KINEMATIC SYNTHESIS OF MECHANISMS

[Graphical & Analytical Methods]

Kinematics ⇒ kinematics defined as study of Motion of Mechanism & Methods of creating them.

Kinematic Analysis ⇒ It is study of Motion of mechanism and determine the performance of given Mechanism

Kinematic Synthesis ⇒ It is used to design a mechanism to satisfy the Motion characteristics like displacement, Velocity and acceleration

⇒ It Mathematically determine geometry of member of mechanism such as to produce, desired performance (Set of position, angular velocities linear/angular acceleration at definite points of time)

⇒ kinematic synthesis is the Reverse problem of kinematic Analysis

Stages of kinematic synthesis

- Type synthesis
- Number synthesis
- Diemensional synthesis

Type Synthesis

⇒ It is used to select the kind of mechanism with gear combination or belt pulley combinations or cam mechanism and so on by considering design aspects like space consideration, Safety aspects, economy considerations, Manufacturing process & so on.

Number Synthesis

⇒ It based on external characteristics of a kinematic chain.

⇒ It used to find no: of links and number of connections required to permit necessary movability.

Diemensional synthesis

⇒ It is used to determine diemensions of parts (lengths & angles) necessary to generate mechanism to obtain desired motion.

Significant diemensions
⇒ Length of all links
⇒ Gear ratios
⇒ Angle b/w arm of bell crank lever
⇒ Eccentricities
⇒ Cam contour diemensions

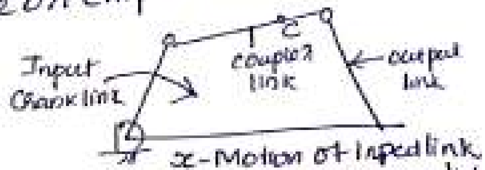
Type of Diemensional Synthesis

① Path Generation

⇒ In path generation particular mechanism is designed so that a point on the coupler link follows a path with a prescribed shape.

⇒ Generally portion of path is an arc of circle or ellipse and a straight Line.

Eg: when crank rotates C on coupler link follows path $\boxed{y = f(x)}$



x-Motion of input link
y-Motion of coupler link

② Function Generation

⇒ Generally the output link may either oscillate, rotate, reciprocate as a function of motion of input link. This is called Function Generation.

Eg: linkage mechanism which adjust steering angles of axes of front wheel of automobile to avoid skidding wheels.

Motion of output link = f(Motion of input link)

$\boxed{y = f(x)}$

x = Motion of input link
y = Motion of output link

③ Motion Generation

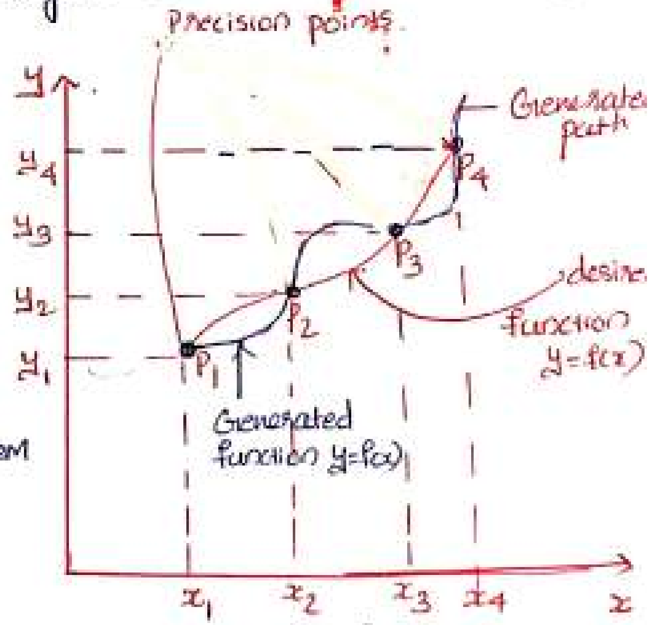
⇒ By using motion generation, the rigid body is guided through a pre determined sequence of motion.

⇒ The path generation is concerned with path of a trace point, while Motion generation is concerned with guidance of entire motion of rigid body.

Precision Points [Approximate & Exact synthesis]

⇒ In function generation problem output is related to input function $y = f(x)$ & It is required to obtain diemension of a linkage to satisfy this relation.

⇒ In general a linkage synthesis problem doesnt have exact solution over its entire range of travel.



⇒ However it is usually possible to design a linkage which exactly satisfy the desired function at a few selected problem known as <u>precision / Accuracy points</u>

Here P_1, P_2, P_3, P_4 are <u>precision points</u>

Approximate Synthesis Approach	Method of Exact Synthesis
⇒ There should be max. accuracy point as possible ⇒ There should be Min. deviation (error) b/w desired & Actual path	⇒ It can handle only few arbitrary function ⇒ Min six link is required for path generation

Error Present in design of linkage for path & Function Generation

① Structural Error ⇒ Difference b/w generated function and desired function

② Mechanical Error ⇒ occur due to Mechanical defects such as improper Machining / casting component of linkage, clearance overload & rubbing in components.

③ Graphical Error

Structural Error

⇒ At precision points the desired & generated function agree & structural error will be zero.

⇒ Other point structural error will have values.

⇒ Exact analysis to minimize structural error is difficult.

⇒ Chebyshev's spacing of precision points can always be take as first approximation to minimize structural error.

Chebyshev's Spacing for Precision point

⇒ For 'n' point in the range $x_s \le x \le x_f$. The precision points x_j according to chebyshevs spacing given as

$$\boxed{x_j = a - b\cos\left(\frac{2j-1}{2n}\pi\right)}$$

where $a = \frac{x_s + x_f}{2}$

$b = \frac{x_f - x_s}{2}$

where $n \rightarrow$ No. of accuracy points, $j \rightarrow 1, 2, 3 \ldots n$

① $y = x^{1.7}$ in the range of $1 \le x \le 4$. Determine the dependent variable y with chebyshev's spacing with 3 number of precision points?

An: using Chebyshev's spacing $n = 4-1 = 3$

$$\boxed{x_j = \frac{1}{2}(x_s + x_f) - \frac{1}{2}(x_f - x_s) \cdot \cos\pi\left(\frac{2j-1}{2n}\right)}$$

$x_s \le x \le x_f$

$1 \le x \le 4$

$x_s = 1,\ x_f = 4$

$x_1 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1) \cdot \cos\pi\frac{(2\times1-1)}{2\times3} = 1.2$ $[j=1]$

$x_2 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1) \cdot \cos\pi\left(\frac{2\times2-1}{2\times3}\right) = 2.5$ $[j=2]$

$x_3 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1) \cdot \cos\pi\left(\frac{2\times3-1}{2\times3}\right) = 3.799$ $[j=3]$

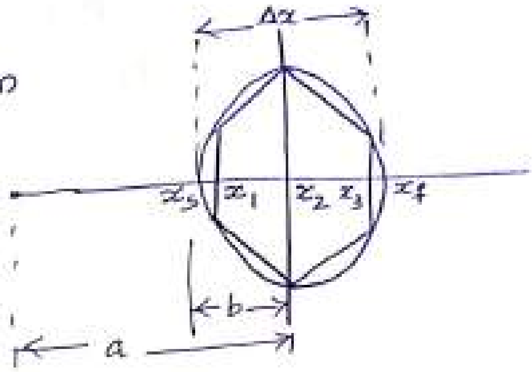
$y_1 = x_1^{1.7} = 1.2^{1.7} = 1.3639$

$y_2 = x_2^{1.7} = 2.5^{1.7} = 4.7478$

$y_3 = x_3^{1.7} = 3.799^{1.7}$
$= 9.6702$

Graphical synthesis for Motion Generation

• Graphical / Geometric construction for obtain Chebyshevs Accuracy points



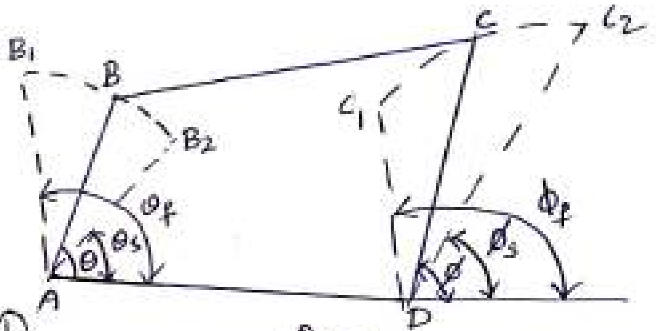
⇒ Draw a circle of diameter $= \Delta x = x_f - x_s$

⇒ Inscribe a regular polygon of 2n sides in the circle such than 2 side of polygon are perpendicular to the x-axis

⇒ Draw projections of vertices of the polygon on the x-axis. The perpendicular intersect diameter Δx at the precision points.

Angular Relationship for Function Generation

⇒ consider 4-bar Mechanism



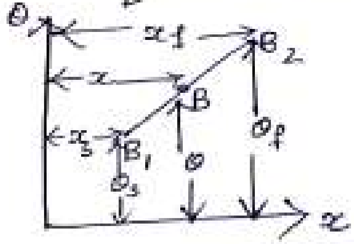
$y = f(x)$

$x \Rightarrow x_f - x_s \qquad y \Rightarrow y_f - y_s$

$\theta \rightarrow \theta_f - \theta_s \qquad \phi \Rightarrow \phi_f - \phi_s$

Linear Relationship b/w x and θ

$$\frac{\theta - \theta_s}{x - x_s} = \frac{\theta_f - \theta_s}{x_f - x_s} = \frac{\Delta\theta}{\Delta x} = r_x \quad -①$$



Linear Relation ship b/w y & ϕ

$$\frac{\phi - \phi_s}{y - y_s} = \frac{\phi_f - \phi_s}{y_f - y_s} = r_y \quad -②$$

For n points in the range $[j = 1, 2 \ldots n]$

eq.① ⇒ $$\boxed{\frac{\theta_j - \theta_s}{x_j - x_s} = \frac{\theta_f - \theta_s}{x_f - x_s}}$$ eq② ⇒ $$\boxed{\frac{\phi_j - \phi_s}{y_j - y_s} = \frac{\phi_f - \phi_s}{y_f - y_s}}$$

$\Delta x = x_f - x_s \qquad \Delta\theta = \theta_f - \theta_s$

$\Delta y = y_f - y_s \qquad \Delta\phi = \phi_f - \phi_s$

② A fourbar Mechanism is to be designed by using 3 precision points generate the function $y = x^{1.5}$ for the range $1 \le x \le 5$. Assuming $40°$ starting position and $120°$ finishing portion for the input link and $100°$ starting position and $180°$ finishing position for output link. Find values of x, y, θ and ϕ corresponding 3 precision point?

G.D

$n = 3$

$y = x^{1.5}$

range $= 1 \le x \le 5 \quad (x_s \le x \le x_f)$, $x_s = 1, x_f = 5$, $\Delta x = x_f - x_s = 5 - 1 = 4$

4.

$\theta_s = 40^\circ, \theta_f = 120^\circ$

$\phi_s = 100^\circ, \phi_f = 180^\circ$

using chebyshev's spacing $x_j = \frac{1}{2}(x_s + x_f) - \frac{1}{2}(x_f - x_s)\cos\pi\left(\frac{2j-1}{2n}\right)$

$x_1 = \frac{1}{2}(1+5) - \frac{1}{2}(5-1)\cos\pi\left(\frac{2\times1-1}{2\times3}\right)$ [j=1]

$= \underline{1.27}$

$x_2 = \frac{1}{2}(1+5) - \frac{1}{2}(5-1)\cos\pi\left(\frac{2\times2-1}{2\times3}\right)$ (j=2)

$= \underline{\underline{3}}$

$x_3 = \frac{1}{2}(1+5) - \frac{1}{2}(5-1)\cos\pi\left(\frac{2\times3-1}{2\times3}\right)$ | j=3

$= \underline{\underline{4.73}}$

$y_1 = 1.27^{1.5} = 1.43$, $y_2 = 3^{1.5} = 5.19$, $y_3 = 4.73^{1.5} = 10.28$

Value of θ

$$\boxed{\frac{\theta_j - \theta_s}{x_j - x_s} = \frac{\theta_f - \theta_s}{x_f - x_s}}$$

If $j=1 \Rightarrow \frac{\theta_1 - 40}{1.27-1} = \frac{120-40}{5-1} \Rightarrow \theta_1 = \underline{\underline{48.4}}$

If $j=2 \Rightarrow \frac{\theta_2 - 40}{3-1} = \frac{120-40}{5-1} \Rightarrow \theta_2 = \underline{\underline{80}}$

If $j=3 \Rightarrow \frac{\theta_3 - 40}{4.73-1} = \frac{120-40}{5-1} \Rightarrow \theta_3 = \underline{\underline{114.6^\circ}}$

Value ϕ

$$\boxed{\frac{\phi_j - \phi_s}{y_j - y_s} = \frac{\phi_f - \phi_s}{y_f - y_s}}$$

$y = x^{1.5}$

$y_f = x_f^{1.5} \Rightarrow y_f = 5^{1.5} = \underline{11.180}$

$y_s = x_s^{1.5} \Rightarrow y_s = 1^{1.5} = \underline{1}$

If $j=1 \Rightarrow \frac{\phi_1 - 100}{1.43-1} = \frac{180-100}{11.180-1} \Rightarrow \phi_1 = 100 + \overset{3.38}{\cancel{}} = \overset{103.38}{\cancel{}}$

If $j=2 \Rightarrow \frac{\phi_2 - 100}{5.19-1} = \frac{180-100}{11.180-1} \Rightarrow \phi_2 = 182.93$

If $j=2 \Rightarrow \frac{\phi_3 - 100}{10.28-1} = \frac{180-100}{11.180-1} \Rightarrow \phi_3 = 172.9408$

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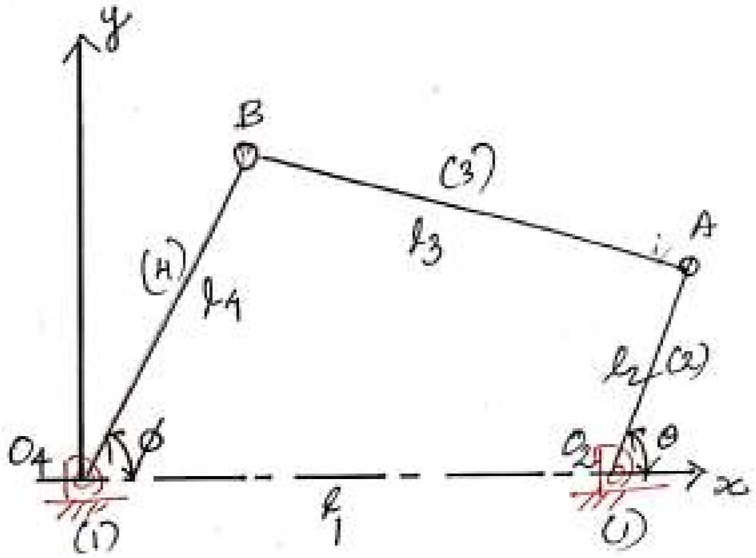
Analytical Methods for Function Generation

- Analytical Method of Synthesis of 4-bar mechanism for function generation aims at establishing a displacement eq. which relates the output angle ϕ to the input crank angle θ. To establish the displacement eq: consider 4-bar mechanism O_2ABO_4 as shown below.

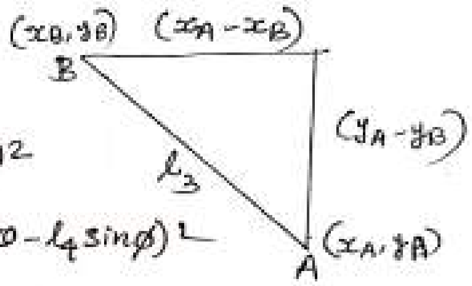
Freudensteins Method

4R linkage Function generator
3-position synthesis

θ & ϕ Angle Made by input & output link



$x_A = l_1 + l_2\cos\theta$, $x_B = l_4\cos\phi$
$y_A = l_2\sin\theta$, $y_B = l_4\sin\phi$



From Figure $\Rightarrow$ $l_3^2 = (x_A - x_B)^2 + (y_A - y_B)^2$

$$l_3^2 = (l_1 + l_2\cos\theta - l_4\cos\phi)^2 + (l_2\sin\theta - l_4\sin\phi)^2$$

$$\cancel{l_3^2 = l_1^2 + l_2^2 + l_4^2 + 2l_1l_2\cos\theta}$$

$$l_3^2 = l_1^2 + 2l_1l_2\cos\theta + l_2^2\cos^2\theta + l_4^2\cos^2\phi - 2(l_1 + l_2\cos\theta)l_4\cos\phi + l_2^2\sin^2\theta - 2l_2l_4\sin\theta\sin\phi + l_4^2\sin^2\phi$$

$$l_3^2 = l_1^2 + 2l_1l_2\cos\theta + \underline{l_2^2\cos^2\theta} + \underline{l_4^2\cos^2\phi} - 2l_1l_4\cos\phi - 2l_2l_4\cos\theta\cos\phi + \underline{l_2^2\sin^2\theta} - 2l_2l_4\sin\theta\sin\phi + \underline{l_4^2\sin^2\phi}$$

$$l_3^2 = l_1^2 + l_2^2 + l_4^2 + 2l_1l_2\cos\theta - 2l_1l_4\cos\phi - 2l_2l_4\cos(\theta - \phi)$$

$$2l_2l_4\cos(\theta - \phi) = l_1^2 + l_2^2 + l_4^2 + 2l_1l_2\cos\theta - 2l_1l_4\cos\phi - l_3^2$$

$$\cos(\theta - \phi) = \frac{l_1^2}{2l_2l_4} + \frac{l_2^2}{2l_4l_2} + \frac{l_4^2}{2l_2l_4} + \frac{l_1\cos\theta}{l_4} - \frac{l_1\cos\phi}{l_2} - \frac{l_3^2}{2l_2l_4}$$

$$\cos(\theta - \phi) = \frac{l_1}{l_4}\cos\theta - \frac{l_1}{l_2}\cos\phi + \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2l_2l_4}$$

4.

$$\cos(\theta-\phi) = k_1\cos\theta - k_2\cos\phi + k_3 \quad \text{(1)}$$

where $k_1 = \frac{l_1}{l_4}$, $k_2 = \frac{l_1}{l_2}$, $k_3 = \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2 l_2 l_4}$

Solving above eq, we can get link length

Very Very important

- A 4-bar mechanism is required such that the input and output angles are co-ordinated as given in the table. Synthesise the 4-bar Mechanism

Input crank angle : 30° 50° 80° (θ)

output crank angle : 0° 30° 60° (ϕ)

Note : For problems

Solution

The displacement eq: given by Freudenstein Method is under

Relace θ by ϕ as well as l_2 by l_4 the right han side of eq (1)

$$\cos(\theta-\phi) = \frac{l_1}{l_2}\cos\phi - \frac{l_1}{l_4}\cos\theta + \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2 l_1 l_4}$$

Where $k_1 = \frac{l_1}{l_2}$, $k_2 = l_1/l_4$, $k_3 = \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2 l_2 l_4}$

$$\cos(\theta-\phi) = k_1\cos\phi - k_2\cos\theta + k_3$$

used for problem

$$k_1\cos\phi - k_2\cos\theta + k_3 = \cos(\theta-\phi)$$

Sub. Value of θ and ϕ in above eq

$k_1\cos 0 - k_2\cos 30 + k_3 = \cos(80-0)$ — (1)

$k_1\cos 30 - k_2\cos 50 + k_3 = \cos(50-30)$ — (2)

$k_1\cos 60 - k_2\cos 80 + k_3 = \cos(80-60)$ — (3)

(1) ⇒ $k_1 - 0.866\,k_2 + k_3 = 0.866$ — (4)

(2) ⇒ $0.866\,k_1 - 0.6427\,k_2 + k_3 = 0.9396$ — (5)

(3) ⇒ $0.5\,k_1 - 0.1736\,k_2 + k_3 = 0.9396$ — (6)

Solving above eq we get $k_1 = 1.8321$, $k_2 = 1.4294$, $k_3 = 0.2718$

Assume link 2 length is unit ($l_2 = 1$)

$$k_1 = \frac{l_1}{l_2} \Rightarrow l_1 = 1.8321\, l_2 = 1.8321$$

$$k_2 = \frac{l_1}{l_4} = 1.4294 \Rightarrow l_4 = \frac{l_1}{1.4294} = \frac{1.8321}{1.4294} = 1.2817$$

$$k_3 = \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2 l_2 l_4} = \frac{1.8321^2 + 1^2 - l_3^2 + 1.2817^2}{2 \times 1 \times 1.2817} = 0.2718$$

$$l_3 = 2.3039$$

Draw kinematic Diagram

Generation of Instantaneous kinematic relationship

- For the 4-bar linkage the following data are given

$\theta_2 = 60^\circ$ $\quad \theta_4 = 90^\circ$ $\quad \alpha_2 = -1\ rad/s^2$

$\omega_2 = 3\ rad/s$ $\quad \omega_4 = 2\ rad/s$ $\quad \alpha_4 = 0$

Determine the linkage length ratios?

Solution

Freudensteins eq given by

$$k_1\cos\theta_4 - k_2\cos\theta_2 + k_3 = \cos(\theta_2-\theta_4) \text{ —— } (1)$$

Take 1st derivative of eq (1)

$$k_1\sin\theta_4 \cdot \frac{d\theta_4}{dt} - k_2\sin\theta_2\frac{d\theta_2}{dt} = \sin(\theta_2-\theta_4)\left(\frac{d\theta_2}{dt} - \frac{d\theta_4}{dt}\right)$$

$$k_1\omega_4\sin\theta_4 - k_2\omega_2\sin\theta_2 = \sin(\theta_2-\theta_4)(\omega_2-\omega_4) \text{ —— } (2)$$

Take Second derivative of eq (2)

$$\Rightarrow k_1\omega_4\cos\theta_4\frac{d\theta_4}{dt} + k_1\sin\theta_4\frac{d\omega_4}{dt} - \left[k_2\omega_2\cos\theta_2\frac{d\theta_2}{dt} + k_2\sin\theta_2\frac{d\omega_2}{dt}\right]$$
$$= \sin(\theta_2-\theta_4)\left(\frac{d\omega_2}{dt} - \frac{d\omega_4}{dt}\right) + (\omega_2-\omega_4)\cos(\theta_2-\theta_4)\left(\frac{d\theta_2}{dt} - \frac{d\theta_4}{dt}\right)$$

$$\Rightarrow k_1\omega_4^2\cos\theta_4 + k_1\alpha_4\sin\theta_4 - \left[k_2\omega_2^2\cos\theta_2 + k_2\alpha_2\sin\theta_2\right]$$
$$= \sin(\theta_2-\theta_4)\left[\alpha_2-\alpha_4\right] + (\omega_2-\omega_4)^2\cos(\theta_2-\theta_4)$$

$$\Rightarrow k_1\left[\omega_4^2\cos\theta_4 + \alpha_4\sin\theta_4\right] - k_2\left(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2\right)$$
$$= (\alpha_2-\alpha_4)\sin(\theta_2-\theta_4) + (\omega_2-\omega_4)^2\cos(\theta_2-\theta_4) \text{ —— } (3)$$

$\Rightarrow$ Sub: Value in eq —(1), eq—(2), eq—(3)

eq-(1) $\Rightarrow$ $k_1\cos 90 - k_2\cos 60 + k_3 = \cos(60-90)$

$$\Rightarrow -0.5k_2 + k_3 = 0.866 \text{ —— } (a)$$

eq—(2) $\Rightarrow$ $k_1 \times 2 \times \sin 90 - k_2 \times 3\sin 60 = \sin(60-90)(3-2)$

$$2k_1 - 2.598k_2 = -0.5 \text{ —— } (b)$$

eq-(3) $\Rightarrow$ $k_1\left[2^2\cos 90 + 0\times\sin 90\right] - k_2\left[3^2\cos 60 - 1\times\sin 60\right]$

$$= (-1-0)\sin(60-90) + (3-2)^2\cos(60-90)$$

$$k_2 = -0.3758 = \underline{-0.376}$$

Solving eqs $\Rightarrow$ $k_1 = -0.738$, $k_3 = \underline{0.678}$

$k_1 = \frac{l_1}{l_2} = -0.738$, Assume $l_1 = 1$ $\Rightarrow$ $l_2 = \frac{l_1}{-0.738} = \frac{1}{-0.738} = \underline{-1.355}$

$k_2 = \frac{l_1}{l_4} = -0.376 \Rightarrow l_4 = \frac{1}{-0.376} = \underline{-2.659}$

$$k_3 = \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2 l_2 l_4} = 0.678$$

$$k_3 = \frac{1^2 + (-1.355)^2 - l_3^2 + (-2.659)^2}{2 \times -1.355 \times -2.659} = 0.678$$

$$\therefore \quad \underline{\underline{l_3 = 2.240}}$$

More Problem of Function generation

- A 4bar Mechanism is to be designed, by using 3 precision points to generate the function $y = x^{1.5}$ for the range $1 \leq x \leq 4$. Assume 30° starting position and 120° finishing position for the input link and 90° starting position and 180° finishing position for the output link. Find the values of x, y, θ & ϕ corresponding to 3 precision points.

G.D

$$\left.\begin{array}{l} x_s \leq x \leq x_f \\ 1 \leq x \leq 4 \end{array}\right\} \begin{array}{l} x_s = 1 \\ x_f = 4 \end{array} \qquad \begin{array}{l} \theta_s = 30^\circ \\ \theta_f = 120^\circ \end{array} \qquad \begin{array}{l} \phi_s = 90^\circ \\ \phi_f = 180^\circ \end{array}$$

3 values of x corresponding 3 precision points $(n=3)$

According to chebyshevs spacing $\quad x_j = a - b\cos\left[\frac{2j-1}{2n}\pi\right]$

where $a = \frac{x_s + x_f}{2} = \frac{4+1}{2} = \underline{\underline{2.5}}$, $b = \frac{x_f - x_s}{2} = \frac{4-1}{2} = \underline{\underline{1.5}}$

n = No. of precision points = 3

$(j=1)$ $x_1 = 2.5 - 1.5\cos\left[\frac{2\times1-1}{2\times3}\pi\right] = \underline{\underline{1.2}}$

$(j=2)$ $x_2 = 2.5 - 1.5\cos\left[\frac{2\times2-1}{2\times3}\pi\right] = \underline{\underline{2.5}}$

$(j=3)$ $x_3 = 2.5 - 1.5\cos\left[\frac{2\times3-1}{2\times3}\pi\right] = 3.8$

$y = x^{1.5}$

$y_1 = x_1^{1.5} = 1.2^{1.5} = \underline{\underline{1.316}}$

$y_2 = x_2^{1.5} = (2.5)^{1.5} = 3.952$

$y_3 = x_3^{1.5} = (3.8)^{1.5} = 7.41$

Note $y_s = (x_s)^{1.5} = 1$, $y_f = (y_f)^{1.5} = 4^{1.5} = 8$

Range x, $\Delta x = x_f - x_s = 4 - 1 = \underline{3}$, $\Delta y = y_f - y_s = \underline{7}$

Value of θ

$$\boxed{\frac{\theta_j - \theta_s}{x_j - x_s} = \frac{\theta_f - \theta_s}{x_f - x_s}}$$

If $j=1 \Rightarrow \frac{\theta_1 - 30}{1.2 - 1} = \frac{120 - 30}{4 - 1} \Rightarrow \theta_1 = \underline{\underline{36^\circ}}$

If $j=2 \Rightarrow \frac{\theta_2 - 30}{2.5 - 1} = \frac{120 - 30}{4 - 1} \Rightarrow \theta_2 = 75^\circ$

If $j=3 \Rightarrow \frac{\theta_3 - 30}{3.8 - 1} = \frac{120 - 30}{4 - 1} \Rightarrow \underline{\underline{\theta_3 = 114^\circ}}$

Value of ϕ

$$\frac{\phi_j-\phi_s}{y_j-y_s}=\frac{\phi_f-\phi_s}{y_f-y_s} \Rightarrow j=1 \Rightarrow \frac{\phi_1-90}{1.316-1}=\frac{180-90}{8-1} \Rightarrow \phi_1=94.05^\circ$$

$$\Rightarrow j=2 \Rightarrow \frac{\phi_2-90}{3.952-1}=\frac{180-90}{8-1} \Rightarrow \phi_2=127.95^\circ$$

$$\Rightarrow j=3 \Rightarrow \frac{\phi_3-90}{7.41-1}=\frac{180-90}{8-1} \Rightarrow \phi_3=172.41^\circ$$

- Synthesise a 4 bar linkage using freudensteins eq to generate the function $y=x^{1.5}$ for the interval $1\le x\le 4$. The input crank is to start from $\theta_2=30^\circ$ and is to have a range of 90°. Take 3 accuracy points. Take output follower angle from 0 to 90°

Solution

$$\left.\begin{aligned}1\le x\le 4\\ x_s\le x\le x_f\end{aligned}\right\} \quad x_s=1 \quad \theta_s=30^\circ \quad \phi_s=0$$
$$x_f=4 \quad \theta_f=\cancel{90}\ 120^\circ \quad \phi_f=90^\circ$$

$$a=\frac{x_s+x_f}{2}=\frac{1+4}{2}=2.5$$
$$b=\frac{x_f-x_s}{2}=\frac{4-1}{2}=1.5$$

3 precision points ($n=3$) using chebyshevs spacing

$$\boxed{x_j=a-b\cos\left(\frac{2j-1}{2n}\pi\right)}$$

$$y=x^{1.5}$$

$$j=1 \Rightarrow x_1=2.5-1.5\cos\left(\frac{2\times1-1}{2\times3}\pi\right)=1.201 \quad \Rightarrow y_1=x_1^{1.5}=1.2^{1.5}=1.316$$

$$j=2 \Rightarrow x_2=2.5-1.5\cos\left(\frac{2\times2-1}{2\times3}\pi\right)=2.5 \quad \Rightarrow y_2=x_2^{1.5}=2.5^{1.5}=3.952$$

$$j=3 \Rightarrow x_3=2.5-1.5\cos\left(\frac{2\times3-1}{2\times3}\pi\right)=3.8 \quad \Rightarrow y_3=x_3^{1.5}=3.8^{1.5}=7.41$$

Value of θ

$$\boxed{\frac{\theta_j-\theta_s}{x_j-x_s}=\frac{\theta_f-\theta_s}{x_f-x_s}} \Rightarrow j=1 \Rightarrow \frac{\theta_1-30}{1.201-1}=\frac{120-30}{4-1} \Rightarrow \theta_1=36^\circ$$

$$\boxed{\begin{aligned}y_s&=x_s^{1.5}=1^{1.5}=1\\ y_f&=x_f^{1.5}=4^{1.5}=8\end{aligned}}$$

$$\Rightarrow j=2 \Rightarrow \frac{\theta_2-30}{2.5-1}=\frac{120-30}{4-1} \Rightarrow \theta_2=75^\circ$$

$$\Rightarrow j=3 \Rightarrow \frac{\theta_3-30}{3.8-1}=\frac{120-30}{4-1} \Rightarrow \theta_3=114^\circ$$

Value of ϕ

$$\boxed{\frac{\phi_j-\phi_s}{y_j-y_s}=\frac{\phi_f-\phi_s}{y_f-y_s}} \Rightarrow j=1 \Rightarrow \frac{\phi_1-0}{1.316-1}=\frac{90-0}{8-1} \Rightarrow \phi_1=4.06^\circ$$

$$\Rightarrow j=2 \Rightarrow \frac{\phi_2-0}{3.952-1}=\frac{90-0}{8-1} \Rightarrow \phi_2=37.467^\circ$$

$$\Rightarrow j=3 \Rightarrow \frac{\phi_3-0}{7.41-1}=\frac{90-0}{8-1} \Rightarrow \phi_3=82.50^\circ$$

Value of θ : 36.03 75° 113.97°

Value of ϕ : 4.063 37.967° 82.35°

Displacement eq given by Freudenstein eq written as

$$\boxed{\cos(\theta-\phi) = k_1\cos\phi - k_2\cos\theta + k_3}$$

$\cos(36.03-4.063) = k_1\cos 4.063 - k_2\cos 36.03 + k_3$ — ①

$\cos(75-37.967) = k_1\cos 37.967 - k_2\cos 75 + k_3$ — ②

$\cos(113.97-82.35) = k_1\cos 82.35 - k_2\cos 113.97 + k_3$ — ③

① ⇒ $0.848 = 0.997k_1 - 0.808k_2 + k_3$ — ④

② ⇒ $0.7982 = 0.788k_1 - 0.2588k_2 + k_3$ — ⑤

③ ⇒ $0.8515 = 0.133k_1 + 0.406k_2 + k_3$ — ⑥

Solving above eq ⇒ we get $k_1 = -0.2838$, $k_2 = -0.198$, $k_3 = 0.97$

Assuming $l_1 = 1$, we get ⇒ $k_1 = \frac{l_1}{l_2} = 0.2838 \Rightarrow l_2 = \frac{-1}{0.2838} = -3.52$

$k_2 = \frac{l_1}{l_4} = -0.198 \Rightarrow l_4 = \frac{-1}{0.198} = -5.02$

$k_3 = \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2l_2l_4} = 0.97 \Rightarrow \frac{1^2 + (-3.52)^2 - l_3^2 + (-5.02)^2}{2\times -3.52\times -5.02}$

$\Rightarrow l_3 = 2.07$

* Synthesis a 4-bar linkage that will in one of its position, satisfy the following values of angular displacement $y = x^{1.2}$ for $1 \le x \le 5$ using chebyshev spacing for 3 precision points Take $\phi_0 = 30^\circ$, $\psi_0 = 60^\circ$ $\Delta\phi = \Delta\psi = 90^\circ$ and $r_1 = 10cm$.

Solution

$x_s \le x \le x_f \Rightarrow 1 \le x \le 5$

$x_s = 1, x_f = 5, n = 3$

using chebyshev spacing

$$\boxed{x_j = a - b\cos\left(\frac{2j-1}{2n}\pi\right)}$$

$\boxed{y = x^{1.2}}$

$a = \frac{x_s + x_f}{2} = \frac{1+5}{2} = 3$

$b = \frac{x_f - x_s}{2} = \frac{5-1}{2} = 2$

$j=1 \Rightarrow x_1 = 3 - 2\cos\left(\frac{2\times1-1}{2\times3}\pi\right) = 1.2679 \Longrightarrow y_1 = x_1^{1.5} = 1.2679^{1.2} = 1.3295$

$j=2 \Rightarrow x_2 = 3 - 2\cos\left(\frac{2\times2-1}{2\times3}\pi\right) = 3 \Longrightarrow y_2 = x_2^{1.5} = 3^{1.2} = 3.7372$

$j=3 \Rightarrow x_3 = 3 - 2\cos\left(\frac{2\times3-1}{2\times3}\pi\right) = 4.732 \Longrightarrow y_3 = x_3^{1.5} = 4.732^{1.2} = 6.4573$

Note ⇒ $x_s = 1$, $y_s = x_s^{1.2} = 1^{1.2} = 1$

$x_f = 5$, $y_f = x_f^{1.2} = 5^{1.2} = 6.8986$

Here ϕ is input angle

$$\frac{\phi_1-\phi_s}{x_j-x_s} = \frac{\phi_f-\phi_s}{x_f-x_s} \quad \text{or} \quad \boxed{\frac{\phi_1-\phi_s}{x_j-x_s} = \frac{\Delta\phi}{x_f-x_s}}$$

$\phi_0 = \phi_s = 30$

$j=1 \Rightarrow \frac{\phi_1-30}{1.2679-1} = \frac{90}{5-1} \Rightarrow \phi_1 = \underline{\underline{36.027^\circ}}$

$j=2 \Rightarrow \frac{\phi_2-30}{3-1} = \frac{90}{5-1} \Rightarrow \phi_2 = \underline{\underline{75^\circ}}$

$j=3 \Rightarrow \frac{\phi_3-30}{4.732-1} = \frac{90}{5-1} \Rightarrow \phi_3 = \underline{\underline{113.97^\circ}}$

$\psi_s = \psi_0 = 60^\circ$

Here ψ output angle

$$\boxed{\frac{\psi_1-\psi_s}{y_j-y_s} = \frac{\Delta\psi}{y_f-y_s}}$$

$j=1 \Rightarrow \frac{\psi_1-60}{1.3295-1} = \frac{90}{6.8986-1} \Rightarrow \psi_1 = \underline{\underline{65.0274^\circ}}$

$j=2 \Rightarrow \frac{\psi_2-60}{3.7372-1} = \frac{90}{6.8986-1} \Rightarrow \psi_2 = \underline{\underline{101.763^\circ}}$

$j=3 \Rightarrow \frac{\psi_3-60}{6.4573-1} = \frac{90}{6.8986-1} \Rightarrow \psi_3 = \underline{\underline{143.2664}}$

Input angle	ϕ	36.027°	75°	113.97°
output angle	ψ	65.0274°	101.763°	143.2664°

using Freudenstein's eq $\cos(\phi-\psi) = k_1\cos\psi - k_2\cos\phi + k_3$

$\cos(65.0274-36.027) = k_1\cos 36.027 - k_2\cos 65.0274 + k_3$ —— (1)

$\cos(75-101.763^\circ) = k_1\cos 101.763 - k_2\cos 75 + k_3$ —— (2)

$\cos(143.97-143.2664) = k_1\cos 143.2664 - k_2\cos 113.97 + k_3$ —— (3)

Solving above eq we get

$k_1 = -0.2688,\ k_2 = -0.2729,\ k_3 = 0.7675$

Assume input link $r_1 = l_1 = 10cm$

$k_1 = \frac{l_1}{l_2} \Rightarrow \frac{10}{l_2} = -0.2688 \Rightarrow l_2 = \underline{\underline{-37.2cm}}$

$k_2 = \frac{l_1}{l_4} \Rightarrow \frac{10}{l_4} = -0.2729 \Rightarrow l_4 = \underline{-36.64cm}$

$k_3 = \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2l_2l_4} = 0.7675 \Rightarrow \frac{1^2+(-37.2)^2-l_3^2+(-36.64)^2}{2\times-37.2\times-36.64} = 0.7675$

$l_3 = \underline{\underline{27.78cm}}$

* Synthesis a 4 bar mechanism to generate a function $y = \sin x$ for $0 \le x \le 90°$. The range of the output crank may be chosen as 60° while that of input crank be 120°. Assume three precision points which are to be form Chebyshevs spacing. Length of fixed link = 52.5mm and $\phi_1 = 105°$, $\psi_1 = 66°$

Solution

From Chebyshev's spacing $\boxed{x_j = a - b\cos\left(\frac{2j-1}{2n}\pi\right)}$

where $x_f = 90$; $x_s = 0$ $\Rightarrow a = \frac{x_f + x_s}{2} = \frac{90+0}{2} = 45°$, $b = \frac{x_f - x_s}{2} = \frac{90-0}{2} = 45°$

n = No. of precision points = 3

$[j=1] \Rightarrow x_1 = 45 - 45\cos\left(\frac{2\times1-1}{2\times3}\pi\right) = 6°$

$[j=2] \Rightarrow x_2 = 45 - 45\cos\left(\frac{2\times2-1}{2\times3}\pi\right) = 45°$

$[j=3] \Rightarrow x_3 = 45 - 45\cos\left(\frac{2\times3-1}{2\times3}\pi\right) = 84°$

corresponding value of y

$y_1 = \sin x_1 = \sin 6 = 0.1045$

$y_2 = \sin x_2 = \sin 45 = 0.7071$

$y_3 = \sin x_3 = \sin 84 = 0.9945$

Also $y_s = \sin x_s = \sin 0 = 0$

$y_f = \sin 90 = 1$

Range of x, $\Delta x = x_f - x_s = 90$

Range of y $\Delta y = y_f - y_s = 1$

Range of input ϕ & output ψ

$\Delta\phi = 120°$, $\Delta\psi = 60°$

$\phi_1 = 105°$, $\psi_1 = 66°$

$\boxed{\frac{\phi_2 - \phi_s}{x_2 - x_s} = \frac{\Delta\phi}{\Delta x}} \rightarrow \boxed{\frac{\phi_2 - \phi_1}{x_2 - x_1} = \frac{\Delta\phi}{\Delta x}}$

when $\frac{\phi_2 - \phi_1}{x_2 - x_1} = \frac{\Delta\phi}{\Delta x} \Rightarrow \frac{\phi_2 - 105}{45 - 6} = \frac{120}{90} \Rightarrow \phi_2 = 157°$

$\frac{\phi_3 - \phi_2}{x_3 - x_2} = \frac{\Delta\phi}{\Delta x} \Rightarrow \frac{\phi_3 - 157}{84 - 45} = \frac{120}{90} \Rightarrow \phi_3 = 209°$

Similarly $\boxed{\frac{\psi_2 - \psi_1}{y_2 - y_1} = \frac{\Delta\psi}{\Delta y}} \Rightarrow \frac{\psi_2 - 66}{0.7071 - 0.1045} = \frac{60}{1} \Rightarrow \psi_2 = 102.2°$

$\frac{\psi_3 - \psi_2}{y_3 - y_2} = \frac{\Delta\psi}{\Delta y} \Rightarrow \frac{\psi_3 - 102.2°}{0.9945 - 0.7071} = \frac{60}{1} \Rightarrow \psi_3 = 119.4°$

$\phi_1 = 105°$	$\phi_2 = 157°$	$\phi_3 = 209°$
$\psi_1 = 66°$	$\psi_2 = 102.2°$	$\psi_3 = 119.4°$

Function Generation – Slider crank Mechanism [3R-1P Mechanism]

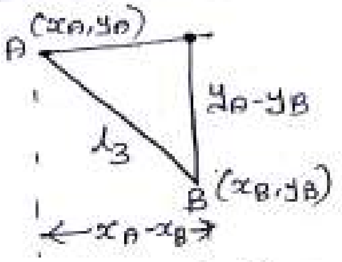
Three position Synthesis

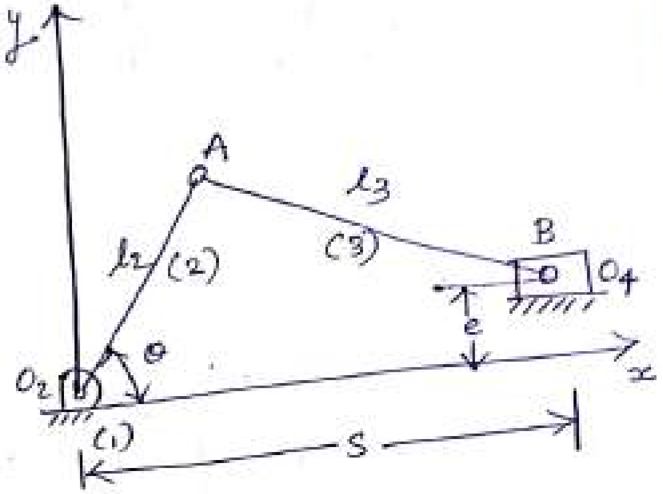
Kinematic diemension $\Rightarrow l_1, l_2$ & e

$x_A = l_2\cos\theta$, $y_A = l_2 \sin\theta$

$x_B = S$, $y_B = e$ (offset)

$$\boxed{AB^2 = l_3^2 = (x_A - x_B)^2 + (y_A - y_B)^2}$$





$$\Rightarrow l_3^2 = (l_2\cos\theta - s)^2 + (l_2\sin\theta - e)^2$$

$$\Rightarrow l_3^2 = l_2^2\cos^2\theta - 2l_2 s\cos\theta + s^2 + l_2^2\sin^2\theta - 2l_2 e\sin\theta + e^2$$

$$\Rightarrow l_3^2 = l_2^2 + s^2 + e^2 - 2sl_2\cos\theta - 2el_2\sin\theta$$

$$\Rightarrow s^2 = +2sl_2\cos\theta + 2el_2\sin\theta + l_3^2 - l_2^2 - e^2$$

$$\boxed{s^2 = D_1 s\cos\theta + D_2\sin\theta + D_3}$$

Where $D_1 = 2l_2$, $D_2 = 2el_2$, $D_3 = l_3^2 - l_2^2 - e^2$

1. Synthesise an offset slider crank mechanism so that displacement of slider is propotional to the square of crank rotation in the interval of $45 \le \theta \le 135$. The distance of slider from crank shaft (S) should be 10cm for $\theta = 45°$ and 3cm for $\theta = 135°$. use 3 chebyshev's accuracy points?

Solution

Displacement eq^n's $\Rightarrow$ $\boxed{s^2 = D_1 s\cos\theta + D_2\sin\theta + D_3}$

D_1, D_2 & D_3 the design variables

[i & f denote intial & Final values]

$S_i = 10\text{cm}$, $S_f = 3\text{cm}$

$S_f - S_i \propto (\theta_f - \theta_i)^2$

$S_f - S_i = c\,(\theta_f - \theta_i)^2$

$$c = \frac{S_f - S_i}{(\theta_f - \theta_i)^2} = \frac{3-10}{(135-45)^2} = \frac{-7}{90^2}$$

c = constant of propotionality

using Chebyshev's spacing?

$$\theta_j = \boxed{x_j = a - b\cos\left[\frac{2j-1}{2n}\pi\right]}$$

$a = \frac{\theta_i+\theta_f}{2} = \frac{45+135}{2} = 90^\circ$, $b = \frac{\theta_f-\theta_i}{2} = \frac{135-45}{2} = \underline{\underline{45^\circ}}$

$j=1 \Rightarrow \theta_1 = 90 - 45\cos\left(\frac{2\times1-1}{2\times3}\pi\right) = \underline{\underline{51.3^\circ}}$

$j=2 \Rightarrow \theta_2 = 90 - 45\cos\left(\frac{2\times2-1}{2\times3}\pi\right) = \underline{\underline{90^\circ}}$

$j=3 \Rightarrow \theta_3 = 90 - 45\cos\left(\frac{2\times3-1}{2\times3}\pi\right) = \underline{\underline{128.97^\circ}}$

Relation b/w s & θ $\boxed{s - s_i = c(\theta-\theta_i)^2}$

Sub $s_f = s_1$, $s_i = 10$, $c = \frac{-7}{8100}$, $\theta_f = \theta_1 = 51.3^\circ$, $\theta_i = 45^\circ$

$(s=s_1) \Rightarrow s_1 - 10 = \frac{-7}{8100}(51.3-45)^2 \Rightarrow s_1 = \underline{\underline{9.97\,cm}}$

$(s=s_2) \Rightarrow s_2 - 10 = \frac{-7}{8100}(90-45)^2 \Rightarrow s_2 = \underline{\underline{8.25\,cm}}$

$(s=s_3) \Rightarrow s_3 - 10 = \frac{-7}{8100}(128.97-45)^2 \Rightarrow s_3 = \underline{3.9\,cm}$

Displacement eq $\Rightarrow$ $\boxed{s^2 = D_1 s\cos\theta + D_2\sin\theta + D_3}$

$s_1^2 = D_1 s_1\cos\theta_1 + D_2\sin\theta_1 + D_3$

$9.97^2 = D_1 \times 9.97 \times \cos 51.3 + D_2 \sin 51.3 + D_3$ —— (1)

$s_2^2 = D_1 s_2\cos\theta_2 + D_2\sin\theta_2 + D_3$

$8.25^2 = D_1 \times 8.25\cos 90 + D_2\sin 90 + D_3$ —— (2)

$s_3^2 = D_1 s_3\cos\theta_3 + D_2\sin\theta_3 + D_3$

$3.91^2 = D_1 \times 3.91 \times \cos 128.97 + D_2 \times \sin 128.97 + D_3$ —— (3)

Solving eq (1), (2) & (3), we get $\Rightarrow D_1 = 9.61$, $D_2 = 130.75$, $D_3 = 62.75$

$D_1 = 2l_2 \Rightarrow 9.61 = 2l_2 \Rightarrow l_2 = 4.805\,cm$

$D_2 = 2el_2 \Rightarrow 130.75 = 2e \times 4.805 \Rightarrow e = 13.606\,cm$

$D_3 = l_3^2 - l_2^2 - e^2 \Rightarrow 62.75 = l_3^2 - 4.805^2 - 13.606^2 \Longrightarrow \underline{\underline{l_3 = 13.606\,cm}}$

Q. Synthesis a function generate to solve the eq $y = \frac{1}{x}$, $1 \le x \le 2$ using 3 precision points?

Hints Assumption $\Delta\phi = 90^\circ$ $\phi_s = 30^\circ$
$\Delta\psi = 90^\circ$ $\psi_s = 200^\circ$

- Synthesise a 4-bar linkage to generate $y = \log_{10} x$ in the interval $1 \le x \le 10$. The input crank length is to be 5cm. The input crank is to rate from 45° to 105° while the output crank move from 135° to 225°. Use three accuracy points with Chebyshevs spacing.

ϕ - input crank angle
ψ - output crank angle

Solution

Here $\phi_s = 45$, $\phi_f = 105^\circ \Rightarrow \Delta\phi = 105 - 45^\circ = 60^\circ$

$\psi_s = 135^\circ$, $\psi_f = 225^\circ \Rightarrow \Delta\psi = 225 - 135 = 90^\circ$

Input crank length = 5cm

$x_s = 1, x_f = 10 \Rightarrow \Delta x = x_f - x_s = 10 - 1 = \underline{\underline{9}}$

$y = \log_{10} x, \; \Delta y = \log_{10} x_f - \log_{10} x_s = \log_{10} 10 - \log_{10} 1 = \underline{\underline{1}}$

Here $\boxed{x_j = a - b\cos\left(\frac{2j-1}{2n}\pi\right)}$

$a = \frac{x_s + x_f}{2} = \frac{10+1}{2} = 5.5$

$b = \frac{x_f - x_s}{2} = \frac{10-1}{2} = 4.5$

when $j = 1 \Rightarrow x_1 = 5.5 - 4.5\cos\left(\frac{2\times1-1}{2\times3}\right) = \_\_\_\_ \Rightarrow y_1 = \log_{10} x_1$

$j = 2 \Rightarrow x_2 = 5.5 - 4.5\cos\left(\frac{2\times2-1}{2\times3}\right) = \_\_\_\_ \Rightarrow y_2 = \log_{10} x_2 =$

$j = 3 \Rightarrow x_3 = 5.5 - 4.5\cos\left(\frac{2\times3-1}{2\times3}\right) = \_\_\_\_ \Rightarrow y_3 = \log_{10} x_3 =$

$$\frac{\phi_j - \phi_s}{x_j - x_s} = \frac{\phi_f - \phi_s}{x_f - x_s}$$

& using $\frac{\psi_j - \psi_s}{y_j - y_s} = \frac{\psi_f - \psi_s}{y_f - y_s}$

$\Downarrow$

Solve Yourself

Answers $\Rightarrow$ $\phi_1 = 49.02^\circ$ $\quad \phi_2 = 75^\circ$ $\quad \phi_3 = 100.98^\circ$

$\psi_1 = 153.441^\circ$ $\quad \psi_2 = 201.641^\circ$ $\quad \psi_3 = 222.57^\circ$

$k_1 = 2.004, \; k_2 = -0.6989, \; k_3 = 1.085$

Then solve for the link lengths

Graphical Method of Diemensional Synthesis

→ Graphical Method have advantage in that they are relatively fast in producing results and at the same time, they maintain touch with physical reality to a much larger extend than do the algebraic methods.

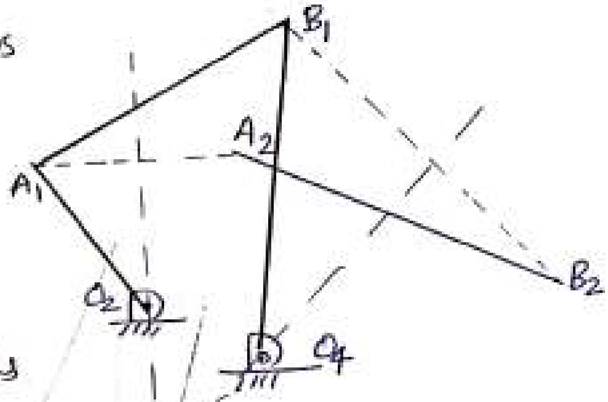
⇒ Also geometric Methods are easier to understand and the degree of accuracy is adequate for all purposes.

Graphical synthesis

Motion Generation (Rigid body Guidence)

Two position synthesis [coupler as the output]

The couple has to move for prescribed position ① to another prescribed position ②



Step1 : Draw link AB in two desribed positions A_1B_1 and A_2B_2

Step2: Connect A_1 to A_2 and B_1 to B_2

Step3: Draw two lines ⊥ular to A_1A_2 and B_1B_2 at the midpoint

Step4 : select two fixed pivot points O_2 and O_4 anywhere on the two midnormals

Step 5: Measure the length of all links

$$\boxed{\begin{array}{l} O_2A = \text{link }2,\ AB = \text{link }3 \\ O_4B = \text{link }4 \ \ \& \ O_2O_4 = \text{link }1 \end{array}}$$

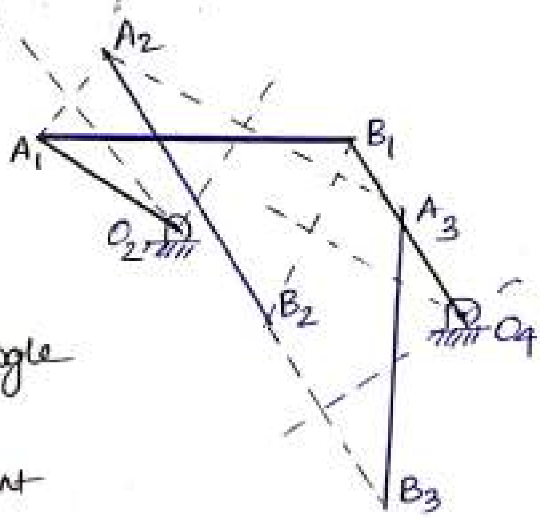
Three positions, coupler as the output

① Draw link AB in the 3 desired positions

② Draw Midnormals to AA_1 and A_2A_3 the intersection locates the fixed pivot point O_2. Same for point B to obtain second pivot poin O_4

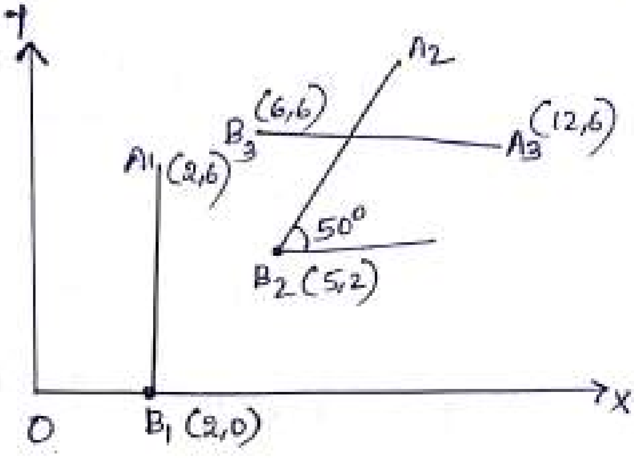
③ Check the accuracy of the mechanism Grashof condition and transmission angle

④ Change the second position of link AB to Vary the locations of fixed point

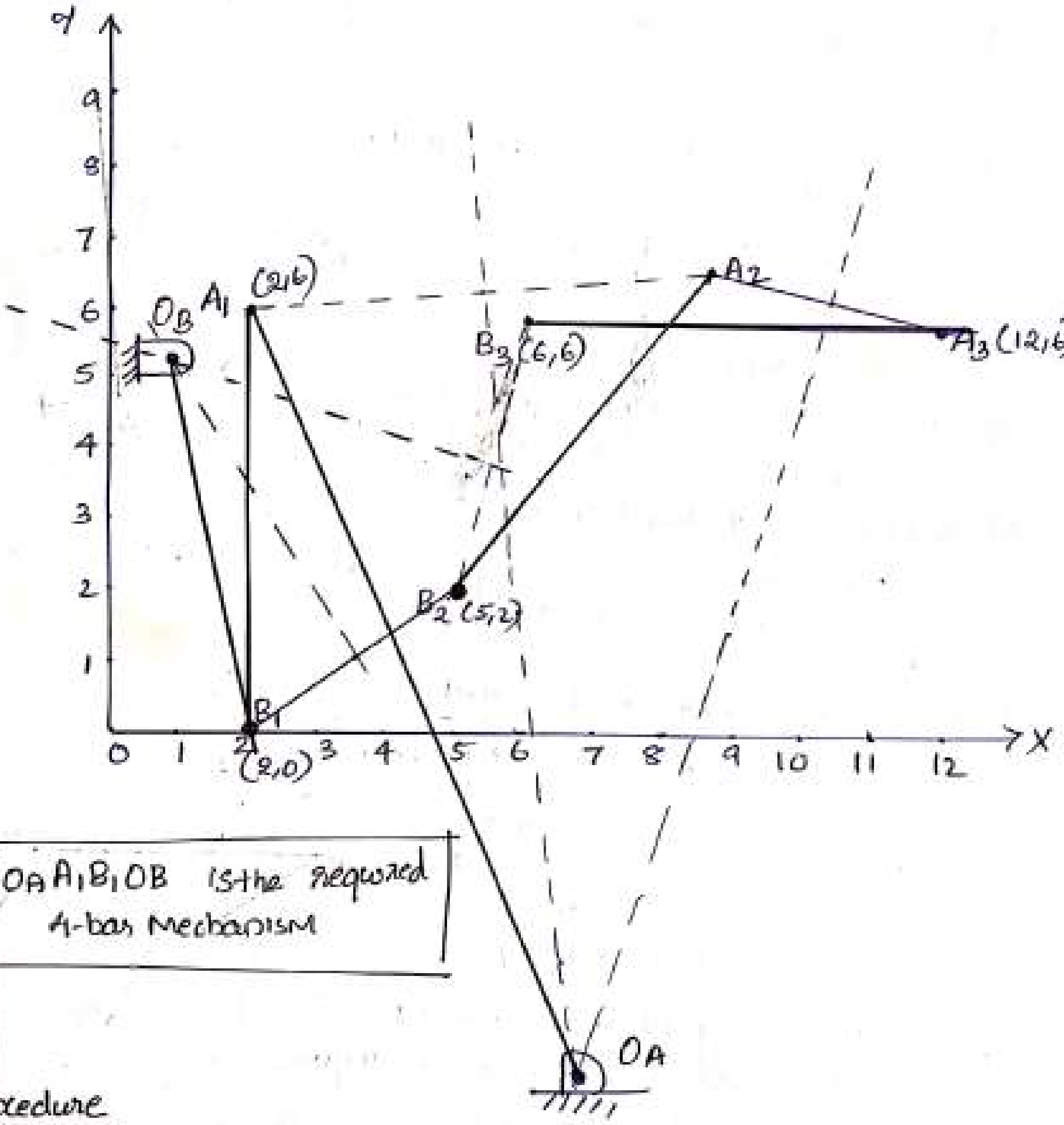


• Synthesis a 4 bar Mechanism to guide a rod AB through three Consecutive positions. A_1B_1, A_2B_2 and A_3B_3 as shown in figure below.

(Imp)



Answer



Here $O_A A_1 B_1 O_B$ is the required 4-bar Mechanism

procedure

- Here draw A_1B_1, A_2B_2, A_3B_3 in the suitable dimension using co-ordinate mension in fig.
- Draw Midnormal by Join A_2 & A_3 and Draw Midnormal By Joining A_1 & A_2. Both Midnormal coincide at O_A.
- Similarly Join B_3B_2 and B_1B_2 Draw Midnormals which Joint at O_B

Scanned by CamScanner

FUNCTION GENERATION
BY INVERSION METHOD

Graphical Method for synthesizes function generation problem is similar to one used for motion generation. Here too, kinematic inversion and intersection of Midnormals used to locate poles.

FOUR-BAR MECHANISM

(a) Two position synthesis

Distance b/w fixed link pivot A & D known. Angle θ_{12} and ψ_{12} known

Steps

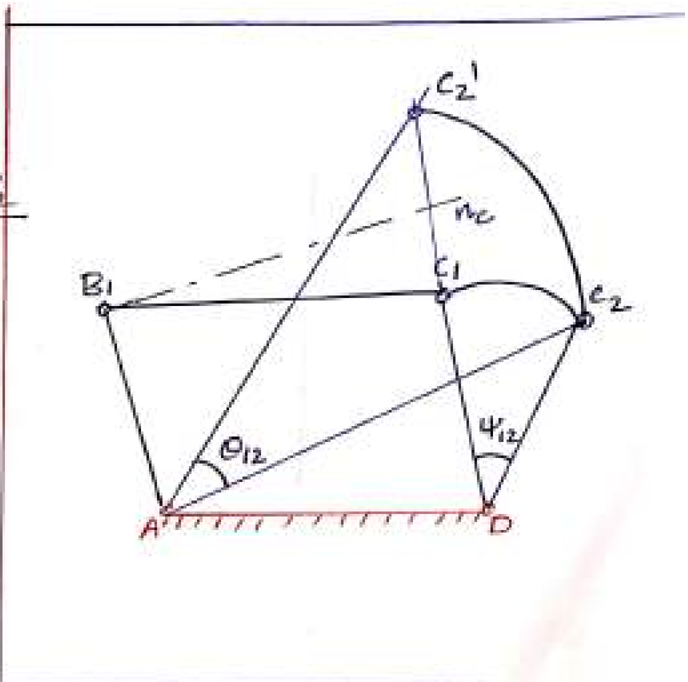
① Draw segment AD of length equal to the distance b/w fixed pivots.

② At point D, construct an angle $\angle C_1DC_2 = \psi_{12}$ clockwise at arbitary position, selecting a suitable output crank length $DC_1 = DC_2$

③ Rotate point C_2 in the C.C.W direction through angle θ_{12} with A as centre and obtain point C_2'.

④ Joint C_1C_2' and draw its Midnormal. Select a suitable point B_1 on it

AB_1C_1 is required 4 link Mechanism in which B_1C_1 is the coupler



V.imp

problem

① Design a 4-link Mechanism to co-ordinate 3 position of the input and of the output link for the following angular displacement by inversion Method.

$\theta_{12} = 35°$ $\psi_{12} = 50°$

$\theta_{13} = 80°$. $\psi_{13} = 80°$

(b) Three position Synthesis

Two angular displacement of input link $(\theta_{12}, \theta_{13})$

Two angular displacement of output link (ψ_{12}, ψ_{13})

Steps

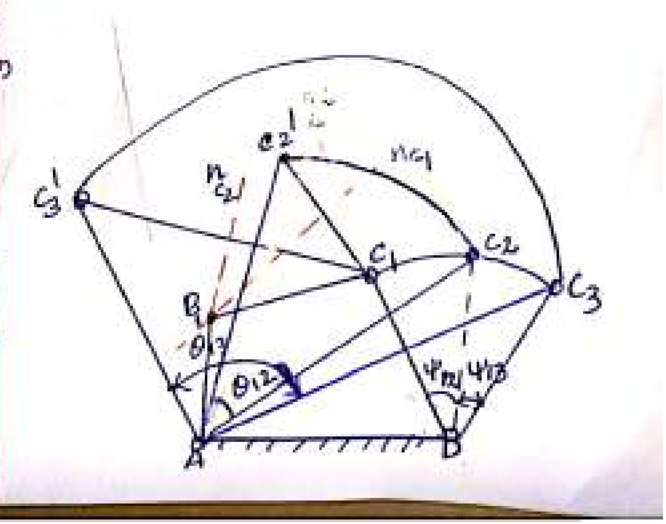
① Draw segment AD of length equal to distance b/w fixed pivots.

② Choose some suitable length of output link DC. Draw it some suitable angle with fixed link AD. Locate 3 position DC_1, DC_2 & DC_3 as its angular displacement known

③ Find point C_2' and C_3' after rotating AC_2 and AC_3 about A through angle θ_{12} and θ_{13} r.ply in CCW direction.

④ Intersection of Midnormal of C_1C_2' & C_1C_3' locate point B_1

$AB_1C_1D_1$ is required 4-bar Mechanism



SLIDER-CRANK MECHANISM

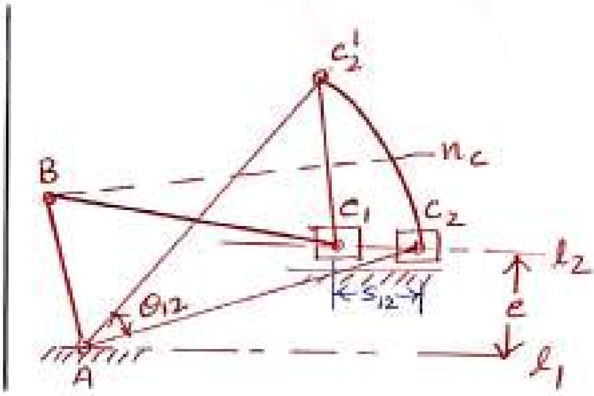
* Two position Synthesis

Angular displacement of input link θ_{12} – known
Linear displacement of Slider S_{12} – known
Eccentricity, e – known

Imp

Steps

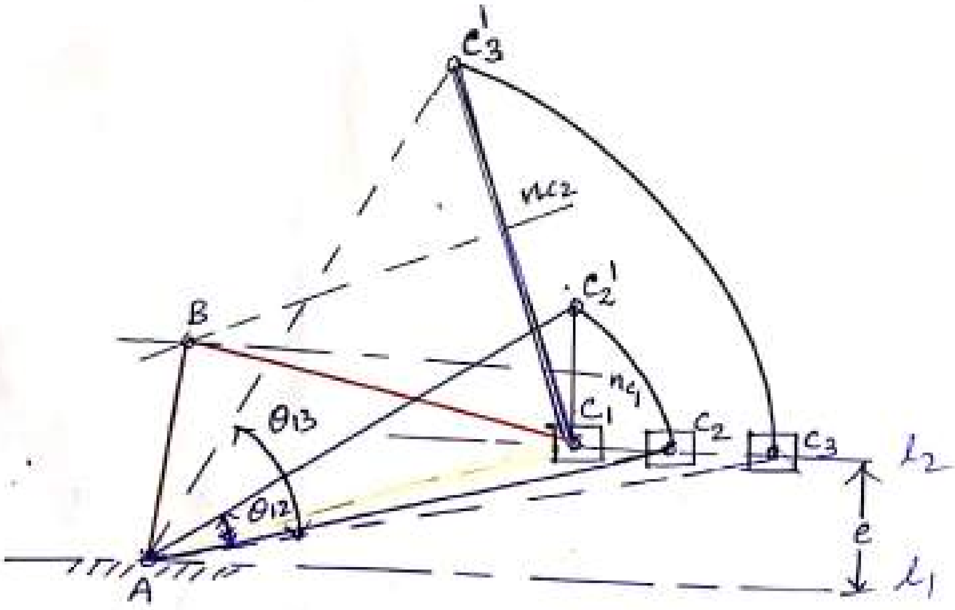
1. Draw two parallel lines l_1 & l_2 at a distance 'e' apart.
2. Take an arbitary point A on the line l_1 for the fixed pivot and two points C_1 and C_2 on the line l_2, a distance S_{12} apart for the intial and final position of Slider.
3. Rotate the point C_2 about A through an angle θ_{12} in the C.C.W direction to obtain point C_2'
4. Joint C_1C_2' and draw its Midnormal n_c. Take an arbitary, but Covenient point B on it. ABC$_1$ is the required slider crank Mechanism



* Three position synthesis

For 3 position synthesis of the input & three position of slider.
Find C_2' & C_3' as usual. Then Midpoint of C_1C_2' and C_1C_3' intersect at point B.

Imp



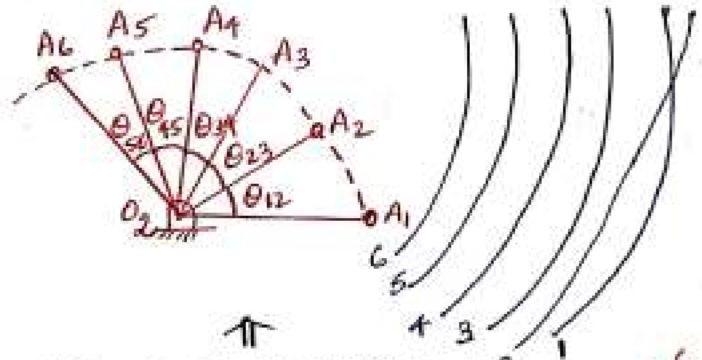
ABC$_1$ is required slider crank Mechanism

Functional Synthesis Overlay Method

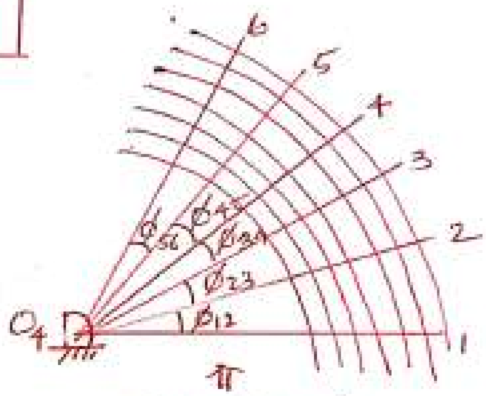
- It is graphical trial and error procedure for the synthesis of mechanism to generate a sequence of specified positions of the output link.
- Overlay Method is extremely versatile, easiest and quickest of all Method to use and widely applicable in design of mechanism

Suppose we have 6 different position of input link and uniform spacing of output rocker.

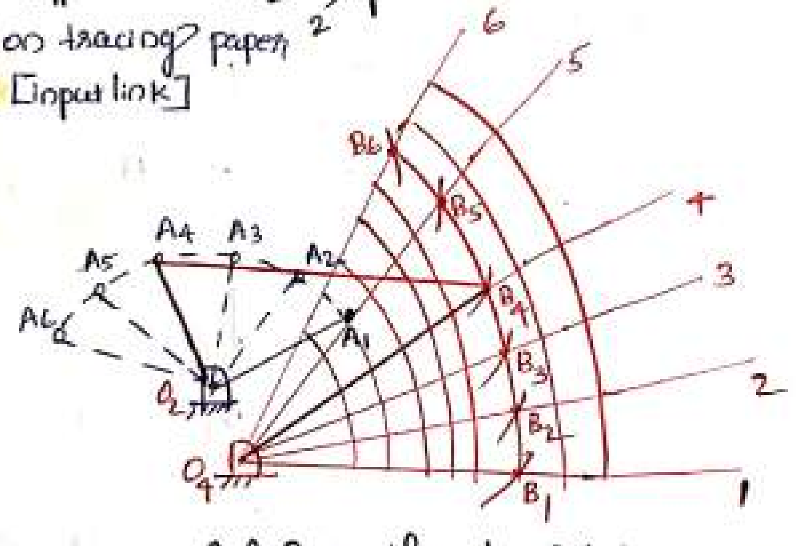
→ Given length of input link O_2A
→ Input angles $\theta_{12}, \theta_{23}, \theta_{34}, \theta_{45}, \theta_{56}$
→ output angles $\phi_{12}, \phi_{23}, \phi_{34}, \phi_{45}, \phi_{56}$



Drawing on tracing paper [input link]



Output rocker



$O_2A_4B_4O_4$ four-bar Mechanism

procedure

① Construct the input rocker O_2A in all its given positions on a sheet of tracing paper.

② choose an arbitary length of the coupler AB and draw arcs numbered 1 to 6 using A_1 to A_6 as centre respectively and coupler length AB as the arc radius

Scanned by CamScanner

③ On another sheet of paper construct output rocker whose length is unknown in all its given positions.

④ Number the positions of output rocker from 1 to 6 as ϕ_{12}, ϕ_{23}

⑤ Through O_4 as centre draw a no. of arbitrary spaced arc intersecting the line $O_4 1$, $O_4 2$ and so on. These arc represent the possible length of the output rocker.

⑥ place tracing paper over the drawing and Manipulate a proper fitting such that the arc 1 to 6 on the tracing paper cut the same arc (from 1 to 6) of the output rocker and that too at the points of intersection of the lines $O_4 1, O_4 2, O_4 3, O_4 4, O_4 5, O_4 6$ the arc of output rocker.

- It Means that arc 1 of the input link cut the line O_4^1 at point B_1 and so on arc 2 of input link cuts line O_4^2 at point B_2

⑨ Joint any point (A_1 to A_6) on input rocker end to the corresponding output rocker end (B_1 to B_6)

⑧ Figure Shows $O_2 O_4 B_4 O_4$ is the required mechanism.

Analytical Synthesis Techniques

① Explain & derive Freudensteins Equation.

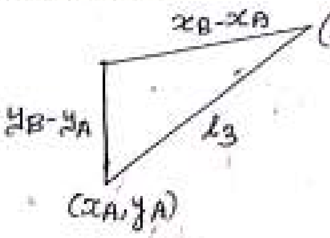
An:

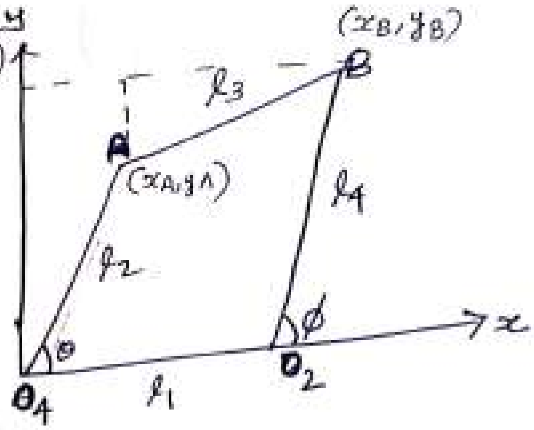
$x_B = l_1 + l_4 \cos\phi$

$x_A = l_2 \cos\theta$

$y_A = l_2 \sin\theta$

$y_B = l_4 \sin\phi$





$$l_3^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

$$l_3^2 = (l_1 + l_4\cos\phi - l_2\cos\theta)^2 + (l_4\sin\phi - l_2\sin\theta)^2$$

$$l_3^2 = (l_1^2 + 2l_4 l_1\cos\phi + l_4^2\cos^2\phi - 2(l_1 + l_4\cos\phi) l_2\cos\theta + l_2^2\cos^2\theta) + (l_4^2\sin^2\phi - 2l_2 l_4 \sin\theta\sin\phi + l_2^2\sin^2\theta)$$

$$l_3^2 = l_1^2 + 2l_4 l_1\cos\phi + \underline{l_4^2\cos^2\phi} - 2l_1 l_2\cos\theta - 2l_2 l_4\cos\theta\cos\phi + \underline{l_2^2\cos^2\theta} + \underline{l_2^2\sin^2\theta} + \underline{l_4^2\sin^2\phi} - 2l_2 l_4\sin\theta\sin\phi$$

$$l_3^2 = l_1^2 + l_2^2 + l_4^2 + 2l_4 l_1\cos\phi - 2l_1 l_2\cos\theta - 2l_2 l_4\cos(\theta - \phi)$$

$$2l_2 l_4\cos(\theta - \phi) = l_1^2 + l_2^2 + l_4^2 + 2l_4 l_1\cos\phi - 2l_1 l_2\cos\theta - l_3^2$$

$$\cos(\theta-\phi) = \frac{l_1}{l_2}\cos\phi - \frac{l_1}{l_4}\cos\theta + \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2l_2l_4}$$

where $k_1 = \frac{l_1}{l_2}$, $k_2 = \frac{l_1}{l_4}$, $k_3 = \frac{l_1^2+l_2^2-l_3^2+l_4^2}{2l_2l_4}$

$$\boxed{\cos(\theta-\phi) = k_1\cos\phi - k_2\cos\theta + k_3}$$

Velocity and Acceleration Synthesis using Complex Number Method

6.12 VELOCITY AND ACCELERATION SYNTHESIS USING COMPLEX NUMBER METHODS

The problems of synthesis have been solved by using the displacement equation. The complex number method follows the same general pattern, but the displacement equation will be written in terms of complex numbers. The complex number method considers angles of cranks and distances of sliders, in addition to vectors, to express the arbitrary motions of points in a plane analytically.

Consider four-bar linkage $O_A ABO_B$ as shown in **Fig. 6.10**. Here the link 4 is stationary, but the other three links (1, 2, and 3) have angular velocities ω_1, ω_2 and ω_3 and angular accelerations α_1, α_2 and α_3. This method is used to find not only the link lengths a_1, a_2, a_3 and a_4 but also the relative positions of the links satisfying angular velocity and acceleration specifications.

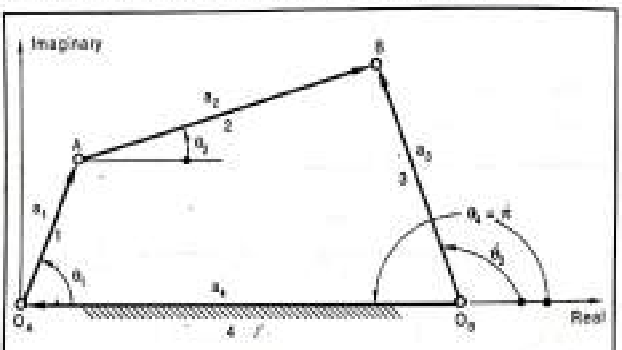


Fig: 6.10 Four - bar linkage and vector polygon

The four-bark linkage $O_A ABO_B$ is considered as four vectors. The linkage is considered as a closed vector polygon, Fig. and we can write

$$a_4 + a_1 + a_2 = a_3$$

$$a_1 + a_2 - a_3 + a_4 = 0 \qquad (1)$$

The vectors can be written as $a = ae^{i\theta}$, where $a =$ distance and $\theta =$ counter clockwise angle measured from the real axis; Now vector equation of the polygon in complex-number form is

$$a_1 e^{i\theta_1} + a_2 e^{i\theta_2} - a_3 e^{i\theta_3} + a_4 e^{i\pi} = 0$$

Here $e^{i\pi} = -1$,

So $$a_1 e^{i\theta_1} + a_2 e^{i\theta_2} - a_3 e^{i\theta_3} - a_4 = 0 \qquad (2)$$

This equation represents the space relationship of the points O_A, A, B and O_B, the points of connection between links.

On differentiating (2) with respect to time, considering $\frac{d\theta}{dt} = \omega$

$$i(a_1 \omega_1) e^{i\theta_1} + i(a_2 \omega_2) e^{i\theta_2} - i(a_3 \omega_3) e^{i\theta_3} - a_4(0) = 0 \qquad (3)$$

This equation (3) is defining the linear velocities of the points and it represents the velocity-vector diagram.

A second differentiation considering $\frac{d\theta}{dt} = \omega$ and $\frac{d\omega}{dt} = \alpha$ gives,

$$i(a_1 \alpha_1) e^{i\theta_1} + i^2(a_1 \omega_1^2) e^{i\theta_1} + i(a_2 \alpha_2) e^{i\theta_2} + i^2(a_2 \omega_2^2) e^{i\theta_2}$$

$$- i(a_3 \alpha_3) e^{i\theta_3} - i^2(a_3 \omega_3^2) e^{i\theta_3} - (a_4)\,0 = 0 \qquad (4)$$

Equation (4) represents the acceleration-vector diagram.

On dividing by i and rearranging,

$$(\alpha_1 + i\,\omega_1^2)\, a_1 e^{i\theta_1} + (\alpha_2 + i\,\omega_2^2)\, a_2 e^{i\theta_2}$$

$$- (\alpha_3 + i\,\omega_3^2)\, a_3 e^{i\theta_3} - (a_4)\,0 = 0 \qquad (5)$$

Assembling Equations (2), (3) and (5) as a group and replacing each $ae^{i\theta}$ by its vector a yields

$$1a_1 + 1a_2 - 1a_3 + 1a_4 = 0$$

$$\omega_1 a_1 + \omega_2 a_2 - \omega_3 a_3 + 0a_4 = 0 \qquad (6)$$

$$(\alpha_1 + i\,\omega_1^2)\,a_1 + (\alpha_2 + i\,\omega_2^2)\,a_2 - (\alpha_3 + i\,\omega_3^2)\,a_3 + 0a_4 = 0$$

This is a system of three homogeneous equations consisting of four unknowns. Hence one of the unknowns, a_4 can be chosen arbitrarily and the system can be rewritten as

$$1a_1 + 1a_2 - 1a_3 = -a_4 \quad (7)$$

$$\omega_1\,a_1 + \omega_2\,a_2 - \omega_2\,a_3 = 0$$

$$(\alpha_1 + i\,\omega_1^2)\,a_1 + (\alpha_2 + i\,\omega_2)^2\,a_2 - (\alpha_3 + i\,\omega_3^2)\,a_3 = 0$$

The solution is obtained by determinants,

$$D = \begin{vmatrix} 1 & 1 & -1 \\ \omega_1 & \omega_2 & -\omega_3 \\ \alpha_1 + i\,\omega_1^2 & \alpha_2 + i\,\omega_2^2 & -(\alpha_3 + i\,\omega_3^2) \end{vmatrix} \quad (8)$$

The unknowns a_1, a_2, a_3 are expressed in complex-number form as

$$a_1 = \frac{-a_4\,[-\omega_2\,(\alpha_3 + i\,\omega_3^2) + \omega_3\,(\alpha_2 + i\,\omega_2^2)]}{D}$$

$$a_2 = \frac{-a_4\,[-\omega_3\,(\alpha_1 + i\,\omega_1^2) + \omega_1\,(\alpha_3 + i\,\omega_3^2)]}{D} \quad (9)$$

$$a_3 = \frac{-a_4\,[\omega_1\,(\alpha_2 + i\,\omega_2^2) - \omega_2\,(\alpha_1 + i\,\omega_1^2)]}{D}$$

Now the arbitrary a_4 is taken proportional to the determinant D as

$$a_4 = -D \quad (10)$$

and the above values of a_1, a_2, a_3 become independent of D and are expressed in simple form as shown in **Table 6.1.**

$$\text{Now } a_4 = -a_1 - a_2 + a_3 \quad (11)$$

In **Table 6.1**, the links are defined as complex numbers of the form $a = c + id$, whose real and imaginary parts are defined by the angular velocities and accelerations specified for the links.

Table 6.1: Four-bar Linkage

Vector or link (a)	Real component (c)	i component (d)	Complex form $a = c + id$	Length $a = \sqrt{c^2 + d^2}$
a_1	$c_1 = \omega_3\,\alpha_2 - \omega_2\,\alpha_3$	$d_1 = \omega_2\,\omega_3\,(\omega_2 - \omega_3)$	$a_1 = c_1 + id_1$	$a_1 = \sqrt{c_1^2 + d_1^2}$
a_2	$c_2 = \omega_1\,\alpha_3 - \omega_3\,\alpha_1$	$d_2 = \omega_3\,\omega_1\,(\omega_3 - \omega_1)$	$a_2 = c_2 + id_2$	$a_2 = \sqrt{c_2^2 + d_2^2}$
a_3	$c_3 = \omega_1\,\alpha_2 - \omega_2\,\alpha_1$	$d_3 = \omega_1\,\omega_2\,(\omega_2 - \omega_1)$	$a_3 = c_3 + id_3$	$a_3 = \sqrt{c_3^2 + d_3^2}$
a_4	$c_4 = c_3 - c_1 - c_2$	$d_4 = d_3 - d_1 - d_2$	$a_4 = c_4 + id_4$	$a_4 = \sqrt{c_4^2 + d_4^2}$

Problem 6.4: *Determine the links of a four-bar mechanism which satisfies the following specifications in one of its positions.*

Solution

$\omega_1 = 8$ rad/sec

$\omega_2 = 2$ rad/sec

$\omega_3 = -4$ rad/sec

$\alpha_1 = 0$

$\alpha_2 = 40$ rad/sec^2

$\alpha_3 = 0$

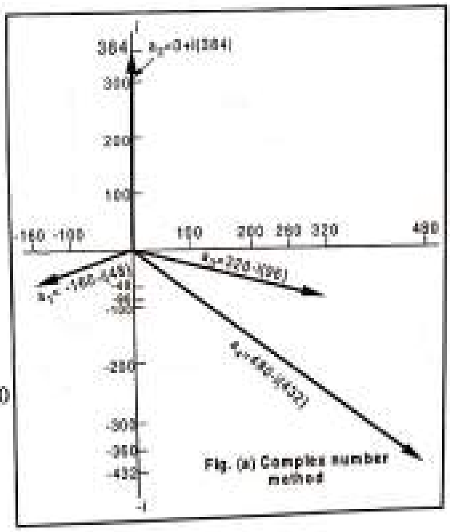


Fig. (a) Complex number method

Substituting values into the definitions of Table

$$c_1 = \omega_3\,\alpha_2 - \omega_2\,\alpha_3$$

$$= -4 \times 40 - 2 \times 0 = -160$$

$$c_1 = -160$$

$$d_1 = \omega_2\,\omega_3\,(\omega_2 - \omega_3)$$

$$= -2 \times 4\,(2 + 4) = -48$$

$a_1 = c_1 + id_1$

$a_1 = -160 - i\,(48)$

$a_1 = \sqrt{c_1^2 + d_1^2}$

$a_1 = \sqrt{160^2 + 48^2}$ = **167 units**

$c_2 = \omega_1\,\alpha_3 - \omega_3\,\alpha_1$

$= 8 \times 0 - [-4 \times 0] = 0$

$d_2 = \omega_3\,\omega_1\,(\omega_3 - \omega_1)$

$= -4 \times 8\,(-4 - 8) = 384$

$a_2 = c_2 + id_2$

$a_2 = 0 + i\,(384)$

$a_2 = \sqrt{384^2} = 384$ units

$a_2 = \sqrt{c_2^2 + d_2^2}$

$c_3 = \omega_1\,\alpha_2 - \omega_2\,\alpha_1$

$= 8 \times 40 - 2 \times 0 = 320$

$d_3 = \omega_1\,\omega_2\,(\omega_2 - \omega_1)$

$= 8 \times 2\,(2 - 8) = -96$

$a_3 = c_3 + id_3 = 320 - i\,(96)$

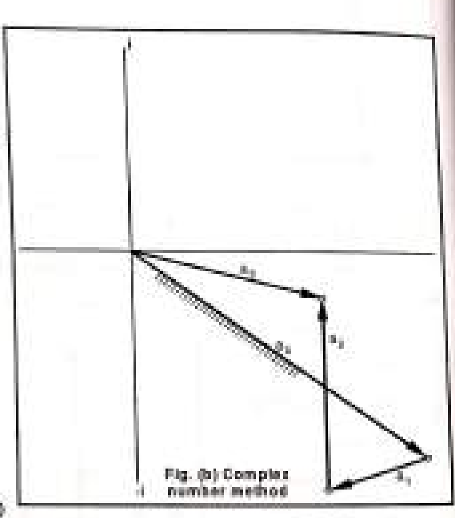


Fig. (b) Complex number method

$a_3 = a_3 = \sqrt{c_3^2 + d_3^2} = \sqrt{(320)^2 + (-96)^2} = 334$ units

$c_4 = c_3 - c_1 - c_2 = 320 - (-160) - (0) = 480$

$d_4 = d_3 - d_1 - d_2 = -96 - (-48) - (384) = -432$

$a_4 = c_4 + id_4 = 480 - i\,(432)$

$a_4 = \sqrt{c_4^2 + d_4^2} = \sqrt{480^2 + (-432)^2}$

$= 645.8$

The vectors represented by the complex numbers are shown in **Fig (a)**. The mechanism is formed by assembling the vectors in sequence, starting with a_4 (**Fig (b)**). The relative lengths of the bars and their terminal points are established as functions of ω and α and the bar a_4 is the fixed link.

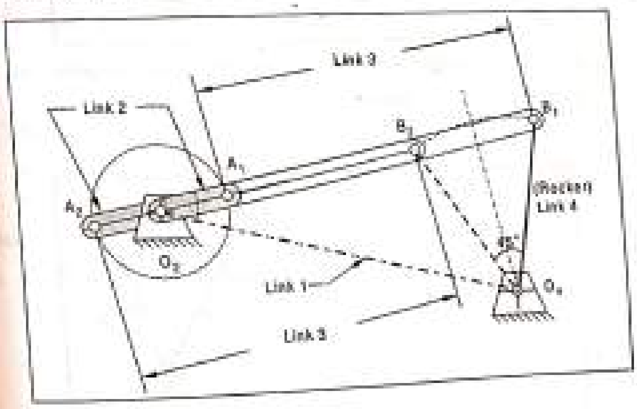
6.12.1 Dead points

A dead point occurs when the follower is at rest at a moment just prior to reversing its direction of rotation, ie when $\omega_3 = 0$. In continuously rotating crank, the crank and coupler are either (1) extended in a straight line or (2) folded over each other into a straight line at the dead points.

6.13 GRAPHICAL SYNTHESIS

6.13.1 Two position synthesis - Overlay method - Problems

Problem 6.5: *Rocker output - Two position with Angular Displacement (Function): Design a four bar Grashof crank-rocker speed motor input to give 45° of rocker motion with equal time forward and back, from a constant speed motor input.*



6.9 COUPLER CURVES

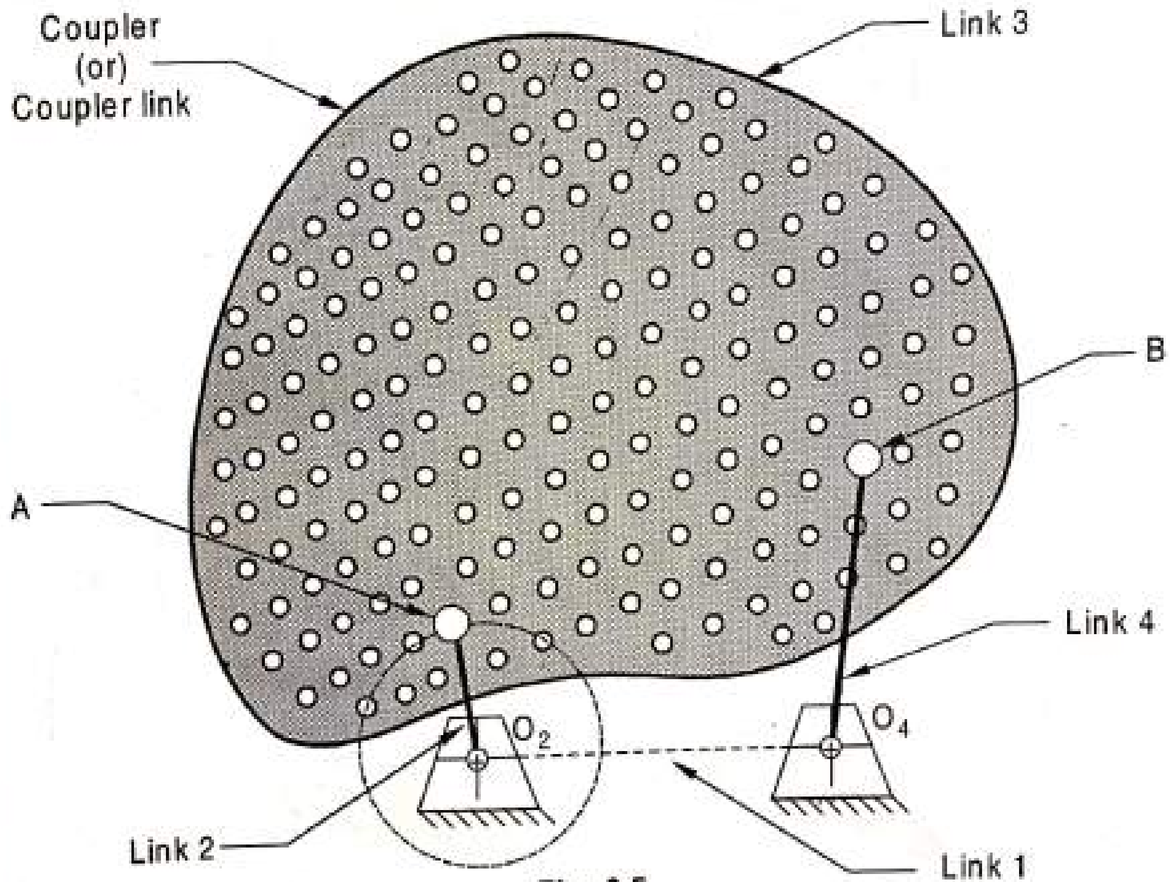


Fig. 6.5

Coupler curve is a curve (or) path traced out by a point located in the plane of the coupler link. Coupler link connects the input and output links through turning pairs. When the input link rotates, any point on coupler generates a path (or) curve called coupler curve.

In the Fig 6.5, the point A will trace a curve and point B will trace another curve. Here A and B are called coupler points. If the length of link 2 is increased, then the point A and B will trace different curves. Similarly, if the link 4 is changed, then the point A and B will trace different curves. Hence various coupler curve shapes are obtained by varying the lengths of link 2 and 3.

Various coupler curve shapes are shown in **Fig. 6.6.**

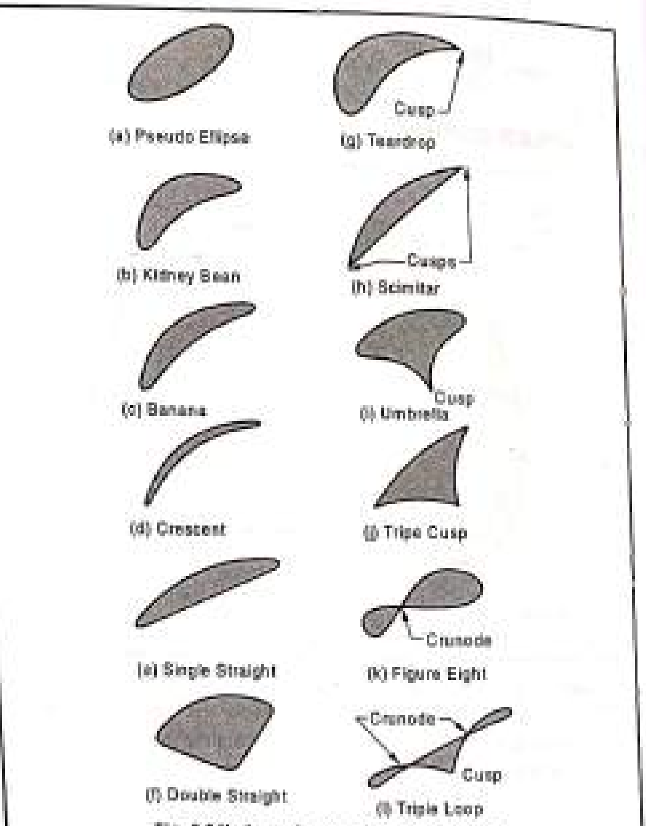


Fig. 6.6 Various shapes of coupler curves

Generally, the crank length (Link 2) is taken as unity and the length of other links are varied to get different mechanisms. The designer of machine components has to visualize the motion of machine components and hence these coupler curves are very useful and invaluable to the machine designer who needs a mechanism to generate a curve with specified characteristics.

Normally a sixth order algebraic equation is required for coupler curves. The coupler curves have variety of shapes like

- Ellipse
- Circle
- Straight line
- Banana
- Arcs
- Cusps and so on as shown in **Fig 6.7**

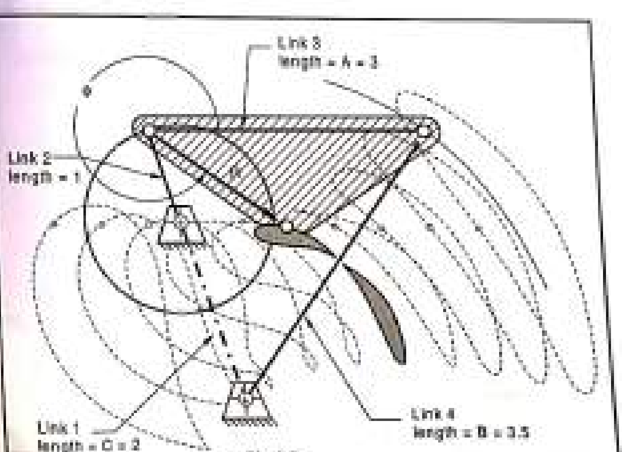


Fig:6.7

Roberts Chebychev's Theorem of Cognate linkages states that the same coupler curve can always be generated by three different linkages called **cognates.**

Chapter - 6

6.1 *What is Kinematic Synthesis?*

Kinematic analysis is the study of motion of mechanism and it determines the performance of given mechanism. Kinematic synthesis is used to design a mechanism to satisfy the motion characteristics like displacement, velocity and acceleration-either individually (or) combined.

Designing a cam profile to achieve different types of follower motion like SHM, Uniform velocity and Cycloidal motion and so on is called Kinematic synthesis.

Hence kinematic synthesis is the reverse problem of kinematic analysis. It is the design of mechanism to produce desired output motion for a given input motion. In otherwords, synthesis mechanism determines the proportions of a mechanism for the given input and output motion.

6.2 *What are the tasks of kinematic synthesis?*

Tasks of kinematic synthesis can be completed by taking decisions on

(i)	The type of mechanism →	**Type synthesis**
(ii)	The number of links and the nature of connections required to permit necessary movability →	**Number synthesis**
(iii)	The proportional lengths of links necessary to complete the specified motion →	**Dimensional synthesis**

6.3 *Define structure and mechanism.*

The combination of different links to meet certain requirements for obtaining specified motion is called as mechanism.

If there is no relative motion between links, then it is called structure.

6.4 *Define degree of freedom.*

The number of degrees of freedom (n) is the number of independent variables needed to define completely the condition of the system.

6.5 State Grashof's law.

Grashof's law states that the sum of the shortest and longest link length should not be greater than the sum of the remaining two links length, if there is to be continuous relative motion between the two links.

6.6 Define dimensional synthesis and the uses of it?

It is used to determine the dimensions of parts - lengths and angles - necessary to generate mechanism to obtain desired motion.

Generating a cam profile to obtain follower motion is dimensional synthesis.

Dimensional synthesis determines

- The distance between hinge pins
- Length of links
- Angle between adjacent links
- Cam profile dimension
- Radius of roller follower
- Gear ratios
- Eccentricities

6.7 Name the problems which can be accomplished by dimensional synthesis.

Dimensional synthesis is used to solve the following problems in a scientific way.

1.	Path generation	–	Guiding along a specified path with prescribed timing.
2.	Function generation	–	Coordinating the positions of input and output links.
3.	Motion generation (Rigid body Guidance)	–	Guiding the rigid body through a number of prescribed positions.

6.8 Briefly explain function generation.

Generally, the output link may either

- Oscillate
- Rotate (or)
- Reciprocate as a function of motion of input link. This is called function generation. In the above 4 bar mechanism, the relationship between motion of output link and input link will be a function as given here.

Motion of output link = f (Motion of input link)

$$y = f(x)$$

where x represents motion of input link

y represents motion of output link

6.9 Define precision points.

In any mechanism, the points of intersection of **desired path with actual path** is known as **Precision points** (or) **Accuracy points**.

6.10 Write the Chebyshev's spacing expression of precision points.

$$\boxed{x_j = \frac{1}{2}(x_s + x_f) - \frac{1}{2}(x_f - x_s)\cos\frac{\pi(2j-1)}{2n}}$$

6.11 Define coupler curves.

Coupler curve is a curve (or) path traced out by a point located in the plane of the coupler link. Coupler link connects the input and output links through turning pairs. When the input link rotates, any point on coupler generates a path (or) curve called coupler curve.

6.12 Name the various shapes of coupler curves.

The coupler curves have variety of shapes like

- Ellipse
- Circle
- Straight line
- Banana
- Arcs
- Cusps

6.13 ***State Roberts Chebychev's theorem of Cognate linkages.***

Roberts Chebychev's Theorem of Cognate linkages states that the same coupler curve can always be generated by three different linkages called **Cognates.**

6.14 ***Write the general four bar coupler-curve equation.***

The **general four-bar coupler-curve equation**, is

$$4k^2 \sin^2 \alpha \sin^2 \beta \sin^2 \gamma \, [x(x-p) - y - py \cot \gamma]^2$$

6.15 ***Write the expression for Freudenstein's equation and position of output link.***

$$k_1 \cos \phi - k_2 \cos \theta + k_3 = \cos(\theta - \phi) \qquad (7)$$

This equation is known as ***Freudenstein's equation***.

From this equation, we can find the position of output link (*i.e.* angle ϕ) if the length of the links and position of the input link (*i.e.* angle θ) is known.

$$\phi = 2 \tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

6.16 ***State the condition of a dead point to occur.***

A dead point occurs when the follower is at rest at a moment just prior to reversing its direction of rotation.

6.17 ***State the expression for time ratio of quick return mechanism.***

Time ratio (T_R) defines the degree of quick-return of the linkage.

$$T_R = \frac{\alpha}{\beta}; \quad \alpha + \beta = 360$$

$$\delta = |180 - \alpha|$$

$$= |180 - \beta|$$

12. (a) Discuss overlay method. (4)

Over lay method

- The over lay method is another graphical method often used for kinematic synthesis.
- It consists basically of constructing a part of the solution to a problem on transparent paper and another part of solution on a separate sheet.
- The overlay is placed over the separate sheet.
- A search is made by moving the transparency until precision points are matched between the transparency (over lay) and the separate sheet.
- This technique can be used for mechanisms involving 2 to 5 positions.
- It is demonstrated by way of a five-precision-point design.
- A four bar function generator is to be designed for the following precision points.

Precision point number	Crank rotation from starting position (deg)	
	Input (cw)	Output (cw)
1	0	0
2	$\phi_2 = 15°$	$\Psi_2 = 20°$
3	$\phi_3 = 30°$	$\Psi_3 = 35°$
4	$\phi_4 = 45°$	$\Psi_4 = 50°$
5	$\phi_5 = 60°$	$\Psi_5 = 60°$

Method

1. On tracing paper layout the input crank positions and select lengths for the input and coupler links (see Fig. 5)

Draw a family of circular arcs with centre at successive crank pin position with a radius equal to the arbitrarily chosen coupler length.

2. On a second piece of paper (Fig.6) layout the output crank positions and add several arcs, indicating possible lengths of link 4.
3. Place the first layout on the second and move until the family of arcs of (fig. 5) falls on the respective positions of the output crank as shown in figure 7.

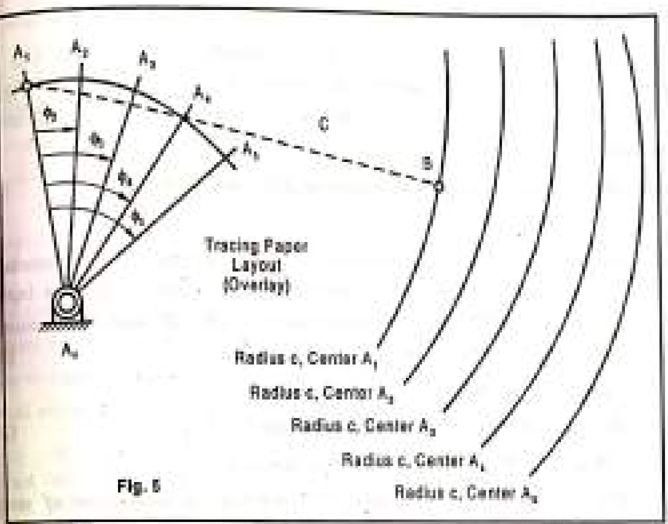


Fig. 5

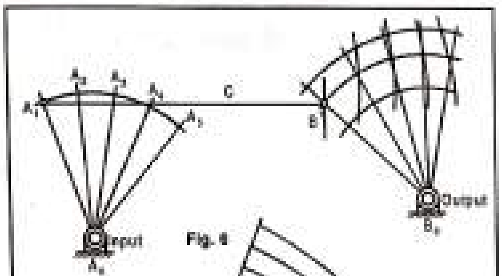


Fig. 6

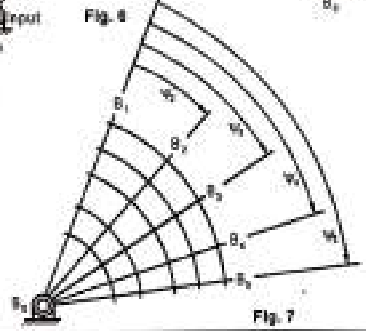


Fig. 7