

MODULE -6

VI	Mohr's circles of stress – plane state of strain – analogy between stress and strain transformation – strain rosettes	3	20%
	Compound stresses: Combined axial, flexural and shear loads – eccentric loading under tension/compression - combined bending and twisting loads.	4	
	Theory of columns: Buckling theory -Euler's formula for long columns – assumptions and limitations – effect of end conditions - slenderness ratio – Rankin's formula for intermediate columns.	3	

Mohr's Circle Module-6

Mohr's circle is graphical Method of finding normal (σ_n), tangential or shear (σ_t) and resultant stresses on oblique plane.

Mohr's circle drawn for following cases

Case I : A body subjected to two mutually perpendicular principal tensile stresses of unequal intensities.

Case II : A body subjected to two mutually perpendicular principal stresses which are unequal and unlike (one tensile and other compressive)

Case III : A body subjected to two mutually perpendicular tensile stresses accompanied by simple shear stress.

Case I : Mohr's circle when body is subjected to two mutually perpendicular principal tensile stresses of unequal intensities

- The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm^2 and 60 N/mm^2 . Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of the minor stress.

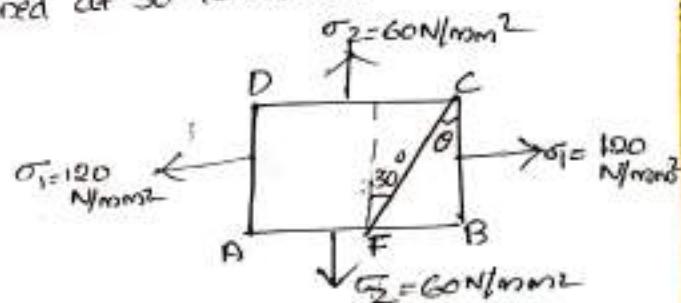
Solution

$$\begin{aligned}\sigma_1 &= 120 \text{ N/mm}^2 \\ \sigma_2 &= 60 \text{ N/mm}^2 \\ \theta &= 30^\circ\end{aligned}$$

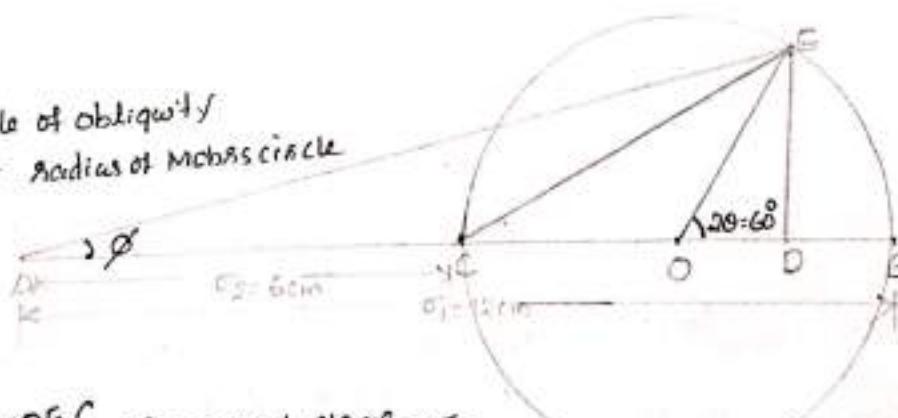
Scale $1 \text{ cm} : 10 \text{ N/mm}^2$

$$\sigma_1 = 12 \text{ cm}$$

$$\sigma_2 = 6 \text{ cm}$$



ϕ = Angle of obliquity
Max shear = radius of Mohr's circle



Consider $\triangle ADE$

$$\text{The normal stress} = \sigma_n$$

$$= \text{length } AD = 10.5 \text{ cm} = 10.5 \times 10 = 105 \text{ N/mm}^2$$

Tangential / shear stress

$$\sigma_t = \text{length of } ED = 2.6 \text{ cm} = 2.6 \times 10 = 26 \text{ N/mm}^2$$

$$\text{Length } AE = \text{Resultant Stress} = 10.8 \text{ cm} = 108 \text{ N/mm}^2$$

Step① : Draw $AB = \sigma_1 = 12 \text{ cm}$
Draw $AC = \sigma_2 = 6 \text{ cm}$

Step② : Locate centre of BC at O . Draw a circle OC or OB radius and centre O . That circle is called Mohr's circle.

Step③
Through O , draw line OE at angle $2\theta = 60^\circ$ with OB
From E draw $ED \perp$ to CB

Step④ Joint AE and EC

Case 2 : Mohr's Circle when a body is subjected to two mutually perpendicular principal stresses which are unequal and unlike (one tensile and other is compressive)

- (2) The stresses at a point in a bar are 200 N/mm^2 (tensile) and -100 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the maximum intensity of shear stress in the material at the point.

Solution

$$\sigma_1 = 200 \text{ N/mm}^2$$

$$\sigma_2 = -100 \text{ N/mm}^2$$

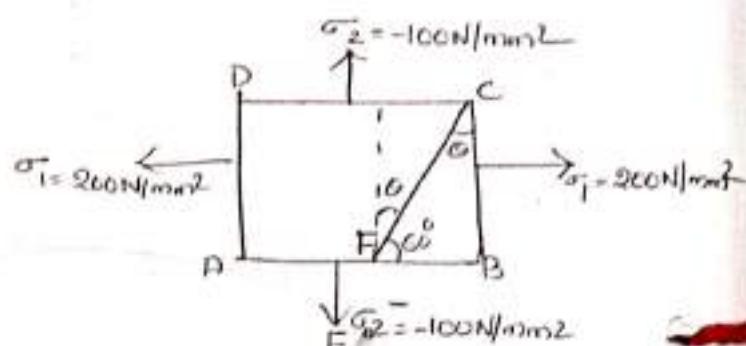
$$\theta = 90 - 60 = 30^\circ$$

Scale

$$1 \text{ cm} = 20 \text{ N/mm}^2$$

$$\sigma_1 = 10 \text{ cm}$$

$$\sigma_2 = -5 \text{ cm}$$



Procedure

Step ① → Draw AB = $\sigma_1 = 10 \text{ cm}$
(towards sign A)

Draw AC = $\sigma_2 = -5 \text{ cm}$
(towards left A)

Step ② → Locate centre of BC
at O. Draw circle OC = OB radius
and O centre AE. This circle called
Mohr's circle.

$$\begin{aligned} \text{Max Shear Stress} &= \text{Radius of Mohr's circle} \\ &= 15 \text{ cm} = 150 \text{ N/mm}^2 // \end{aligned}$$

Step 3: OE Make angle $2\theta = 60^\circ$ with OB. From E draw ED \perp OB. Join AE & CE

- Normal Stress = $\sigma_n = AD = 6.25 \text{ cm} = 6.25 \times 20 = 125 \text{ N/mm}^2$
- Tangential / Shear Stress = $\sigma_t = DE = 6.5 \text{ cm} = 6.5 \times 20 = 130 \text{ N/mm}^2$
- Resultant Stress = $AE = 9 \text{ cm} = 9 \times 20 = 180 \text{ N/mm}^2$
- $\phi = \text{angle of obliquity} = 46^\circ$

Mohr's circle when Body is subjected to two mutually perpendicular principal tensile stresses accompanied by simple shear stress?

- ③ A point in a strained material is subjected to stresses shown in oblique plane check answer analytically

Solution

$$\sigma_1 = 65 \text{ N/mm}^2$$

$$\sigma_2 = 25 \text{ N/mm}^2$$

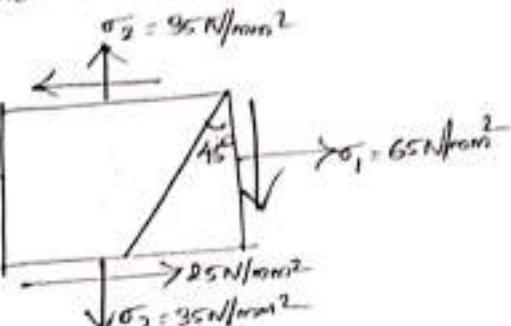
$$\theta = 45^\circ$$

$$T = 25 \text{ N/mm}^2$$

$$\sigma_1 = 65 \text{ N/mm}^2$$

$$\sigma_2 = 25 \text{ N/mm}^2$$

$$\sigma_2 = 35 \text{ N/mm}^2$$



Scale

$$1 \text{ cm} = 10 \text{ N/mm}^2$$

$$T = 2.5 \text{ cm}$$

$$\theta = 3.5 \text{ cm}$$

$$T = 2.5 \text{ cm}$$

Steps

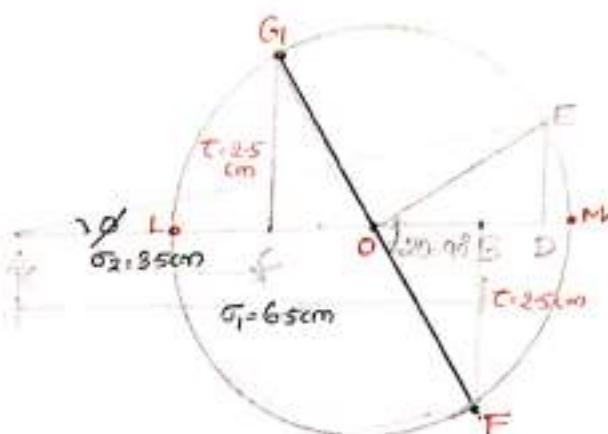
- Step ① • Draw AB = σ₁ = 6.5 cm

Draw AC = σ₂ = 2.5 cm

(towards right of A)

- Step ② • Draw ⊥ lines from BC
BF and CG equal to shear
stress T = 2.5 cm
- Bisect BC at O as centre
and radius = OF = OG. Draw
a circle it is called Mohr's circle

- Step ③ • Draw line OE making
an angle 90° with OF
from E draw ED ⊥ line
to AB produced
- Join AE



$$\text{Normal Stress} = \sigma_n = AD = 7.5 \text{ cm} = 75 \text{ N/mm}^2$$

$$\text{Tangential Stress} = \sigma_t = DE = 1.5 \text{ cm} = 15 \text{ N/mm}^2$$

$$\text{Resultant Stress} = AE = 7.5 \text{ cm} = 75 \text{ N/mm}^2$$

$$\phi = \text{angle of obliquity} = 120^\circ$$

$$\text{Max Shear Stress} = \text{Radius of Mohr's circle} = 2.8 \text{ cm} = 28 \text{ N/mm}^2$$

$$AL = \sigma_{n1} = \text{Major Principal Stress} = 9.1 \text{ cm} = 91 \text{ N/mm}^2$$

$$OM = \sigma_{n2} = \text{Major Principal Stress} = 7.8 \text{ cm} = 78 \text{ N/mm}^2$$

Homework

(Compose Yourself Analytically)

- ④ At a certain point in a stressed material, the intensities of stresses on two planes at right angles to each other are 20 N/mm^2 and 10 N/mm^2 both tensile. They are accompanied by shear stress of magnitude 10 N/mm^2 . Find graphically, location of principal planes and evaluate the principal stresses.

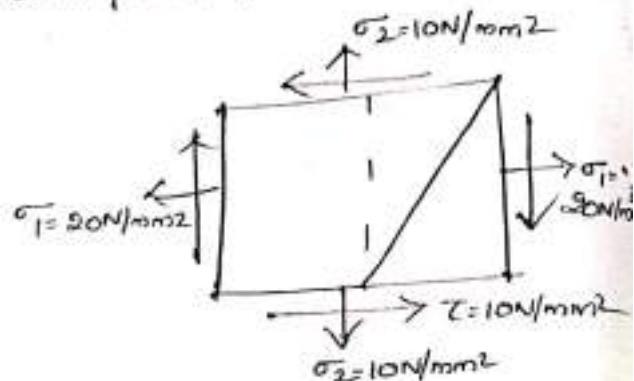
Solution

$$\theta = \text{Not given}$$

$$\sigma_1 = 20 \text{ N/mm}^2$$

$$\sigma_2 = 10 \text{ N/mm}^2$$

$$T = 10 \text{ N/mm}^2$$



Scale

$$1 \text{ cm} = 2 \text{ N/mm}^2$$

$$\sigma_1 = 10 \text{ cm}$$

$$\sigma_2 = 5 \text{ cm}$$

$$T = 5 \text{ cm}$$

Steps

Step ① : Draw $AB = 10 \text{ cm} (\sigma_1)$

Draw $AC = 5 \text{ cm} (\sigma_2)$

towards right of A

Step ② : Draw \perp circular from B and C (cd' off BF = BG)

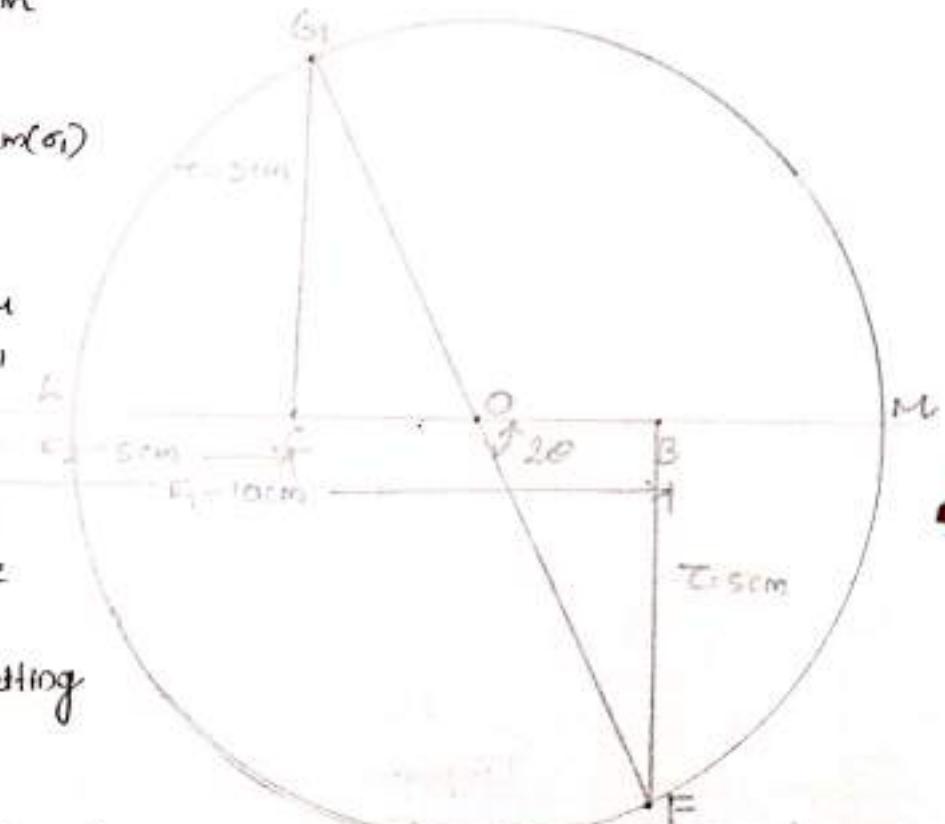
$$T = 5 \text{ cm}$$

- Bisect BC at O

- Draw circle O as centre

$OG_1 = OF$ radius. This circle called Mohr's circle

- Draw horizontal line cutting circle at point M



$$\text{Major principal stress} = \sigma_{1\perp} = 10 + 5 = 15 \text{ N/mm}^2$$

$$\text{Minor principal stress} = \sigma_{2\perp} = AM = 13 \text{ cm} = 13 \times 2 = 26 \text{ N/mm}^2$$

$$\text{Shear stress} = \sigma_{12} = \text{length AL} = 19 \text{ cm} = 19 \times 2 = 3.8 \text{ N/mm}^2$$

Location of principal plane

$$2\theta = 63^\circ$$

$$\theta = 31^\circ$$

$$\text{Second principal plane} = \theta + 90^\circ = 121^\circ$$

(3)

⑤ An elemental cube is subjected to tensile stress of 200 N/mm^2 and compressive of -150 N/mm^2 acting on two mutually perpendicular planes and shear stress of 80 N/mm^2 on these planes. Draw Mohrs circle of stresses and hence otherwise determine magnitudes and direction of principal stresses and also the greatest shear stress.

Solution

$$\sigma_1 = 200 \text{ N/mm}^2$$

$$\sigma_2 = -150 \text{ N/mm}^2$$

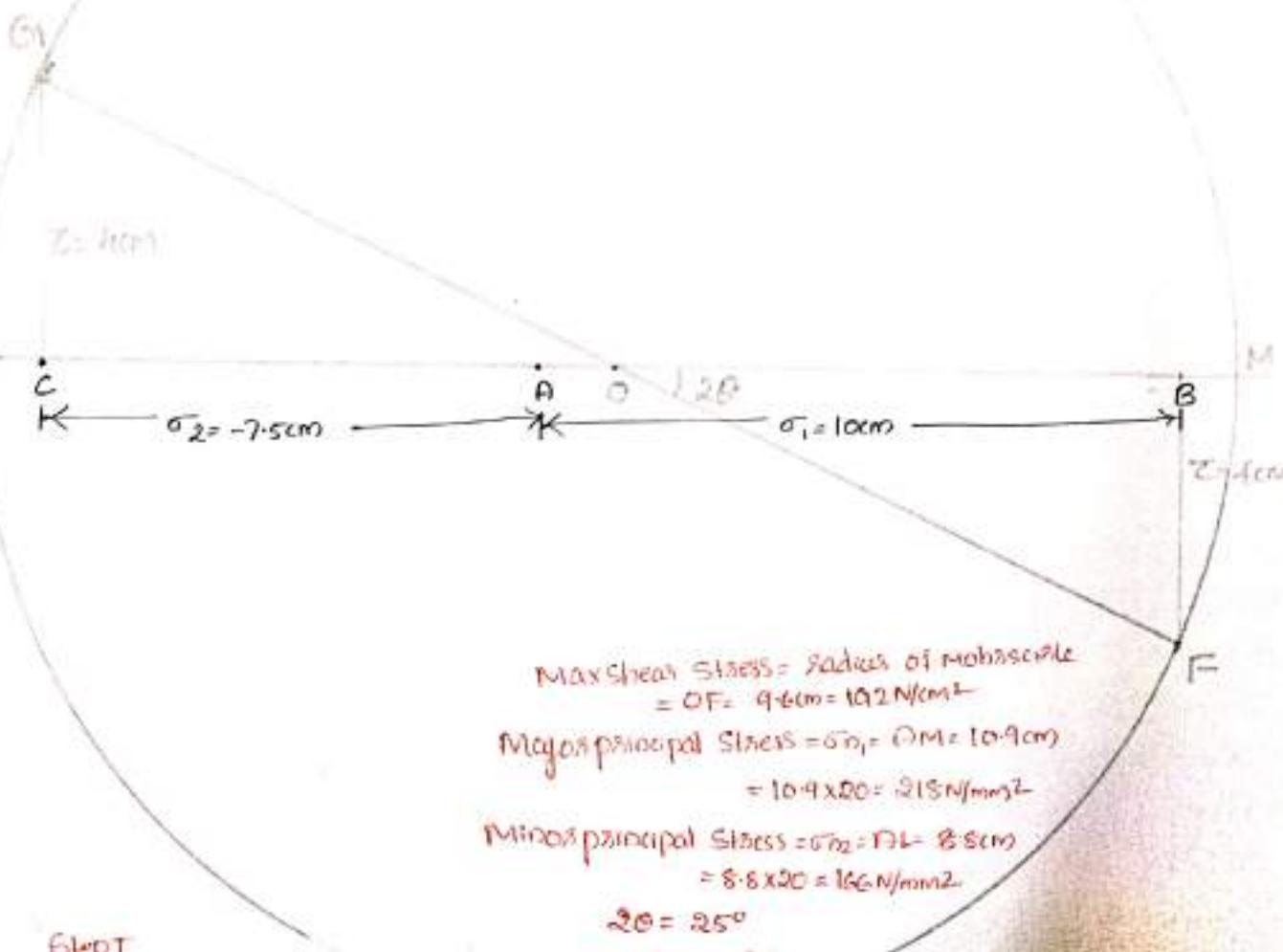
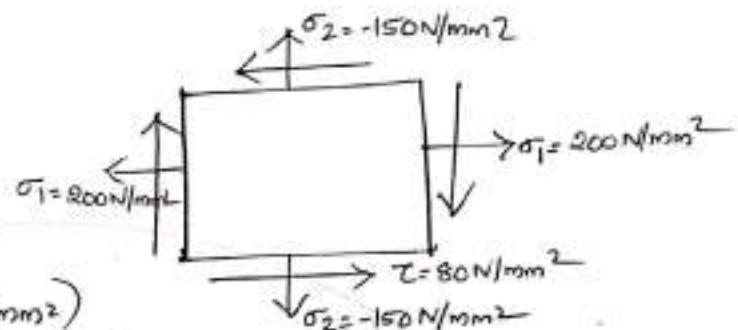
$$T = 80 \text{ N/mm}^2$$

Scale ($1\text{cm} = 20\text{N/mm}^2$)

$$\sigma_1 = 10\text{cm}$$

$$\sigma_2 = -7.5\text{cm}$$

$$T = 4\text{cm}$$



Step 1

- Draw $OB = \sigma_1 = 10\text{cm}$ (towards right)

- Draw $OC = -\sigma_2 = -7.5\text{cm}$ (towards left)

Step 2

- Draw Lateral $CGAOF$ from horizontal plane

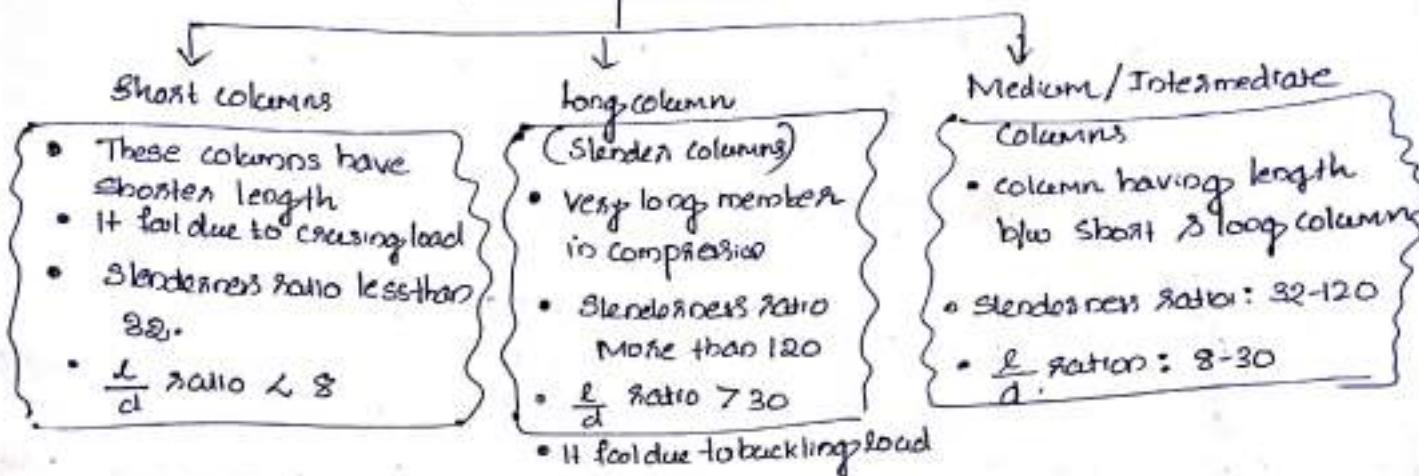
Step 3

- OF bisect at O-Draw arc circle O centre OG=OF radius
This circle called Mohrs circle

① COLUMNS

- Any structure (or) Machine member loaded in compression is called a column or stiel or pillar.
- Vertical Member in compression is called column while others are stiel.
Example: piston rod, connecting rod

Columns classification



Slenderness ratio of the columns

Ratio of the length of the column to the least radius of gyration.

of cross-sectional area.

$$\text{Slenderness ratio} = \frac{l}{k}$$

l = length of column

k = least (minimum radius) of gyration of cross sectional area

Short column

- Short column uniform c/s Area subjected to (compressive) axial load?
- compressive stress = $\frac{P}{A}$
- If compressive load increase column failure by crushing



Long column

l = length of column

P = compressive load

$$A = \text{c/s Area}$$

e = Maximum Bending at centre

$$G = \frac{P}{A} \quad (\text{stress due to } P)$$

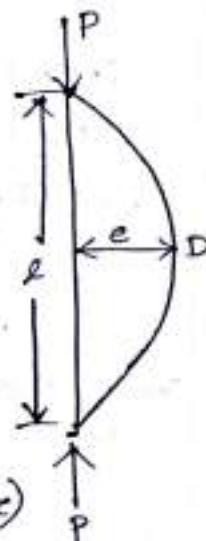
$$\sigma_b = \frac{Pe}{Z} \quad (\text{stress due to bending at centre})$$

Z = Section Modulus

$$\text{Max Stress} = G + \sigma_b$$

$$\text{Min Stress} = G - \sigma_b$$

It fails due to bending (buckling load)



(Compressive) axial load?

- compressive stress = $\frac{P}{A}$
- If compressive load increase column failure by crushing

$$\text{Crushing Stress} = \frac{P_c}{A}$$

P_c = crushing load

Buckling load / Crippling load

Buckling (critical) load is defined as minimum limiting load at which the column tends to buckle.

Equivalent length

Equivalent length for a column is the length which gives the same buckling load as given by both ends hinged column.

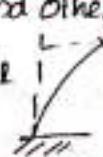
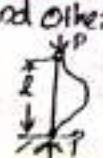
Buckling Factor = $\frac{\text{Equivalent length of column}}{\text{Min. radius of gyration}}$

Safe load = $\frac{\text{Buckling load}}{\text{F.O.S}}$ (F.O.S: Factor of Safety)

Euler Equation

Assumption

- ① Column initially perfectly straight and is axially loaded
- ② Cross sectional area of column is uniform
- ③ Column material is perfectly elastic, homogeneous and isotropic and obeys hook's law.
- ④ Length of column is very large compared to breadth and width/diameter.
- ⑤ The direct stress is very small compared to bending stress.
- ⑥ Column will fail by buckling alone.
- ⑦ Self weight of column is negligible.

S.NO	End conditions of column	Crippling load in terms of		Reaction between effective length and actual length
		Actual length	Effective length	
1	Both ends hinged 	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = l$
2	One end fixed and other is free 	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = 2l$
3	Both ends fixed 	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = \frac{l}{2}$
4	One end fixed and other is hinged 	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = \frac{l}{\sqrt{2}}$

$$\text{Slenderness ratio } \kappa = \frac{l}{k} \quad (2)$$

$$\sigma_{cr} = \frac{P_{cr}}{A} \quad \text{where } P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 EI}{Al^2} = \frac{\pi^2 EA k^2}{Al^2} \quad \left| \begin{array}{l} I = Ak^2 \\ A = \frac{\pi d^2}{4} \end{array} \right.$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} = \frac{\pi^2 E}{\text{Slenderness ratio}^2}$$

Limitation of Euler's Formula

we know $P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 EA k^2}{l^2}$

Critical stress $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$ where $\frac{l}{k}$ = Slenderness ratio

- For Mild Steel Column
- Direct Stress (Crushing Stress) for Mild Steel $F_d = 330 \text{ N/mm}^2$
 $E_{\text{Mild Steel}} = 2.1 \times 10^5 \text{ N/mm}^2$
- $$\sigma_{cr} = F_d \Rightarrow \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} = 330 \Rightarrow \frac{\pi^2 \times 2.1 \times 10^5}{\left(\frac{l}{k}\right)^2} = 330$$
- $$\frac{l}{k} = 79.25 \approx 80$$

Here slenderness ratio is less than 120. The column will not be considered as long column. Hence Euler's formula should not be used.

- ① A round steel rod of diameter 15mm and length 2m is subjected to a gradually increasing axial compressive load. Using Euler's formula. Find buckling load. Find also the maximum lateral deflection corresponding to the buckling condition. Both end of the rod may be taken as hinged.

$$E = 2 \times 10^5 \text{ N/mm}^2, \text{ Yield stress of Steel} = 250 \text{ N/mm}^2$$

Solution

$$d = 15 \text{ mm}$$

$$l = 2 \text{ m} = 2000 \text{ mm}$$

(l = l_e) $l_e = 2000 \text{ mm} \Rightarrow$ Both end hinged

$$P = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 2 \times 10^5 \times 2485.04}{2000^2}$$

$$= \underline{\underline{1287.64 \text{ N}}}$$

$$\left| I = \frac{\pi d^4}{64} = \frac{\pi \times 15^4}{64} \right.$$

$$\left. = 2485.05 \text{ mm}^4 \right.$$

$$\sigma_c = \frac{M_y}{I} = \frac{P \times d/2}{\frac{\pi d^4}{64}} \Rightarrow 250 = \frac{1287.64 \times 8 \times 2.5}{\frac{\pi \times 15^4}{64}}$$

$$S = 64.98 \text{ mm}$$

- ② The Euler stress of column whose end are hinged 40 MPa. Calculate the slenderness ratio of the column. $E = 2 \times 10^5 \text{ N/mm}^2$

Solution

Crippling stress

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 E A k^2}{l^2} = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

$$40 \text{ MPa} = \frac{\pi^2 \times 2 \times 10^5}{\left(\frac{l}{k}\right)^2}$$

$$\left(\frac{l}{k}\right)^2 = 19298$$

$$\text{Slenderness Ratio} = \frac{l}{k} = \underline{\underline{222.091}}$$

③ A Solid Rod has 3m long and 5cm in diameter is used as a strut with both end hinged. Determine Crippling Load. Take $E = 2 \times 10^5 \text{ N/mm}^2$

Solution Also determine Crippling load in other 3 cases

$$l = 3\text{m}, d = 5\text{cm} = 50\text{mm}, E = 2 \times 10^5 \\ = 2000\text{mm}$$

* Both end of has hinged Crippling Load $P_{cr} = \frac{\pi^2 EI}{l_e^2}$

$$I = \frac{\pi d^4}{64} \\ = \frac{\pi \times 50^4}{64} \\ = 30.68 \times 10^4 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2} = 67295\text{N}$$

* Crippling load when one end fixed and other free

$$P_{cr} = \frac{\pi^2 EI}{4l_e^2} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{4 \times 3000^2} = 16822\text{N}$$

OR

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} \text{ where } l_e = 2l \Rightarrow P_{cr} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{(2 \times 3000)^2} = 16822\text{N}$$

* Crippling load when both end are fixed

$$P_{cr} = \frac{4\pi^2 EI}{l_e^2} = \frac{4\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2} = 269.152\text{KN}$$

OR

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} \text{ where } l_e = \frac{l}{2} \Rightarrow P_{cr} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{\left(\frac{3000}{2}\right)^2} = 269.152\text{KN}$$

* Crippling load when one end fixed and other end hinged

$$P_{cr} = \frac{2\pi^2 EI}{l_e^2} = \frac{2\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2} = 134576\text{N}$$

OR

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} \text{ where } l_e = \frac{l}{\sqrt{2}} \Rightarrow P_{cr} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{\left(\frac{3000}{\sqrt{2}}\right)^2} = 134576\text{N}$$

④ A hollow Mild Steel tube 6m long 4cm internal dia and 5mm thick used as strut with ends hinged. Find Crippling load and safe load taking Factor of Safety 3. $E = 2 \times 10^5 \text{ N/mm}^2$

Solution

$$l = 6\text{m} = 6000\text{mm} = l_e$$

$$d = 4\text{cm} = 40\text{mm}$$

$$t = 5\text{mm} \Rightarrow D = d + 2t = 40 + 2 \times 5 = 50\text{mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 2 \times 10^5 \times \frac{\pi}{64} (50^4 - 40^4)}{6000^2} = 9429.9\text{N}$$

- (5) A column of timber section 150mm x 200mm is 6m long, both end fixed.

$$E = 17.5 \text{ kN/mm}^2$$

Determine (a) crippling load

(b) Safe load, F.O.S = 3

Solution

$$L = 6\text{m} = 6000\text{mm}$$

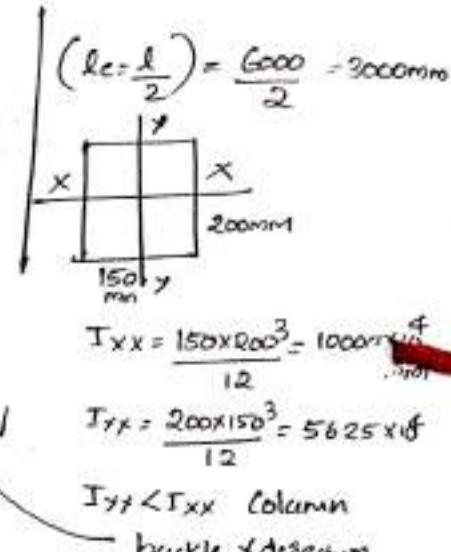
$$F = 17.5 \times 10^3 \text{ N/mm}^2$$

$$(a) \text{ Crippling load } P_{cr} = \frac{\pi^2 EI}{l_e^2} \quad (1)$$

$$\text{Best } I = 5625 \times 10^4 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 \times 17.5 \times 10^3 \times 5625 \times 10^4}{3000} \\ = 1079.48 \times 10^3 \text{ N}$$

$$(b) \text{ Safe Load} = \frac{P_{cr}}{\text{F.O.S}} = \frac{1079.48 \times 10^3}{3} = 359.8 \times 10^3 \text{ N}$$



- (6) A SSB of length 4m is subjected to UDL of 30kN/m over whole span and deflect 15mm at the centre. Determine Crippling load.

when beam assume column (a) one end fixed, other end hinged
(b) both pin-jointed

Solution

$$\text{SSB with UDL} \Rightarrow \delta = \frac{5}{384} \frac{Wl^4}{EI} \Rightarrow 15 = \frac{5}{384} \frac{30 \times 4000^4}{EI} \Rightarrow EI = 6.667 \times 10^{12}$$

(a) P_{cr} (one end fixed other end hinged)

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.667 \times 10^{12}}{(4000)^2} = \frac{8224.5 \text{ kN}}{\left(\frac{l_e}{\sqrt{2}}\right)^2} = \frac{4000}{\sqrt{2}}$$

(b) Both end pin-jointed

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.667 \times 10^{12}}{(4000)^2} \quad (l = l_e = 4000\text{mm}) \\ = 4112.25 \text{ kN}$$

- ⑦ A solid round bar 4m long and 5cm diameter was found to extend 4.6mm under tensile load of 50kN. The bar is used as a strut with both ends hinged. Determine buckling load for the bar. Determine Safe Load (FOS=4) (4)

Solution

$$l = 4000\text{mm}$$

$$d = 50\text{mm}$$

$$A = \frac{\pi}{4} d^2 = \underline{\underline{625\pi \text{mm}^2}}$$

$$SL = 4.6\text{mm}$$

$$\bullet P = W = 50 \times 10^3 \text{N}$$

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{P/l}{SL} = \frac{50 \times 10^3}{\frac{625\pi}{4000}} = \underline{\underline{2.214 \times 10^9 \text{N/mm}^2}}$$

$$\text{Clipping Load } P_{cl} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 2.214 \times 10^9 \times \pi (50)^4}{\frac{64}{4000^2}} (l_e = l) \\ = \underline{\underline{4189.94 \text{N}}}$$

$$\text{Safe load} = \frac{P_{cl}}{4} = \frac{4189.94}{4} = \underline{\underline{1047.5 \text{N}}}$$

(Hollow) \Rightarrow do yourself

- ⑧ An I-Section Joint 450mmx950mmx15mm and 6m long is used as column both end hinged. what is Euler's Clipping load of column. $E = 200 \text{GPa}$

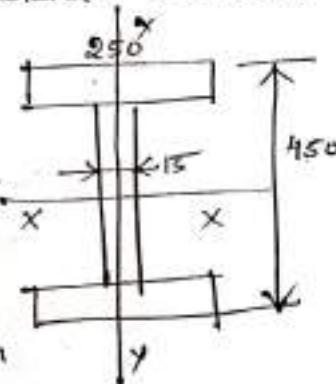
Solution

$$I_{xx} = \frac{250 \times 450^3}{12} - \frac{235 \times 420^3}{12} = \underline{\underline{447.55 \times 10^6 \text{mm}^4}}$$

$$I_{yy} = \frac{450 \times 250^3}{12} - \frac{420 \times 235^3}{12} = \frac{137.71 \times 10^6 \text{mm}^4}{12} = \underline{\underline{137.71 \times 10^6 \text{mm}^4}}$$

$I_{yy} < I_{xx} \Rightarrow$ That joint buckle in yy direction

$$P_{cl} = \frac{\pi^2 EI}{l_e} = \frac{\pi^2 \times 200 \times 10^9 \times 137.71 \times 10^6}{(6)^2} (l = l_e) \\ = \underline{\underline{2403.5 \text{kN}}}$$



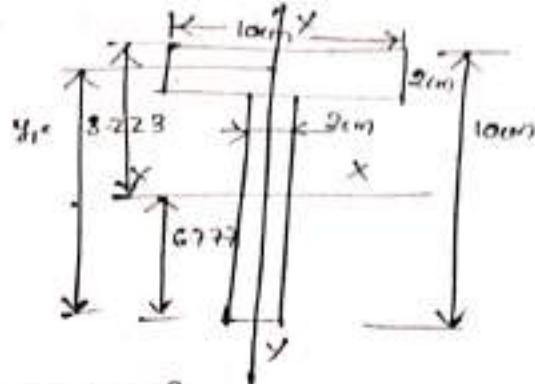
- ⑨ Determine the crippling load for T-section dimension 100x100x9cm and length 5m when it is used straight with both of its end hinged

$$E = 2 \times 10^5 \text{ N/mm}^2$$

Solution

$$\bar{A} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 100 \text{ cm}^2$$

$$= \frac{20 \times 9 + 16 \times 4}{20+16} = \underline{\underline{6.777 \text{ cm}^2}}$$



$$I_{xx} = \frac{10 \times 83 + 20 \times 2.333^2 + 2 \times 3^3 + 16 \times 2.777^2}{12} = \underline{\underline{314.221 \text{ cm}^4}}$$

$$I_{yy} = \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12} = \underline{\underline{172 \text{ cm}^4}}$$

$I_{yy} < I_{xx}$ \Rightarrow H buckle in y direction

($l = l_c$)

$$P_{cr} = \frac{\pi^2 EI}{l_c^2} = \frac{\pi^2 \times 2 \times 10^5 \times 172 \times 10^6}{5000^2} = \underline{\underline{135805.7 \text{ N}}}$$

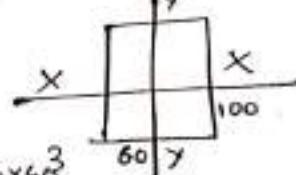
Refer problem (7.12, 7.13) in Avravat text book

- ⑩ Find shortest length l for steel column both end hinged having 60mm x 100mm for Euler's formula. $E = 2 \times 10^5 \text{ N/mm}^2$

Critical Stress = 250 N/mm²

Solution

$$I_{xx} = \frac{60 \times 100^3}{12} = 5 \times 10^6 \text{ mm}^4, I_{yy} = \frac{100 \times 60^3}{12} = \underline{\underline{1.8 \times 10^6 \text{ mm}^4}}$$



$I_{yy} < I_{xx}$ \Rightarrow H buckle in y direction

$$P_{cr} = \frac{\pi^2 EI}{l_c^2} \Rightarrow P_{cr} = \frac{\pi^2 EI}{A l_c^2} \Rightarrow 250 = \frac{\pi^2 \times 2 \times 10^5 \times 1.8 \times 10^6}{60 \times 100 \times l_c^2}$$

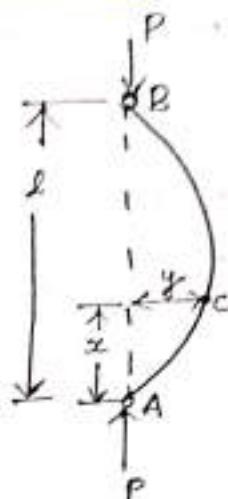
$$l_c = l = 1.5 \text{ m} //$$

Derivation of Euler's Equation

(5)

Case I : When both end are pinned or hinged

- Load at which column just buckles (or bend) is called crippling load.
- Consider column AB of length l and uniform C/S Area, hinged at both ends A and B
- P = Crippling load
- Due to crippling load, column will deflect into curved form ACB



Consider a section x from A

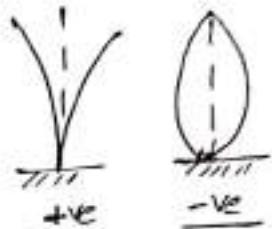
y = deflection at section

Moment due to crippling load = $-Py$

$$\text{Moment} = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -Py \Rightarrow EI \frac{d^2y}{dx^2} + Py = 0$$

$$\boxed{\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0}$$



Solution of above differential eq:

$$y = C_1 \cos(x\sqrt{\frac{P}{EI}}) + C_2 \sin(x\sqrt{\frac{P}{EI}}) \quad \text{--- (1)}$$

where C_1 and C_2 = Constant of integration

(1) At $x=0, y=0$ (at A)

$$0 = C_1 \cos 0 + C_2 \sin 0 \Rightarrow \boxed{C_1 = 0} \quad \text{--- (2)}$$

(2) At B ($x=l, y=0$)

$$0 = C_1 \cos l\sqrt{\frac{P}{EI}} + C_2 \sin l\sqrt{\frac{P}{EI}} \quad \text{--- (3)}$$

$$0 = 0 + C_2 \sin l\sqrt{\frac{P}{EI}} \Rightarrow \boxed{C_2 = 0}$$

From eq(3) $C_2 = 0$ OR $\sin(l\sqrt{\frac{P}{EI}}) = 0$

If C_1 and $C_2 = 0$ Mean column will not bend (actually bend)

$$\sin(l\sqrt{\frac{P}{EI}}) = 0 \quad (\sin 0 = 0) \text{ or } (\sin \pi = 0)$$

$$l\sqrt{\frac{P}{EI}} = 0 \quad \text{OR}$$

$$l\sqrt{\frac{P}{EI}} = \pi \Rightarrow \frac{l^2 P}{EI} = \pi^2$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

(Drive crippling load
very important)

Case 2 : when one end is fixed and other is free

- Consider a column of length l whose lower end A is fixed and upper end B is free
- Due to critical load P , column just buckle
- Let a be the deflection at A
- Take any section x from fixed end A

$$B.M = +P(a-y)$$

$$B.M = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = P(a-y) \Rightarrow EI \frac{d^2y}{dx^2} + Py = Pa$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}}$$

General Solution of above eq: $y = C_1 \cos(x\sqrt{\frac{P}{EI}}) + C_2 \sin(x\sqrt{\frac{P}{EI}}) + a$ ①

C_1 and C_2 constant of integration

Deflection at A

$$x=0, y=0 \Rightarrow 0 = C_1 \cos 0 + C_2 \sin 0 + a$$

$$\boxed{C_1 = -a}$$

$$\text{at A} \Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -C_1 \sin(x\sqrt{\frac{P}{EI}}) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos(x\sqrt{\frac{P}{EI}}) \cdot \sqrt{\frac{P}{EI}} + 0$$

$$\text{at A } (x=0) \text{ and } \frac{dy}{dx} = 0$$

$$0 = 0 + C_2 \times \sqrt{\frac{P}{EI}} \Rightarrow \boxed{C_2 = 0}$$

$$\text{Sub } C_1 = -a, C_2 = 0 \text{ in eq ①}$$

$$y = -a \cos(x\sqrt{\frac{P}{EI}}) + a$$

Deflection at B ($x=l, y=a$)

$$a = -a \cos l \sqrt{\frac{P}{EI}} + a$$

$$\cos l \sqrt{\frac{P}{EI}} = 0$$

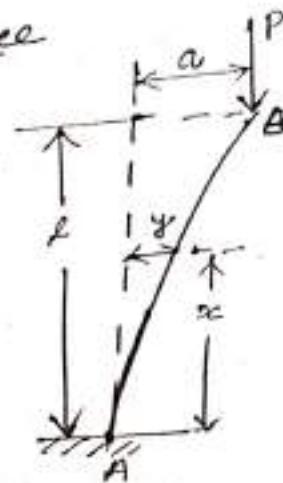
$$\begin{aligned} & (\cos 90^\circ = 0) \\ & \cos \frac{\pi}{2} = 0 \end{aligned}$$

$$l \sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\frac{l^2 P}{EI} = \frac{\pi^2}{4}$$

Crushing load

$$\boxed{P = \frac{\pi^2 EI}{4 l^2}}$$



Case 3 : One end is fixed and other end is pinned (or) binged

(6)

- Consider column AB of length l whose lower end A fixed, upper end B hinged
- M_0 = Restant Moment at lower fixed end
- existence of Restored moment balanced by Moment due to horizontal force H at B
- Consider Section X at distance x from lower fixed end A

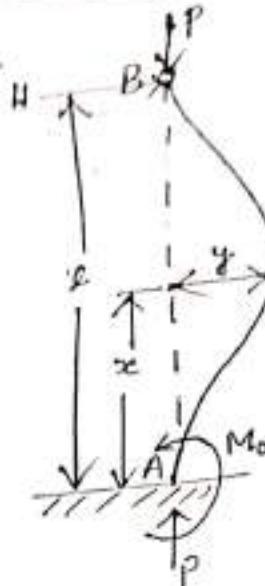
$$B.M = -Py + H(l-x)$$

$$BM = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -Py + H(l-x)$$

$$EI \frac{d^2y}{dx^2} + Py = H(l-x)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{H}{EI} (l-x) \Rightarrow \boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} \frac{H(l-x)}{P}}$$



General Solution of above eq

$$y = c_1 \cos\left(x \sqrt{\frac{P}{EI}}\right) + c_2 \sin\left(x \sqrt{\frac{P}{EI}}\right) + \frac{H(l-x)}{P} \quad \boxed{①}$$

c_1 and c_2 constant of integration

- at fixed end A ($x=0, y=0, \frac{dy}{dx}=0$)
- at fixed end B ($x=l, y=0$)

Sub: eq ①

$$0 = c_1 x + 0 + \frac{Hl}{P}$$

$$c_1 = -\frac{Hl}{P}$$

$$\frac{dy}{dx} = -c_1 \sin\left(x \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} + c_2 \cos\left(x \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$0 = -\frac{Hl}{P} \times 0 + c_2 \times \cos 0 \times \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$c_2 = \frac{H}{P} \sqrt{\frac{EI}{P}}$$

Sub: value of c_1 and c_2 in eq ① at point B ($x=l, y=0$)

$$0 = -\frac{Hl}{P} \cos\left(l \sqrt{\frac{P}{EI}}\right) + \frac{H}{P} \sqrt{\frac{EI}{P}} \sin\left(l \sqrt{\frac{P}{EI}}\right) + \frac{H}{P} (l-x)$$

$$\frac{Hl}{P} \cos\left(l \sqrt{\frac{P}{EI}}\right) = \frac{H}{P} \sqrt{\frac{EI}{P}} \sin\left(l \sqrt{\frac{P}{EI}}\right) \quad (1+5=45)$$

$$\tan\left(l \sqrt{\frac{P}{EI}}\right) = l \sqrt{\frac{P}{EI}} \Rightarrow l \sqrt{\frac{P}{EI}} = 45 \quad \text{cosec } 45^\circ = 2\sqrt{2}$$

$$l^2 \frac{P}{EI} = 20.25 \Rightarrow \boxed{P = \frac{2\sqrt{2}EI}{l^2}}$$

crimping load

Case 4 when both end of the column are fixed

- Consider AB of length l
- whose ends A and B are fixed
- there will be restrained moment M_0
- consider any section X a distance x from lower end A

$$BM = M_0 - Py$$

$$BM = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} + Py = M_0$$

$$\boxed{\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{M_0}{EI}} \Rightarrow \boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} \times \frac{M_0}{P}}$$

General Solution $\boxed{y = C_1 \cos(x\sqrt{\frac{P}{EI}}) + C_2 \sin(x\sqrt{\frac{P}{EI}}) + \frac{M_0}{P}} \quad \text{--- (1)}$

C_1 and C_2 constant of integration

$$\text{at A } (x=0, y=0) \Rightarrow 0 = C_1 \cos 0 + C_2 \sin 0 + \frac{M_0}{P} \Rightarrow C_1 = -\frac{M_0}{P}$$

$$\frac{dy}{dx} = -C_1 \sin(x\sqrt{\frac{P}{EI}}) \sqrt{\frac{P}{EI}} + C_2 \cos(x\sqrt{\frac{P}{EI}}) \sqrt{\frac{P}{EI}} + 0$$

$$(x=0, y=0, \frac{dy}{dx}=0) \Rightarrow 0 = -C_1 \times 0 + C_2 \times 1 \times \sqrt{\frac{P}{EI}}$$

$$0 = C_2 \sqrt{\frac{P}{EI}} \Rightarrow \boxed{C_2 = 0}$$

Sub: value C_1 and C_2 in eq (1) at point B ($x=l, y=0$)

$$0 = -\frac{M_0}{P} \cos(l\sqrt{\frac{P}{EI}}) + 0 + \frac{M_0}{P}$$

$$\cancel{\frac{M_0}{P} \cos(l\sqrt{\frac{P}{EI}})} = \cancel{\frac{M_0}{P}}$$

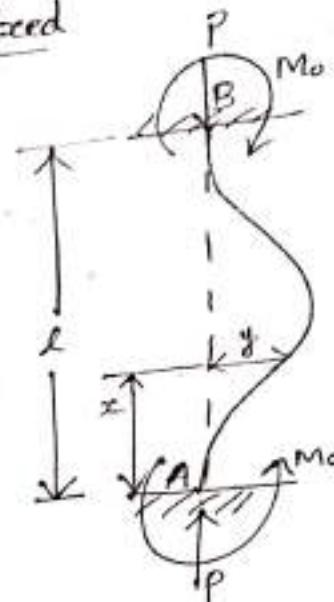
$$\cos(l\sqrt{\frac{P}{EI}}) = 1$$

$$\begin{aligned} (\cos 0 &= 1) \\ \cos 2\pi &= 1 \end{aligned}$$

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

$$l^2 \frac{P}{EI} = (2\pi)^2$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$



(7)

Rankine's Formula

- Short columns fail by direct causing load $\Rightarrow P_d = i_{c,1} \sigma_c A$
 σ_c = direct causing stress

$$\text{Buckling Load} = P_{cr} = \frac{\pi^2 EI}{l_e^2}$$

- The failure of members will due to combined effect of direct and bending (buckling) stresses. Rankine derive a Empirical formula for short and long column.

P = Actual crippling Load

$$\text{Rankine Formula} \quad \frac{1}{P} = \frac{1}{P_d} + \frac{1}{P_{cr}}$$

$$\frac{1}{P} = \frac{P_d + P_{cr}}{P_d P_{cr}} \Rightarrow P = \frac{P_d P_{cr}}{P_d + P_{cr}} = \frac{P_d}{1 + \frac{P_d}{P_{cr}}}$$

$$P = \frac{\sigma_c A}{\frac{1 + \sigma_c A}{\frac{\pi^2 EI}{l_e^2}}} \Rightarrow P = \frac{\sigma_c A}{1 + \frac{\sigma_c A l_e^2}{\pi^2 E A k^2}} \quad (J = A k^2)$$

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k}\right)^2} = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k}\right)^2}$$

$$\boxed{P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k}\right)^2}}$$

where $\alpha = \frac{\sigma_c}{\pi^2 E}$ = Rankine constant

- The external and internal diameter of a hollow cast iron column are 5cm and 4cm respectively. If the length of this column is 3m and both of its ends are fixed. Determine Crippling Load using Rankine formula. $\sigma_c = 500 \text{ N/mm}^2$ and $\alpha = \frac{1}{1600}$

Solution

$$D = 5 \text{ cm}$$

$$d = 4 \text{ cm}$$

$$l_e = \frac{l}{2} \text{ (both end fixed)} \\ = \frac{3000}{2} = 1500 \text{ mm}$$

$$A = \frac{\pi}{4} (5^2 - 4^2) = 28.5 \text{ cm}^2 = \underline{\underline{225 \pi \text{ mm}^2}}$$

$$I = \frac{\pi}{64} (5^4 - 4^4) = 5.7656 \pi \text{ cm}^4 = \underline{\underline{57656 \pi \text{ mm}^4}}$$

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k}\right)^2} = \frac{500 \times 225 \pi}{1 + \frac{1}{1600} \left(\frac{1500}{\frac{225.025}{1600}}\right)^2} = \underline{\underline{54.473 \times 10^3 \text{ N}}}$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{57656 \pi}{225 \pi}} \\ = \underline{\underline{16007 \text{ mm}}}$$

- (2) A hollow cylindrical cast iron column 4m long, with both end fixed. Determine maximum diameter of the column. If it has carry a safe load 250N with F.O.S=5. Take internal dia as 0.8 times of external diameter. $\sigma_c = 550 \text{ N/mm}^2$, $\alpha = \frac{1}{1600}$

Solution

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$125 \times 10^4 = \frac{550 \times A \times 0.01 D^2}{1 + \frac{1}{1600} \left(\frac{2000}{0.8D} \right)^2}$$

$$\frac{125 \times 10^4}{550 \times \pi \times 0.09 D^2} = \frac{D^2}{1 + 2444}$$

$$D^4 - 6083 D^2 - 196234700 = 0$$

$$D^2 = \frac{8083 \pm \sqrt{8083^2 + 4 \times 196234700}}{2}$$

$$= 18542.5 \text{ mm}^2 \Rightarrow D = 196.3 \text{ mm}$$

$$\text{Total dia} = \underline{\underline{196 \text{ mm}}}$$

- (3) 1.5m long, column has a circular cross-section 5cm diameter. One the end of column, is fixed in direction and position and other end is free. Take F.O.S=3

Calculate Safe load using (a) Rankine formula, $\sigma_c = 560 \text{ N/mm}^2$

$$\alpha = \frac{1}{1600}$$

Solution

(a)

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$P = \frac{560 \times 19.625 \times 10^{-4}}{1 + \frac{1}{1600} \left(\frac{2000}{12.5} \right)^2}$$

$$= \underline{\underline{29708.1 \text{ N}}}$$

$$\text{Safe Load} = \frac{P}{\text{F.O.S}} = \frac{29708.1}{3} = \underline{\underline{9902.7 \text{ N}}}$$

le = $\frac{l}{2}$ (both end fixed)

$$= \frac{1000}{2} = 2000 \text{ mm}$$

$$\sigma_c = 550 \text{ N/mm}^2$$

$$A = \frac{\pi}{4} (D^2 - d^2) \quad | \quad d = 0.8D$$

$$A = \frac{\pi}{4} (D^2 - (0.8D)^2) = \underline{\underline{\pi \times 0.09 D^2}}$$

$$\alpha = \frac{1}{1600}, J = \frac{\pi}{64} (D^4 - (0.8D)^4)$$

$$k = \sqrt{\frac{J}{A}} = \frac{1}{0.009225 \times \pi \times D^2}$$

$$= \sqrt{\frac{0.009225 \times D^4}{\pi \times 0.09 \times D^2}} = \underline{\underline{0.32D}}$$

P.s = F.O.S x Safe Load

$$= 5 \times 250 \times 10^3 = \underline{\underline{125 \times 10^4 \text{ N}}}$$

(b) Euler's formula $E = 1.2 \times 10^5 \text{ N/mm}^2$

$$\sigma_c = 560 \text{ N/mm}^2, J = \frac{\pi}{64} l^4 = 30.7 \text{ cm}^4$$

$$A = \frac{\pi}{4} \times 5^2 = 19.625 \text{ cm}^2 = 19.625 \times 10^{-4} \text{ m}^2$$

$$\alpha = \frac{1}{1600}, l_e = 2l = 2 \times 1500 = 3000 \text{ mm}$$

$$k = \sqrt{\frac{J}{A}} = \sqrt{\frac{30.7 \times 10^4}{19.625 \times 10^{-4}}} = 12.5 \text{ mm}$$

(b) Euler's formula

$$P_{cr} = \frac{\pi^2 EI}{le} = \frac{\pi^2 \times 1.2 \times 10^5 \times 30.7 \times 10^6}{30002} = 40200 N$$

$$\text{Safe load} = \frac{P_{cr}}{F.O.S} = \underline{\underline{1340 N}}$$

① Find crippling load for a hollow steel column 50mm internal diameter and 5mm thick. The column is 5m long with one end fixed and other end hinged. Use Rankine formula. Rankine constant $\frac{1}{7500} \times 10^5 = 235 \text{ N/mm}^2$

Solution Internal dia, $d = 50\text{mm}$, $t = 5\text{mm}$

External dia $D = d + 2t = 50 + 2 \times 5 = 60\text{mm}$

$$l_e = 5m = 5000 \text{ mm}$$

One end fixed other end hinged

$$\sigma_c = 235 \text{ N/mm}^2$$

$$\alpha = \frac{1}{7500}$$

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$I = \frac{\pi}{4} (60^4 - 50^4)$$

$$= 3.29 \times 10^5 \text{ mm}^4$$

$$P = \frac{395 \times 863.5}{1 + \frac{1}{7500} \left(\frac{3535539}{19.519} \right)^2}$$

$$= \underline{\underline{53828154 N}}$$

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (60^2 - 50^2)$$

$$= 863.5 \text{ mm}^2$$

$$l_e = \frac{l}{\sqrt{2}} = \frac{5000}{\sqrt{2}} = 3535539 \text{ mm}$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.29 \times 10^5}{863.5}} = \underline{\underline{19.519 \text{ mm}}}$$

- ⑥ A short length of tube, 4cm internal diameter and 5cm external diameter, failed in compression at load of 240KN. When a 2m length of same tube was tested as strut with fixed end, the load failure was 158KN. Assume that σ_c in Rankine formula is given by the first test, find the value of the constant α in the same formula. What will be the crippling load of this tube if it used as a strut 3m long with one end fixed and other end hinged?

$$D = 50\text{mm} = 50\text{mm}$$

P-crushing load = 158KN Solution

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$158 \times 10^3 = \frac{839.5 \times 225 \pi}{1 + \alpha \left(\frac{1000}{16} \right)^2}$$

$$\therefore \alpha = 0.001328/\pi$$

$$\sigma_c = \frac{P_{cr}}{A} = \frac{240}{\frac{\pi}{4} (50^2 - 40^2)} = \frac{240 \times 10^3}{\frac{\pi}{4} (50^2 - 40^2)}$$

$$A = \frac{\pi}{4} (50^2 - 40^2) = 225 \pi \text{ mm}^2$$

$$= \underline{\underline{339.5 \text{ N/mm}^2}}$$

$$l_e = \frac{l}{2} = \frac{2000}{2} = 1000 \text{ mm} \quad (\text{Both end fixed})$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{31650 \pi}{225 \pi}} = 16 \text{ mm} \quad I = \frac{\pi}{64} (60^4 - 40^4)$$

$$= \underline{\underline{57650 \pi \text{ mm}^2}}$$

(b) Euler's formula

(8)

$$P_{cr} = \frac{\pi^2 EI}{l_e} = \frac{\pi^2 \times 12 \times 10^5 \times 30.7 \times 10^6}{3000^2} = 10200 \text{ N}$$

$$\text{Safe load} = \frac{P_{cr}}{F.O.S} = \underline{1340 \text{ N}}$$

- ① Find crippling load for a hollow steel column 50mm internal diameter and 5mm thick. The column is 5m long with one end fixed and other end hinged. Use Rankine formula. Rankine constant: $\frac{1}{7500} \times 2 = 335 \text{ N/mm}^2$

Solution

$$\text{Internal dia, } d = 50 \text{ mm, } t = 5 \text{ mm}$$

$$\text{External dia, } D = d + 2t = 50 + 2 \times 5 = 60 \text{ mm}$$

$$l = 5 \text{ m} = 5000 \text{ mm}$$

One end fixed other end hinged

$$\sigma_c = 335 \text{ N/mm}^2$$

$$d = \frac{1}{7500}$$

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$I = \frac{\pi}{64} (60^4 - 50^4)$$

$$= 3.29 \times 10^5 \text{ mm}^4$$

$$P = \frac{335 \times 863.5}{1 + \frac{1}{7500} \left(\frac{3535539}{14.319} \right)^2}$$

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (60^2 - 50^2)$$

$$= 863.5 \text{ mm}^2$$

$$= 53828154 \text{ N}$$

$$l_e = \frac{l}{\sqrt{2}} = \frac{5000}{\sqrt{2}} = 3535.539 \text{ m}$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.29 \times 10^5}{863.5}} = 19.579 \text{ mm}$$

- ② A short length of tube, 4cm internal diameter and 5cm external diameter, failed in compression at load of 240kN. When a 2m length of same tube was tested as strut with fixed end, the load failure was 158kN. Assume that σ_c in Rankine formula is given by the first test, find the value of the constant α in the same formula. What will be the crippling load of this tube if it used as a strut 3m long with one end fixed and other end hinged?

$$D = 5 \text{ cm} = 50 \text{ mm}$$

$$P_{cr} = 240 \text{ kN} \quad \text{Solution}$$

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k} \right)^2}$$

$$158 \times 10^3 = 334.5 \times 225 \pi$$

$$\alpha = 0.0001328 \frac{1}{l_e^2}$$

$$\sigma_c = \frac{P_{cr}}{A} = \frac{240 \times 10^3}{\frac{\pi}{4} (50^2 - 40^2)} = \frac{240 \times 10^3}{\frac{\pi}{4} (50^2 - 40^2)} \quad d = 40 \text{ mm} \\ A = \frac{\pi}{4} (50^2 - 40^2) = 225 \pi \text{ mm}^2$$

$$l_e = \frac{l}{2} = \frac{2000}{2} = 1000 \text{ mm} \quad (\text{Ballhead fixed})$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{57656 \pi}{225 \pi}} = 16 \text{ mm} \quad I = \frac{\pi}{64} (50^4 - 40^4) = 57656 \pi \text{ mm}^4$$

(i) Capping load for beam of length 6m when one end is fixed & other end hinged

$$l = 6\text{m} = 6000\text{mm}$$

$$l_e = \frac{l}{\sqrt{2}} = \frac{6000}{\sqrt{2}}$$

$$P_c = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{K}\right)^2} = \frac{\pi^2 E I}{1 + \alpha \left(\frac{l_e}{K}\right)^2} = \frac{\pi^2 \times 1.2 \times 10^5 \times 53689300}{1 + 0.001828 \left(\frac{6000}{62.5}\right)^2} = 7197 \times 10^3 \text{N}$$

(6) Find Euler crushing load for a hollow cylindrical cast iron column 20cm external dia and 25mm-thick. If it is 6m long and hinged at both ends. $E = 1.2 \times 10^5 \text{ N/mm}^2$

Compare load with crushing load as given by Rankine formula $\sigma_c = 550 \text{ N/mm}^2$. $\alpha = \frac{1}{1600}$, for what length of the column would these two formula give the crushing load?

Solution

$$\text{Euler's crushing load } P_c = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 53689300}{6000^2} = 1366307 \text{N}$$

Crushing load by Rankine formula

$$P_c = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{K}\right)^2}$$

$$= \frac{550 \times 13744}{1 + \frac{1}{1600} \left(\frac{6000}{62.5}\right)} = 1118224.8 \text{N}$$

Length of columns which two formula give same crushing load

$$\frac{\pi^2 EI}{l_e^2} = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{K}\right)^2}$$

$$\frac{\pi^2 \times 1.2 \times 10^5 \times 53689300}{6000^2} = \frac{550 \times 13744}{1 + \frac{1}{1600} \left(\frac{l_e}{62.5}\right)^2}$$

Solve \rightarrow

$$\text{we get } l_e = \sqrt{\frac{-811800}{0.346}}$$

$$E = 1.2 \times 10^5 \text{ N/mm}^2$$

$$D = 20\text{cm} = 200\text{mm}$$

$$d = 200 - 2 \times 25 = 150\text{mm}$$

$$l = 6\text{m} = 6000\text{mm}$$

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (200^4 - 150^4) = 53689300 \text{mm}^4$$

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (200^2 - 150^2) = 13744 \text{mm}^2$$

$$l_e = l = 6000\text{mm} \quad (\text{Both end hinged})$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{53689300}{13744}} = 62.5 \text{mm}$$

$$= 62.5 \text{mm}$$

$$= 62.5 \text{mm}$$

Plane State of a Stress

- Normal Stress in x and y direction assume +ve
- Shear Stress in + Side or same

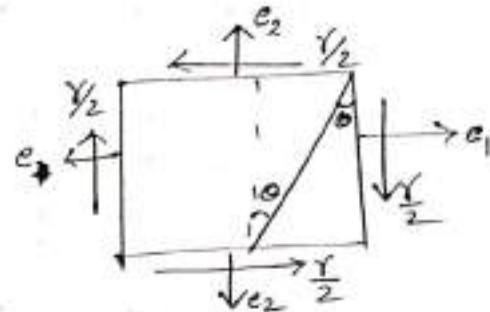
$$\gamma_{xy} = \gamma_{yx}$$

Stress transformation eq: identical to stress transformation

① Replace σ on e

② Replace γ

Combined Normal & Shear Strain
Shear and Shear Stress



Normal Strain

$$e_n = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} + \frac{\gamma}{2} \sin 2\theta$$

$$e_t = \frac{e_1 - e_2}{2} \sin 2\theta - \frac{\gamma}{2} \cos 2\theta$$

Tangential Strain

Principal Strain (only normal strain, no shear strain)

Major principal Strain

$$e_{n1} = \frac{e_1 + e_2}{2} + \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$$

Minor principal Strain

$$e_{n2} = \frac{e_1 + e_2}{2} - \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$$

$$\tan 2\theta = \frac{\gamma}{e_1 - e_2}$$

Max. Shear Strain

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$$

If $\sigma_1, \sigma_2, \sigma_3$ principal stresses acting three principal plane

principal strain $e_1 = \frac{\sigma_1 - \mu \sigma_2 + \sigma_3}{E}, e_2 = \frac{\sigma_2 - \mu \sigma_1 + \sigma_3}{E}$

$$e_3 = \frac{\sigma_3 - \mu \sigma_1 + \sigma_2}{E} \quad (\mu = \text{Poisson's ratio})$$

If two principal stresses

$$e_1 = \frac{\sigma_1 - \mu \sigma_2}{E}$$

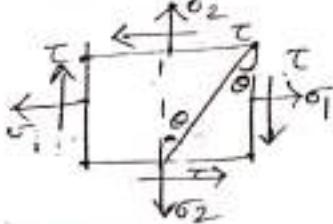
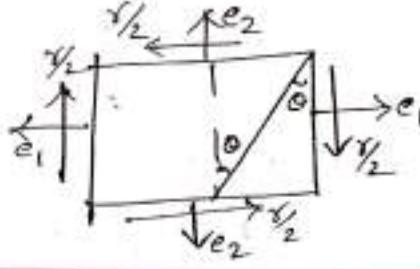
$$e_2 = \frac{\sigma_2 - \mu \sigma_1}{E}$$

Plane State of Stress

Analogy between Stress and Strain (Transformation)

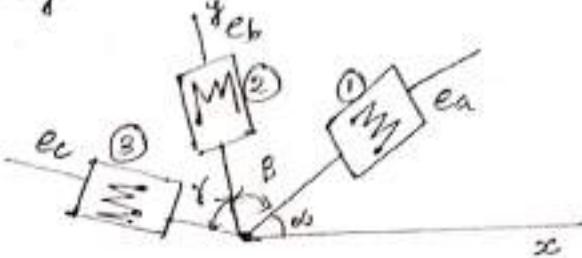
- Stress and Strain (transformation) eq: almost identical
- Stress σ replaced by e in strain
- Shear stress τ replaced by γ in shear strain

Ans:

Stress	Strain
<ul style="list-style-type: none"> • <u>Principal Stress</u>: A plane having only normal stress no shear stress. That plane called <u>principal plane</u> and normal stress called <u>principal stress</u> • <u>Max. Shear Stress</u> & <u>Shear Stress</u> 	<ul style="list-style-type: none"> • <u>Principal Strain</u>: A plane having only normal strain no shear strain. That plane is called <u>principal plane</u>. That normal strain called <u>principal strain</u> • <u>Max. Tensile Strain</u> & <u>Shear Strain</u> 
$\text{Normal Stress} \sigma_{\text{m}} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$	$\text{Normal Strain} e_{\text{m}} = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 2\theta + \frac{\gamma}{2} \sin 2\theta$
$\text{Shear/Tangential Stress} \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$	$\text{Shear/Tangential Strain} e_t = \frac{e_1 - e_2}{2} \sin 2\theta - \frac{\gamma}{2} \cos 2\theta$
<u>Major principal Stress</u> $\sigma_{\text{m1}} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$	<u>Major principal Strain</u> $e_{\text{m1}} = \frac{e_1 + e_2}{2} + \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$
<u>Minor principal Stress</u> $\sigma_{\text{m2}} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$	<u>Minor principal Strain</u> $e_{\text{m2}} = \frac{e_1 + e_2}{2} - \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$
<u>Max Shear Stress</u> $\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$	<u>Max Shear Strain</u> $\gamma_{\text{max}} = \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$
$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$	$\tan 2\theta = \frac{\gamma}{e_1 - e_2}$

Strain Rosette

- Strain gauge measure strain only one direction
- To measure strain in both x and y directions two/three strain gauge required.
- Consider 3 strain gauges
 - to make angle strain gauge ① with x-axis
 - Other strain gauge ②, ③ make internal angle β & γ
- Strain Measure strain gauge are e_a, e_b, e_c



Transformation eq: Convert longitudinal strain from strain gauge into strain expressed x-y co-ordinate

$$e_n = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 2\alpha + \frac{\gamma_{12}}{2} \sin 2\alpha$$

Apply e_n : 3 strain gauge

$e_1 \rightarrow$ strain in x-direction
 $e_2 \rightarrow$ strain in y-direction

$$e_a = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 2\alpha + \frac{\gamma_{12}}{2} \sin 2\alpha$$

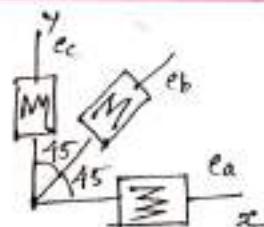
$$e_b = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 2(\alpha + \beta) + \frac{\gamma_{12}}{2} \sin 2(\alpha + \beta)$$

$$e_c = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 2(\alpha + \beta + \gamma) + \frac{\gamma_{12}}{2} \sin 2(\alpha + \beta + \gamma)$$

Special cases

Strain rosette - 45°

$$\alpha = 0, \beta = \gamma = 45^\circ$$



$$e_a = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 0^\circ + \frac{\gamma_{12}}{2} \sin 0^\circ = \frac{e_1 + e_2 + e_1 - e_2}{2} = \underline{e_1}$$

$$e_b = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} \cos 90^\circ + \frac{\gamma_{12}}{2} \sin 90^\circ \Rightarrow e_c = \frac{e_1 + e_2}{2} + \frac{e_1 - e_2}{2} (-1) = \underline{e_2}$$

Similarly calculate other case

$$\underline{\alpha = 30, \beta = 60, \gamma = 60}$$

Plane State of Stress

- ① A rectangular strain rosette gives following reading in a strain measurement task.

$$\epsilon_{11} = 1000 \times 10^{-6}$$

$$\epsilon_{22} = 800 \times 10^{-6}$$

$$\epsilon_{33} = 600 \times 10^{-6}$$

Determine

The direction of Major principal strain ϵ_1 with respect to gauge ①

Solution $(\alpha = 0, \beta = 45, \gamma = 45)$

Direction of Major principal strain

$$\tan 2\theta = \frac{\gamma_{12}}{\epsilon_1 - \epsilon_2}$$

where -

$$\epsilon_1 = \text{Strain in x direction} = \epsilon_{11}$$

$$= 1000 \times 10^{-6}$$

$$\epsilon_2 = \text{Strain in y direction} = \epsilon_{33}$$

$$= 600 \times 10^{-6}$$

$$\epsilon_{22} = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\beta + \frac{\gamma}{2} \sin 2\beta$$

$$800 \times 10^{-6} = \frac{1000 \times 10^{-6} + 600 \times 10^{-6}}{2} + \frac{1000 \times 10^{-6} - 600 \times 10^{-6}}{2} \cos 90 + \frac{\gamma}{2} \sin 90$$

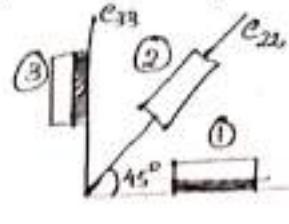
$$800 \times 10^{-6} = 800 \times 10^{-6} + \frac{\gamma}{2} \Rightarrow \frac{\gamma}{2} = 0$$

$$\tan 2\theta = 0 \Rightarrow \theta = 0, 90^\circ$$

Major principal strain

$$\epsilon_{D1} = \frac{\epsilon_1 + \epsilon_2}{2} + \sqrt{\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2}$$

$$= \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} = \frac{2\epsilon_1}{2} = \underline{\underline{\epsilon_1 = 1000 \times 10^{-6}}}$$

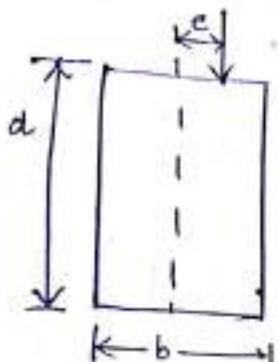


COMBINED BENDING & DIRECT (NORMAL) STRESS

~~Eccentric loading~~

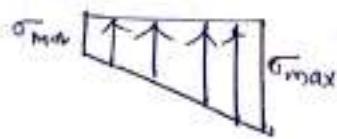
Compressive Load = P cause direct
Compressive Stress.

$$\sigma = \frac{P}{A}$$



e = eccentricity

- An eccentric load will produce a direct stress as well as bending stress.
- By adding these two stresses algebraically a single resultant stress can be obtained



$$\sigma_{\max} = \frac{P + My}{A} \Rightarrow \sigma_{\max} = \frac{P}{A} + \frac{Pe}{bd^2} \quad \text{Eqn I}$$

$$\sigma_{\min} = \frac{P - My}{A} \Rightarrow \sigma_{\min} = \frac{P}{A} - \frac{Pe}{bd^2} \quad \text{Eqn II}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{6Pe}{bd^2} = \frac{P}{A} + \frac{6Pe}{ab^2} \quad \text{Eqn III}$$

$$\sigma_{\min} = \frac{P}{A} - \frac{6Pe}{bd^2} = \frac{P}{A} - \frac{6Pe}{ab^2} \quad \text{Eqn IV}$$

- (1) A rectangular column of width 200mm and of thickness 150mm carrying a point load of 240kN at an eccentricity of 10mm as shown in fig. Determine the maximum and minimum to-be section.

Solution

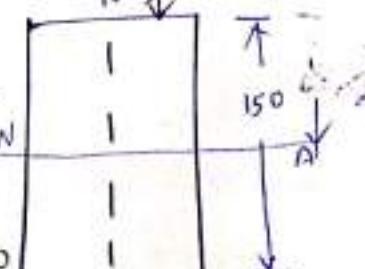
$$b = 200\text{mm}$$

$$d = 150\text{mm}$$

$$e = 10\text{mm}$$

$$A = 200 \times 150 \\ = 30000 \text{ mm}^2$$

10 240kN



$$\sigma_{\max} = \frac{P}{A} + \frac{My}{I}$$

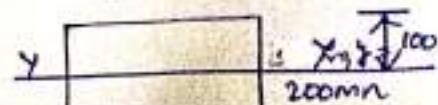
$$\sigma_{\max} = \frac{240 \times 10^3}{30,000} + \frac{240 \times 10^3 \times 100}{160 \times 200} \\ = 10.4 \text{ N/mm}^2$$

OR

$$M = Pe \\ = 240 \times 10^3 \times 10$$

$$y = \frac{800}{2}$$

$$I = \frac{bd^3}{12} = \frac{150 \times 200^3}{12}$$



$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b}\right) = \frac{240 \times 10^3}{30,000} \left(1 + \frac{6 \times 10}{200}\right) \\ = 10.4 \text{ N/mm}^2$$

$$\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6e}{b}\right) = \frac{240 \times 10^3}{30,000} \left(1 - \frac{6 \times 10}{200}\right) = 5.6 \text{ N/mm}^2$$

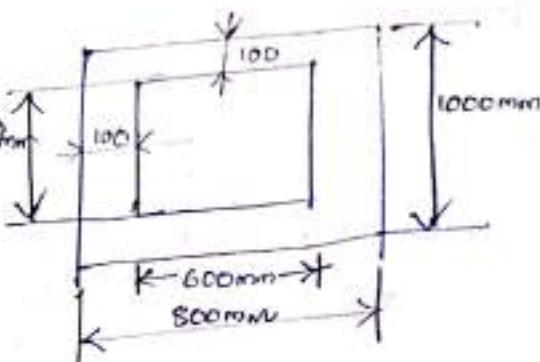
- (2) A hollow rectangular column of external depth 100mm and external width 0.8m is 10cm thick. Calculate the maximum and minimum stresses in the section of the column if a vertical load of $800 \times 10^3 \text{ N}$ acting within eccentricity of 15cm.

$$\left\{ \begin{array}{l} M = Pe + 2 \times 10^3 x \\ 150 = 300 \times 10^3 \times 100 \end{array} \right.$$

Solution

$$\left\{ \begin{array}{l} P = 8 \times 10^3 \text{ N} \\ A = Bd = bd \\ = 800 \times 1000 - 600 \times 800 \\ = 32 \times 10^4 \text{ mm}^2 \end{array} \right.$$

$$\sigma_{\max} = \frac{P + My}{A}$$



$$\begin{aligned} I &= \frac{Bd^3 - bd^3}{12} \\ &= \frac{1000 \times 800^3 - 800 \times 600^3}{12} \\ &= 28.25 \times 10^9 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} &= \frac{2 \times 10^3}{32 \times 10^4} + \frac{3 \times 10^6 \times 100}{28.28 \times 10^9} \\ &= 0.00625 + \frac{3 \times 10^6 \times 100}{28.28 \times 10^9} \\ &= 1.0496 \text{ N/mm}^2 \end{aligned}$$

$$\sigma_{\min} = \frac{P - My}{I} = \frac{0.00625 - 3 \times 10^6 \times 100}{28.28 \times 10^9} = 0.2004 \text{ N/mm}^2$$



- (3) A circular column of diameter 250mm carries a vertical load of 600kN at a distance of 30mm from Y-Y axis. Find Maximum & minimum value of stresses induced in sections.

Solution

$$d = 250 \text{ mm}$$

$$P = 600 \text{ kN} = 600 \times 10^3 \text{ N}$$

$$e = 30 \text{ mm}$$

$$\sigma_{\max} = \frac{P + My}{A}$$

$$= \frac{600 \times 10^3}{\frac{\pi}{4} 250^2} + \frac{600 \times 10^3 \times 30 \times \frac{250}{2}}{\frac{\pi}{4} 250^4}$$

$$= 12.929 + 11.74$$

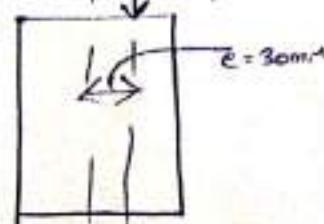
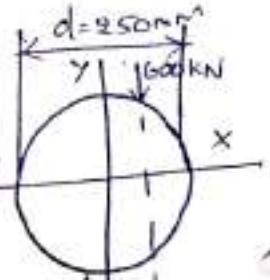
$$= 23.969 \text{ N/mm}^2$$

$$\text{Where } A = \frac{\pi D^2}{4}$$

$$= \frac{\pi}{4} \times 250^2$$

$$I = \frac{\pi D^4}{64} = \frac{\pi}{64} 250^4$$

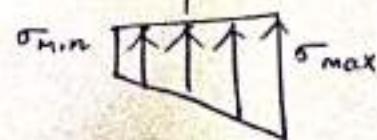
$$J = \frac{D}{2} = \frac{250}{2}$$



$$\sigma_{\min} = \frac{P - My}{A}$$

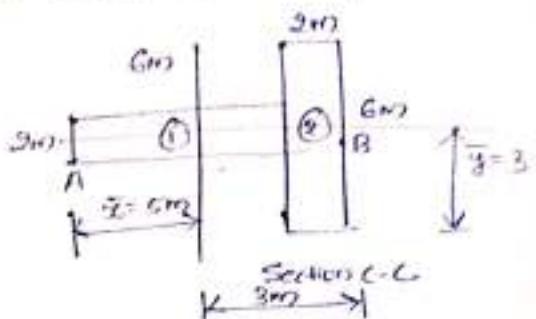
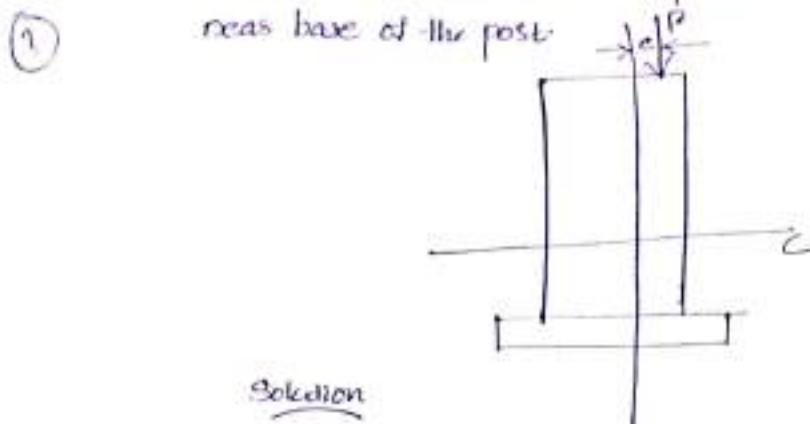
$$= 12.929 - 11.74$$

$$= 0.489 \text{ N/mm}^2$$



① T-Section shown in fig. is used as short post to support a compression load P of 150kN. The load applied on centerline of the stem at a distance $e = 3\text{m}$ from the centroid of the cross section.

Determine normal stress at points A and B on a transverse plane C-C near base of the post.



Solution

$$A_1 = a_1 + a_2 = 6 \times 2 + 2 \times 6 = 24\text{m}^2$$

$$\bar{y} = 3 \Rightarrow \bar{x} = \frac{a_1 \bar{x}_1 + a_2 \bar{x}_2}{a_1 + a_2} = \frac{6 \times 2 \times 3 + 6 \times 2 (7)}{6 \times 2 + 6 \times 2} = 5\text{m} \quad (\text{from MA})$$

$$I_{xx} = I_1 + I_2 = \frac{6 \times 2^3}{12} + 2 \times 6 \times 2 + \frac{2 \times 6^3 + 2 \times 6 \times 2}{12} = 88\text{m}^4$$

P=150kN

$$M = Pe = 150 \times 2 = 300\text{kNm}$$

$$\sigma_A = \frac{-P + My^2}{I} = \frac{-150 + 300 \times 5}{88} = \frac{10.74}{88} \text{ kN/m}^2$$

$$\sigma_B = \frac{-P - My^2}{I} = \frac{-150 - 300 \times 3}{88} = \frac{16.47}{88} \text{ kN/m}^2$$

Combined Bending and Tension

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{My}{I}$$

$$\frac{T}{A} = \frac{\tau}{r} \quad \text{and} \quad \tau = \frac{T}{J} r \quad \text{--- (2)}$$

$$\sigma_{\max} = \frac{My^2}{I} = \frac{32M}{\pi D^3} \quad \left| \begin{array}{l} y = D/2 \\ I = \pi d^4 / 64 \end{array} \right.$$

$$\sigma_{\max} = \frac{T r}{A} = \frac{T \times D/2}{\pi D^4 / 32} = \frac{16T}{\pi D^3}$$

$$\text{Major principal stress} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{32M}{2\pi D^3} + \sqrt{\left(\frac{32M}{2\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

Major
principal
stress

$$\sigma_1 = \frac{16}{\pi D^3} [M + \sqrt{M^2 + T^2}]$$

Minor
principal
stress

$$\sigma_2 = \frac{16}{\pi D^3} [M - \sqrt{M^2 + T^2}]$$

$$\text{Max shear stress} = \frac{16}{\pi D^3} (\sqrt{M^2 + T^2})$$

- ① A solid shaft of diameter 80mm subjected to ~~bending~~^{twisting} moment of 8MN-mm and bending moment of 5MN/mm at a point. Determine

(a) principal stresses

(b) position of plane on which they act

Solution

$$\text{Major principal stress} \quad \sigma_1 = \frac{16}{\pi D^3} [M + \sqrt{M^2 + T^2}]$$

$$= \frac{16}{\pi \times 80^3} [5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}]$$

$$= \frac{16 \times 10^6}{\pi \times 80^3} [5 + \sqrt{25 + 64}] = \underline{19357 \text{ N/mm}^2}$$

$$\text{Minor principal stress} \quad \sigma_2 = \frac{16}{\pi D^3} [M - \sqrt{M^2 + T^2}] = \frac{16 \times 10^6}{\pi \times 80^3} [5 - \sqrt{25 + 64}]$$

$$\tan 2\theta = \frac{\tau}{M} \Rightarrow \tan 2\theta = \frac{8 \times 10^6}{5 \times 10^6} = 1.6 \Leftrightarrow \theta = \underline{44.1 \text{ N/mm}^2}$$

Find θ
~~using~~

(6)

COMBINED BENDING & TORSION

Let d : shaft diameter

$$M = B.M$$

T = Twisting moment

- bending stress $\sigma_b = \frac{My}{I}$

- shear stress from torsion eqn $\tau = \frac{T}{J} r$

Location of principal plane [Given by]

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

- Major principal stress $\Rightarrow \sigma_{11} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}]$
- $\Rightarrow \sigma_{22} = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{16}{\pi d^3} [M - \sqrt{M^2 + T^2}]$

where

$$\text{Now } \sigma = \frac{M}{Z} \text{ or } \sigma = \frac{My}{I}$$

$$\tau = \frac{Tr}{J}$$

$$\sigma = \frac{M \times d/2}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3}$$

$$\tau = \frac{T}{\frac{\pi d^4}{32}} \times \frac{d/2}{2} = \frac{16T}{\pi d^3}$$

- ① A 200mm diameter shaft with built bracket is fixed to wall and loaded as shown in figure. Determine the principal stresses at top extremity of vertical diameter for section marked A.

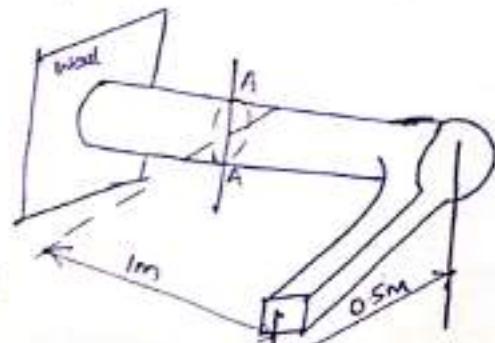
Solution

Bending moment at A

$$M = 400 \times 1 = 400 \text{ Nm}$$

$$= 400 \times 10^3 \text{ Nmm}$$

(Torque at A) $T = 400 \times 0.5 = 200 \text{ Nm} = 200 \times 10^3 \text{ Nmm}$



Major Principal Stress:

$$\sigma_{11} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{400 \times 10^3 \times 300}{2} + \frac{200 \times 10^3}{\pi d^3} = 0.509 + \frac{(200 \times 10^3)^2 + (200 \times 10^3)^2}{\pi d^3} = 0.509 + \frac{800 \times 10^6}{\pi d^3} = 0.509 + \frac{800 \times 10^6}{0.509 \times 10^6} = 0.509 + 1.59 = 2.009 \text{ N/mm}^2$$

$$\sigma_{11} = 0.509 \text{ N/mm}^2$$

$$\tau = \frac{T}{J} = \frac{200 \times 10^3}{\frac{\pi d^4}{32}} = \frac{200 \times 10^3}{\pi d^4} = 0.54 \text{ N/mm}^2$$

Minor Principal Stress:

$$\sigma_{22} = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{400 \times 10^3 - \sqrt{(400 \times 10^3)^2 + (200 \times 10^3)^2}}{2} = 0.509 - \frac{200 \times 10^3}{\pi d^3} = 0.509 - \frac{200 \times 10^3}{0.509 \times 10^6} = 0.509 - 0.403 = 0.106 \text{ N/mm}^2$$

$$\sigma_2 = \frac{16}{\pi d^3} \left[M + \sqrt{M^2 + T^2} \right] = \frac{16}{\pi \times (200)^3} \left[400 \times 10^3 + \sqrt{(400 \times 10^3)^2 + (200 \times 10^3)^2} \right] = 0.030 \text{ N/mm}^2$$

- (2) At a certain section of shaft 16mm diameter, there is a bending moment of 7kNm and torque moment of 10kNm. Find maximum direct stresses at fiber and bending moment of 10kNm. Specify the position of the plane on which it occurs. Take $\mu = 0.28$. Total shear stress acting along can produce same maximum stress.

Solution

$$M_{\text{max}} = 7 \text{ kNm}$$

$$T = 10 \text{ kNm} = 10 \times 10^6 \text{ Nmm}, \mu = 0.28$$

Major principal stress

$$\sigma_1 = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}] = \frac{16}{\pi \times (16)^3} [7 \times 10^6 + \sqrt{(7 \times 10^6)^2 + (10 \times 10^6)^2}]$$

$$= 23.87 \text{ N/mm}^2$$

Minor principal stress

$$\sigma_2 = \frac{16}{\pi d^3} [M - \sqrt{M^2 + T^2}] = \frac{16}{\pi \times 16^3} [7 \times 10^6 - \sqrt{(7 \times 10^6)^2 + (10 \times 10^6)^2}]$$

$$= -6.472 \text{ N/mm}^2$$

position of principal plane

$$\tan 2\theta = \frac{T}{M} = \frac{10}{7} = 1.4286 \Rightarrow \theta = 27.5^\circ$$

MAX. Stress

$$\epsilon = \frac{\sigma_1 - \sigma_2}{E} = \frac{1}{E} [23.87 - (-6.472 \times 0.28)] \Rightarrow \epsilon = \frac{25.682}{E}$$

$$\text{Stress} = \epsilon \times E \Rightarrow \sigma = \frac{25.682}{E} \times E = \underline{\underline{\underline{25.682 \text{ N/mm}^2}}}$$

Note

Equivalent Bending Moment

$$Me = \frac{\pi d^3}{32} \sigma_1 = \frac{\pi d^3}{32} \times \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}] = \frac{1}{2} [M + \sqrt{M^2 + T^2}]$$

$$\boxed{Me = \frac{1}{2} [M + \sqrt{M^2 + T^2}]}$$

Equivalent Torque

$$Te = \frac{\pi d^3}{16} T_{\text{max}} = \frac{\pi d^3}{16} \frac{16}{\pi d^3} \sqrt{M^2 + T^2}$$

$$\boxed{Te = \sqrt{M^2 + T^2}}$$

- (3) To views of crank are shown. A 2000N is applied to the crank pin in the direction shown. At a distance 150mm from centre of adjacent bearing. The crank shaft is solid section of 75mm dia. Calculate max. principal stress & shear stress in the section of shaft at centre of bearing.

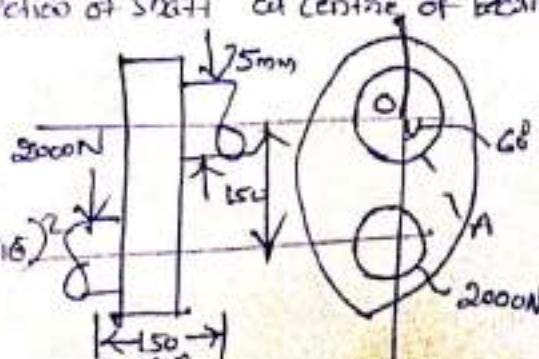
Solution

$$T = 2000 \times OA = 2000 \times 150 \cos 60^\circ = 2.59 \times 10^5 \text{ Nmm}$$

$$M = 2000 \times 150 = 3 \times 10^5 \text{ Nmm}$$

$$Me = \frac{1}{2} [M + \sqrt{M^2 + T^2}] = \frac{1}{2} [3 \times 10^5 + \sqrt{(3 \times 10^5)^2 + (2.59 \times 10^5)^2}]$$

$$= 3.48 \times 10^5 \text{ Nmm}$$



$$\sigma_1 = \frac{Me}{\frac{\pi d^3}{32}} = \frac{32 Me}{\pi d^3} = \frac{32 \times 3.48 \times 10^5}{\pi \times 75^3} = \underline{\underline{\underline{8.4 \text{ N/mm}^2}}}$$

$$Te = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^5)^2 + (2.59 \times 10^5)^2} \\ = 3.96 \times 10^5 \text{ Nmm}$$

$$T_{\text{max}} = Te / \frac{\pi d^3}{16} / 16 = 4.78 \text{ N/mm}^2 //$$

Compound Stresses

Combined Axial, Flexural and shear loads

Locate Max. Stress in this case

- ① Longitudinal & transverse shearing stress maximum where σ is maximum. It located at centroidal axis of section where

$$\tau_{\text{maximum}} = \frac{\sigma_{\text{max}} V_{\text{max}}}{I_b}$$

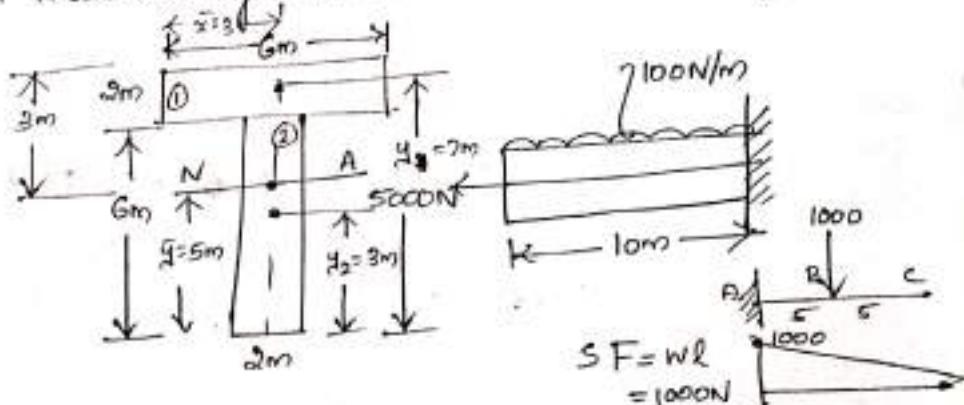
- ② Flexural stress is maximum at the greatest distance (effet) from centroidal axis, section where M maximum

$$\sigma = \frac{M_{\text{max}} Y_{\text{max}}}{I}$$

- ③ Superposition Method used to combine stresses on any given plane at any specific point, provided that the stresses are below proportion of limit of material.

- ④ Stress transformation eq: 3 Mohrs circle used.

- ① The dimension of a T-shaped beam and the load on it are shown in fig. The horizontal 5000N tensile force acts through the centroid of the cross-sectional area. Determine Max principal, normal stress and max. shearing stress in the beam, assuming elastic behavior.



Solution

$$\bar{x} = 3m$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{2 \times 6 \times 7 + 2 \times 6 \times 3}{2 \times 6 + 2 \times 6} = 5m$$

$$J_1 = \frac{G \times 2^3}{12} + 2 \times 6 \times 2, \quad J_2 = \frac{2 \times 6^3}{12} + 2 \times 6 \times 2$$

$$I = J_1 + J_2 = 28 + 60 = 88 \text{ m}^4$$

$$\tau_{\text{max}} = \tau_{NA} = \frac{F A \bar{y}}{I_b} = \frac{1000 \times (6 \times 2 \times 2 + 1 \times 2 \times 0.5)}{88 \times 2} = 183.82 \text{ N/m}^2$$

Longitudinal stress upper most fiber

$$A = A_1 + A_2 = 12 + 12 = 24$$

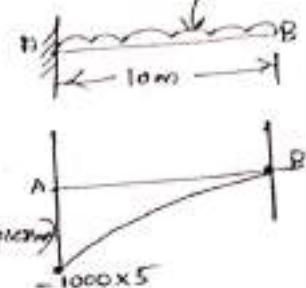
$$\sigma = \frac{P}{A} + \frac{My}{I} = \frac{5000}{24} + \frac{(5000 \times 10) \times 3}{68} = \frac{428.92 \text{ N/mm}^2}{68}$$

Longitudinal stress lower most fiber

$$\sigma = \frac{P}{A} - \frac{My}{I}$$

$$= \frac{5000}{24} - \frac{(5000 \times 5)}{68}$$

$$= -159.31 \text{ N/mm}^2 = 159.3 \text{ (compression)}$$



$$T_{\max} = \frac{\sigma_{\max} \cdot I_{\text{min}}}{2}, \quad \frac{428.92 \cdot 0}{2} = 214.46 \text{ N/mm}^2 \Rightarrow M = 5000 \text{ Nm}$$

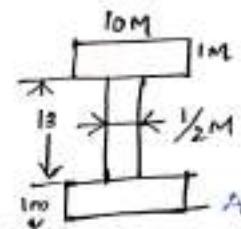
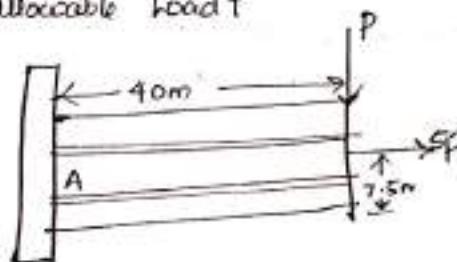
- (2) A Cantilevered beam has the cross section shown in figure. The allowable stresses are 10,000 N/mm² shear and 16000 N/mm² tension and compression at point A (Just below flange). Find Max. allowable load?

Solution

$$F = 5P$$

$$V = P$$

$$M_A = 40P: \underline{40P}$$



$$I = \frac{10 \times 15^3}{12} - \frac{9.5 \times 13^3}{12} = \frac{1073 \text{ mm}^4}{1073}$$

$$A = 10 \times 1 + 10 \times 1 + 13 \times \frac{1}{2} = 26.5 \text{ mm}^2$$

$$\sigma_A = \frac{P}{A} + \frac{My}{I} = \frac{5P}{26.5} + \frac{40P \times 6.5}{1073} = 0.4310P$$

$$\tau_A = \frac{FA}{Ib} = \frac{Px(10x1) \times 6.5}{1073 \times 10} = 6.057 \times 10^{-3} P \text{ N/mm}^2$$

$$\text{At joint of web \& flange } \tau_B = 6.057 \times 10^3 \frac{10}{0.5} = 0.121P$$

$$\sigma_{\max} = \sigma_1 = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau^2}$$

$$= \frac{0.4310P}{2} + \sqrt{\left(\frac{0.4310P}{2}\right)^2 + (0.121P)^2}$$

$$= 0.2155P + 0.247P$$

$$= 0.4628P < 16000 \text{ N/mm}^2$$

$$P < 34583.37 \text{ N}$$

$$T_{\max} = 0.247P \times 10000 \text{ N/mm}^2 \Rightarrow P < 40485.82 \text{ N}$$

$$P = 34583 \text{ N (Smaller value)}$$