

# Module-2

V-imp

Tensor: A tensor is multidimensional array of numerical values that can be used to describe physical state.

Rank of tensor  $\Rightarrow$  ① Zero rank tensor (Scalar) ② 1st rank tensor (Vector)

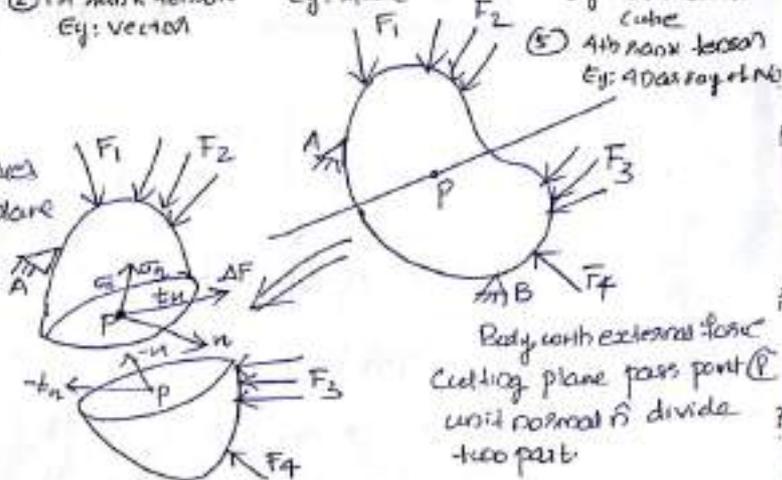
- ③ 2nd rank tensor (Eg: Square matrix)
- ④ 3rd rank tensor (Eg: 3D matrix cube)
- ⑤ 4th rank tensor (Eg: 4D array of No)

## Stress at a point

- Stress at a point is the ratio of resultant of distributed internal forces to area around point on cutting plane

Stress vector at P

$$t_n = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$



## Stress Tensor

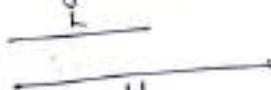
- Definition
- Infinite no. of plane can be pass through point P to obtain infinite of stress vector
  - Set of stress vector acting on every plane passing through a point describe State of stress at a point / stress tensor.

$$\text{Stress tensor } \sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

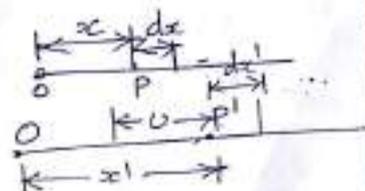
## Strain at a point

Strain - Ratio of elongation to original length

$$e = \frac{dl}{l} = \frac{l' - l}{l}$$



Strain at a point Means when load applied P to P1 dx change dx'



$$e = \frac{dx' - dx}{dx} = \frac{du}{dx}$$

Similarly  $e_y = \frac{dv}{dy}$ ,  $e_z = \frac{dw}{dz}$ , shear strain  $\frac{\gamma_{xy}}{2} = \frac{1}{2} \left( \frac{dv}{dz} + \frac{du}{dy} \right)$ ,  $\frac{\gamma_{yz}}{2} = \frac{1}{2} \left( \frac{dw}{dy} + \frac{dv}{dz} \right)$

$$\frac{\gamma_{zx}}{2} = \frac{1}{2} \left( \frac{du}{dz} + \frac{dw}{dx} \right)$$

## Strain Tensor

- Definition
- Similar to stress tensor
  - Replace stress by strain

$$E = \begin{bmatrix} e_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yz}}{2} & e_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & e_z \end{bmatrix}$$

## Poisson's Ratio ( $\mu$ )

If a body is stressed within elastic limit, the ratio of lateral strain to longitudinal strain is a constant called Poisson's ratio

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\delta l / l}{\delta L / L}$$

- ① A steel bar 3000mm long, 50mm wide and 30mm thick is subjected to an axial pull of 300kN in the axial direction. Find the change in length, width and thickness of the bar.  $E = 2 \times 10^5 \text{ N/mm}^2$  and poisson ratio = 0.3

Solution

$$L = 3000 \text{ mm}, w = 50 \text{ mm}, t = 30 \text{ mm}$$

$$P = 300 \times 10^3 \text{ N}, E = 2 \times 10^5 \text{ N/mm}^2, \mu = 0.3$$

$$\text{Change in length } \delta L = \frac{PL}{AE} = \frac{300 \times 10^3 \times 3000}{20 \times 50 \times 2 \times 10^5} = 2 \text{ mm}$$

$$\text{Linear strain} = \frac{\delta L}{L} = \frac{2}{3000} = 6.667 \times 10^{-4}$$

$$\text{Lateral strain} = \mu \times \text{Linear strain} = 0.3 \times 6.667 \times 10^{-4} = 2 \times 10^{-4}$$

$$\text{Change in width } \delta b = b \times \text{Lateral strain} = 50 \times 2 \times 10^{-4} = 0.01 \text{ mm}$$

$$\text{Change in thickness } (\delta t) = t \times \text{Lateral strain} = 30 \times 2 \times 10^{-4} = 0.006 \text{ mm}$$

## Elastic Constants

- ① Modulus of Elasticity (E) / Young's Modulus

upto elastic limit axial stress to axial strain is constant called Young's Modulus

$$E = \frac{\text{Axial stress}}{\text{Axial strain}} \quad (\text{unit: N/mm}^2)$$

- ② Rigidity Modulus / Shear Modulus (G)

Shear stress to shear strain is constant upto elastic limit.

$$G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

- ③ Bulk Modulus (K) - Ratio of direct stress to corresponding volumetric strain is constant upto elastic limit.

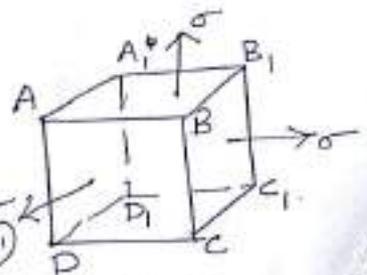
$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_v}$$

## Relation b/w Elastic Constants

- (a) Relation b/w Bulk Modulus (K) & Young's Modulus (E)

• Tensile strain  $\frac{\sigma}{E}$  (due to forces BB<sub>1</sub>, CC<sub>1</sub> & AA<sub>1</sub>, DD<sub>1</sub>)

• Compressive lateral strain =  $\mu \times \frac{\sigma}{E}$  (due to forces on AA<sub>1</sub>, BB<sub>1</sub> & DD<sub>1</sub>, CC<sub>1</sub>)



$$\text{Net tensile strain } \delta L = \frac{\sigma}{E} - \mu \frac{\sigma}{E} - \mu \frac{\sigma}{E} \Rightarrow \frac{\delta L}{L} = \frac{\sigma}{E} (1 - 2\mu) \quad \text{--- (1)}$$

$$V = L^3, \quad \frac{\partial V}{\partial L} = 3L^2 \Rightarrow \partial V = 3L^2 \partial L - 3L^3 \times \frac{\partial L}{L}$$

$$\partial V = 3L^3 \times \frac{\sigma}{E} (1 - 2\mu) \Rightarrow \frac{\partial V}{V} = \frac{3L^3}{L^3} \times \frac{\sigma}{E} (1 - 2\mu)$$

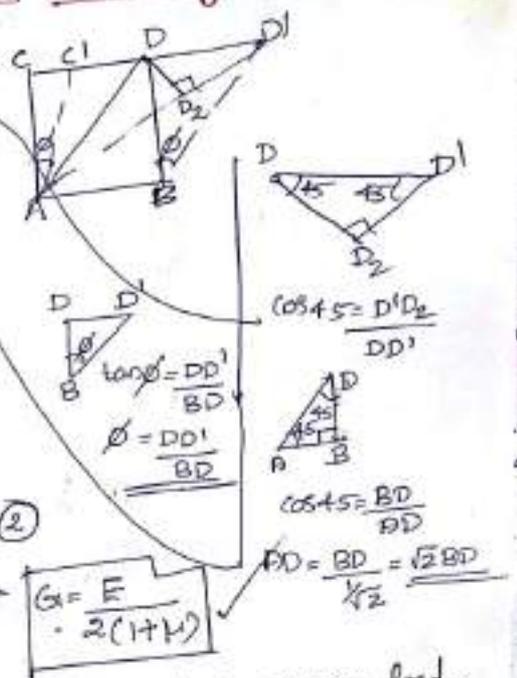
$$\frac{\partial V}{V} = 3 \times \frac{\sigma}{E} (1 - 2\mu) \Rightarrow$$

$$\text{Bulk Modulus } K = \frac{\sigma}{\frac{\partial V}{V}} = \frac{\sigma}{\frac{3\sigma}{E} (1 - 2\mu)} = \frac{E}{3(1 - 2\mu)}$$

$$K = \frac{E}{3(1 - 2\mu)}$$

(By heart)

## Relation b/w Modulus of Elasticity (E) and Modulus of Rigidity (G) (2)



Strain  $AD = \frac{AD' - AD}{AD} = \frac{D'D_2}{AD} = \frac{DD' \cos 45}{BD \sqrt{2}}$

$= \frac{DD' \times \frac{1}{\sqrt{2}}}{BD \sqrt{2}} = \frac{DD'}{2BD} = \frac{\phi}{2}$

Linear Strain of  $AD = \frac{\phi}{2} = \frac{\tau}{2G}$  — (1)  $G = \frac{\tau}{\phi}$

Tensile strain on  $AD = \frac{\tau}{E}$

Tensile strain on  $BC = \mu \times \frac{\tau}{E}$

Total strain on  $AD = \frac{\tau}{E} + \mu \frac{\tau}{E} = \frac{\tau}{E} (1 + \mu)$  — (2)

Equat (1) & (2)  $\Rightarrow \frac{\tau}{2G} = \frac{\tau}{E} (1 + \mu) \Rightarrow \boxed{G = \frac{E}{2(1 + \mu)}}$

- ① A bar of circular cross section 20mm dia is subjected to axial compressive load of 100kN. The increase in dia found to be 0.082mm. Calculate poisson ratio & Modulus of elasticity. Modulus of rigidity =  $8 \times 10^4 \text{ N/mm}^2$

Solution

$d = 20 \text{ mm}, \delta d = 0.082 \text{ mm}, P = 100 \times 10^3 \text{ N}, G = 8 \times 10^4 \text{ N/mm}^2$

$\mu = \frac{\delta d/d}{\delta l/l} \Rightarrow \text{where } \frac{\delta l}{l} = \frac{e}{l}, e = \frac{P}{AE} = \frac{100 \times 10^3}{\frac{\pi \times 20^2}{4} \times \frac{2(1 + \mu) \times 8 \times 10^4}{E}}$

$\mu = \frac{\delta d/d}{e} = \frac{0.082/20}{\frac{\pi \times 20^2 \times 2(1 + \mu) \times 8 \times 10^4}{4 \times E}} \Rightarrow \text{Solving we get } \underline{\mu = 0.259}$

$E = 2(1 + \mu)G = 2(1 + 0.259) \times 8 \times 10^4 = \underline{2.015 \times 10^5 \text{ N/mm}^2}$

- ② A cylindrical bar is 20mm dia and 800mm long. During a tensile test it is found that the longitudinal strain is 4 times the lateral strain. Calculate the modulus of rigidity and Bulk Modulus, if elastic modulus  $1 \times 10^5 \text{ N/mm}^2$ . Find change in volume when bar subjected to hydrostatic pressure of  $100 \text{ N/mm}^2$

Solution

$d = 20 \text{ mm}, l = 800 \text{ mm}, \left(\frac{\delta l}{l}\right) = \left(\frac{4 \delta d}{d}\right) \Rightarrow \mu = \frac{\delta d/d}{\delta l/l} = \frac{\delta d/d}{4 \delta d/d} = 0.25$

$E = 1 \times 10^5 \text{ N/mm}^2$

$G = \frac{E}{2(1 + \mu)} = \frac{1 \times 10^5}{2(1 + 0.25)} = 4 \times 10^4 \text{ N/mm}^2, k = \frac{E}{3(1 - 2\mu)} = \frac{1 \times 10^5}{3(1 - 2 \times 0.25)} = \frac{6.67 \times 10^4}{3} \text{ N/mm}^2$

$k = \frac{\sigma}{\frac{\delta v}{v}} \Rightarrow \frac{\delta v}{v} = \frac{\sigma}{k} \Rightarrow \delta v = \frac{\sigma}{k} \times v = \frac{100}{6.67 \times 10^4} \times \frac{\pi \times 10^2 \times 800}{\pi \times 2^2} = \underline{376.61 \text{ mm}^3}$

- ③ A metallic bar 30mm dia is subjected to axial tensile load of 60kN. The measured extension on gauge length of 150mm is 0.075mm and change in diameter is 0.00375mm. Calculate poisson's ratio, Young's modulus & Modulus of Rigidity.

Solution

$$d = 30\text{mm}, P = 60 \times 10^3 \text{N}, l = 150\text{mm}, \delta l = 0.075\text{mm}, \delta d = 0.00375\text{mm}$$

$$\mu = \frac{\delta d/d}{\delta l/l} = \frac{0.00375/30}{0.075/150} = 0.25 \Rightarrow E = \frac{P/l}{\delta l/l} = \frac{60 \times 10^3}{\frac{0.075}{150}} = 1.699 \times 10^5 \text{N/mm}^2$$

$$G = \frac{E}{2(1+\mu)} = \frac{1.699 \times 10^5}{2(1+0.25)} = 6.796 \times 10^4 \text{N/mm}^2, k = \frac{E}{3(1-2\mu)} = \frac{1.69 \times 10^5}{3(1-2 \times 0.25)} = 11.27 \times 10^4 \text{N/cm}^2$$

- ④ A rod which tapers uniformly from 50mm dia to 30mm dia length of 1.5m subjected to axial force of 150kN. Determine the elongation of rod  $E = 200 \text{GPa}$

Solution

$$d_1 = 50\text{mm}, d_2 = 30\text{mm}$$

$$l = 1.5\text{m} = 1.5 \times 10^3 \text{mm}, P = 150 \times 10^3 \text{N}, E = 200 \times 10^3 \text{N/mm}^2$$

$$\text{Deformation } \Delta = \frac{4Pl}{\pi E d_1 d_2} = \frac{4 \times 150 \times 10^3 \times 1.5 \times 10^3}{\pi \times 200 \times 10^3 \times 50 \times 30} = 0.955 \text{mm}$$

### Bi-axial and Tri-axial Deformation (Volumetric Strain)

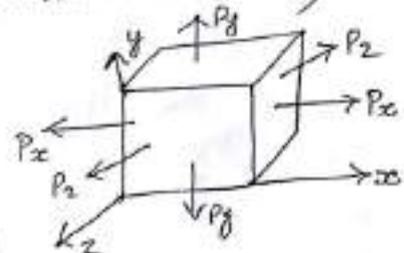
$V_{up}$

$$\sigma_x = \frac{P_x}{A_x}, \sigma_y = \frac{P_y}{A_y}, \sigma_z = \frac{P_z}{A_z}$$

$$\frac{\Delta V}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \mu(\sigma_y + \sigma_z)), \epsilon_y = \frac{1}{E} (\sigma_y - \mu(\sigma_x + \sigma_z))$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \mu(\sigma_x + \sigma_y))$$



- ① An Al alloy plate of size 50mm x 20mm with thickness 5mm is loaded shown find change in thickness? What must be loaded to be applied to have same change in thickness if load applied only along thickness direction.  $E = 1 \times 10^5 \text{N/mm}^2$

$$\mu = 0.25$$

Solution

$$E = 1 \times 10^5 \text{N/mm}^2, l = 50\text{mm}, W = 20\text{mm}, T = 5\text{mm}$$

$$\mu = 0.25, \sigma_x = \frac{P_x}{A_x} = \frac{20 \times 10^3}{20 \times 5} = 200 \text{N/mm}^2$$

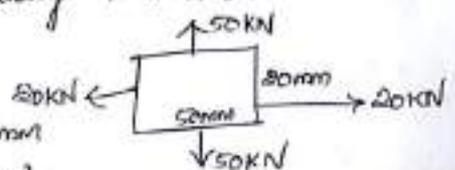
$$\sigma_y = \frac{P_y}{A_y} = \frac{50 \times 10^3}{50 \times 5} = 200 \text{N/mm}^2$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} = \frac{1}{E} (\sigma_x - \mu \sigma_y) = \frac{1}{1 \times 10^5} (200 - 0.25 \times 200) = 1.5 \times 10^{-3}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \mu \sigma_x) = \frac{1}{1 \times 10^5} (200 - 0.25 \times 200) = 1.5 \times 10^{-3}, \epsilon_z = -\mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$= -0.25 \times \frac{200}{1 \times 10^5} \times 2 = -1 \times 10^{-3}$$

$$\epsilon_z = \frac{\Delta T}{T} = \frac{\Delta T}{T} \times 1 \times 10^3 \Rightarrow \Delta T = 0.005 \text{mm}$$



$$\epsilon_z = \frac{\sigma_z}{E} = \frac{P_z / 50 \times 20}{1 \times 10^5} = 1 \times 10^{-8} P_z = \frac{dT}{T}$$

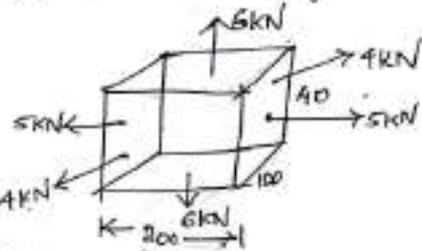
$$\frac{0.005}{5} = 1 \times 10^{-8} P_z \Rightarrow P_z = 100000 \text{ N} = 100 \text{ kN}$$

- ② A metallic bar 300mm x 100mm x 40mm is subjected to a force of 5kN (tensile) 6kN (tensile) and 4kN (tensile) along x, y, z s-ply. Determine change in volume of block  $E = 2 \times 10^5 \text{ MPa}$ ,  $\mu = 0.25$

Solution

$$\frac{\Delta V}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

$$\sigma_x = \frac{P_x}{A_x} = \frac{5 \times 10^3}{100 \times 40} = 1.25 \text{ N/mm}^2, \sigma_y = \frac{P_y}{A_y} = \frac{6 \times 10^3}{300 \times 100} = 0.2 \text{ N/mm}^2$$



$$\sigma_z = \frac{P_z}{A_z} = \frac{4 \times 10^3}{200 \times 40} = 0.5 \text{ N/mm}^2$$

$$\frac{\Delta V}{V} = \frac{1}{2 \times 10^5} (1.25 + 0.2 + 0.5) (1 - 2 \times 0.25) = 4.45 \times 10^{-6}$$

$$\Delta V = 4.45 \times 10^{-6} \times 300 \times 100 \times 40 = 5.34 \text{ mm}^3$$

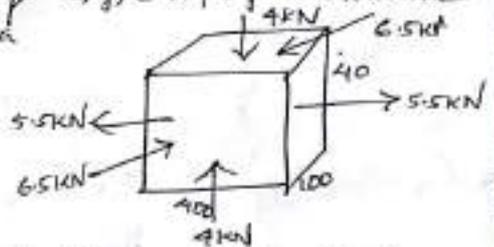
- ③ A rectangular block 400mm x 100mm x 40mm is subjected to force of 5.5kN (tensile) 6.5kN (compressive) and 4kN (compressive) along x, y, z s-ply. Determine change in volume of block.  $\mu = 0.3$ ,  $E = 200 \text{ GPa}$

Solution

$$\sigma_x = \frac{P_x}{A_x} = \frac{5.5 \times 10^3}{100 \times 40} = 1.375 \text{ N/mm}^2$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{-4 \times 10^3}{400 \times 100} = -0.1 \text{ N/mm}^2$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{-6.5 \times 10^3}{400 \times 40} = -0.406 \text{ N/mm}^2$$



$$\frac{\Delta V}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu) V$$

$$= \frac{1}{200 \times 10^3} (1.375 - 0.1 - 0.406) (1 - 2 \times 0.3) \times 1600000$$

$$= 2.780 \text{ mm}^3$$

## Torsion

Torque  $\Rightarrow$  Product of force applied and radius of shaft  $T = F \times r$

Torsion Equation (Analysis of torsion of c/s)

Assumption of theory of Torsion

- ① Material of shaft uniform
- ② Twist along shaft is uniform
- ③ Shaft is uniform circular c/s
- ④ Material obey hook's law
- ⑤ plane before twist remain plane after twist

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}$$

$$P = \frac{2\pi T l}{60}$$

Solid shaft

$$\text{Area} \Rightarrow A = \frac{\pi}{4} d^2$$

$$\text{polar M-I} \Rightarrow J = \frac{\pi}{32} d^4$$

Hollow shaft

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$J = \frac{\pi}{32} (D^4 - d^4)$$

T = Torque (Nm)

$$J = \text{polar M-I} = \frac{\pi}{32} d^4$$

G = Shear Modulus (N/m<sup>2</sup>)

$\theta$  = angle of twist ( $\frac{\theta \times \pi}{180}$ )

l = length of shaft

$\tau$  = Shear Stress (N/mm<sup>2</sup>)

To determine dia of shaft

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \text{find } d \quad / \quad \frac{T}{J} = \frac{\tau}{r} \Rightarrow \text{Find } d$$

Take Max: dia

- ① A shaft of 50mm dia is made of material having allowable shear stress of 120MPa. If the shaft runs at 3000 rpm. What is maximum power that can be carried by the shaft before failure?

Solution

$$d = 50 \text{ mm}; \tau = 120 \text{ N/mm}^2, N = 3000 \text{ rpm}$$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \frac{\tau}{r} J = \frac{\tau}{r} \times \frac{\pi}{32} d^4 = \frac{120}{25} \times \frac{\pi}{32} 50^4 = 2945243.11 \text{ Nmm}$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 3000 \times 2945243.11}{60} = 92527.5 \text{ Watts}$$

- ② The propeller shaft for a small ship is of solid steel bar 100mm diameter. If the permissible shear stress is 50MPa and angle of twist 0.8°/m length. Calculate Max torque that can be applied on shaft.  $G = 80 \text{ GPa}$

Solution

$$d = 100 \text{ mm}, \tau = 50 \times 10^3 \text{ N/mm}^2, \theta = 0.8 \times \frac{\pi}{180} = 0.014 \text{ radian}$$

$$G = 80 \times 10^3 \text{ N/mm}^2, L = 1 \text{ m} = 1000 \text{ mm}$$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \frac{\tau}{r} J = \frac{50 \times 10^3}{50} \times \frac{\pi}{32} 100^4 = 98.12 \times 10^8 \text{ Nmm}$$

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow T = \frac{G\theta}{L} J = \frac{80 \times 10^3 \times 0.014}{1000} \times \frac{\pi}{32} 100^4 = 10.99 \times 10^6 \text{ Nmm}$$

$$T_{\text{max}} = 98.12 \times 10^8 \text{ Nmm}$$

- ③ A solid circular shaft transmit 75kW power at 200 rpm. Calculate the shaft dia. If twist in the shaft is not exceed 1° in 2m length of shaft and shear stress is limited to 50 N/mm<sup>2</sup>. Take  $C = 1 \times 10^5 \text{ N/mm}^2$

Solution

$$P = 75 \times 10^3 \text{ W}, N = 200 \text{ rpm}, G = 1 \times 10^5 \text{ N/mm}^2, \tau_{\text{max}} = 50 \text{ N/mm}^2, L = 2 \text{ m} = 2000 \text{ mm}$$

$$\theta = 1^\circ = 1 \times \frac{\pi}{180} = 0.017 \text{ radian}$$

$$P = \frac{2\pi NT}{60} = 75 \times 10^3 \Rightarrow T = \frac{75 \times 10^3 \times 60}{2\pi \times 200} = 3580.9862 \text{ Nm}$$

Consider angle of twist ( $\theta$ )

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$J = \frac{TL}{G\theta} \Rightarrow \frac{\pi}{32} d^4 = \frac{3580.9862 \times 10^3 \times 2000}{1 \times 10^5 \times 0.017} \Rightarrow d = 80.94 \text{ mm}$$

Consider shear stress ( $\tau$ )

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \frac{\tau}{r} J \Rightarrow d = \left( \frac{3580.9862 \times 16}{\pi \times 50} \right)^{\frac{1}{3}} = 71.46 \text{ mm}$$

Required dia = 80.94 mm (larger value)

- ④ A solid shaft has to transmit 100kW at 1600rpm taking allowable shear stress as 70MPa. Find suitable dia of shaft, The max torque transmitted in each revolution exceeds mean by 20%.

Solution

$$P = 100 \times 10^3 \text{ W}, N = 1600 \text{ rpm}, \tau = 70 \text{ N/mm}^2$$

$$T_{\text{max}} = T_{\text{mean}} + \frac{20}{100} T_{\text{mean}} \Rightarrow T_{\text{max}} = 1.2 T_{\text{mean}}$$

$$P = \frac{2\pi NT}{60} \Rightarrow T = \frac{P \times 60}{2\pi N} \Rightarrow T_{\text{mean}} = \frac{10^5 \times 60}{2\pi \times 1600} = 5968.31 \text{ Nm} = \underline{5968.31 \times 10^3 \text{ Nmm}}$$

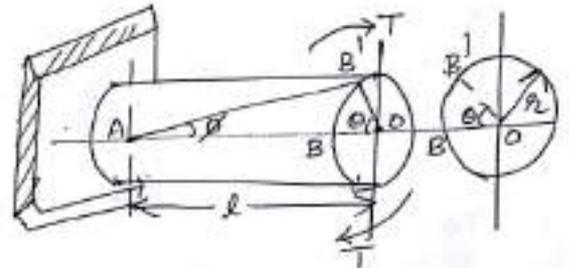
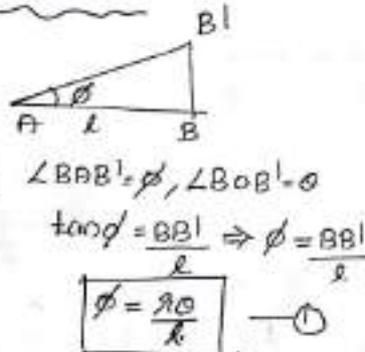
$$T_{\text{max}} = 1.2 T_{\text{mean}} = 7161.97 \times 10^3 \text{ Nmm}$$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow \frac{T_{\text{max}}}{J} = \frac{\tau}{r} \Rightarrow \frac{7161.97 \times 10^3}{\frac{\pi}{32} d^4} = \frac{70}{\frac{d}{2}} \Rightarrow \underline{d = 80.47 \text{ mm}}$$

⑤

Derivation of Torsion Eq:

AB deformed to A'B'  
OB to O'B'



$$\phi = \frac{\tau}{G} \quad \text{--- (2)} \quad \text{Equate (1) \& (2)} \Rightarrow \frac{\tau}{G} = \frac{r\theta}{l} \Rightarrow \boxed{\frac{\tau}{r} = \frac{G\theta}{l}} \quad (\text{stiffness eq})$$

$$\frac{\tau_x}{\tau} = \frac{x}{r} \Rightarrow \tau_x = \frac{x}{r} \tau$$

$$\text{Shear Force} = \frac{x}{r} \tau \times 2\pi x dx = \frac{2\pi \tau x^2 dx}{r}$$

Torque/Torsion moment

$$dT = \frac{2\pi \tau x^2 dx}{r} \times x = \frac{2\pi \tau x^3 dx}{r}$$

$$T = \int_0^r \frac{2\pi \tau x^3 dx}{r} = \frac{2\pi \tau}{r} \left( \frac{x^4}{4} \right)_0^r = \frac{\tau}{r} \frac{2\pi r^4}{4}$$

$$T = \frac{\tau}{r} \times 2\pi \times \left( \frac{d}{2} \right)^4 \Rightarrow T = \frac{\tau}{r} \left( \frac{\pi d^4}{32} \right) \Rightarrow T = \frac{\tau J}{r}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{r}}$$

(Strength eq)

$$\text{Hence } \boxed{\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}} \quad \text{Torsion eq}$$

- Polar Modulus ( $Z_p$ ): Ratio of polar M-I to Max radius

$$Z_p = \frac{\text{polar M-I}}{\text{Max radius}} = \frac{J}{r}$$

- Torsional rigidity  $\Rightarrow \frac{T}{J} = \frac{G\theta}{l} \Rightarrow \theta = \frac{TL}{GJ}$

Product of  $G$  and  $J$  called torsional rigidity ( $= GJ$ )

Stiffness shaft: If a shaft more stiff, given Torque  $\theta$ /unit length within allowable limit

polar modulus of solid shaft  $Z_p = \frac{\pi d^3}{16}$

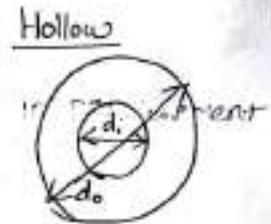
polar modulus of hollow shaft  $Z_p = \frac{\pi}{16} (D^4 - d^4)$

# Problem on Replacing a solid shaft by hollow shaft

① A solid shaft of length \$l\$ and diameter \$d\$ is to be replaced by a tubular steel shaft of same length and same outer diameter \$d\_o\$ such that each of the two shaft could have same angle of twist per unit length. What must be inner diameter of tubular steel shaft be?



$$T = \frac{\tau}{r} \cdot \frac{\pi d^3 \tau}{16}$$



$$T = \frac{\tau}{r} \cdot \frac{\pi (d_o^4 - d_i^4) \tau}{16}$$

Modulus of rigidity of steel is three times that of aluminium.

Solution

\$d = 50\text{mm}\$ (given)

$$\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \frac{\theta}{T} = \frac{l}{GJ}$$

Angle of twist/unit torsional moment is same for two shaft of equal length. \$GJ\$ equal for both shaft.

$$\frac{1}{GJ_s} = \frac{l}{3 \times 61 \times J_h} \Rightarrow 3J_h = J_s \Rightarrow 3 \times \frac{\pi}{32} (50^4 d^4) = \frac{\pi}{32} 50^4$$

$$\Rightarrow d = 45.18\text{mm}$$

② Compare the strength of hollow shaft of diameter ratio 0.75 to that of solid shaft by considering permissible shear stress. Both the shafts are of same material of same length and weight.

Solution

Solid shaft  $T = \frac{\pi}{16} d^3 \tau$

Hollow shaft  $T = \frac{\pi}{16} \frac{(d_o^4 - d_i^4) \tau}{d_o}$

$$\frac{d_i}{d_o} = 0.75$$

weight same

$$W_s = W_h \Rightarrow \frac{\pi}{4} d^2 l \rho = \frac{\pi}{4} (d_o^2 - (0.75d_o)^2) l \rho$$

$$\Rightarrow d^2 = 0.4375 d_o^2 \Rightarrow d = 0.661 d_o$$

$$\frac{T_h}{T_s} = \frac{\frac{\pi}{16} \frac{(d_o^4 - (0.75d_o)^4) \tau}{d_o}}{\frac{\pi}{16} (0.661 d_o)^3 \tau} = \frac{0.684 d_o^3}{0.661^3 \times d_o^3} = 2.3636$$

③ A solid steel shaft of 60mm dia is to be replaced by hollow shaft whose internal dia 0.5 of external dia. Find dia of hollow shaft and % saving material.

Solution

Solid shaft  $T = \frac{\pi}{16} 60^3 \tau$

Hollow shaft  $T = \frac{\pi}{16} \frac{(d_o^4 - (0.5d_o)^4) \tau}{d_o}$

$$T_s = T_h \Rightarrow \frac{\pi}{16} 60^3 \tau = \frac{\pi}{16} \frac{(d_o^4 - (0.5d_o)^4) \tau}{d_o} \Rightarrow \frac{15 d_o^4}{16 d_o} = 60^3 \Rightarrow d_o = 61.30\text{mm}$$

$$d_i = 30.65\text{mm}$$

$$\% \text{ saving material (weight)} = \frac{W_s - W_h}{W_s} \times 100 = \frac{\frac{\pi}{4} d^2 l \rho - \frac{\pi}{4} (d_o^2 - d_i^2) l \rho}{\frac{\pi}{4} d^2 l \rho} = \frac{60^2 - (61.3^2 - 30.65^2)}{60^2} \times 100$$

(Refer more problem on main note) = 21.71%