

Module-I

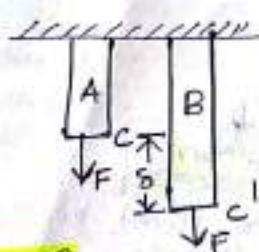
Introduction to Analysis of Deformed Bodies

- **Deformation** - Change in the shape / size of body under application of force

Deformed Bodies

- **Elastic Material**
 - Body subjected external loading. Deformation disappears on removal of load.
- **Plastic Material**
 - Body undergo continuous deformation
 - Body does not regain its original dimension on the removal of loading.
- **Rigid Bodies** - Does not undergo deformation when subjected to external loading

Eg:



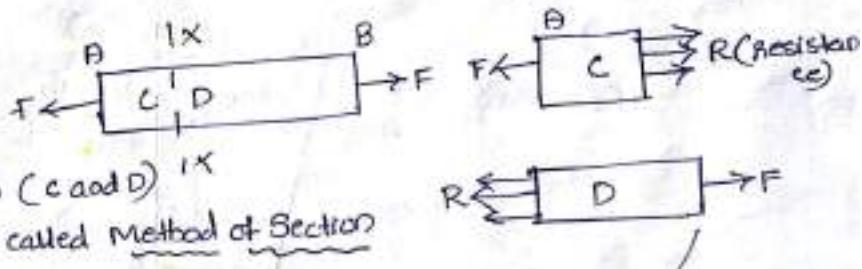
$$\text{Change in Length } CC' = s = \text{Deflection}$$

Internal Forces

- ① **Strength**: Internal resistance offered by body against deformation due to external load.

Method of Section

- Section X-X cut homogeneous bar ABC
- Applied load F is to two (C and D)
- It's for analysis. This is called Method of Section



Assumption

- ① Body is homogeneous
- ② Body is equilibrium after application of load
- ③ Sum of force on body = Internal resistance
- ④ Friction b/w various layers neglected
- ⑤ Magnetic, electric, vibration neglected

Limitation

- ① Complicated for irregular object
- ② used only body static equilibrium
- ③ Approximate value in case of irregular object
- ④ Not accurate. Miscellaneous external forces.

CLASSIFICATION OF STRESSES & STRAINS

Stress (σ)

Definition? Force of resistance per unit area against deformation

$$\sigma = \frac{P}{A} = \frac{\text{Load}}{\text{Area}}$$

Unit: N/m^2 or N/mm^2 , $N/m^2 = Pa$
 $1kPa = 10^3 Pa$, $1MPa = 10^6 Pa$, $1GPa = 10^9 Pa$
 $[Mpa = N/mm^2]$
 (Very important)

Stresses

Normal Stress

- Stress normal to c/s of Member

$$\sigma = \frac{P}{A}$$

Tensile Stress

Resistance offered by body against increase in length

$$\sigma = \frac{P}{A}$$

Compressive Stress

Resistance offered by body against decrease in length

$$P \rightarrow \boxed{\quad} \leftarrow P \quad \sigma = \frac{P}{A}$$

Tangential Stress

Shear Stress (τ)

- Stress induced when two equal & opposite forces (cables) are tangential across resisting section

$$P \leftarrow \boxed{\quad} \rightarrow P$$

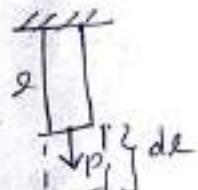
$$\tau = \frac{\text{Shear resistance}}{\text{Shear area}}$$

$$\tau = \frac{P}{A}$$

Strain (e)

Definition? Ratio of change in length to original length

$$\text{Strain} = \frac{\Delta L}{L} = \frac{\text{Change in length}}{\text{Original length}}$$



Strain

Tensile Strain

Ratio of increase in length of body to original length.

$$\text{Tensile Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

$$e = \frac{\Delta L}{L}$$

$$P \leftarrow \boxed{\quad} \rightarrow P$$

compressive strain

Ratio of decrease in length to original length

$$\text{Compressive Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

$$e = -\frac{\Delta L}{L}$$

$$P \rightarrow \boxed{\quad} \leftarrow P$$

Shear Strain (γ)

$\gamma = \frac{\text{Transverse displacement}}{\text{Displacement from lower surface}}$

$$\gamma = \frac{\Delta d}{L}$$

Lateral Strain

$$\gamma = \frac{\Delta d}{d}$$

$$\gamma = \frac{\text{change in dia}}{\text{Original dia}}$$

Volumetric Strain

$$\gamma = \frac{\Delta V}{V}$$

$$\gamma = \frac{\text{Change in } V}{\text{Original } V}$$

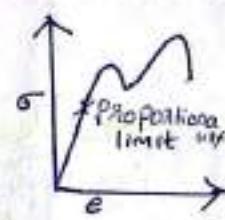
(Refer Simple problems
Muniruzzaman)

Hooke's Law

(2)

- Hooke's Law for Linear Isotropic Material (what's Young's Modulus (E))

Statement { within elastic limit Stress is proportional to strain
 $\frac{\sigma}{e} = \frac{\text{Stress}}{\text{Strain}} = \text{constant} = E$, where E = Young's Modulus (Modulus of elasticity)



- Hooke's Law for Shear deformation (what's Modulus of Rigidity (G) = ?)

{ up-to elastic limit Shear Stress proportional to Shear Strain

$$\frac{\tau}{\gamma} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \text{constant} = G$$

FOS

$$\text{Factor of Safety (FOS)} = \frac{\text{Ultimate Stress}}{\text{Permissible Stress}} \quad (\text{FOS} \geq 1)$$

Concrete	3
Steel	1.85
Timber	1-6

- ① A brass rod of 25mm dia and 1.8m long subjected to axial pull of 4KN. Find the stress, strain & elongation of the bar. $E = 1 \times 10^5 \text{ N/mm}^2$

Solution

$$P = 4000 \text{ N}, d = 25 \text{ mm}$$

$$E = 1 \times 10^5 \text{ N/mm}^2, l = 1.8 \text{ m} = 1800 \text{ mm}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{4000}{\frac{\pi}{4} \times 25^2} = 8.15 \text{ N/mm}^2$$

$$E = \frac{\sigma}{\epsilon} \Rightarrow \epsilon = \frac{\sigma}{E} = \frac{8.15}{1 \times 10^5} = 8.2 \times 10^{-6}$$

$$\epsilon = \frac{\Delta l}{l} \Rightarrow \Delta l = \epsilon l = 8.2 \times 10^{-6} \times 1800 = 0.106 \text{ mm}$$

Deformation of body due to force acting on it

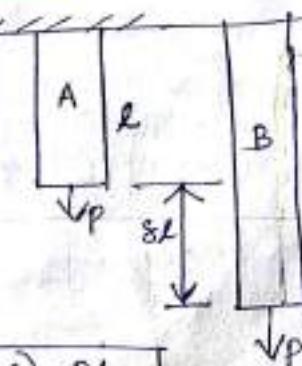
- A bar (②) deflected from length l to $l + \Delta l$ due to P .

$$\sigma = \frac{P}{A}, \quad F = \frac{\sigma}{\epsilon}$$

$$\epsilon = \frac{\sigma}{E} = \frac{P/A}{E} = \frac{P}{AE} \quad \text{--- (1)}$$

$$\text{we know } \epsilon = \frac{\Delta l}{l} \quad (\text{sub: eq-1})$$

$$\frac{\Delta l}{l} = \frac{P}{AE} \Rightarrow \boxed{\text{Deflection } (\Delta l) = \frac{PL}{AE}}$$



Ques ①

- Two plates of thickness 2mm each are joined using single rivet plate are subjected to tensile load of 314 N. If the material of rivet is having allowable shear strength of 100 mpa. Determine dia of rivet pin?

Solution

$$P = 314 \text{ N}$$

$$\sigma = 100 \text{ mpa} = 100 \text{ N/mm}^2$$

$$\sigma = \frac{P}{\frac{\pi}{4} d^2} \quad 100 = \frac{314}{\frac{\pi}{4} d^2} \Rightarrow d = 0.5 \text{ mm}$$



STRESS-STRAIN DIAGRAM

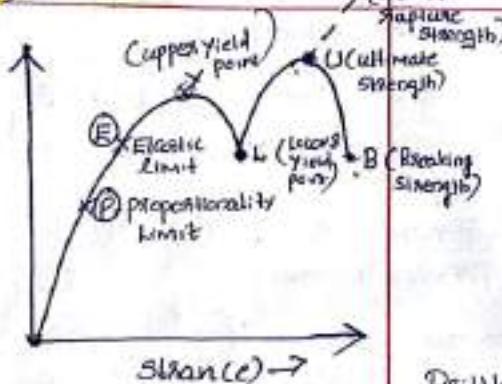
A curve plotted up to the point at which the specimen fails. Hence curve known as Stress-Strain Curve.
elast. apperably before failure

Stress-Strain Curve for ductile Material

(Or Elasto Stress-Strain diagram of Mild Steel)

- P- Proportionality Limit

Obey Hooke's Law
upto P known
Limit of proportionality



- Elastic limit (E)

upto E load remove
It return its original shape.

- Y-Upper Yield point

plastic state permanent deformation upto Y
Corresponding stress is upper yield stress

- L-Lower Yield point

Load decrease with increase in strain upto L
Corresponding stress is lower yield stress

- U-Ultimate Strength

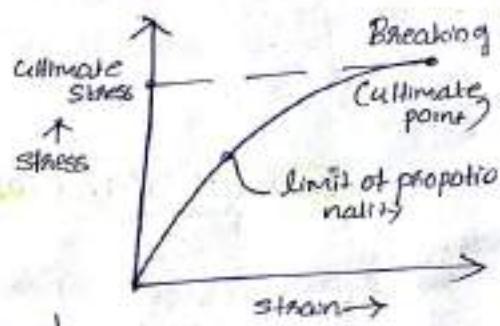
From L Stress increases & reaches Max value
Corresponding stress called ultimate tensile stress

- B-Breaking point

Stress decrease, strain increase upto B &
Fracture occurs. Corresponding stress
called breaking stress

Stress-Strain Curve for Brittle Material

(Draw Stress-Strain dia
gram of Cast iron, high grade steel, concrete)



Brittle Material

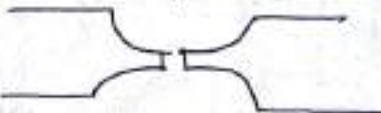
- fails suddenly without noticeable deformation too.
- No yield point.
- when strain increase stress increase and fail at Breaking point without any noticeable deformation.

- Necking - cup & cone failure

(Reduce dia of specimen due to layer yielding)

- Brittle Failure

Increase brittleness cup & cone disappears



- Isootropic Material

Same property in every direction

- Ostroisotropic Material

Different property in different orthogonal axis

- Anisotropic Material

Property change along different direction.

- ① In a tension test of Mildsteel Specimen 10mm dia & 250mm long, gauge length, with following observations made.

Elongation under 16kN Load = 0.2mm Length b/w gauge mark
 Load at Yield point = 27kN after fracture = 290mm
 Ultimate load = 51kN Dia of Neck = 7.5mm
 Breaking load = 36kN

- Calculate (a) Nominal Yield stress (d) Young's Modulus
 (b) Nominal ultimate stress (e) % elongation
 (c) Nominal Breaking stress (f) % reduction area

Solution

$$(a) \text{Nominal Yield Stress} = \frac{\text{Load at Yield point}}{\text{Area}} = \frac{27 \times 10^3}{\frac{\pi}{4} \times 10^2} = 343.7 \text{ N/mm}^2$$

$$(b) \text{Nominal ultimate Stress} = \frac{\text{Ultimate Load}}{\text{Area}} = \frac{51 \times 10^3}{\frac{\pi}{4} \times 10^2} = 649.35 \text{ N/mm}^2$$

$$(c) \text{Nominal Breaking Stress} = \frac{\text{Breaking Load}}{\text{Area}} = \frac{36 \times 10^3}{\frac{\pi}{4} \times 10^2} = 458.37 \text{ N/mm}^2$$

$$(d) \text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}} = \frac{P/L}{S/L} = \frac{P \cdot L}{A \cdot S \cdot L} = \frac{16 \times 10^3 \times 250}{\frac{\pi}{4} \times 10^2 \times 0.2} = 2.46 \times 10^5 \text{ N/mm}^2$$

$$(e) \% \text{ elongation} = \frac{l - l_0}{l_0} \times 100 = \frac{290 - 250}{250} \times 100 = 16\%$$

$$(f) \% \text{ reduction area} = \frac{A_0 - A}{A_0} = \frac{\frac{\pi}{4} \times 10^2 - \frac{\pi}{4} \times 7.5^2}{\frac{\pi}{4} \times 10^2} \times 100 = 43.57\%$$

reproblem

Note } If external dia $D = 220\text{mm}$ FOS = 6
 } (Us) ultimate stress = 540 N/mm^2 Working stress ($\sigma_{100\%K}$) = ? , $d = ?$

$$\text{FOS} = \frac{0.8}{\sigma_{100\%K}} \quad \sigma_{100\%K} = \frac{0.8}{\text{FOS}} = \frac{540}{6} = 90 \text{ N/mm}^2 \quad \sigma_{100\%K} = \frac{P/A}{\frac{\pi}{4}(220 - d)^2} = 90$$

$$d = 200\text{mm}$$

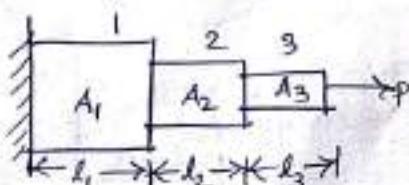
Principle of Superposition

Deflection } Total change in length will be obtained by adding the change in
 } length of individual sections.

$$\Delta l_1 = \frac{P_1 L_1}{A_1 E_1}, \quad \Delta l_2 = \frac{P_2 L_2}{A_2 E_2}, \quad \Delta l_3 = \frac{P_3 L_3}{A_3 E_3}$$

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

$$\boxed{\Delta l = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}}$$



Case-I (Problems)

- ① A straight bar 450mm long is 40mm dia for first 250mm length and 20mm dia for remaining length. If bar is subjected to an axial pull of 15KN. Find Max & Min Stresses and total extension. $E = 2 \times 10^5 \text{ N/mm}^2$

Ans

Solution

If load
Tensile/compressive
Same as

$$\text{Max stress} \Rightarrow \sigma_2 = \frac{15 \times 10^3}{\frac{\pi}{4} 20^2} \left(\frac{P}{A_2} \right) = \frac{15 \times 10^3}{\frac{\pi}{4} 20^2} \frac{15 \text{ KN}}{11.94 \text{ mm}^2} = 17.78 \text{ N/mm}^2$$

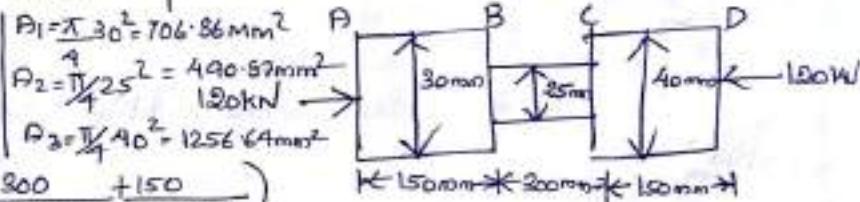
$$\text{Min stress} \Rightarrow \sigma_1 = \frac{15 \times 10^3}{\frac{\pi}{4} 40^2} \left(\frac{P}{A_1} \right) = \frac{15 \times 10^3}{\frac{\pi}{4} 40^2} \frac{15 \text{ KN}}{502.65 \text{ mm}^2} = 11.94 \text{ N/mm}^2$$

$$\text{Total extension } \Delta l = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} = \frac{P}{E} \left(\frac{l_1}{A_1} + \frac{l_2}{A_2} \right) = \frac{15 \times 10^3}{2 \times 10^5} \left(\frac{250}{\frac{\pi}{4} 40^2} + \frac{200}{\frac{\pi}{4} 20^2} \right) = 0.123 \text{ mm}$$

- ② A bar ABCD of steel 600mm long and two end AB & CD are 30mm & 40mm dia and each 150mm length. Middle portion being 25mm dia. Determine total elongation when bar is subjected axial compressive load of 120KN. $E = 2.1 \times 10^5 \text{ N/mm}^2$

Solution

$$\begin{aligned} \Delta l &= \frac{P}{E} \left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right) \\ &= \frac{120 \times 10^3}{2.1 \times 10^5} \left(\frac{150}{706.86} + \frac{300}{490.87} + \frac{150}{1256.64} \right) \\ &= 0.54 \text{ mm} \quad (l = l - \Delta l) = 600 - 0.54 = 599.46 \text{ mm} \end{aligned}$$

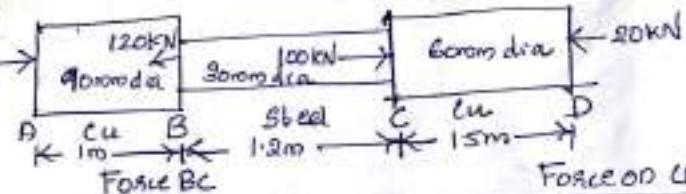


Case-2

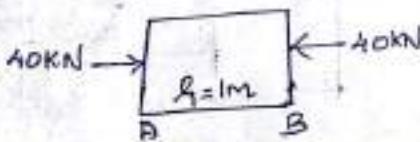
- ① A steel bar is fastened b/w to copper bar in fig. The assembly is subjected to loads at position o fig. Calculate total deformation of bar.

$$E_s = 200 \text{ GPa} \quad E_c = 110 \text{ GPa}$$

Solution



Force AB



$$P_1 = \text{Force on AB} = -40 \text{ KN (compressive)} \quad P_2 = \text{Force on BC} = 80 \text{ KN (tensile)} \quad P_3 = \text{Force on CD} = 20 \text{ KN}$$

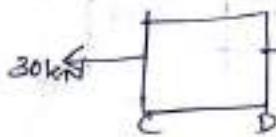
$$\begin{aligned} \Delta l &= \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3} \\ &= \frac{-40 \times 10^3 \times 1000}{\frac{\pi}{4} 90^2 \times 110 \times 10^3} + \frac{80 \times 10^3 \times 1.2 \times 10^3}{\frac{\pi}{4} 30^2 \times 200 \times 10^3} + \frac{(-20) \times 10^3 \times 1.5 \times 10^3}{\frac{\pi}{4} 60^2 \times 110 \times 10^3} \\ &= -0.571 + 0.679 - 0.096 \\ &= 0.025 \text{ mm} \end{aligned}$$

- ② A stepped bar is loaded as shown. Determine the total extension of the bar. If Young's Modulus is 200 GPa?

Solution

Divide fig \Rightarrow ABCD

Section CD
 $A_1 = 90 \text{ mm}^2, A_2 = 50 \text{ mm}^2, A_3 = 80 \text{ mm}^2$
 $l_1 = 150 \text{ mm}, l_2 = 200 \text{ mm}$
 $l_3 = 150 \text{ mm}$

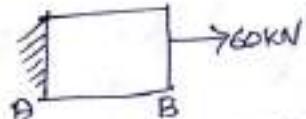


$$\text{Force on } CD = 30 \text{ kN (Tensile)}$$



$$\text{Force on } BC = 30 - 20 = 10 \text{ kN}$$

Section AB



$$\text{Force on } AB = 30 + 20 + 50 = 60 \text{ kN}$$

$$\delta = \frac{1}{E} \left(\frac{P_1 l_1}{A_1} + \frac{P_2 l_2}{A_2} + \frac{P_3 l_3}{A_3} \right) = \frac{1}{200 \times 10^9} \left(\frac{60 \times 10^3 \times 150}{90} + \frac{10 \times 10^3 \times 200}{50} + \frac{30 \times 10^3 \times 150}{80} \right) = 1 \text{ mm}$$

Deformation of a body due to Self weight



* Vertical bar length L and uniform c/s Area A

* 'w' be weight of material

* Slab PQR of length dx at distance z from free end
 $\text{Weight of slab} = \text{Density} \times g \times V = \rho g V$

$$\text{Tensile Stress of } PQ = \frac{\text{weight of } PQ}{A} = \frac{W/z}{A} = Wz = \frac{\rho g z}{A}$$

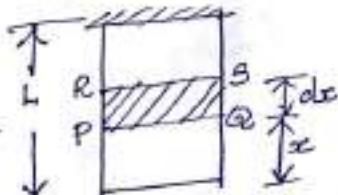
$$\text{Tensile Stress} = \frac{\rho g z}{E}, \quad \text{Elongation of slab} = \frac{\rho g z \cdot dz}{E}$$

$$\text{Total elongation } \delta L = \int_0^L \frac{\rho g z \cdot dz}{E} = \frac{\rho g}{E} \left(\frac{z^2}{2} \right)_0^L$$

$$= \frac{\rho g L^2}{2E} = \frac{WL^2}{2AE}$$

$$\boxed{\delta L = \frac{WL^2}{2AE}}$$

$$\begin{aligned} N &= \text{Total weight} \\ &= wAL \\ w &= \frac{N}{AL} \end{aligned}$$



$$\begin{cases} \text{where} \\ w = \rho g z \\ V = Az \end{cases}$$

Analysis of Composite Bars

Composite Bar \Rightarrow It composed of two or more different material
 \Rightarrow Joined together, and elongate single unit

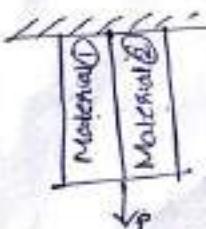
① Total elongation must be same

$$\delta L_{(1)} = \delta L_{(2)}$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2} \quad (1)$$

② Total Load = Load of Material (1) + Load of Material (2)

$$\boxed{P = P_1 + P_2} \quad (2)$$



- ① A Steel rod of 25mm dia place inside Cu tube and 30mm internal dia and 5mm thick end cap rigidly connected. This assembly subjected to compressive load of 250kN. Determine Stress induced in Steel & Cu tube.
 $E_S = 200 \text{ GPa}$, $E_{Cu} = 80 \text{ GPa}$

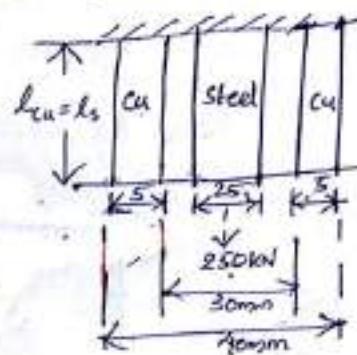
Solution

$$8l_S = 8l_{Cu}$$

$$\frac{P_S l_S}{A_S E_S} = \frac{P_{Cu} l_{Cu}}{A_{Cu} E_{Cu}} \Rightarrow$$

$$\Rightarrow \frac{P_S}{\frac{\pi}{4}(25 \times 10^3)^2 \times 200 \times 10^9} = \frac{P_{Cu}}{\frac{\pi}{4}((40 \times 10^3)^2 - (20 \times 10^3)^2) \times 80 \times 10^9}$$

$$\Rightarrow P_S = 2.2 P_{Cu} \quad \dots \textcircled{1} \quad P_S + P_{Cu} \Rightarrow 250 \times 10^3 = 2.2 P_{Cu} + P_{Cu}$$



$$P_{Cu} = \frac{250 \times 10^3}{3.2} = 77.4 \text{ kN}, P_S = 172.6 \text{ kN}$$

$$\text{Stress in Cu} = \frac{P_{Cu}}{A_{Cu}} = \frac{172.6 \times 10^3}{\frac{\pi}{4}(40^2 - 20^2)} = 140.86 \text{ N/mm}^2$$

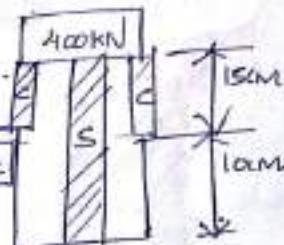
$$\text{Stress in Steel} = \frac{P_S}{A_S} = \frac{172.6 \times 10^3}{\frac{\pi}{4} \times 25^2} = 351.72 \text{ N/mm}^2$$

- ② A Steel rod and two Copper rods together support a load of 400kN as shown in fig. The C/S Area of Rod is 2000 mm^2 and each Cu Rod is 1500 mm^2 . Find stress in the rods. $E_S = 2 \times 10^5 \text{ N/mm}^2$, $E_C = 1 \times 10^5 \text{ N/mm}^2$

Solution

$$l_{Cu} = 15 \text{ cm}, l_S = 20 \text{ cm}, A_S = 2000 \text{ mm}^2, A_{Cu} = 1500 \text{ mm}^2$$

$$\frac{P_S l_S}{A_S E_S} = \frac{P_{Cu} l_{Cu}}{A_{Cu} E_C} \Rightarrow \frac{P_S \times 250}{2000 \times 2 \times 10^5} = \frac{P_{Cu} \times 150}{3000 \times 1 \times 10^5} \Rightarrow P_S = 0.8 P_{Cu}$$



$$P_S = P_{Cu} + P_C \Rightarrow 400 \times 10^3 = 0.8 P_C + P_C \Rightarrow P_C = 22222.2 \text{ N}$$

$$P_C = 177777.78 \text{ N}$$

$$\text{Stress in Steel} \sigma_S = \frac{P_S}{A_S} = \frac{177777.78}{2000} = 88.889 \text{ N/mm}^2$$

$$\text{Stress in Copper} \sigma_C = \frac{P_C}{A_C} = \frac{22222.22}{3000} = 7.4074 \text{ N/mm}^2$$

- ③ Three bars made of Cu, Zn, Al are equal length has C/S 500, 700 & 1000 mm². This compound member is subjected to longitudinal pull of 250kN. Estimate the proportion of the load carries on each bar & induced stress.
 $E_{Cu} = 1.8 \times 10^5 \text{ N/mm}^2$, $E_{Zn} = 1 \times 10^5 \text{ N/mm}^2$, $E_{Al} = 0.8 \times 10^5 \text{ N/mm}^2$

Solution

$$8l_{Cu} = 8l_{Zn} = 8l_{Al}$$

$$\frac{P_{Cu} l_{Cu}}{A_{Cu} E_{Cu}} = \frac{P_{Zn} l_{Zn}}{A_{Zn} E_{Zn}} = \frac{P_{Al} l_{Al}}{A_{Al} E_{Al}}$$

$$\frac{P_{Cu}}{500 \times 1.8 \times 10^5} = \frac{P_{Zn}}{700 \times 1 \times 10^5} = \frac{P_{Al}}{1000 \times 0.8 \times 10^5}$$

$$P_{Zn} = 1.154 P_{Cu}$$

$$P = P_{Cu} + P_{Zn} + P_{Al}$$

$$P_{Al} = 1.281 P_{Cu}$$

$$250 \times 10^3 = P_{Cu} + 1.154 P_{Cu} + 1.281 P_{Cu}$$

$$P_{Cu} = 73860.279 \text{ N}, P_{Zn} = 85234.762 \text{ N}, P_{Al} = 90922.003 \text{ N}$$

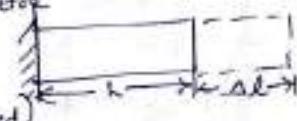


$$\frac{P_{Cu}}{E_{Cu}} = \frac{147.7 \text{ N/mm}^2}{E_{Cu}}, \sigma_{Zn} = \frac{P_{Zn}}{E_{Zn}} = 113.64 \text{ N/mm}^2, \sigma_{Al} = \frac{P_{Al}}{E_{Al}} = 90.92 \text{ N/mm}^2$$

Thermal Stress & Thermal Effect

(5)

- Consider a body allowed to increase in temperature result expansion. (No stress developed)
- When cooled result contract (No stress developed)
- If such body are fixed to support. No expansion prevented & develop stress. It is called thermal stress and corresponding strain is called thermal strain.



Case I Body heated up to $T^\circ\text{C}$ of length L extension, $\Delta L \propto T\ell$ d = Coefficient of thermal expansion

$$\frac{\Delta L}{L} = \alpha T = \text{Strain}$$

$$\frac{\sigma}{E} = \epsilon \Rightarrow \sigma_{\text{thermal}} = \epsilon E = \alpha ET$$

Case II Stress & strain when supports rigidly

$$\Delta L = \alpha TL - \delta, \text{ Strain} = \frac{\Delta L}{L} = \frac{\alpha TL - \delta}{L}$$

$$\sigma_{\text{thermal}} = \left(\frac{\alpha TL - \delta}{L} \right) E$$

- ① A steel rod of ~~2cm~~ diameter and ~~5m~~ long is connected to two grips and rod is maintained at temperature of ~~95^\circ\text{C}~~. Determine the strain & pull exerted when temperature fall to ~~30^\circ\text{C}~~. If (a) Ends do not yield $E = 2 \times 10^5 \text{ MN/m}^2$ (b) ends yield by 0.125mm $\alpha = 12 \times 10^{-6}/\text{C}$

Solution:

$$d = 20\text{mm}, L = 5000\text{mm}, \Delta T = 95 - 30 = 65^\circ\text{C}, E = 2 \times 10^5 \text{ N/mm}^2 \quad | A = \frac{\pi}{4} d^2 = 225\text{mm}^2$$

(a) $\sigma = \kappa TE = 12 \times 10^{-6} \times 65 \times 2 \times 10^5 = 156 \text{ N/mm}^2$

(b) $\delta = 0.12 \times 10\text{mm} \Rightarrow \sigma = \left(\frac{\alpha TL - \delta}{L} \right) E = \frac{(12 \times 10^{-6} \times 65 \times 5000 - 1.2) \times 2 \times 10^5}{5000} = 108 \text{ N/mm}^2 \Rightarrow \text{pulling load} = \sigma \times A = 108 \times 225 = 76340.7\text{N}$

Thermal Stresses in Composite Bars

$$\sigma_1, \sigma_2 = \text{Stresses}$$

$$\epsilon_1, \epsilon_2 = \text{Strains}$$

$$\alpha_1, \alpha_2 = \text{Coefficient of expansion}$$

Tensile Load of Material 1 = compressive load of Material 2

$$\sigma_1 F_1 = \sigma_2 F_2$$

$$\text{Eq: } \alpha_1 T l_1 + \frac{\sigma_1}{E_1} l_1 = \alpha_2 T l_2 + \frac{\sigma_2}{E_2} l_2$$

In question which material having α value higher put +ve sign to $\frac{\sigma}{E}$

Here σ_2 having high α $\Rightarrow \alpha_1 T l_1 + \frac{\sigma_1}{E_1} l_1 = \alpha_2 T l_2 - \frac{\sigma_2}{E_2} l_2$

If σ_1 having high α $\Rightarrow \alpha_1 T l_1 - \frac{\sigma_1}{E_1} l_1 = \alpha_2 T l_2 + \frac{\sigma_2}{E_2} l_2$

- (3) A brass bar 20mm dia is enclosed in steel tube. Both bars having same length & fastened rigidly at their ends. The composite bar is free of stress at 20°C. What temperature assembly must be heated to generate compressive stress of 48MPa in brass bar? Determine stress in steel tube. $E_s = 200 \text{ GPa}$, $E_b = 84 \text{ GPa}$, $\alpha_s = 12 \times 10^{-6}/\text{C}$, $\alpha_b = 18 \times 10^{-6}/\text{C}$

Solution

$$d_b = 20 \text{ mm}, d_s = 25 \text{ mm}, D_s = 50 \text{ mm}$$

$$\sigma_b A_s = \sigma_b A_b$$

$$\frac{\sigma_s \times \pi}{4} (50^2 - 25^2) = \sigma_b \times \pi \frac{20^2}{4} \Rightarrow \sigma_s = 0.213 \sigma_b$$

Here α_b is high
 α_b put negative
 $\frac{\sigma_b}{E} \propto \alpha_b$ term

$$\alpha_s T l_s + \frac{\sigma_s}{E_s} l_s = \alpha_b T l_b - \frac{\sigma_b}{E_b} l_b \quad (l = l_s = l_b)$$

$$\frac{12 \times 10^{-6} \times T + 0.213 \sigma_b}{200 \times 10^3} = \frac{18 \times 10^{-6} \times T - \sigma_b}{84 \times 10^3}$$

$$\text{Given } \sigma_b = 48 \text{ MPa} \Rightarrow \frac{12 \times 10^{-6} \times T + 0.213 \times 48}{200 \times 10^3} = \frac{18 \times 10^{-6} \times T - 48}{84 \times 10^3}$$

$$\Rightarrow 12 \times 10^{-6} T + 5.112 \times 10^{-5} = 18 \times 10^{-6} T - 5.714 \times 10^{-4}$$

$$\Rightarrow 6 \times 10^{-6} T = 6.2252 \times 10^{-4} \Rightarrow T = 103.75^\circ\text{C}$$

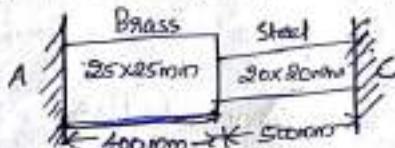
$$(\text{Increase in temp}) = 20 + \text{Temp of assembly} \Rightarrow 103.75 - 20 + \text{Temp of assembly}$$

$$= 123.75^\circ\text{C}$$

$$\sigma_b = 0.213 \times 48 = 10.224 \text{ N/mm}^2$$

- (4) A bar made of brass & steel as shown in figure is held b/w two rigid supports A and C. Find the stresses in each material if the temperature raised by 40°C. $E_b = 1 \times 10^5 \text{ N/mm}^2$, $\alpha_b = 19 \times 10^{-6}/\text{C}$, $E_s = 2 \times 10^5 \text{ N/mm}^2$, $\alpha_s = 12 \times 10^{-6}/\text{C}$

Care



Solution

$$T = 40^\circ\text{C}$$

$$\sigma_b A_b = \sigma_s A_s$$

$$\sigma_b \times 25 \times 25 = \sigma_s \times 20 \times 20 \Rightarrow \sigma_b = 0.64 \sigma_s$$

$$\delta = \alpha_s T l_s + \alpha_b T l_b$$

$$= 12 \times 10^{-6} \times 40 \times 500 + 19 \times 10^{-6} \times 40 \times 400 = 0.544 \text{ mm}$$

$$\delta l = \frac{\sigma_s l_s + \sigma_b l_b}{E_s} \Rightarrow 0.544 = \frac{\sigma_s \times 500}{2 \times 10^5} + \frac{0.64 \sigma_s \times 400}{1 \times 10^5}$$

$$\Rightarrow 0.544 = \frac{2.5 \times 10^{-5}}{2.5 \times 10^3} \Rightarrow \sigma_s = 10.57 \text{ N/mm}^2, \sigma_b = 68.81 \text{ N/mm}^2$$

- (5) A copper strip $20 \times 2.5 \text{ mm}^2$ is section held b/w two strips of steel each $20 \times 2.5 \text{ mm}^2$ in section. Find stresses in steel & copper due to temp rise 6°C. $\alpha_s = 1.2 \times 10^{-5}/\text{C}$, $\alpha_c = 1.85 \times 10^{-5}/\text{C}$, $E_s = 2 \times 10^5 \text{ N/mm}^2$, $E_c = 1.2 \times 10^5 \text{ N/mm}^2$

Solution

$$\sigma_{cu} A_{cu} = \sigma_s A_s$$

$$\sigma_{cu} \times 26 \times 2.5 = \sigma_s \times 2 \times 26 \times 2.5$$

$$\sigma_{cu} = 26 \sigma_s$$

$$\alpha_s T l_s + \frac{\sigma_s}{E_s} l_s = \alpha_c T l_c - \frac{\sigma_{cu} \times l_{cu}}{E_c}$$

$$1.2 \times 10^{-5} \times 6 + \frac{\sigma_s}{2 \times 10^5} = 1.85 \times 10^{-5} \times 6 - \frac{26 \sigma_s}{1.2 \times 10^5}$$

$$2.167 \sigma_s = 0.65 \times 10^{-5} \times 6 \quad \sigma_s = 1.79 \text{ N/mm}^2 \quad \sigma_b = 3.60 \text{ N/mm}^2$$

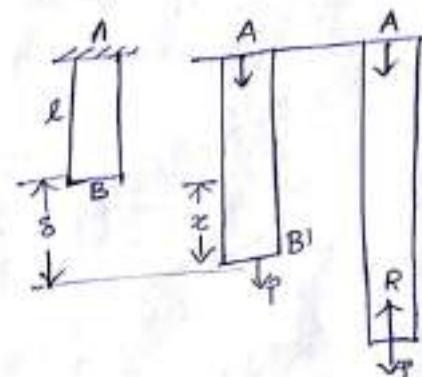
Strain Energy (U)

- Energy absorbed by body due to stretching effect called strain energy.
- Resilience = Total energy stored in body.
- Proof of Resilience → The maximum strain energy stored in body up to elastic limit
- Modulus of Resilience = $\frac{\text{Proof of Resilience}}{\text{Volume of body}}$

Derivation of strain energy

$$\text{Average resistance} = \frac{P}{2} + \frac{R}{2}$$

$$\begin{aligned} \text{Strain energy } U &= \frac{P \times S}{2} \\ &\quad \text{Resilience distance} \\ U &= \frac{\sigma A \times x \times l}{2} = \frac{\sigma A \times \frac{c}{2} l}{2} = \frac{l^2 \sigma A l}{2E} \\ U &= \frac{\sigma^2 V}{2E} \quad |V = Al| \end{aligned}$$



- Strain energy density (u) = $\frac{\text{Strain energy}}{\text{Volume}} = \frac{U}{V} = \int_0^{e_i} \sigma_x d\epsilon_x$
- Strain energy in body when load applied gradually $\Rightarrow U = \frac{1}{2E} \sigma^2 \times \text{Volume}$
- Strain energy stored in body when load is applied suddenly
- Strain energy of body due to impact loading $\sigma = \frac{P}{A} \left[1 \pm \sqrt{1 + 2AEh} \right]$
- Impact by shock $\frac{\sigma^2}{2E} \times Al = \frac{1}{2} mv^2$

$$\sigma_i = \frac{2 \times P}{A} \quad \sigma_i = \text{Instantaneous Stress}$$

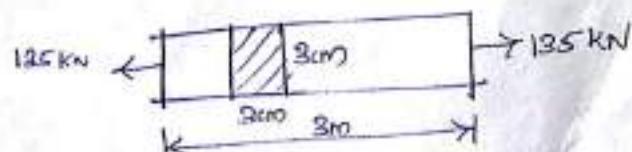
- ① A steel has 3cm x 3cm section 3m long is subjected to an axial pull of 195kN. Calculate the elongation in the length of the bar, energy stored in the bar. $E = 200 \text{ GIN/m}^2$

Solution

$$A = 3 \times 3 = 9 \text{ cm}^2 = 9 \times 10^{-4} \text{ m}^2$$

$$P = 195 \times 10^3 \text{ N}, \quad l = 3 \text{ m}$$

$$E = 200 \times 10^9 \text{ N/m}^2$$



$$\Delta l = \frac{Pl}{PE} = \frac{195 \times 1000 \times 3}{9 \times 10^4 \times 200 \times 10^9} = 0.0022 \text{ m}$$

$$\sigma = \frac{P}{A} = \frac{195 \times 1000}{9 \times 10^4} = 15 \times 10^6 \text{ N/m}^2$$

→ Strain energy $(U = \frac{\sigma^2}{2E} \times V)$

$$\begin{aligned} &= (15 \times 10^6)^2 \times 9 \times 10^{-4} \times 3 \\ &= 1.41 \times 10^9 \text{ J} \end{aligned}$$

Strain Energy

$$U = \frac{\sigma^2}{2E} \times \text{Volume}$$

Modulus of Resilience

$$\overline{U} = \frac{U}{V} = \frac{\sigma^2}{2E}$$

- ① A steel bar of 15mm dia cracks by load of 10KN. If the bar is 850mm long.
 Calculate (a) Strain energy (b) Strain energy / unit volume

$$F = 200 \text{ GPa}$$

Solution

$$P = 10 \text{ kN}, d = 15 \text{ mm}, P = 10 \text{ kN}$$

$$(a) U = \frac{\sigma^2}{2E} V$$

$$= \frac{56.58^2}{2 \times 200 \times 10^3} \times \frac{\pi}{4} \times 15^2 \times 850 = \underline{\underline{44156.25 \text{ mm}^3}}$$

$$= \underline{\underline{353.393 \text{ Nmm}}}$$

$$(b) \overline{U} = \frac{\sigma^2}{2E} = \frac{56.58^2}{2 \times 200 \times 10^3} = \underline{\underline{0.008 \text{ N/mm}^2}}$$

Strain Energy of a body when Load is applied Suddenly

From Figure Strain Energy $\Rightarrow U = P\Delta L$

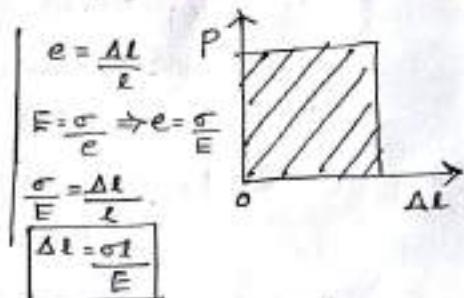
$$U = P \left(\frac{\sigma t}{E} \right) \quad \text{--- (1)}$$

Strain Energy of gradually applied load

$$U = \frac{\sigma^2}{2E} V \quad \text{--- (2)}$$

$$\therefore \text{Equating eq (1) & (2)} \quad \frac{\sigma^2 V}{2E} = P \frac{\sigma t}{E}$$

$$\Rightarrow \frac{\sigma (At)}{2} = Pt \Rightarrow \boxed{\sigma = \frac{2P}{A}}$$



Max: Stress induced due to suddenly applied load = 2x Stress induced in gradually applied load

- ② Calculate the instantaneous Stress produced in a bar 10cm^2 in area and 3m long by the sudden application of tensile load of unknown magnitude. If the extension of bar due to suddenly applied load is 1.5mm . Assume $E = 2 \times 10^5 \text{N/mm}^2$. Also determine suddenly applied load?

Solution

$$G.i.D \Rightarrow A = 10\text{cm}^2 = 10 \times (10\text{mm})^2 = 10 \times 100\text{mm}^2 = 1000\text{mm}^2$$

$$l = 3\text{m} = 3 \times 10^3 \text{mm}$$

$$\Delta l = 1.5\text{mm}$$

$$E = 2 \times 10^5 \text{N/mm}^2$$

$$\frac{\Delta l}{A E} \Rightarrow \Delta l = \frac{\sigma l}{E} \Rightarrow 1.5 = \frac{\sigma \times 3 \times 10^3}{2 \times 10^5} \Rightarrow \sigma = 100 \text{N/mm}^2$$

$$\text{Stress } \sigma = \frac{P}{A} \Rightarrow P = \sigma A = \frac{100 \times 1000}{2} = 50000 \text{N}$$

- ③ A tension bar made up of two part. Length of first part is 900cm and area of 20cm^2 , while second part is of length 200cm , & area 30cm^2 . Axial load of 90kN is gradually applied.

- (a) Find total strain energy produced in the bar.
 (b) Compare this value with that obtained in uniform bar of same length and same volume under same load. $E = 2 \times 10^5 \text{N/mm}^2$

Solution

$$l_1 = 900\text{cm} = 9000\text{mm}, A_1 = 20\text{cm}^2 = 2000\text{mm}^2$$

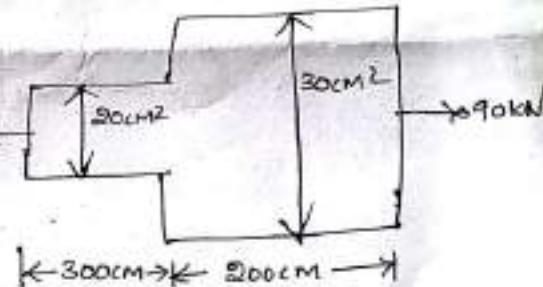
$$V_1 = A_1 l_1 = 2000 \times 9000 = 6 \times 10^6 \text{mm}^3$$

$$l_2 = 200\text{cm} = 2000\text{mm}, A_2 = 30\text{cm}^2 = 3000\text{mm}^2$$

$$V_2 = A_2 l_2 = 3000 \times 2000 = 6 \times 10^6 \text{mm}^3$$

$$P = 90\text{kN} = 90 \times 10^3 \text{N}$$

$$E = 2 \times 10^5 \text{N/mm}^2$$



Part-I

$$\sigma_1 = \frac{P}{A_1} = \frac{90 \times 10^3}{2000} = 45 \text{N/mm}^2$$

$$U_1 = \frac{\sigma_1^2}{2E} V_1 = \frac{45^2}{2 \times 2 \times 10^5} 6 \times 10^6 = 30375 \text{Nm}$$

Part-II

$$\sigma_2 = \frac{P}{A_2} = \frac{90 \times 10^3}{3000} = 30 \text{N/mm}^2$$

$$U_2 = \frac{\sigma_2^2}{2E} V_2 = \frac{30^2}{2 \times 2 \times 10^5} 6 \times 10^6 = 13500 \text{Nm}$$

$$\text{Total strain energy } U = U_1 + U_2 = 43875 \text{Nm}$$

Strain energy of given bar

$$\begin{aligned} \text{Strain energy in uniform bar} \\ = \frac{43875}{42187} &= 1.03 \end{aligned}$$

Strain Energy stored in uniform bar

$$V = V_1 + V_2 = 6 \times 10^6 + 6 \times 10^6 = 12 \times 10^6 \text{mm}^3, L = l_1 + l_2 = 9000 + 2000 = 5000 \text{mm}$$

$$V = A \cdot l \Rightarrow A = \frac{V}{l} = \frac{12 \times 10^6}{5000} = 2400 \text{mm}^2 \Rightarrow U = \frac{\sigma^2}{2E} \times V = \frac{37.5^2}{2 \times 2 \times 10^5} \times 12 \times 10^6 = 42187 \text{Nm}$$

$$\begin{aligned} \sigma &= \frac{P}{A} \\ &= \frac{90 \times 10^3}{2400} \\ &= 37.5 \text{N/mm}^2 \end{aligned}$$

Statically indeterminate problem in Tension & compression

- Some structural system: eq of equilibrium insufficient for find reaction
- Cannot determine internal forces using equilibrium. Such system/problem called Statically indeterminate problems

- ① Consider a bar AB Supported at both ends by fixed support, with an axial force at 12kN applied at C as shown. Find reaction at the ends.

Solution

$$\text{Reaction at A \& B} \Rightarrow \sum F_x = 0$$

$$\Rightarrow R_A + R_B - 12 = 0$$

$$R_A + R_B = 12 \quad ? \text{ Here } 1 \text{ eqn } \& \text{ Single eqn}$$

Given problem is Statically indeterminate

A \& B fixed

$$S_{AB} = 0 \Rightarrow S_{AC} + S_{CB} = 0 \Rightarrow -\frac{R_A \times 500}{AE} + \frac{R_B \times 100}{AE} = 0$$

$$100 R_B = 500 R_A \Rightarrow 400 R_B = 500 (12 - R_B)$$

$$R_B = 6.667 \text{ N}, R_A = 5.333 \text{ N}$$

★ (Refer More problem look Moon Note)

