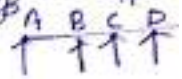
Module-3

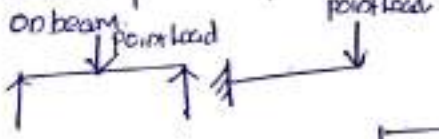
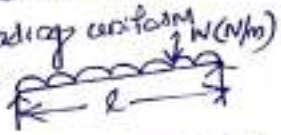
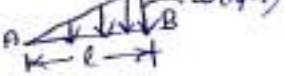
Beam

A beam is defined as a structural member subjected to external loads and right angles to its longitudinal axis.

Classification of Beam

- **Cantilever Beam**
  - : Beam fixed at one end and free other end
  -
- **SSB - Simply Supported Beam**
  - Beam supported by two supports or resting freely both ends
  -
- **Overhanging Beam**
  - End portion of beam is extended beyond the fixed supports
  -
- **Fixed Beam**
  - Both end fixed
  -
- **Continuous Beam**
  - Beam supp on more than two support
  - 

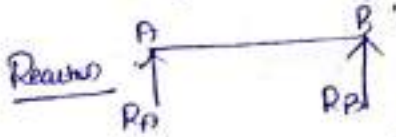
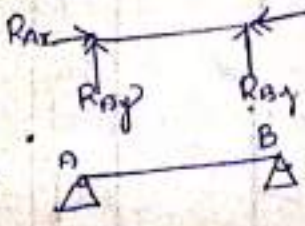
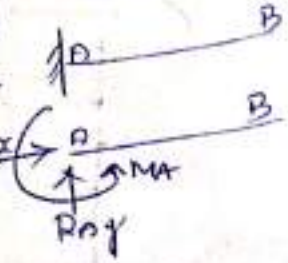
Type of Loading

- **point/concentrated load**
  - load act particular point on beam
  - 
- **(UDL) uniformly distributed load**
  - load spread over beam
  - Rate of loading uniform along length
  - 
- **uniformly varying load (UVL)**
  - spread over beam
  - Rate of loading uniformly increase from zero to W(N/m) through span of length constant rate.
  - 

Sl No	Type of Load	Representation	Point load Conversion
①	point load	W, l	W, l
②	UDL	w(N/m), l	wl, l/2, l/2
③	UVL	w, l	½wl, 2l/3, l/3

Ans.

Supports and Their Reactions

- **Simple support**
  - Support on beam rest freely
  - offer vertical reaction
  - Reaction: 
  - Representation:
- **Roller support**
  - Beam supported by roller points
  -
  - offers vertical reaction
- **Hinged (pin-joint support)**
  - Eg: Single riveted lap joint
  - Reaction either vertical or horizontal/inclined
  - 
- **Fixed support**
  - Beam completly fixed
  - It excert horizontal & vertical reaction
  - 
- **Smooth Support**
  - Contact on smooth surface (1)

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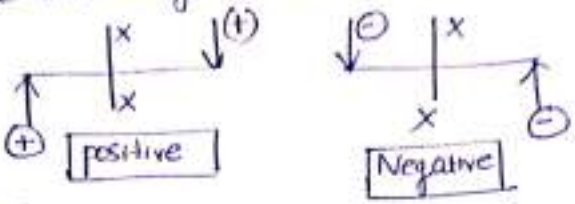
Axial Force

unbalanced horizontal force acting on the section of the beam

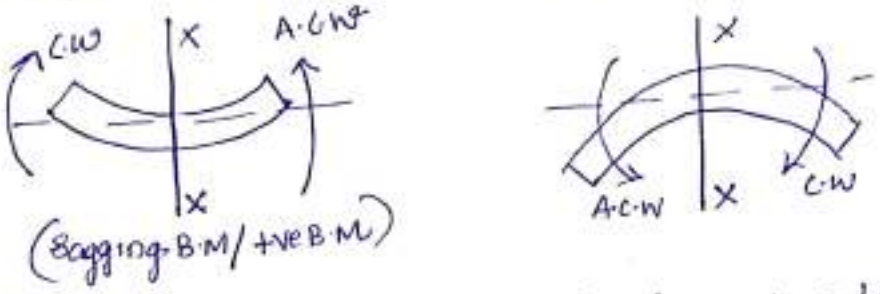
Shear Force in Beams (S.F)

unbalanced vertical force to the right or left of the section of beam defined as Shear force.



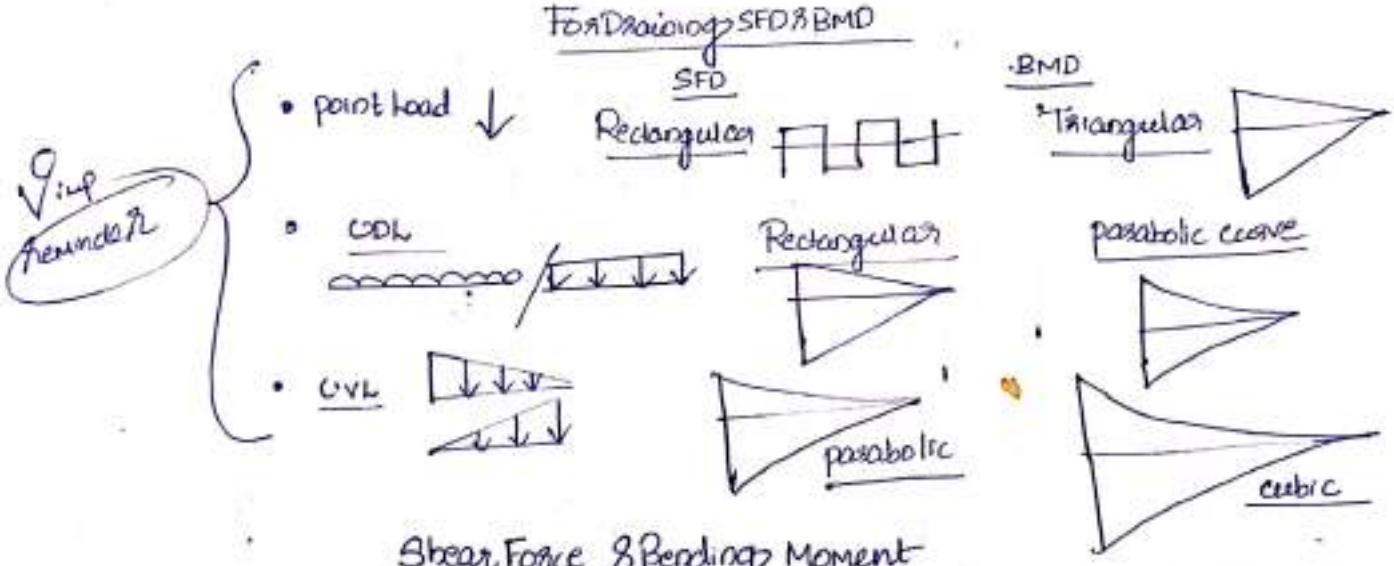
Bending Moment (B.M)

B.M tends to bend beam in the plane of loads and usually referred as B.M

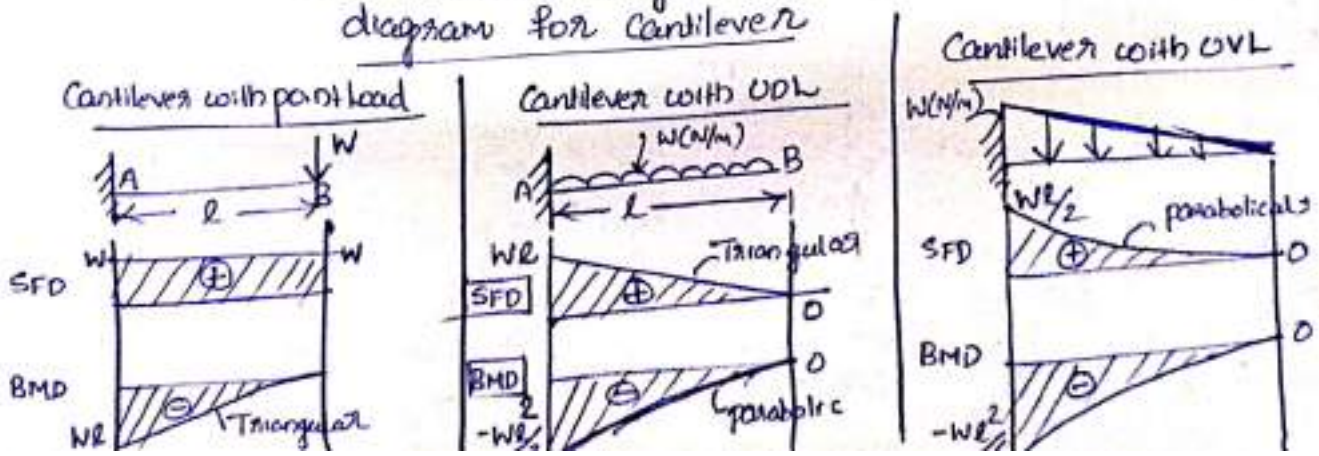


Shear Force diagram (SFD) - Variation of shear force along length of beam called SFD

Bending Moment diagram (BMD) - Diagram showing variation of B.M along length of beam is BMD.



Shear Force & Bending Moment diagram for Cantilever



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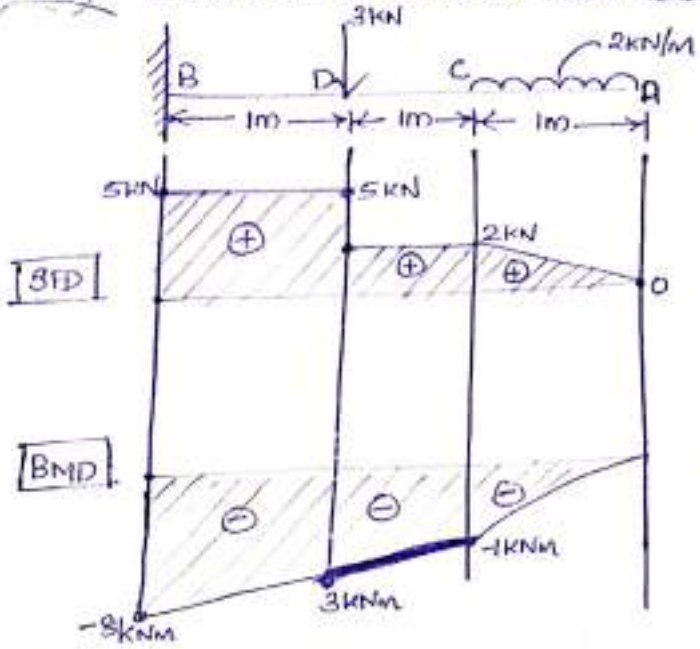
① Draw SFD & BMD of cantilever beam loaded as shown in figure ②



SFD

at CD { at A = SF 0
at C = 2×1 = 2KN

DC { at C = 2KN
at D = 2KN (without 3KN)

BD { at D = 2+3 = 5KN
at B = 2+3 = 5KN

BMD

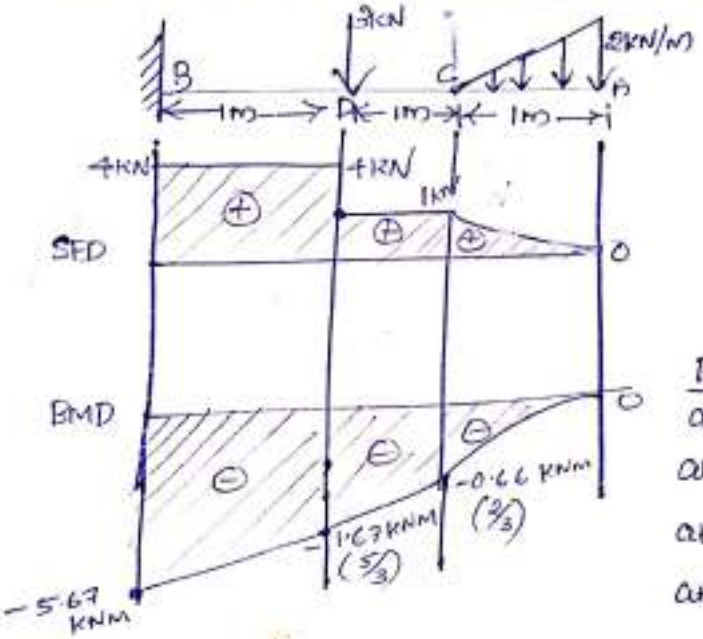
at A ⇒ BM 0

at C ⇒ -2×1×0.5 = -1KNm

at D ⇒ -2×1×1.5 = -3KNM

at B = -2×1×2.5 - 3×1 = -8KNm

② Draw SFD & BMD of cantilever Beam loaded as shown in fig.



SFD SF

CA { at A = 0
at C = 1/2 ×2×1 = 1KN

DC { at C = 1KN
at D = 1KN (without 3KN)

BD { at D = 1+3 = 4KN
at B = 4KN

BMD

at A ⇒ BM = 0

at C ⇒ BM = -1/2 ×2×1×2×1/3 = -4/6 = -2/3

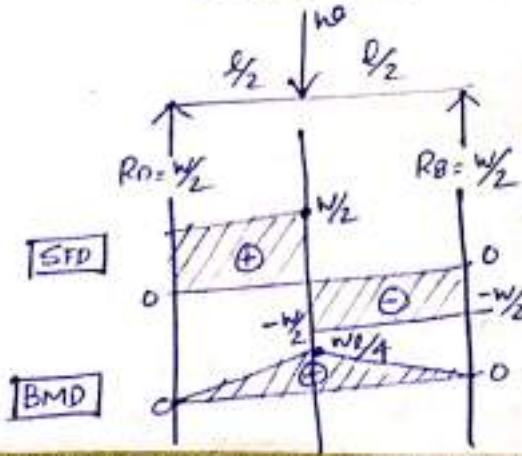
at D ⇒ BM = -1/2 ×2×1×(2/3 + 1) = -5/3

at B ⇒ BM = -1/2 ×2×1(2/3 + 2) - 3×1

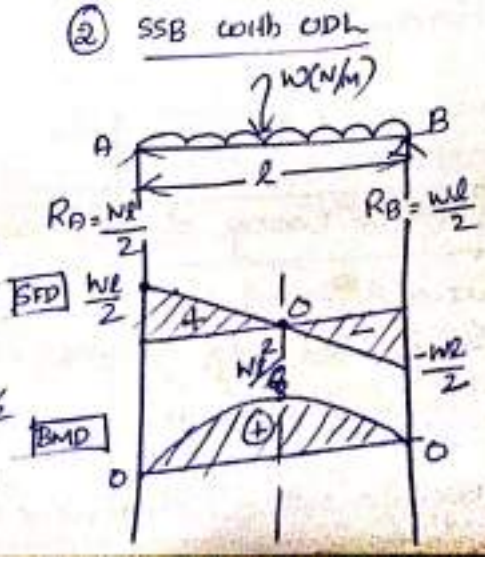
= -8/3 - 3 = -5.67 KNM

Shear Force & Bending Moment Diagram of Simply Supported Beam

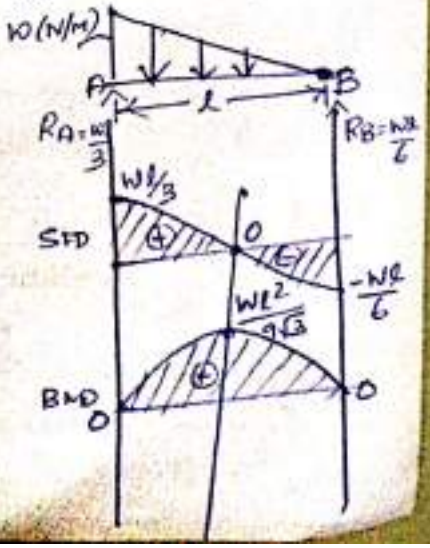
① SSB with point load



② SSB with UDL



③ SSB with UVL



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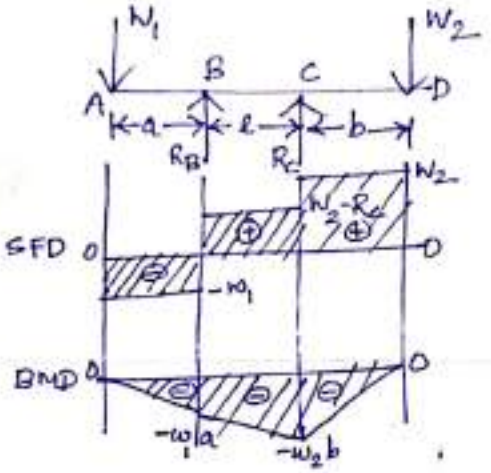
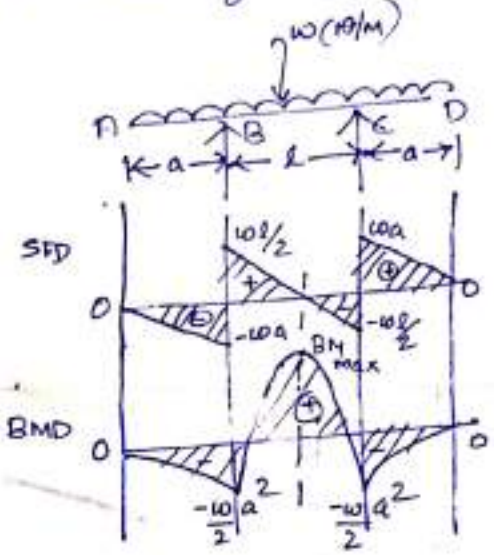
Shear Force and Bending Moment for Overhanging Beam

(a) Overhanging Beam with point load



(b) Overhanging Beam with UDL

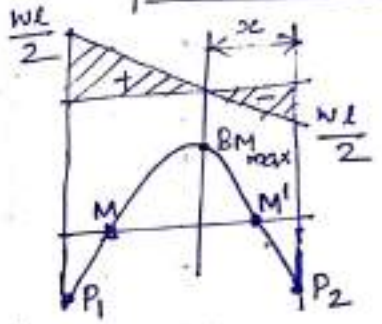


Point of Contraflexure / Inflection

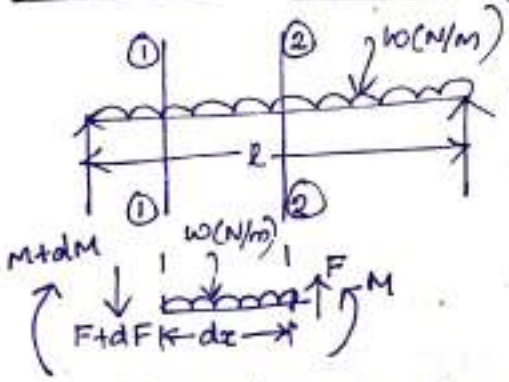
The point where B.M changes from + sign is called point of contraflexure

Eg: From point P_1 to P_2 B.M changes it sign

M/M' is point of contraflexure



Relation b/w Load, Shear Force and Bending Moment



For equilibrium $Wdx - F + F + dF = 0$

$$Wdx = -dF$$

$$\boxed{w = \frac{-dF}{dx}}$$

Rate of change of S.F = Rate of loading.

Moment about Section ①-①

$$M + Fdx - wdx \times \frac{dx}{2} - (M + dM) = 0$$

$$-\frac{wdx^2}{2} + Fdx = dM \Rightarrow Fdx = dM \Rightarrow \boxed{F = \frac{dM}{dx}}$$

(Neglect) 0

Shear Force equal to rate of change of B.M

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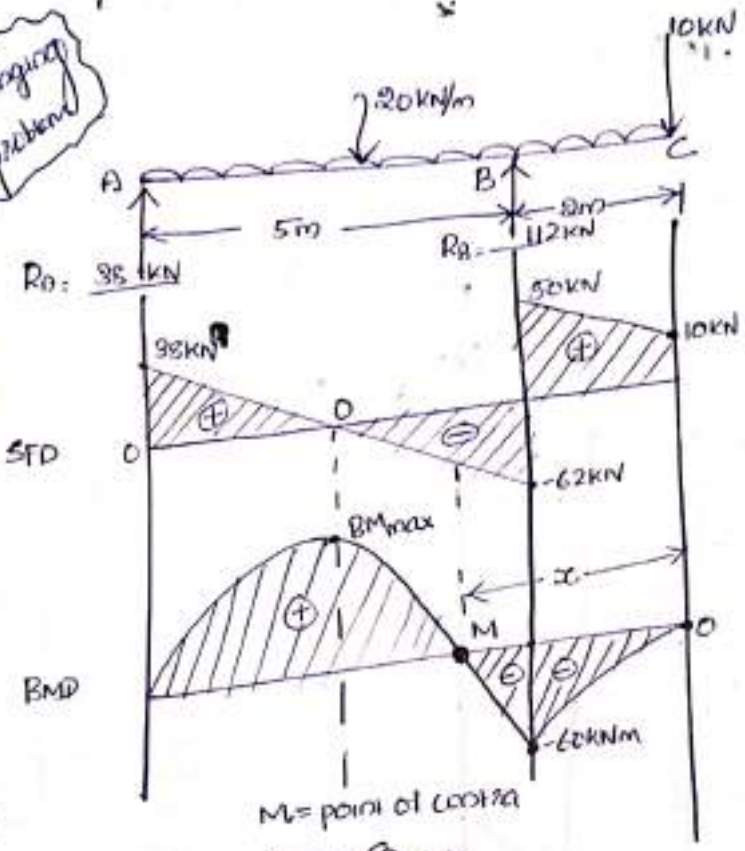
(1) A SSB Beam ABC with supports A, B, 5m apart with an overhanging BC 2m long carries UDL of 20kN/m over whole length. Draw SFD & BMD locate point of contraflexure.

Overhanging Beam problem



Reaction

$\Sigma M_B = 0$

$R_D \times 5 - 20 \times 7 \times 3.5 - 10 \times 7 = 0$

$R_B = 112KN$

$R_D + R_B = 20 \times 7 + 10$

$R_D = 38kN$

SFD SF

BC { at C = 10KN; at B = 10 + 20×2 = 50KN (without 112KN)

DB { at B = 10 + 20×2 − 112 = −62KN; at A = 10 + 112 + 20×7 = 38KN; at A = 0 (with R_D)

BMD BM

at C = 0

at B = −10×2 − 20×2×1 = −20 − 40 = −60KNm

at A = −10×7 + 112×5 − 20×7×3.5 = 0

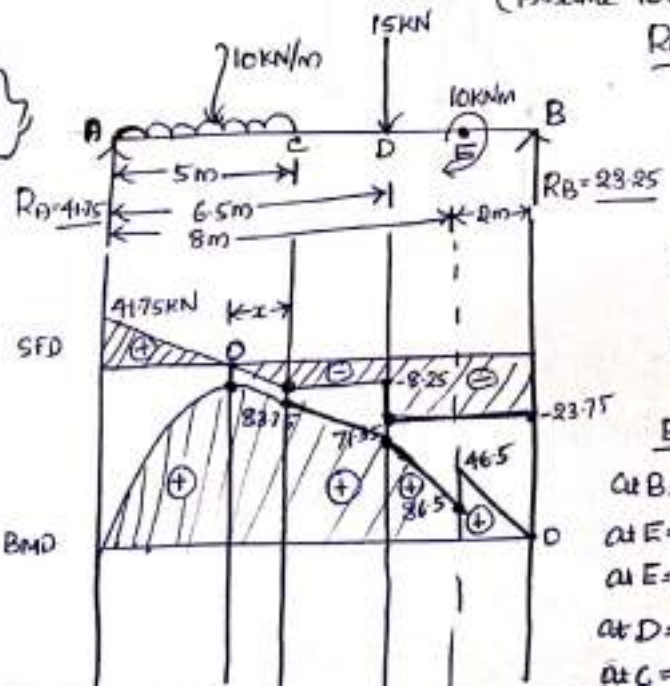
$\Rightarrow -10x + 112(x-2) - 20 \times x \times \frac{x}{2} = 0 \Rightarrow -10x + 112x - 224 - 10x^2 = 0$

$\Rightarrow 102x - 224 - 10x^2 = 0 \Rightarrow 10x^2 - 102x + 224 = 0 \Rightarrow x = 3.2m$

(M) point of contraflexure at distance 3.2m from C / 3.8m from A

(2) A SSB of length 10m carries a UDL of 10kN/m over span of length 5m from left support. A point load of 15KN and a moment of 10KNm acting on the beam at a distance 6.5m & 8m from left support. Draw SFD, BMD. Find BM_{max} & location (Assume 10KNm C.W)

SSB problem



Reaction $\Sigma M_D = 0 \Rightarrow R_B \times 10 - 15 \times 6.5 - 10 \times 5 \times 2.5 - 10 = 0$

$R_B = 23.25KN$

$R_D + R_B = 50 + 15 \Rightarrow R_D = 41.75KN$

SFD

DB { at B = −23.25KN; at D = −23.25KN

CD { at D = −23.25 + 15 = −8.25KN; at C = −8.25KN

AC { at C = −8.25KN; at A = −8.25 + 50 = 41.75KN

at A = 0

BMD BM

at B = 0

at E = 23.25×2 = 46.5KNm

at E = 46.5 − 10 = 36.5KNm

at D = 23.25×3.5 − 10 = 71.375

at C = 23.25×5 − 10 − 15×1.5 = 83.75 KNm

at A = 23.25×10 − 10 − 15×6.5 − 10×5×2.5 = 0

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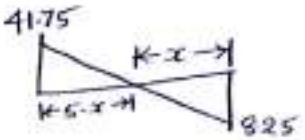


$\frac{41.75}{5-x} = \frac{8.25}{x}$

$41.75x = 41.25 - 8.25x$

$50x = 41.25$

$x = 0.825$ (MAX BM 5-0.825 = 4.175m from left support)

BM at O

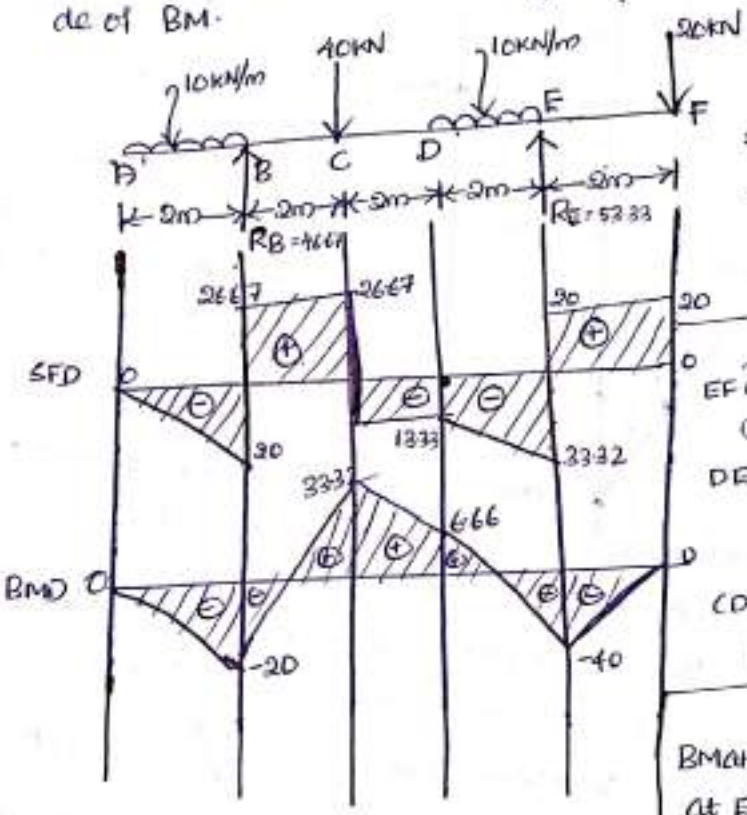
$= 23.25(5+0.825) - 10 \cdot 15(1.5+0.825) - \frac{10 \times 0.825 \times 0.825}{2}$

$= [redacted] - 10 - 34.875 - 3.403125 = 87.15 \text{ kNm}$

195.42125

③ Overhanging problem

Draw SFD & BMD for overhanging shown in figure. Indicate position & magnitude of BM.



Reaction

$\Sigma M_B = 0$

$\Rightarrow R_E \times 6 - 20 \times 8 - 20 \times 5 - 40 \times 2 + 10 \times 2 \times$

$\Rightarrow R_E = 53.33 \text{ KN}$

$R_B + R_E = 20 + 40 + 20 + 20 \Rightarrow R_B =$

SFD

EF { at F = 20KN; at E = 20KN

DE { at E = 20 − 53.33 = −33.33 KN; at D = −33.33 + 20 = −13.33 KN

CD { at D = −13.33 KN; at C = −13.33 KN

BC { at C = −13.33 + 40 = 26.67; at B = 26.67

DB { at B = 26.67 − 46.67 = −20; at A = 0

BMD

at A = 0

BM at F = 0

at E = −20×2 = −40 KNm

at D = −20×4 + 53.33×2 − 20×1 = 6.66 KNM

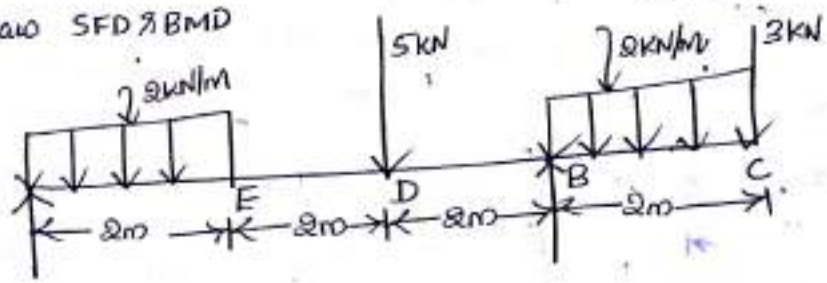
at C = −20×6 + 53.33×4 − 20×3 = 33.32

at B = −20×8 + 53.33×6 − 20×5 − 40×2 = −20.02

Try to do Yourself

④ Draw SFD & BMD

Overhanging problem



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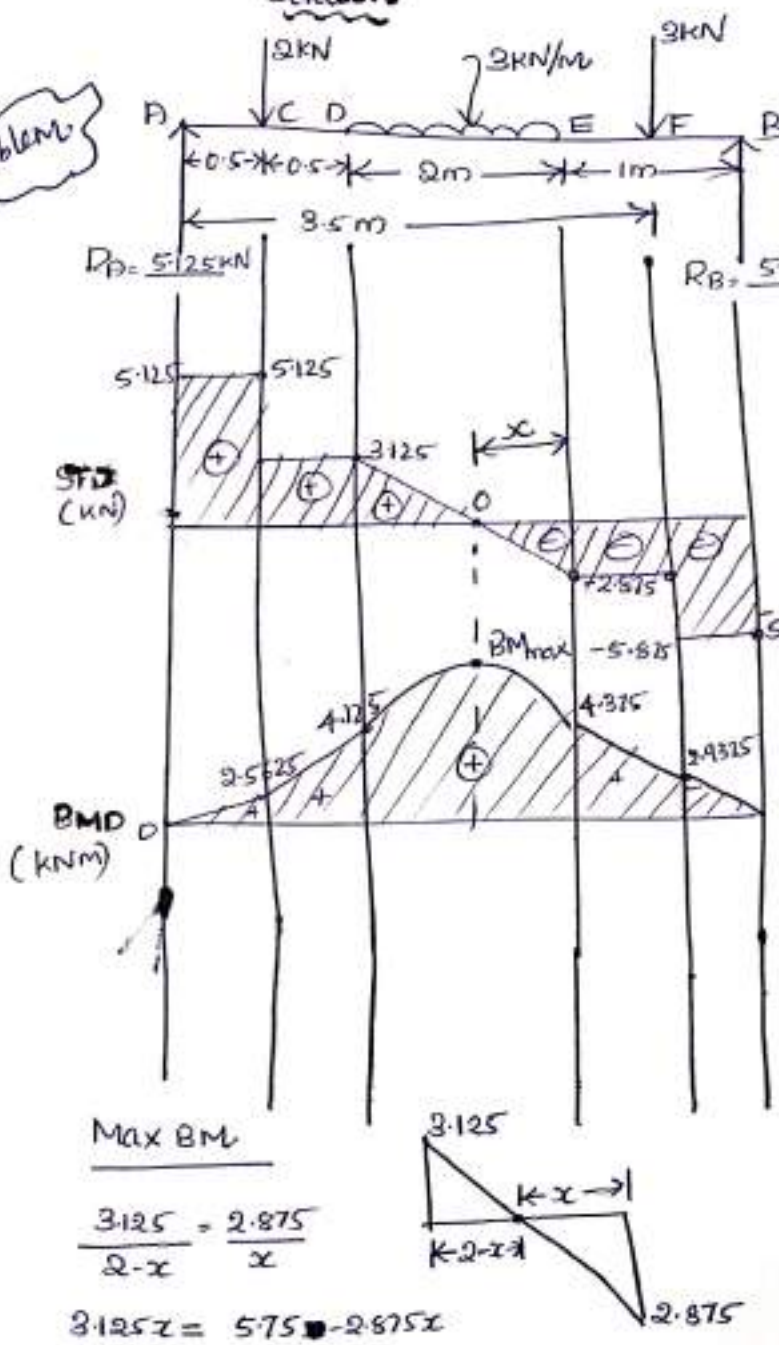
(5) A SSB of length 4m carries UDL of 3KN/m over central 2m of length and two point loads 2KN and 3KN at a distance 0.5m and 3.5m from left Support. Draw SFD & BMD. Locate BM_{max} and find its value.

SSB problem

Solution



Reaction

$\Sigma M_A = 0$

$R_B \times 4 - 3\times 3.5 - 6\times 2 - 2\times 0.5 = 0$

$R_B = 5.875$KN

$R_A + R_B = 2+6+3$

$R_A = 5.125$KN

SFD

FB: at B = -5.875KN; at F = -5.875KN

EF: at F = -5.875+3 = -2.875KN; at E = -2.875KN

DE: at E = -2.875KN; at D = -2.875+6 = 3.125KN

CD: at D = 3.125KN; at C = 3.125KN

AC: at C = 3.125+2 = 5.125KN; at A = 5.125KN; at A = 0

BMD

at B = 0

at F = 5.875×0.5 = 2.9375KNM

at E = $5.875\times 1 - 3\times 0.5$ = 4.375KNM

at D = $5.875\times 3 - 3\times 2.5 - 6\times 1$ = 4.125KNM

at C = $5.875\times 3.5 - 3\times 3 - 6\times 1.5$ = 2.5625

at A = 0

Max BM

$\frac{3.125}{2-x} = \frac{2.875}{x}$

$3.125x = 5.75 - 2.875x$

$2.875x + 3.125x = 5.75$

$x = 0.9583$ m

BM at a distance 1.9583 from B / 2.047 from A

BM at O $\Rightarrow$ BM_{max}

$\Rightarrow 5.875\times(1+0.9583) - 3\times(0.5+0.9583) - 3\times 0.9583\times\frac{0.9583}{2}$

= ~~~ (find value yourself)

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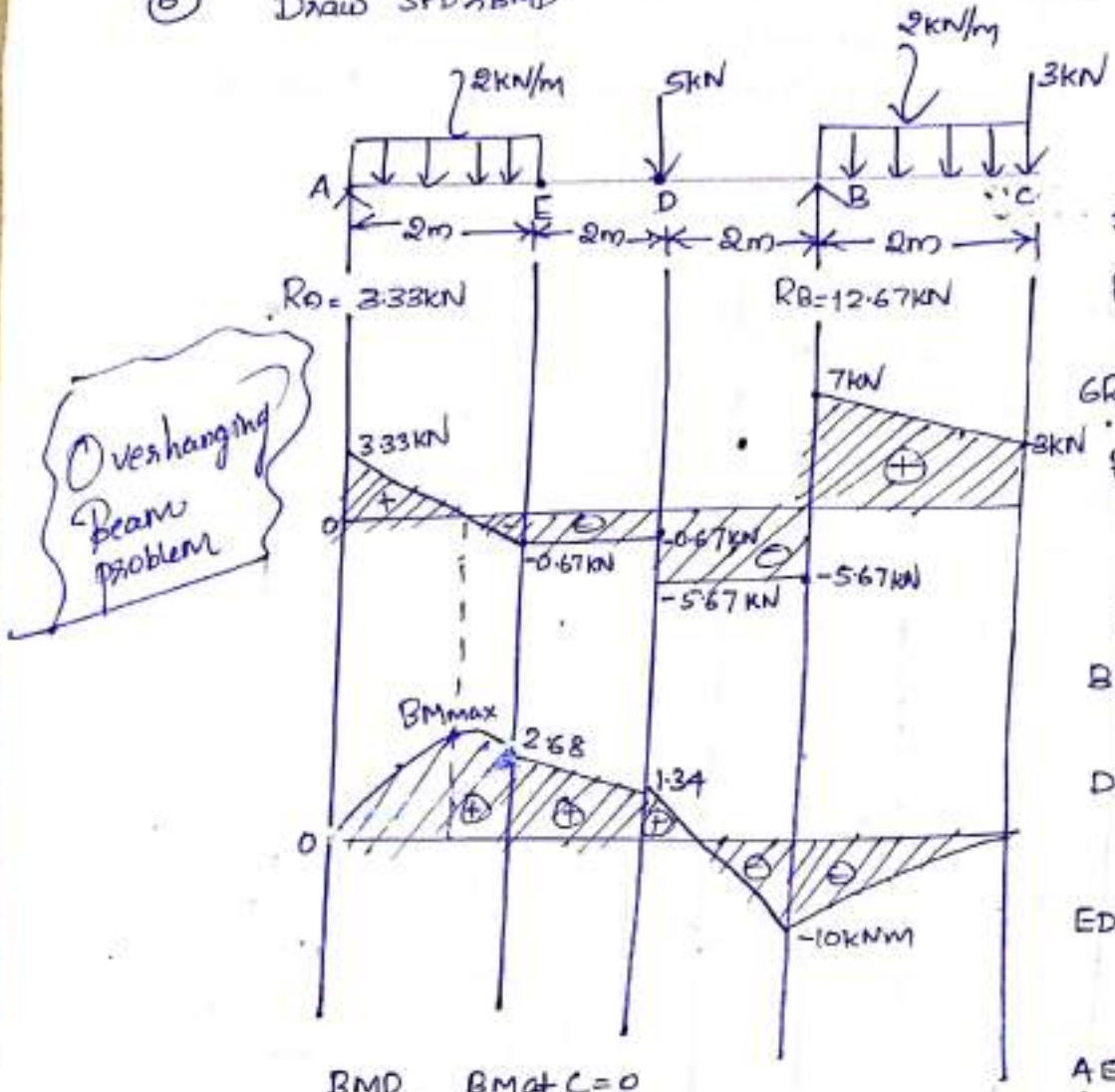
(6) Draw SFD & BMD



Overhanging Beam problem

Reaction

$\Sigma M_D = 0$

$R_B \times 6 - 3\times 8 - 2\times 2 \times 7 - 5\times 4 - 4\times 1 = 0$

$6R_B = 76 \Rightarrow R_B = 12.67\,kN$

$R_A + R_B = 4+5+4+3$

$R_A = 3.33\,kN$

SFD

BC: SF at C = 3kN; SF at B = 3+4 = 7kN

DB: SF at B = 7 − 12.67 = −5.67kN; SF at D = −5.67kN

ED: SF at D = −5.67 + 5 = −0.67kN; SF at E = −0.67kN

AE: SF at E = −0.67kN; SF at A = −0.67 + 4 = 3.33kN

SF at A = 0

BMD

BM at C = 0

BM at B = −3×2 − 4×1 = −6 − 4 = −10kNM

BM at D = $-3\times4 - 4\times3 + 12.67\times2 = 1.34$ kNM

BM at E = $-3\times6 - 4\times5 + 12.67\times4 - 5\times2 = 2.68$ kNM

BM at A = $-3\times8 - 4\times7 + 12.67\times6 - 5\times4 - 4\times1 = 0$ kN

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