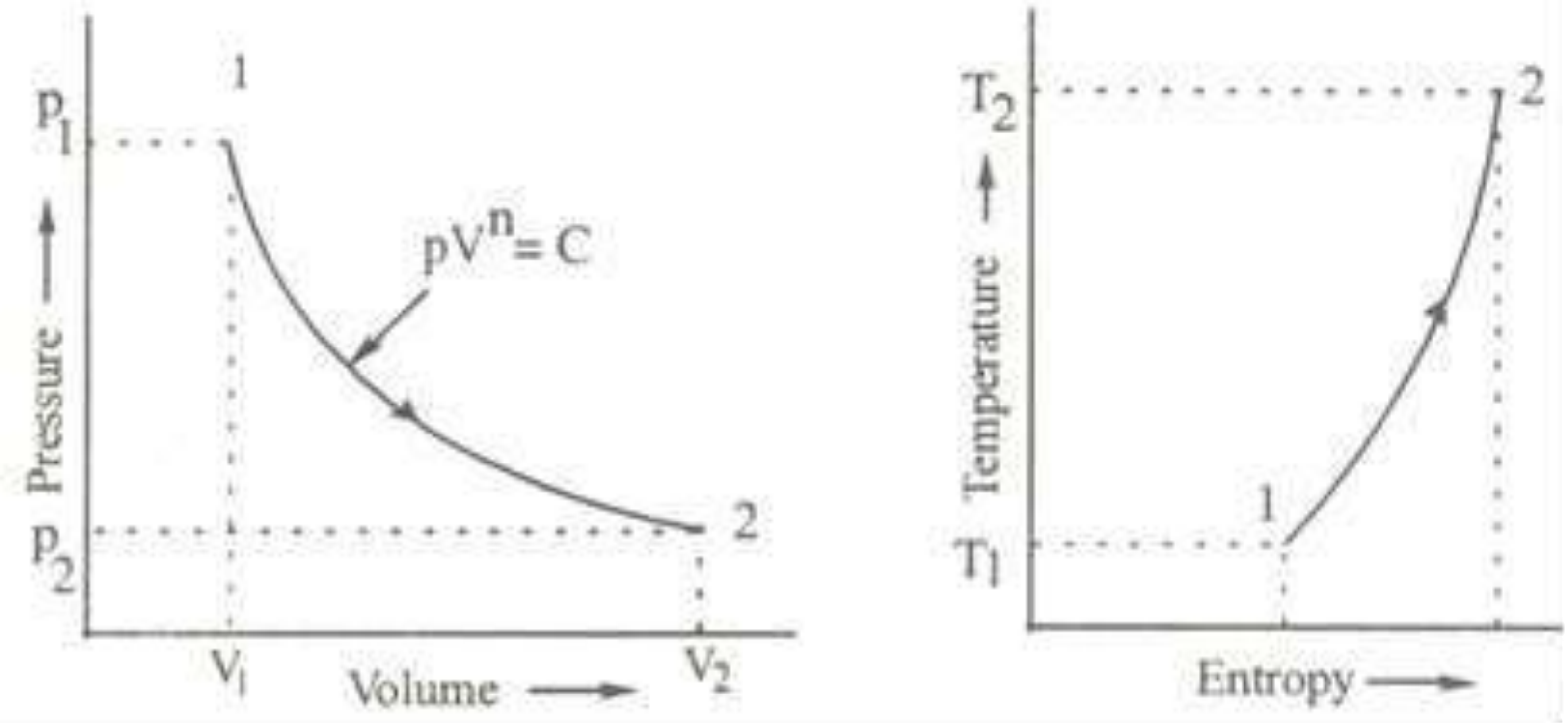
$= mCv (T_1 – T_3)$

$Cp – Cv = R = 8314/M = 8314/2 = 4157 \text{ J/kgK}$

$\therefore Cv = Cp – R = 14300 -4157 = 10143 \text{ J/kgK}$

E) POLYTROPIC PROCESS

- The curve of expansion or compression follows the law PVn = const, where 'n' is a constant called polytropic index of expansion or compression.



- 1) P-V-T Relation

$$\mathbf{P_1/P_2 = (V_2/V_1)^{\,n}}$$

$$\mathbf{(V_2/V_1) = (T_1\ /T_2)^{1/\,n\,-1}}$$

$$\mathbf{T_2/T_1 = (P_1/\ P_2)^{1\,-\,n\,/\,n}}$$

- 2) Work done

$$ {}_1W_2 = {}^{v2}\!\int_{v1} PdV$$

$$P_1V_1^{\,n} = P_2V_2^{\,n} = PV^n = C$$

$$\mathbf{P = C\ /V^n}$$

$$ {}_1W_2 = {}^{v2}\!\int_{v1} C\ dV/V^{\,n}$$

$$= C\ {}^{v2}\!\int_{v1} V^{-\,n}\ dV$$

$$= C \left[\frac{V^{-n+1}}{-n+1} \right]_{V_1}^{V_2}$$

$= C/ (1- n) \{V_2^{- n + 1} - V_1^{-n +1}\}$

$C = P_1V_1^n = P_2V_2^n$

$\square\ {}_1W_2 = 1/ (1- n) \{P_2V_2^n\, V2^{– n + 1} - P_1V_1^{n}\, V_1^{- n +1}\}$

$= 1/ (1- n) \{P_2V_2^{- n + n + 1} - P_1V_1^{n - n +1}\}$

$= 1/ (1- n) \{P_2V_2 - P_1V_1\}$

$\mathbf{{}_1W_2 = (P_2V_2 - P_1 V_1)/ 1 – n}$

$\mathbf{{}_1W_2 = (P_1V_1 – P_2 V_2)/ n -1 = mR (T_1 –T_2) / n - 1}$

- 3) Change in internal energy

$\Delta U = mCv\ \Delta T$

Heat exchanged ${}_1Q_2 = \Delta U + {}_1W_2$

${}_1Q_2 = mCv(T_2 - T_1) + mR/(n-1)\{T_2 - T_1\}$

$= m(T_2 - T_1)\{R/(n-1) - Cv\}$

But $Cv = R/(\gamma - 1)$

$\therefore {}_1Q_2 = m(T_1 - T_2)\{R/(n-1) - R/(\gamma - 1)\}$

$= mR(T_1 - T_2)\{1/(n-1) - 1/(\gamma - 1)\}$

$= mR(T_1 - T_2)\{(\gamma - 1 - n + 1)/(n-1)(\gamma - 1)\}$

$= mR(T_1 - T_2)\ \{(\gamma - n)/(n-1)(\gamma - 1)\}$

$= (\gamma - n) / (\gamma - 1)\ \{mR(T_1 - T_2)/(n-1)\}$

ie ${}_1Q_2 = (\gamma - n) / (\gamma - 1)$ x Work done

- Expression for polytropic index

  $P_1V_1^n = P_2V_2^n$

  $P_1/P_2 = (V_2/V1)^n$

  Taking logarithm,

  $\ln(P_1/P_2) = n \ln (V_2/V_1)$

  $n = \ln (P_1/P_2)/ \ln (V_2/V_1$

- Q) A certain quantity of air has a volume of 0.028 m^3 at a pressure of 1.25 bar and 25^0C. It is compressed to a volume of 0.0042 m^3 according to the law $PV^{1.3}$ constant. Find the final temperature and work done during compression. Also determine the reduction in pressure at a constant volume required to bring the air back to its original volume.
- Solution;
- $V_1 = 0.028\ m^3$, $V_2 = 0.0042\ m^3$, $n = 1.3$, $T_1 = 25^0C = 298K$, $T_3 = T_1 = 298\ K$
- $P_1 = 1.25\ bar = 1.25 \times 10^5\ N/m^2$
- T_2, ${}_1W_2$, $(P_2 – P_3) =?$

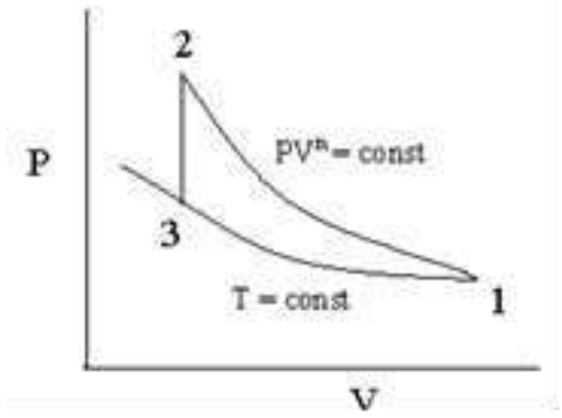
For the polytropic process,

$$\mathbf{(V_2/V_1) = (T_1/T_2)^{1/n-1}}$$

ie, $T_1/T_2 = (V_2/V_1)^{n-1}$

$T_2 = T_1/(V_2/V_1)^{n-1} = 298/\{0.0042/0.028\}^{1.3-1} = 526.49K$

$$\mathbf{P_1/P_2 = (V_2/V_1)^n}$$



$P_2 = P_1/(V_2/V_1)^n = 1.25 \times 10^5/\{0.0042/0.028\}^{1.3} = 14.723 \times 10^5$ N/m^2

$$\mathbf{{}_1W_2 = (P_1V_1 - P_2V_2)/n-1}$$

${}_1W_2 = (1.25 \times 10^5 \times 0.028 - 14.723 \times 10^5 \times 0.0042) / (1.3 - 1) = -8.946$ kJ

- For the constant volume process 2-3,

  $P_3/T_3 = P_2/T_2$

  $P_3 = P_2/T_2 \times T_3 = (14.723 \times 10^5 \times 298)/\ 526.49 = 8.33 \times 10^5\ N/m^2$

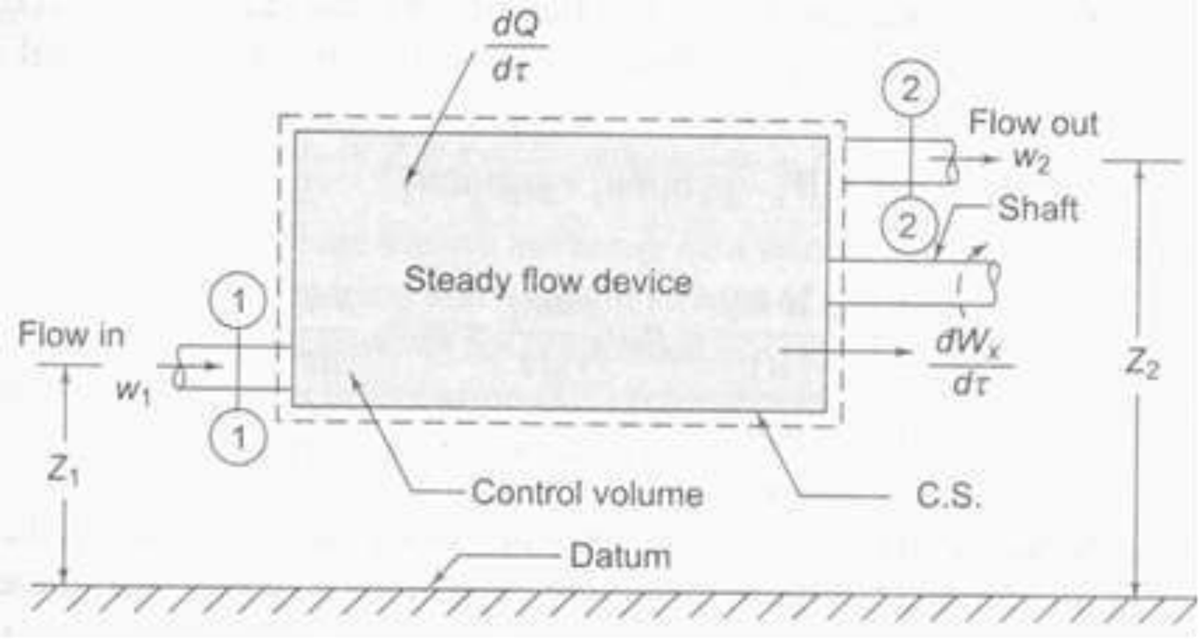
  $P_2 – P_3 = 14.723 \times 10^5 - 8.33 \times 10^5 = 6.393$ bar

- **FIRST LAW OF THERMODYNAMICS APPLIED TO OPEN SYSTEMS**

- A large number of engineering devices such as turbines, compressors, and nozzles operate for long periods of time under the same conditions once the transient start-up period is completed and steady operation is established, and are classified as steady-flow devices.
- Processes involving such devices can be represented reasonably well by a somewhat idealized process, called the steady-flow process.
- That is, the fluid properties can change from point to point within the control volume, but at any point, they remain constant during the entire process.

  (Steady means no change with time.)
- Thus, the volume V, the mass m, and the total energy content E of the control volume remain constant and the total mass or energy entering the control volume must be equal to the total mass or energy leaving it (since m_{cv} = constant and E_{cv} = constant).

- Consider an open system having an inlet at section 1- 1 and outlet at section 2 –2.
- The cross sectional area, pressure, specific volume, mass flow rate at 1 –1 & 2 –2 are;
  - => A_1, P_1, v_1, m_1
- 2 –2 => A_2, P_2, v_2, m_2
- The fluid flowing across the control surface enters & leaves with an amount of energy per unit mass.
- $e_1 = u_1 + ½ C_1^2 + gz_1$
- $e_2 = u_2 + ½ C_2^2 + gz_2$ for 1 –1 & 2 –2 respectively



- Again as the amount of mass flows in there is a pressure at its back surface so that it is being pushed by the mass behind it, which is the surroundings.

- Which is the work flowing in similarly the fluid flowing out must push the surrounding fluid ahead of it doing work on it, which work is leaving the open system.

- Flow work = Force x velocity

  $= \int PdA \text{ x velocity} \qquad (AC = V)$

  $= PV = Pvm$

$\therefore$ Flow work in = $P_1v_1m_1$

- Flow work out = $P_2v_2\ m_2$

- $\therefore$The total energy associated with the flow of mass = [Stored energy + flow energy]

$(e + Pv)\, m = [(u + Pv) + ½\, C^2 + gz]\, m$

ie, $(e + Pv)m = (h + ½\, C^2 + gz)m$

ie, $m_1(e_1+P_1v_1)$ for 1- 1

$m_2\, (e_2+P_2v_2)$ for 2- 2

Let the open system is also having heat & work interactions, Q & W

The change in energy due to heat & work interaction = Q –W.

The fundamental law states that we cannot create or destroy energy such that the change of energy must be caused by energy into or out of the open system (control of volume).

- So the total change in energy for the system

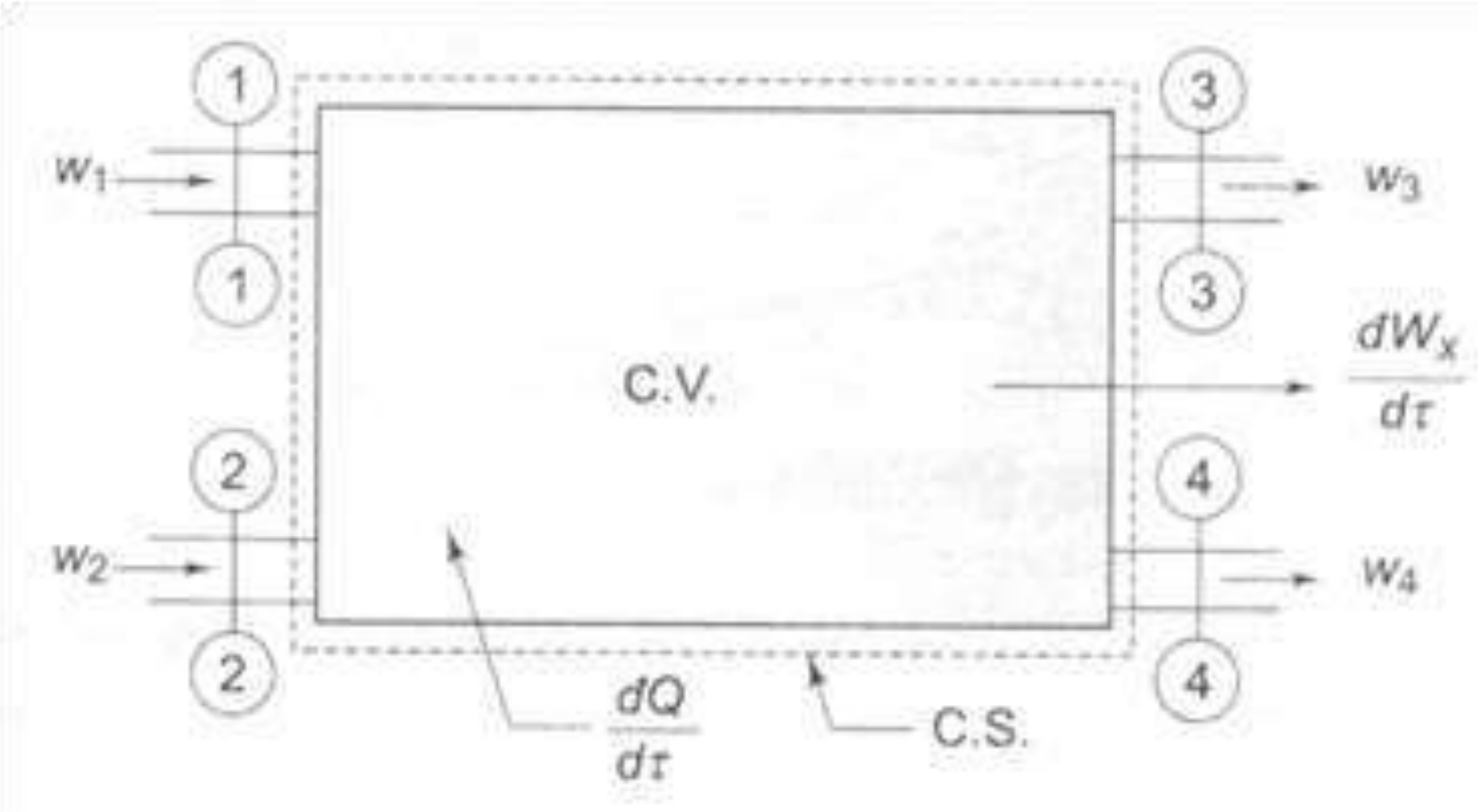
- $\Delta E_{cv} = [Q – W] + [m_1 (e_1 + p_1 v_1) - m_2 (e_2+P_2v_2)]$
- ie, $\Delta E_{cv} = Q – W + m_1(h_1 + ½ C_1^2 + gz_1)- m_2(h_2+½ C_2^2 + gz_2)$

- During a steady- flow process, the total energy content of the control volume is constant ie change in energy $\Delta E_{cv} = 0$, then

- $Q + m_1 (h_1 + ½ C_1^2 + gz_1) = W + m_2 (h_2+½ C_2^2 + gz_2)$

- ie, the amount o energy entering a control volume in all forms (by heat, work & mass) must be equal to the amount of energy leaving it. There is no accumulation of mass or energy within the control volume & the properties of any location within the control volume will not change with time.

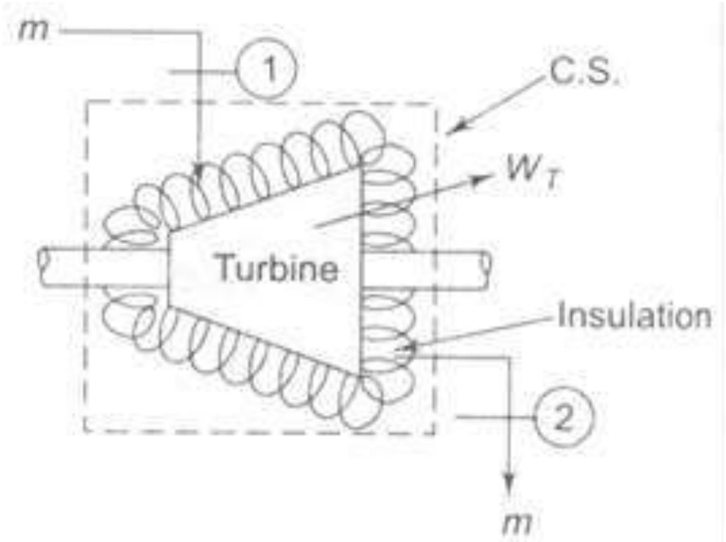
- By equation of continuity, $m_i = m_e$
- ie, $A_1C_1/ v_1 = A_2C_2/v_2$
- For a system of perfectly insulated type, $Q = 0$
- $(h_1 + C_1^2/2 + gz_1)\, m_1 = W + (h_2 + C_2^2/2 + gz_2)\, m_2$
- b) For any system having more than one inlets, outlets and energy interactions.
- Net heat added, $Q = Q_1 – Q_2 + Q_3$
- Net work done, $W = W_1 + W_2$



$$Q + m_1(h_1+C_1^2/2+gz_1) + m_2(h_2+C_2^2/2+gz_2) = W + m_3(h_3+C_3^2/2+gz_3) + m_4(h_4+ C_4^2/2+gz_4)$$

- **TURBINE**

- It is a device which produces shaft work at the expense of the pressure of the working fluid.
- $Q + (h_1 + C_1^2/2 + gz_1)\, m_1 = W + (h_2 + C_2^2/2 + gz_2)\, m_2$
- $m_1 = m_2$
- KE & PE changes are negligible.
- The expansion is assumed to be adiabatic ie; $Q = 0$.



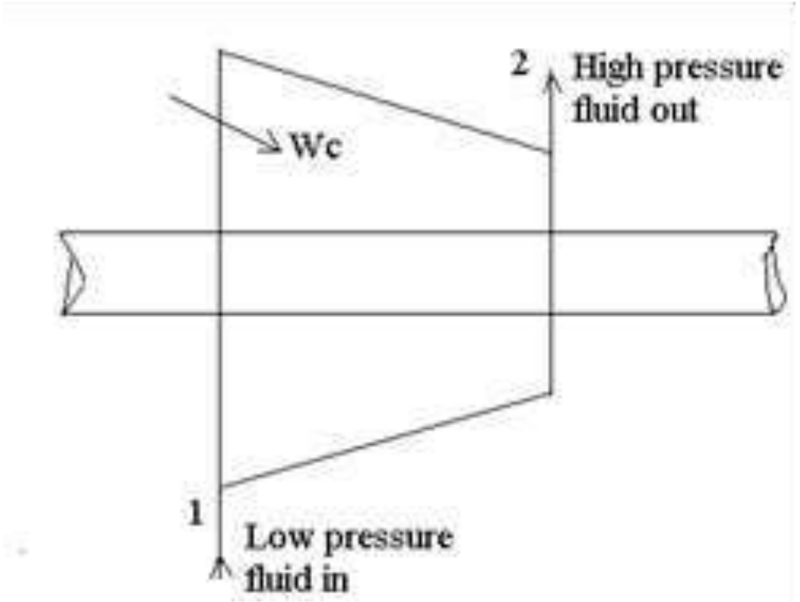
- $\therefore mh_1 = W + mh_2$

- $\mathbf{W = m\,(h_1 - h_2) = m\,Cp\,(T_1 - T_2)}$

- ie. Work is done by the fluid at the expense of its enthalpy.

- **COMPRESSOR AND PUMP**

- In both compressor and pump the pressure of the fluid is increased by putting in shaft work.
- $Q + m_1 (h_1 + C_1^2/2 + gz_1) = W + m_2 (h_2 + C_2^2/2 + gz_2)$
- $\Delta PE = 0$, $Q = 0$, $m_1 = m_2$, C_1 is negligible



- $\therefore\ mh_1 = w + mh_2 + C_2^2/2$
- $\therefore\ W = m (h_1 – h_2) - C_2^2/2$
- If C_2 is also neglected

- $\mathbf{W = m (h_1 – h_2) = mCp (T_1 – T_2)}$

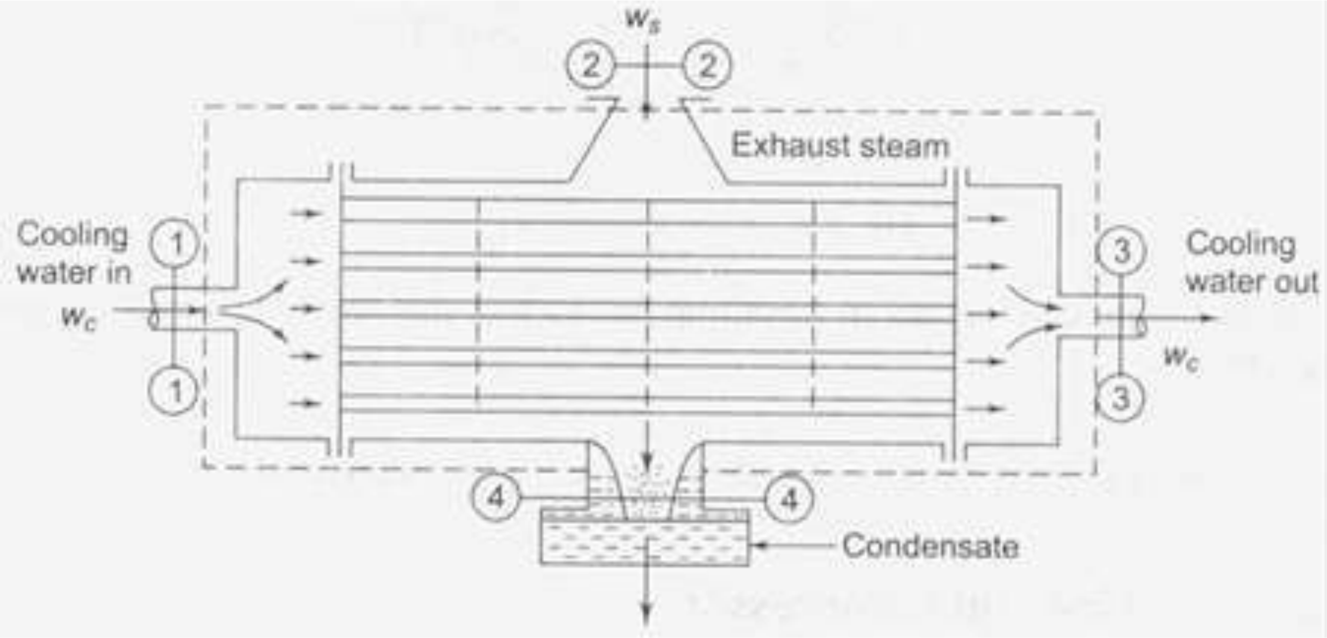
- But W is negative as it is work input in this case

- **HEAT EXCHANGER /CONDENSER**

- A heat exchanger is a device in which heat is transferred from one fluid to another.
- Figure shows a steam condenser, where steam condenses outside the tubes & cooling water flows through the tubes.
- $\Delta KE = 0, \quad \Delta PE = 0, \quad Q = 0, \quad W = 0$

- $w_c h_{1+} + w_s h_2 = w_c h_{3+} + w_s h_4$

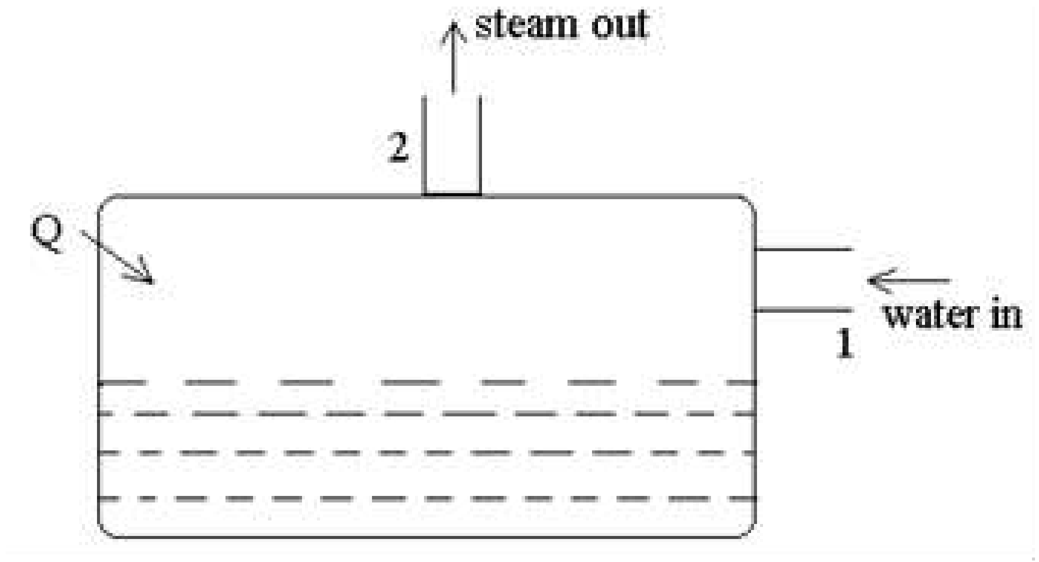
- **ie. $w_c\ (h_1 - h_3) = w_s(h_4 - h_2)$**



- **BOILER**

- Boiler is the device used for steam generation at const pressure. Heat is supplied externally to the boiler for steam generation upon state of steam desired.

- $W = 0, \quad \Delta KE = 0, \quad \Delta PE = 0, \quad m_1 = m_2$
- $Q + mh_1 = mh_2$
- $\mathbf{Q = m (h_2 - h_1) = mCp(T_2 - T_1)}$



- **NOZZLE & DIFFUSER**

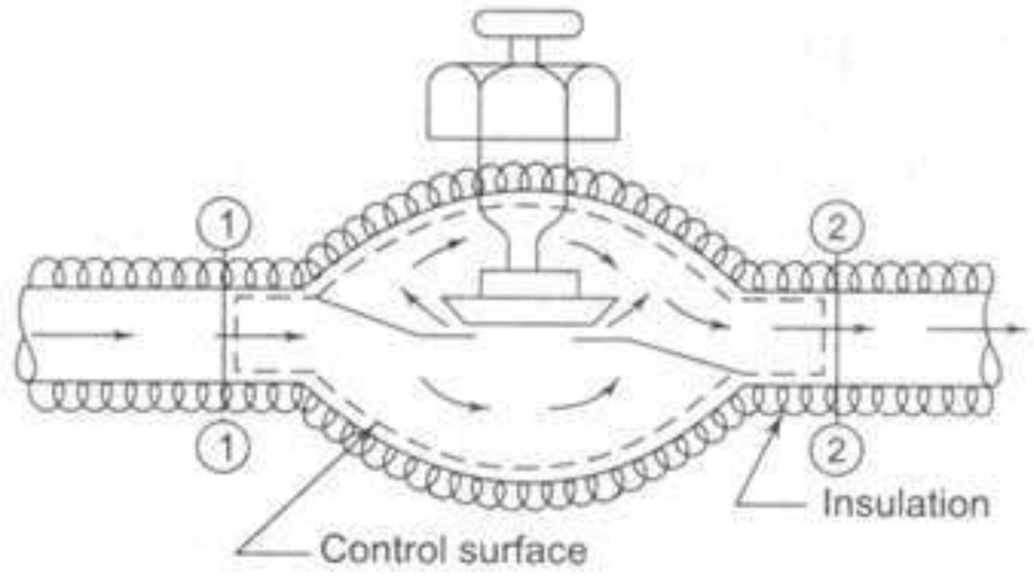
- A nozzle is a device which increases the velocity or KE of a fluid at the expense of its pressure drop, whereas a diffuser increases the pressure of a fluid at the expense of its KE.
- $Q = 0$, $W = 0$, $\Delta PE = 0$, $m_1 = m_2$

- ie, $m (h_1 + C1^2/2) = m (h_2 + C_2^2/2)$
- Compared to exit velocity, inlet velocity is negligible,

  $h_1 = (h_2 + C_2^2/2)$
- ie. $h_1 = h_2 + C_2^2/2$
- or, $C_2 = \sqrt{\{2(h_1 – h_2)\}}$ m/s
- Where $h_1 – h_2$ is in J/kg

- **THROTTLING PROCESS**

- When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure, & the flow is said to be throttled.
- $Q = 0, \quad W = 0, \quad \Delta PE = 0, \quad m_1 = m_2$



- ie. $(h_1 + C_1^2/2) = m\,(h_2 + C_2^2/2)$
- ie. $h_1 + C_1^2/2 = h_2 + C_2^2/2$

- Often the pipe velocities in throttling are so low that the KE terms are also negligible.

  $\Delta\, h_1 = h_2$

- ie. Throttling process is a pressure drop at constant enthalpy unless otherwise mentioned.

- Assuming a throttle to be a constant enthalpy process leads us to define a property called the Joule-Thomson coefficient μ_J as:
-

$$\mu_J = \left(\frac{\partial T}{\partial P} \right)_h$$

- Positive value for μ means that the fluid temperature drops during throttling process.
- Negative value for μ means that the fluid temperature raises during throttling process.

- Q. Steam enters a turbine at 20 m/s & at a specific enthalpy of 3000 kJ/kg and leaves the turbine at 40 m/s and at a specific enthalpy of 2500 kJ/kg. Heat lost to the surroundings is 25 kJ/kg. Steam passes through the turbine with a flow rate of 3,60,000 kg/hr. Determine the output from the turbine in MW.
- Ans.
- $Q + m_1 (h_1 + C_1^2/2 + g_1 z_1) = W + m_2 (h_2 + C_2^2/2 + g_2 z_2)$
- $C_1 = 20$m/s, $C_2 = 40$ m/s, $Q = -25$ kJ/kg
- $h_1 = 3000$ kJ/kg, $h_2 = 2500$ kJ/kg, $m = 360000/3600 = 100$ kg/s, W/s =?

- ie, $-25 \times 10^3 + (3000 \times 10^3 + 20^2) = W + (2500 \times 10^3 + 40^2/2)$
- $W = 3000 \times 10^3 + 20^2/2 - 25 \times 10^3 - 2500 \times 10^3 - 40^2/2 = 474.4 \times 10^3$ J/kg

- Mass flow/sec = 360000/3600 = 100 kg/s
- WD/sec = $474.4 \times 10^3 \times 100$ kW = 47.44 MW

- Q. A steam turbine receives steam at the rate of 22700 kg/hr when it is delivering 500 kW power. The inlet & outlet velocities of steam are 75m/s & 300 m/s respectively. The inlet pipe is 3m above the exhaust pipe. Neglecting the heat lost from the turbine, find the change in enthalpy per kg of steam.
- Ans.
- $Q + m_1 (h_1 + C_1^2/2 + g_1 z_1) = W + m_2 (h_2 + C_2^2/2 + g_2 z_2)$
- $C_1 = 75$m/s, $C_2 = 300$ m/s, $W = 500$ kJ/s
- $z_1 = 3$ m, $z_2 = 0$ m, $m = 22700/3600 = 6.31$ kg/s, $Q = 0$, $(h_1 - h_2) = ?$
- $[(h_1 - h_2) + (C_1^2 - C_2^2)/2 + (z_1 - z_2)\, g]\, m - W = 0$
- Therefore,
- $[(h_1 - h_2) + (75^2 - 300^2)/2 + (3 - 0)\, 9.8]\, 6.31 - 500 \times 10^3 = 0$
- ie, $[\,(h_1 - h_2) - (42187.5) + 29.4]\, 6.31 = 500 \times 10^3$
- $h_1 - h_2 = (500 \times 10^3)/6.31 + (4.2187.5 - 29.4) = 121.397$ kJ/kg

- Q) A centrifugal air compressor used in gas turbine power plant receives air at 100 kPa & 300 K. It discharges air at 400 kPa & 500 K. The velocity of the air leaving the compressor is 100 m/s. Neglecting the velocity at the entry of the compressor, determine the power required to drive the compressor for a mass flow rate of 5 kg/s. Neglect any heat transfer. Take C_P (air) = 1 kJ/kg K.
- Ans.
- $Q + m_1 (h_1 + C_1^2 /2 + gz_1) = W + m_2 (h_2 + C_2^2 /2 + gz_2)$
- $P_1 = 100$ kPa, $P_2 = 400$ kPa, $T_1 = 300$ K, $T_2 = 500$ K, m = 5 kg/s, $C_1 = 0$
- Cp = 1 kJ/kg K, W/s =?

- ie, $[(h_1 – h_2) + (C_1^2 – C_2^2)/2]\, m = W$
- But, $\Delta h = Cp\Delta T$
- ie, $[CP (T_1 – T_2) + (C_1^2 – C_2^2)/2]\, m = W$
- $[1 \times 10^3 (300 - 500) + (0^2\ 100^2)/2]\ 5 = W$
- $(-200 \times 10^3 - 5000)5 = W$
- ie, W/s = -1025000 J/s
- or Power = -1025 kW

- Q. Air flow steadily at the rate of 0.5 kg/s through an air compressor, entering at 7m/s velocity, 100 kPa pressure, and 0.95 m^3/kg volume, and leaving at 5m/s, 700 kPa & 0.19m^3/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of air entering. Cooling water in the compressor jackets absorbs heat from the air at the rate of 58 kW.
- a) Compute the rate of shaft work input to the air in kW.
  b) Find the ratio of the inlet pipe diameter to outlet pipe diameter.
- Ans.
- $C_1 = 7$ m/s, $C_2 = 5$m/s, $Q = -58$ kW, $P_1 = 100$ kPa, $P_2 = 700$ kPa
- $V_1 = 0.95m^3/kg$, $V_2 = 0.19m^3/kg$, $m = 0.5$ kg/s, $d_1/ d_2 =?$ $W =?$

- $Q + m_1 (h_1 + C_1^2 /2+ gz_1) = W + m_2 (h_2 + C_2^2 /2+ gz_2)$
- ie, $Q + m_1(u_1 + p_1v_1 + c_1^2/2 + gz1) = W + m_2 (h_2 + c_2^2/2 + gz_2)$
- $- 58 \times 10^3 + 0.5[-90 + (100 \times 10^3 \times 0.95 – 700 \times 10^3 \times 0.19) + (7^2 – 5^2)/2] = W$

- ie, $-58000 + 0.5 \; [-90 \times 10^3 - 38000 + 12] = W$
- ie, $W = -121.994$ kW

- From mass balance, $A_1C_1/V_1 = A_2C_2/V_2$

- ie $A_1/A_2 = C_2/C_1 \times V_1/V_2 = 5/7 \times 0.95/0.19 = 3.57$

- $A = \pi d^2/4$
- ie $d_1/d_2 = \sqrt{3.57} = 1.89$

- Following two cases only will be discussed : 1. Filling a tank.
- 2. Emptying a tank or tank discharge.
- Filling a tank : Let
- m_1 = Initial mass of fluid, p_1 = Initial pressure, v_1 = Initial specific volume, T_1 = Initial temperature, u_1 = Initial specific internal energy,
- m_2 = Final mass of fluid, p_2 = Final pressure,