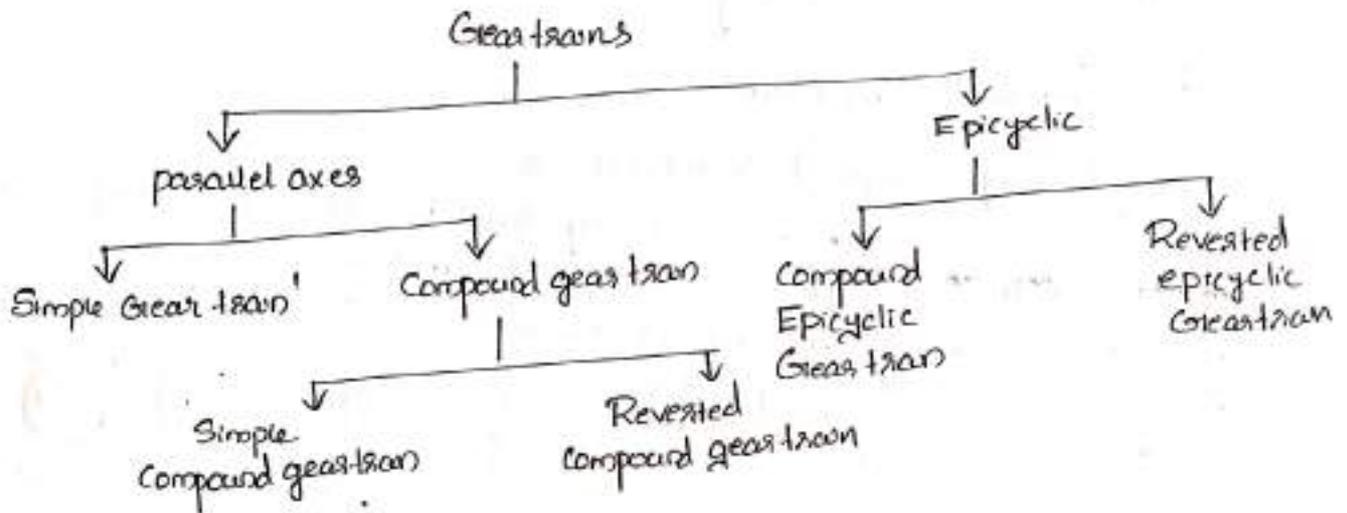


Gear Trains

Definition

- Gear train is a combination of two or more gears meshing with one another to transmit power from one shaft to another. It called gear train or train of toothed wheel. It required to obtain large speed reduction within a small space.
- Gear train are used when
 - (i) A large Velocity reduction is desired.
 - (ii) The distance b/w the two shaft is neither too high nor short

Classification



① Simple gear train

Each shaft carries only one gear. Such gear train called Simple gear train

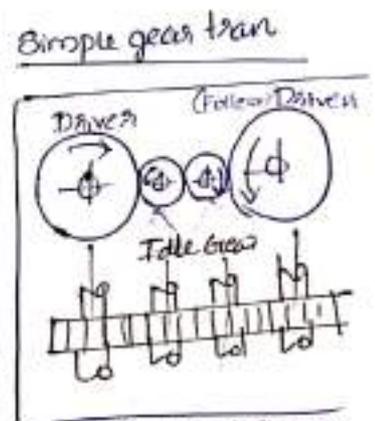
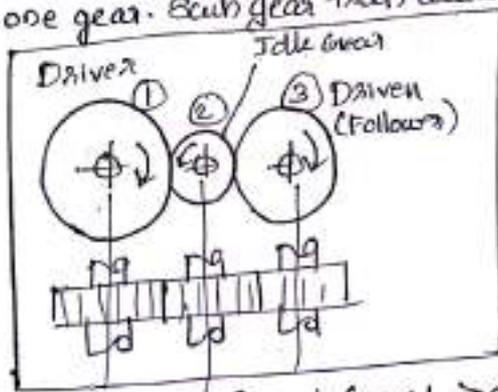
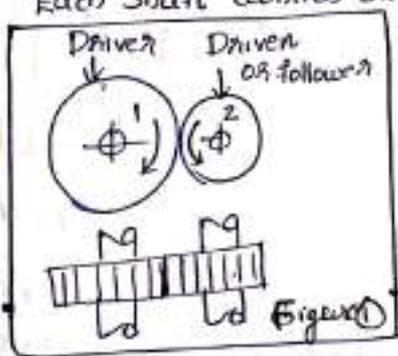


Figure 1 shows Gear 1 drives gear 2. ∴ Gear 1 → driver, Gear 2 → driven (Follower)
Motion of driven opposite to the motion of the driver.

Velocity Ratio (speed ratio)

It is the ratio of speed of the driver to the speed of driven

$$\text{Velocity ratio (speed ratio)} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It is the ratio of no: of teeth on follower to the no: of teeth of driver

Train value

[For Gear ① & ②]

$$\text{Train value} = \frac{1}{\text{Velocity ratio}} \Rightarrow \frac{N_2}{N_1} = \frac{T_1}{T_2} \quad \text{--- (1)}$$

$$\text{For gear ② & ③} \quad \text{Train value} = \frac{N_3}{N_2} = \frac{T_2}{T_3} \quad \text{--- (2)}$$

(Multiplying) Equating L.H.S

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} = \frac{T_1}{T_2} \times \frac{T_2}{T_3}$$

$$\boxed{\text{Train value} = \frac{N_3}{N_1} = \frac{T_1}{T_3}}$$

$$\boxed{\text{Speed ratio} = \frac{N_1}{N_3} = \frac{T_3}{T_1}}$$

$$\boxed{\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of follower}} = \frac{\text{No. of teeth of follower}}{\text{No. of teeth of driver}}}$$

② Compound Gear Train

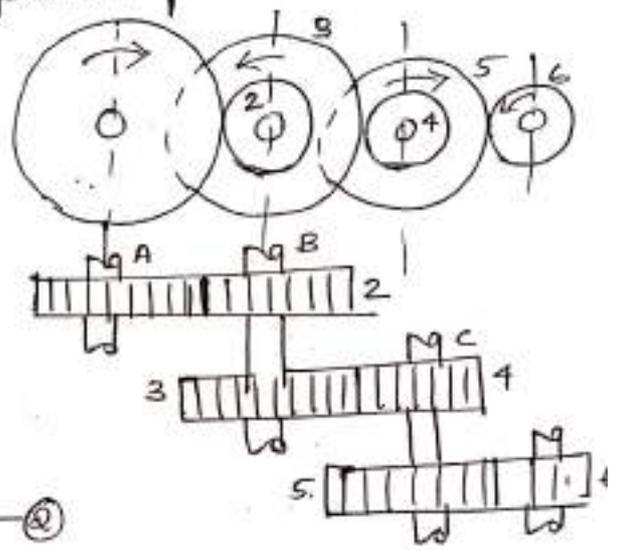
(a) Simple Compound Gear train

When more than one gear is rigidly fixed to a shaft and they rotate together with same speed. This compound gear train.

⇒ Gear 2 and 3 are known compound gears

⇒ Similarly 4 & 5 also compound gear

Gear 1 → driver, 6 → follower



Gear 1 meshing with gear 2

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

Gear 2 and 3 compound gears

$$N_2 = N_3$$

$$\text{Gears 3 & 4} \Rightarrow \text{Speed ratio} = \frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{--- (2)}$$

$$\text{Gear 5 & 6} \Rightarrow \text{Speed ratio} = \frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{--- (3)}$$

$$[N_4 = N_5]$$

$$\text{eq (1) } \times \text{ (2) } \times \text{ (3)} \Rightarrow \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \Rightarrow \boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$

$$\begin{aligned} \text{Speed ratio} &= \frac{\text{Speed of first driver}}{\text{Speed of last follower}} \\ &= \frac{\text{product of No. of teeth on follower}}{\text{product of no. of teeth on drivers}} \end{aligned}$$

⇒ In simple gear train, intermediate gears do not have effect on speed ratio.

⇒ But intermediate compound gears have much effect on speed ratio.

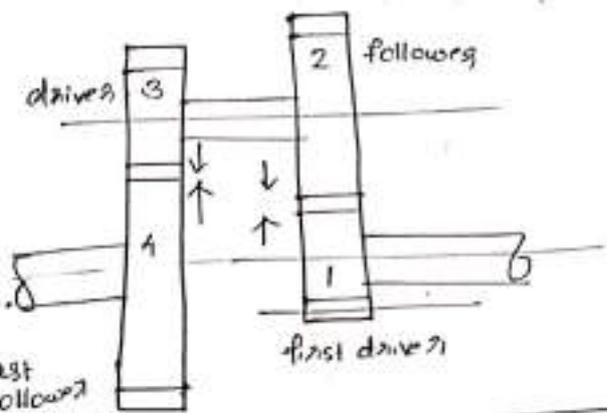
(b) Riveted compound gear trains

⇒ First & last gear coaxial (rotating same direction). It's called riveted compound gear trains

Gears 1 & 2 ⇒ $S.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$ — (1)

Gears 3 & 4 ⇒ $S.R = \frac{N_3}{N_4} = \frac{T_4}{T_3}$ — (2)

(1) × (2) ⇒ $\frac{N_1}{N_4} = \frac{T_2 T_4}{T_3 T_1}$ ($N_2 = N_3$) last follower



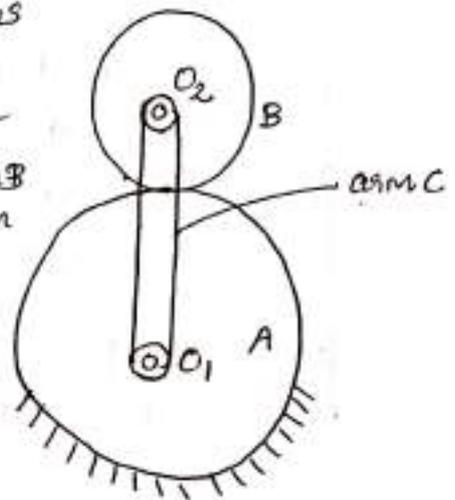
$$\text{Speed ratio} = \frac{\text{Speed of first driver}}{\text{Speed of last follower}} = \frac{\text{Product of no. of teeth on followers}}{\text{Product of no. of teeth on drivers}}$$

(3) Epicyclic gear Train [or planetary gear train]

• If the axis of the shaft over which the gears are mounted moving relative to a fixed axis. This gear train known as Epicyclic gear train

• From fig A gear fixed. Arm C rotates, the gear B rotate about axis O_2 and also it revolves upon and around the gear. It is similar to planets rotates about their own axes and revolves around the sun.

Gear B = planet gear
Gear A = sun gear



Application ⇒ (1) Wrist watches

(2) Back gear of lathe

(3) Differential in Automobiles

(4) Hoists

(5) Pulley Blocks

V.R (Velocity Ratio) of Epicyclic gear train found by

(1) Tabulation Method

(2) Formula or Algebraic Method

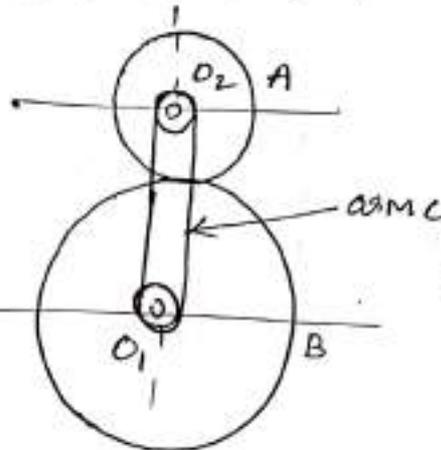
- ① In spur gears A and B of an epicyclic gear train having 24 and 30 teeth respectively. The arm rotates at 100 rpm in the clockwise direction. Find speed of gear B on its own axis when gear A is fixed. The wheel A rotates 200 rpm in the counter clockwise direction. What will be speed of B?

Given

$$T_A = 24$$

$$T_B = 30$$

$$N_c = \text{speed of arm} = 100 \text{ rpm}$$



When A rotate +ve

B rotate -ve

Condition Functions	Arm C (N_c)	Wheel A (N_A)	Wheel B (N_B)
① Arm C is fixed, +1 revolution of gear A	0	+1	$\frac{N_B}{N_A} = \frac{T_A}{T_B}$ $N_B = -\left(1 \times \frac{T_A}{T_B}\right)$
② Arm C is fixed, +x revolution on gear A	0	x	$N_B = -\left(x \times \frac{T_A}{T_B}\right)$
③ Adding +y revolution to all elements	$0+y = y$	$x+y$	$N_B = y - \frac{x T_A}{T_B}$

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

(i) $N_B = ? \Rightarrow N_B = y - \left(\frac{x T_A}{T_B}\right)$ [from table]

$$N_A = x + y \text{ (table)}$$

$$\text{Sub } N_A = 0 \Rightarrow 0 = x + y \Rightarrow x = -y$$

$$N_c = y \text{ (table)} \Rightarrow 100 = y$$

$$\text{If } x = -100, N_B = 100 - \left(-100 \times \frac{24}{30}\right) = 180 \text{ rpm (clockwise)}$$

(ii) If $N_A = 200$ rpm (given)

$$N_A = x + y \Rightarrow 200 = x + y \Rightarrow 200 = x + 100 \Rightarrow x = 100 \text{ rpm}$$

$$N_B = y - \left(\frac{x T_A}{T_B}\right) = 100 - \left(\frac{100 \times 24}{30}\right) = \underline{\underline{80 \text{ rpm}}}$$

Differential Gear

During turning of vehicle, the outer wheel will have to travel greater distance as compared to the inner wheel.

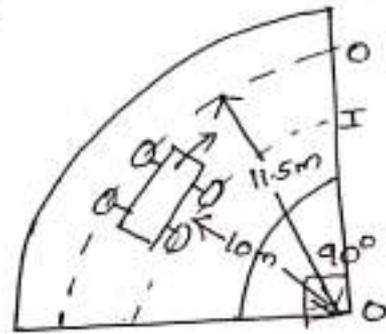
Outer wheel distance = $O \rightarrow O$

Inner wheel distance = $I \rightarrow I$

⇒ If vehicle has solid rear axle then there will be a tendency to skid.

⇒ Hence in order to avoid this skidding and maintaining different speed for

inner and outer wheels, we use a device called Differential



Differential is automobile consist of an epicyclic train of gears designed to permit two or more shaft to rotate at different speeds, by means of which outer wheel run faster than inner wheel while taking a turn.

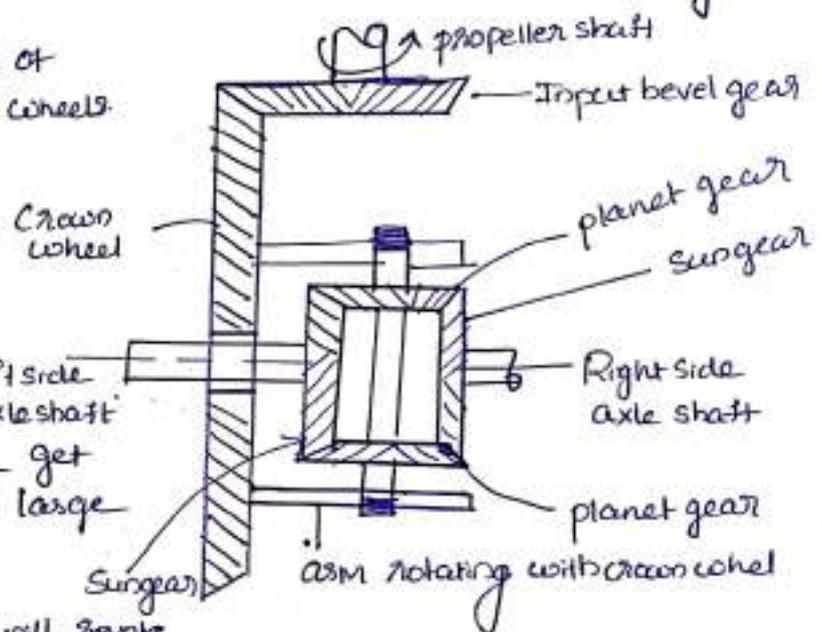
⇒ Differential reduce speed of propeller shaft to that of wheels.

⇒ Crown wheel connected to cage so that it rotate along with crown wheel.

⇒ Small planetary gears freely rotating on the pins on the cage get revolving motion around large sun gear.

⇒ Small planetary gears will revolve around and rotate the large sun gears connected to rear axles.

⇒ Crown wheel give rotary motion to rear axle. Size of crown wheel bigger than that of bevel pinion. ∴ Speed of rear axle (crowned) lower than speed of pinion.

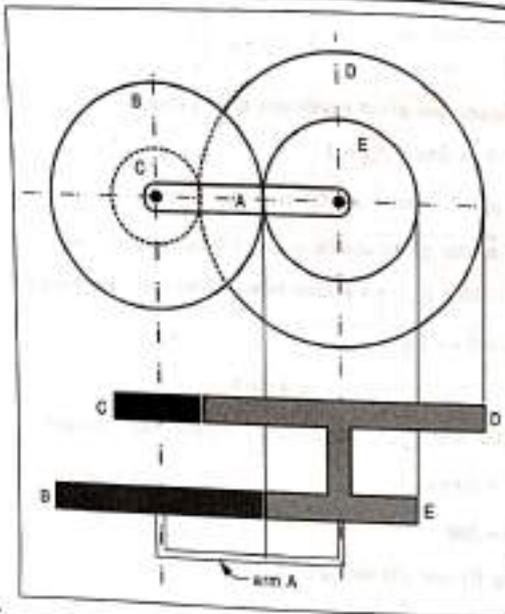


$$N_B = y - x \times \frac{T_A}{T_B}$$

$$N_B = 200 - \left[-200 \times \frac{36}{45} \right]$$

$$= 360 \text{ rpm (anticlockwise)}$$

Problem 5.2: In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D-E. B gears with E and C gears with D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when the gear B is fixed and the arm A makes 200 rpm clockwise.



Solution:

$$T_B = 75; T_C = 30; T_D = 90; N_B = 0 \text{ [} \therefore \text{ fixed]}; N_C = -200$$

$$\text{[} \therefore \text{ clockwise] } N_C = ?$$

To find T_E

From the geometry of the figure, we can find T_E through D_B, D_C, D_D and D_E which are the pitch circle diameters of the gears B, C, D and E respectively.

From geometry of figure, consider the centre to centre distances.

$$\text{We know, } \frac{D_B}{2} + \frac{D_E}{2} = \frac{D_C}{2} + \frac{D_D}{2}$$

$$\text{(or) } D_B + D_E = D_C + D_D$$

$$\text{We know } D_B = mT_B; D_E = mT_E; D_C = mT_C \text{ and } D_D = mT_D$$

[Assume module m for all meshing gears should be same]

$$\text{So, } D_B + D_E = D_C + D_D \text{ can be written as}$$

$$T_B + T_E = T_C + T_D$$

$$75 + T_E = 30 + 90$$

$$T_E = 120 - 75 = 45$$

$$T_E = 45$$

Now we can form tabular column;

Operations	N_A	$N_D = N_E$	N_C	N_B
1. Arm is fixed. Rotate compound gear D-E through +1 revolution (Note: Any gear can be rotated)	$N_A = 0$	$N_D = +1$	$\frac{N_C}{N_D} \times \frac{T_D}{T_C}$ $N_C = -1 \times \frac{T_D}{T_C}$ [- sign for clockwise] (when D is +ve, C should be -ve for external gearing)	$\frac{N_B}{N_D} \times \frac{T_D}{T_E}$ $N_B = -1 \times \frac{T_D}{T_E}$ [When E is +ve, B should be negative. So - sign]
2. Multiply by x	0	x	$-x \frac{T_D}{T_C}$	$-x \frac{T_D}{T_E}$
3. Add y to step 2	y	$x+y$	$y - x \frac{T_D}{T_C}$	$y - x \frac{T_D}{T_E}$

Compare the given conditions and third row of table.

$$N_B = 0 \text{ But } N_B = y - x \frac{T_E}{T_B}$$

$$N_A = -200 \text{ But } N_A = y$$

$$\text{So } N_A = -200 = y$$

$$y = -200$$

$$N_B = 0 = y - x \frac{T_E}{T_B}$$

$$0 = -200 - x \times \frac{45}{75}$$

$$x = -200 \times \frac{75}{45} = -333.33$$

$$x = -333.33$$

Substituting x and y values in the equation

$$N_C = y - x \times \frac{T_D}{T_C} = -200 - \left[-333.33 \times \frac{90}{30} \right]$$

$$= +800 \text{ (ie anti clockwise)}$$

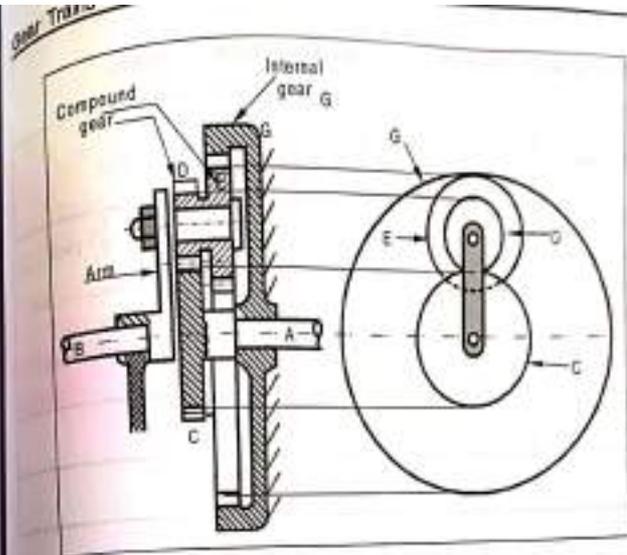
$$N_C = 800 \text{ rpm (anticlockwise).}$$

Problem 5.3: Two shafts A and B are co-axial. The sun gear C (50 teeth) is rigidly mounted on shaft A . The compound gears (planet gears) D - E meshes with C and internal gear G . D (20 teeth) meshes with C . The gear E (35 teeth) meshes with an internal gear G .

The gear G is fixed and is concentric with the shaft axis. The compound gears (planet gears) D - E mounted on a pin which projects from an arm keyed to the shaft B . Sketch the arrangement and find the number of teeth on internal gear G , assuming that all gears have same module. If the shaft rotates at 220 rpm, find the speed of shaft B .

Given:

$$T_C = 50 ; T_D = 20 ; T_E = 35 ; N_A = 200 \text{ rpm}$$



To find Number of teeth on gear G

From geometry of the figure,

$$\frac{D_G}{2} = \frac{D_E}{2} + \frac{D_D}{2} + \frac{D_C}{2}$$

$$(\text{or}) D_G = D_E + D_D + D_C$$

Since module is same for all meshing gears,

$$\text{we can write, } T_G = T_E + T_D + T_C$$

$$= 35 + 20 + 50 = 105$$

$$T_G = 105$$

To find speed of shaft $B = N_B$ from The tabular column.

Operation	N_{arm} (speed of shaft B)	N_c (or) N_A	$N_D = N_E$	N_G
1. Arm is fixed. Rotate gear C through +1 revolution	0	+1	$\frac{N_D}{N_C} = -\frac{T_C}{T_D}$ $N_D = -\frac{T_C}{T_D}$ [D meshes with C]	$\frac{N_G}{N_E} = \frac{T_E}{T_G}$ $N_G = -\frac{T_C}{T_D} \times \frac{T_E}{T_G}$ [Here when E rotates clockwise (-), the G also rotates clockwise (-), because it is an internal gearing.]
Multiply by x	0	x	$-x \frac{T_C}{T_D}$	$-x \frac{T_C}{T_D} \times \frac{T_E}{T_G}$
Add y	y	x+y	$y - x \frac{T_C}{T_D}$	$y - x \frac{T_C}{T_D} \times \frac{T_E}{T_G}$

Compare the third row of table and given conditions,

$$N_G = 0; \quad N_G = y - x \frac{T_C}{T_D} \times \frac{T_E}{T_G}$$

$$N_G = 0 = y - x \frac{T_C}{T_D} \times \frac{T_E}{T_G}$$

$$0 = y - x \frac{50}{20} \times \frac{35}{105}$$

$$0 = y - x \times 0.833 \quad \dots(i)$$

$$y = 0.833x$$

$$N_A = 220; \quad N_A = x + y$$

$$\text{So, } x + y = 220 \quad \dots(ii)$$

$$y - 0.833x = 0$$

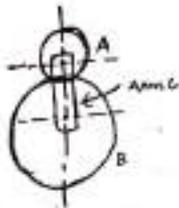
$$y + x = 220$$

Substituting equ (i) into equ (ii)

$$x + 0.833x = 220$$

$$x = 120$$

Q 2 gear gears A and B of an epicyclic gear train have 24 and 30 teeth respectively. The arm rotates at 100 rpm in the clockwise direction. Find speed of gear B with arm axis when the gear A is fixed. If instead of being fixed the wheel A rotates at 200 rpm in the common clockwise direction. What will be speed of B?



Given
 $T_A = 24$ N_A
 $T_B = 30$
 $N_c = 100 \text{ rpm}$

Conditions functions	Arm c	Wheel A	Wheel B
Arm c is fixed, +1 revolution on gear A	0	+1	$-\left(1 \times \frac{T_A}{T_B}\right)$
Arm c is fixed +x revolutions on gear A	0	+x	$-\left(x \times \frac{T_A}{T_B}\right)$
Adding +y revolutions to all elements	+y	+xy	$y - \left(x \times \frac{T_A}{T_B}\right)$

$$\frac{N_B}{N_c} = \frac{T_A}{T_B}$$

(i) $N_B = ?$
 $N_B = y - \left(x \times \frac{T_A}{T_B}\right)$ (from table)
 $N_A = x + y$ (table)
 $0 = x + y$
 $x = -y$

$N_c = y$ (table)

$100 = y$

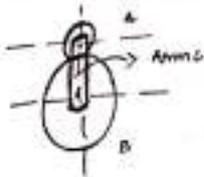
if $x = -100 \text{ rpm}$

$N_B = 100 - \left(-100 \times \frac{24}{30}\right)$
 $= 180 \text{ rpm}$ (clockwise)

(ii) $N_A = 200 \text{ rpm}$ (given)
 $N_A = x + y$ (from table)
 $200 = x + y$
 $y = 100 \text{ rpm}$
 $x = 100 \text{ rpm}$

$N_B = y - \left(x \times \frac{T_A}{T_B}\right)$
 $= 100 - \left(100 \times \frac{24}{30}\right)$
 $= -20 \text{ rpm}$

Q 2 The arm of an epicyclic gear train rotates at 100 rpm in the anticlockwise direction. The arm carries 2 wheels A and B having 30 and 40 teeth respectively. The wheel A is fixed and arm rotates about centre of wheel A. Find speed of wheel B if the wheel A instead of being fixed, makes 200 rpm clockwise.



$N_C = 100 \text{ rpm}$ (antitock wise)
 $T_A = 26$
 $T_B = 15$

Conditions functions	Ann C	Wheel A	Wheel B
Ann C fixed, +1 Revolution on gear A	0	+1	$-\left(1 \times \frac{T_A}{T_B}\right)$
Ann C fixed, wheel A having +2 revolution	0	+2	$-\left(2 \times \frac{T_A}{T_B}\right)$
Adding +y to all elements	+y	+2y	$y - \left(2 \times \frac{T_A}{T_B}\right)$

$N_B = ?$
 $N_B = y - \left(2 \times \frac{T_A}{T_B}\right)$
 $N_A = 0$ (given)
 $N_C = 100$ (table)
 $x + y = 0$
 $N_C = 100 \text{ rpm}$ (given)
 $N_C = +y$ (table)
 $y = 100$
 $x = -100$

$$N_B = 100 - \left(-100 \times \frac{26}{15}\right) = 180 \text{ rpm} \text{ (clockwise)}$$

$$N_B = y - \left(x \times \frac{T_A}{T_B}\right) \text{ (table)}$$

$$N_A = -200 \text{ rpm} \text{ (given)}$$

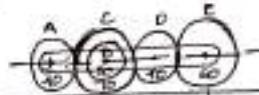
$$x + y = -200$$

$$y = 100 \text{ rpm}$$

$$x = -200 - 100 = -300 \text{ rpm}$$

$$N_B = 100 - \left(-300 \times \frac{26}{15}\right) = 340 \text{ rpm}$$

Figure shows a gear train consisting of gears B, C, D, and E. Gears B, C, and D constitute a compound gear. The no. of teeth are given along with each wheel in the figure. Speed, direction or rotation of wheels A, C, and E if gear B is fixed at 200 rpm clockwise and gear D is fixed.



Conditions of element	Speed of Element				
	N_A	N_B	$N_C = N_D$	N_D	N_E
① Ann A is fixed, wheel B having +1 revolution	0	+1	$-\left(1 \times \frac{T_A}{T_B}\right)$	$\left(\frac{T_B}{T_C} \times \frac{T_C}{T_D}\right)$	$\left(\frac{T_B}{T_D} \times \frac{T_D}{T_E}\right)$
② Ann A is fixed, wheel B having +2 revolution	0	+2	$-\left(2 \times \frac{T_A}{T_B}\right)$	$\left(\frac{T_B}{T_C} \times \frac{T_C}{T_D}\right)$	$-\left(2 \times \frac{T_B}{T_D} \times \frac{T_D}{T_E}\right)$
③ Adding +y to all elements	+y	+2y	$y - \left(2 \times \frac{T_A}{T_B}\right)$	$y \left(\frac{T_B}{T_C} \times \frac{T_C}{T_D}\right)$	$y \left(\frac{T_B}{T_D} \times \frac{T_D}{T_E}\right)$

$$\frac{N_E}{N_D} = \frac{T_D}{T_E}$$

$$N_E - N_D = \frac{T_D}{T_E} = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_D}{T_E}$$

$$= \frac{T_A}{T_B} \times \frac{T_C}{T_E}$$

$N_A, N_E = ?$

$$N_A = 20 \text{ rpm} = y$$

$$N_D = 0$$

$$N_A = x + y$$

$$N_E = y - \left[x \times \frac{T_D}{T_B} \times \frac{T_C}{T_E} \right]$$

$$N_D = y + \left[x \times \frac{T_D}{T_B} \times \frac{T_C}{T_E} \right] = 0$$

$$0 = 20 + \left[x \times \frac{40}{20} \times \frac{70}{70} \right]$$

$$x \times \frac{2}{1} = -210$$

$$x = \frac{-210 \times 1}{2} = -105$$

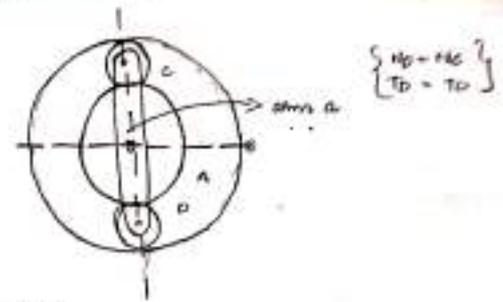
$$N_A = -105 + 20 = \underline{120 \text{ rpm}} \text{ (anticlockwise)}$$

$$N_E = 20 - \left[-105 \times \frac{40}{20} \times \frac{70}{70} \right]$$

$$= \underline{350 \text{ rpm}} \text{ (clockwise)}$$

An epicyclic gear train is shown in figure. The no. of teeth on A and B are 80 and 200. Determine speed of arm A, if A rotates at 100 rpm anticlockwise and B at 50 rpm clockwise. If A rotates at 100 rpm counter clockwise and B is stationary.

ans:



To find T_C and T_D

$$O_B = O_A + r_A + r_B$$

$$r_A = \frac{D}{2}$$

$$r_B = 2r_A$$

$$T \propto D$$

$$T_B = T_A + 2T_C$$

$$200 = 80 + 2T_C$$

$$2T_C = 200 - 80$$

$$2T_C = 120$$

$$T_C = 120/2 = 60 = T_D$$

$$\frac{N_E}{N_A} = \frac{T_A}{T_C}$$

$$N_C = N_A \frac{T_A}{T_C}$$

$$\frac{N_B}{N_C} = \frac{T_C}{T_B}$$

$$N_B = N_C \frac{T_C}{T_B}$$

Conditions of element	Speed of element			
	N_A	N_B	$N_C = N_D$	$N_E = N_D$
① arm A is fixed, wheel A having +1 revolution	0	+1	$-\left(1 \times \frac{T_A}{T_C}\right)$	$-\left(\frac{T_D}{T_B}\right)$
② arm A is fixed, wheel A having +x revolution	0	+x	$-\left(x \times \frac{T_A}{T_C}\right)$	$-x \left(\frac{T_D}{T_B}\right)$
③ Adding +y to all elements	+y	x+y	$y - 2\left(\frac{T_A}{T_C}\right)$	$y - 2\left(\frac{T_D}{T_B}\right)$

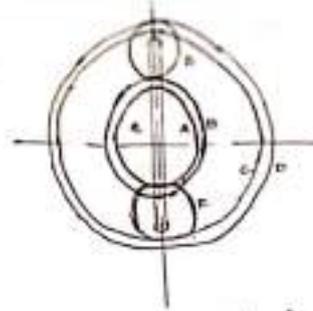
$$\begin{aligned}
 N_A &= 100 \text{ rpm} \\
 N_B &= -100 \text{ rpm} \\
 N_C &= ? \\
 N_D &= ? \\
 N_A &= x + y = 100 \\
 x &= 100 - y \\
 N_B &= y - (x \cdot \frac{T_B}{T_A}) = -50 \\
 -50 &= y - ((100 - y) \cdot \frac{90}{200}) \\
 y &= -11.4 \text{ rpm} = N_C
 \end{aligned}$$

5th Case

$$\begin{aligned}
 N_A &= 100 \text{ rpm} \\
 N_B &= 0
 \end{aligned}$$

$$\begin{aligned}
 0 &= y - ((100 - y) \cdot \frac{90}{200}) \\
 y &= 28.6 \text{ rpm} = N_C
 \end{aligned}$$

- Q In the epicyclic gear train shown in figure. The compound wheels A and B as well as internal wheels E and D rotate multiple revolutions about the axis O. wheels K and F rotate on the pins fixed to the arm C. All the wheels are of the same module. No. of teeth on the wheels are $T_A = 52$, $T_B = 52$, $T_C = T_F = 26$. determine speed of C if
- The wheel D fixed and arm C rotates at 200 rpm clockwise.
 - The wheel D rotates at 200 rpm counter clockwise and arm C rotates at 200 rpm counter clockwise.
- Soln
 $T_A = 52$
 $T_B = 52$
 $T_C = T_F = 26$



$$\begin{aligned}
 \frac{N_B}{N_C} &= \frac{T_C}{T_B} \\
 N_C &= N_B \cdot \frac{T_B}{T_C}
 \end{aligned}$$

From figure

$$R_B + D_F = R_C$$

$$\frac{D_B}{2} + D_F = \frac{D_C}{2}$$

$$\left\{ \begin{matrix} N_B T = D \\ T < D \end{matrix} \right\}$$

$$\frac{T_B}{2} + T_F = \frac{T_C}{2}$$

$$T_C = 2 \left(\frac{T_B}{2} + T_F \right) = 104$$

From figure

$$R_D = R_B + D_C$$

$$P D_D = D_B + D_C$$

$$T_D = 2 \left(\frac{T_B}{2} + T_C \right)$$

$$= 2 \left(\frac{52}{2} + 26 \right)$$

$$= 104$$

Condition of elements	Speed of elements					
	N_A	$N_B = N_C$	N_E	N_F	N_D	N_O
① Arm C fixed, compound wheel rotating +1 rev/clock	0	+1	$-\left(\frac{52}{52}\right)$	$-\left(\frac{52}{26}\right)$	$-\left(\frac{26}{90}\right)$	$-\left(\frac{26}{104}\right)$
② Arm C fixed, wheel A is having +2 rev/clock	0	+2	$-\left(\frac{2 \cdot 52}{52}\right)$	$-\left(\frac{2 \cdot 52}{26}\right)$	$-\left(\frac{2 \cdot 26}{90}\right)$	$-\left(\frac{2 \cdot 26}{104}\right)$
③ Adding +y to all elements	+y	+y	$y \cdot \left(\frac{2 \cdot 52}{52}\right)$	$y \cdot \left(\frac{2 \cdot 52}{26}\right)$	$y \cdot \left(\frac{2 \cdot 26}{90}\right)$	$y \cdot \left(\frac{2 \cdot 26}{104}\right)$

$$\begin{aligned}
 N_D &= 0 \\
 N_A &= 200 \text{ rpm}
 \end{aligned}$$

$$\frac{N_1}{Z} = \frac{d_1 \omega_1}{d_2 \omega_2} \Rightarrow N_1 \omega_1 = N_2 \omega_2$$

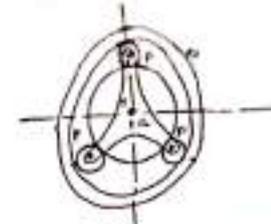
$$N_1 = N_2 \frac{T_2}{T_1}$$

$$N_1 = N_2 \frac{T_2}{T_1}$$

(i) $N_2 = 0$
 $N_1 = 300 \text{ rpm}$
 $y = -200 \text{ rpm}$
 $0 = y - (x \frac{T_2}{T_1})$
 $0 = -200 - (x \frac{50}{124})$
 $200 = x \frac{50}{124}$
 $x = -494.1$
 $N_2 = -200 - (-494.1 \times \frac{50}{124})$
 $= -8.31 \text{ rpm}$

(ii) $N_2 = 300 \text{ rpm}$
 $40 = y - x \left(\frac{T_2}{T_1}\right)$
 $200 = 420 - x \left(\frac{50}{124}\right)$
 $x = -411.428 \text{ rpm}$
 $N_2 = y - x \frac{T_2}{T_1}$
 $= 20 + 411.428 \times \frac{50}{124} = 172.53 \text{ rpm}$

The amount of steel used in each gear is same. The 3 gears spider is driven at 100 rpm. determine no of teeth required on each gear.



$$\frac{N_S}{T_S} = \frac{T_C}{T_S}$$

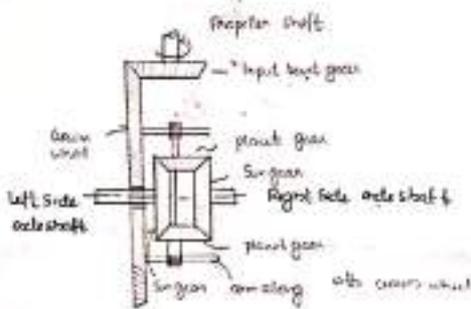
$$N_P = N_C \frac{T_C}{T_P}$$

Conditions	Speed of elements			
	N_A	N_S	N_P	N_C
Planet fixed and spider is having + rotation	0	+1	$-\left(\frac{T_S}{T_P}\right)$	$-\left(\frac{T_C}{T_S}\right)$
sun fixed and spider and planet	0	+2	$-\left(2 \frac{T_S}{T_P}\right)$	$-\left(2 \frac{T_C}{T_S}\right)$
sun and spider adding by both planets	+y	+y	$y \times \frac{T_S}{T_P}$	$y \times \frac{T_C}{T_S}$

$N_A = 300 \text{ rpm}$
 $N_S = y - x \frac{T_S}{T_P}$
 $N_C = 100 \text{ rpm}$
 $200 = 100 - \left(-100 \times \frac{50}{124}\right)$
 $T_2 = 124$
 $N_2 = 100 + 100 \times \frac{50}{124}$
 $= 140.32 \text{ rpm}$

$$v = \frac{v_1}{2} = \frac{v_2}{2} = \frac{v_3}{2}$$

ifferential



Differential in automobile consist of an epicyclic train of gears designed to permit axle shaft to rotate at different speeds, by means of which outer wheels turn faster than inner wheel while taking curves.

12-11-2018
Monday

Module - 6

Kinematic Synthesis of Planar Mechanism

Introduction (Planar mechanism).

Kinematic analysis is the study of motion of mechanism, and it determines the performance of given mechanism.

Kinematic synthesis is used to design a mechanism to satisfy the motion characteristics like displacement, velocity and acceleration either motion dualy or combined. Hence kinematic synthesis is the reverse problem of kinematic analysis.

Tasks of kinematic synthesis

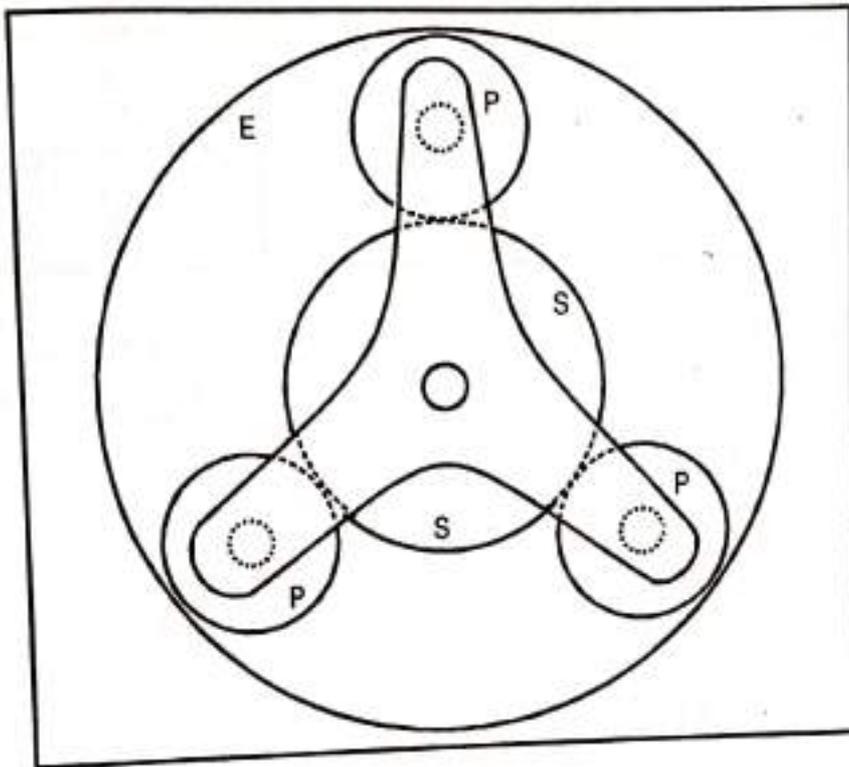
- ① Type synthesis
- ② Number synthesis
- ③ Dimensional synthesis

①: → It is used to select the kind of mechanism with gear combination or belt pulley combination or cam mechanism and so on by considering design aspects like space consideration, safety aspects, economy consideration, manufacturing process and so on.

②: → It is used to find no. of links and nature of connections required to permit necessary mobility.

③: → It is used to determine dimensions of parts (lengths and angle) necessary to generate mechanism to obtain desired motion.

Problem 5.15: An epicyclic gear train for an electric motor is shown in figure. The wheel S has 15 teeth and is fixed to the motor shaft running at 1450 rpm. The planet P has 45 teeth, gears with fixed annulus E and rotates on a spindle carried by an arm A which is fixed to the output shaft. If the motor transmits, 1.5 kw, determine the torque required to fix the annulus E . (FAQ)



Solution

$$T_s = 15; N_s = 1450 \text{ r.p.m}$$

$$T_p = 45; \text{ power} = 1.5 \text{ kW} = 1500 \text{ W} = P_s;$$

Torques on motor shaft

$$T_s = \frac{P_s \times 60}{2\pi N_s} = \frac{1500 \times 60}{2\pi \times 1450} = 9.88 \text{ N-m}$$

From the Geometry of the figure, $D_s + 2D_p = D_E$

\therefore No. of teeth are proportional to pitch circle diameter of Gear.

$$\therefore T_s + 2T_p = T_E$$

$$\therefore T_E = 15 + 2 \times 45 = 105 \therefore T_E = 105$$

S. No.	Condition of motion	Revolutions of Elements			
		Arm N_A	Annulus "E" N_E	Planet gear "P" N_P	Wheel "S" N_S
1.	Arm is fixed. Rotate the Annulus E through +1 revolution	0	+1	$-\frac{T_E}{T_P}$	$-\frac{T_E}{T_S}$
2.	Multiply by x	0	x	$-x \cdot \frac{T_E}{T_P}$	$-x \cdot \frac{T_E}{T_S}$
3.	Add y	y	x+y	$y - x \cdot \frac{T_E}{T_P}$	$y - \left(\frac{T_E}{T_S} \times x\right)$

Given $N_E = 0$ (\therefore fixed; $\therefore x + y = 0 \therefore x = -y$)

From tables, $N_S = y - \left(\frac{T_E}{T_S} \times x\right)$

$$1450 = y - \left\{ \frac{105}{15} \times (-y) \right\} = y + 7y$$

$$\therefore y = \frac{1450}{8} = 181.25$$

$$\therefore x = -181.25$$

$$\text{Torque}_A \times \omega_A = \text{Torque}_S \times \omega_S$$

$$\text{Torque}_A = \text{Torque}_S \times \frac{\omega_S}{\omega_A} = 9.88 \times \frac{N_S}{N_A}$$

$$= 9.88 \times \frac{1450}{181.25}$$

$$= 79.04 \text{ N-m}$$

$$N_A = y = 181.25$$

Fixing torque to fix Annulus E

$$= 79.04 - 9.88 = 69.16 \text{ N-m}$$

So the number of teeth on different wheels of the train

$$T_S = 16 ; T_E = 64 ; T_P = 24$$

Torque necessary to keep the internal gear stationary

$$\text{Torque on } S \times \omega_S = \text{Torque on } C \times \omega_C$$

(or)

$$M_S \times N_S = M_C \times N_C$$

$$100 \times 5 = M_C \times 1$$

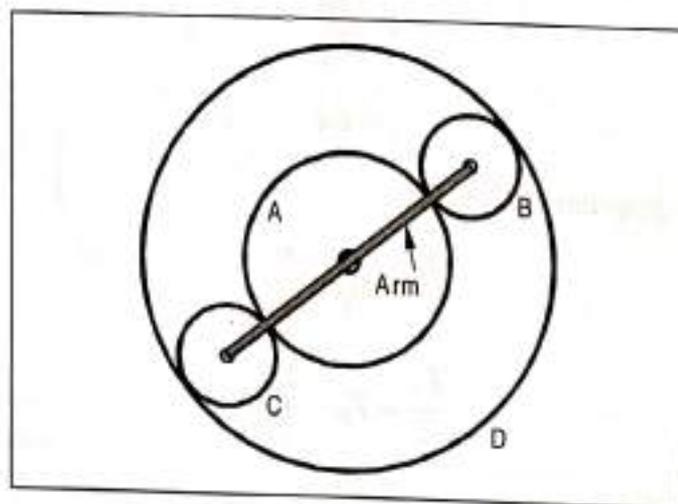
$$M_C = 500 \text{ N-m}$$

Torque necessary to keep the internal gear stationary (ME)

$$= 500 - 100 = 400 \text{ N-m}$$

$$M_E = 400 \text{ N-m}$$

Problem 5.14: An epicyclic gear train is shown in the Fig. How many revolution does the arm makes, (1) When A makes one revolution in clockwise and D makes $\frac{1}{2}$ a revolution in the opposite sense. (2) When A makes one revolution in clockwise and D remains stationary. The number of teeth in gears A and D are 40 and 90 respectively. (FAQ)



Solution: Given: $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears B and C (i.e. T_B and T_C). Let D_A, D_B, D_C and D_D be the pitch circle diameters of gears A, B, C and D respectively. Therefore from the geometry of the figure.

$$D_A + D_B + D_C = D_D \text{ or } D_A + 2D_B = D_D \quad (\because D_B = D_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2T_B = T_D \text{ or } 40 + 2T_B = 90$$

$$\therefore T_B = 25, \text{ and } T_C = 25 \quad (\because T_B = T_C)$$

The table of motions is given here:

Table of motions.

Step No.	Conditions	Revolutions of elements			
		Arm	Gear A	Compound gear B - C	Gear D
1.	Arm fixed, gear A rotates through +1 revolution	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B} \times \frac{T_B}{T_D} = -\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through +x revolutions	0	+x	$-x \times \frac{T_A}{T_B}$	$-x \times \frac{T_A}{T_D}$
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	Total motion	+y	+x+y	$y - x \times \frac{T_A}{T_B}$	$y - x \times \frac{T_A}{T_D}$

I. Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$+x + y = +1 \text{ or } x + y = 1 \quad \dots (i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x \times \frac{T_A}{T_D} - y = \frac{-1}{2} \quad \text{or} \quad y - x \times \frac{40}{90} = \frac{-1}{2},$$

$$90y - 40x = -45$$

$$2.25y - x = -1.125$$

... (ii)

From equations (i) and (ii), $x = 1.04$ and $y = -0.04$

$$\therefore \text{Speed of arm} = +y = +(-0.04) = -0.04$$

$$= -0.04 \text{ revolution anticlockwise}$$

2. Speed of arm when A makes 1 revolution clockwise and D is stationary

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$+x + y = +1 \quad \text{or} \quad x + y = 1$$

... (iii)

Also the gear D is stationary, therefore

$$y - x \times \frac{T_A}{T_D} = 0 \quad \text{or} \quad y - x \times \frac{40}{90} = 0$$

$$90y - 40x = 0$$

$$2.25y - x = 0$$

... (iv)

From equations (iii) and (iv),

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$$\therefore \text{Speed of arm} = +y = +0.308 = 0.308 \text{ revolution clockwise.}$$

So Arm $N_A = y = -300$

Arm $N_A = 300$ rpm (clockwise)

Speed of output member A = 300 rpm (clockwise)

$$\text{Input Power} = \frac{2\pi N_S M_S}{60}$$

$$3000 = \frac{2\pi \times 1000 \times M_S}{60}$$

$$M_S = 28.65 \text{ N-m}$$

Since η is not given, we can take $\eta = 100\%$

So output power = Input power = 3000 W

$$\text{Output power} = \frac{2\pi \times N_A M_A}{60}$$

$$3000 = \frac{2\pi \times 300 \times M_A}{60}$$

$$\text{Output Torque} = M_A = 95.5 \text{ N-m}$$

$$\text{Fixing torque} = 95.5 - 28.65$$

$$M_A = 66.85 \text{ N-m}$$

Problem 5.13: An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star-shaped planet carrier C. The minimum number of teeth on any wheel is 16. The size of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheel S. The driving torque on the sun wheel is 100 N-m. Determine:

- (i) Number of teeth on different wheels of the train and
- (ii) Torque necessary to keep the internal gear stationary. (FAQ)

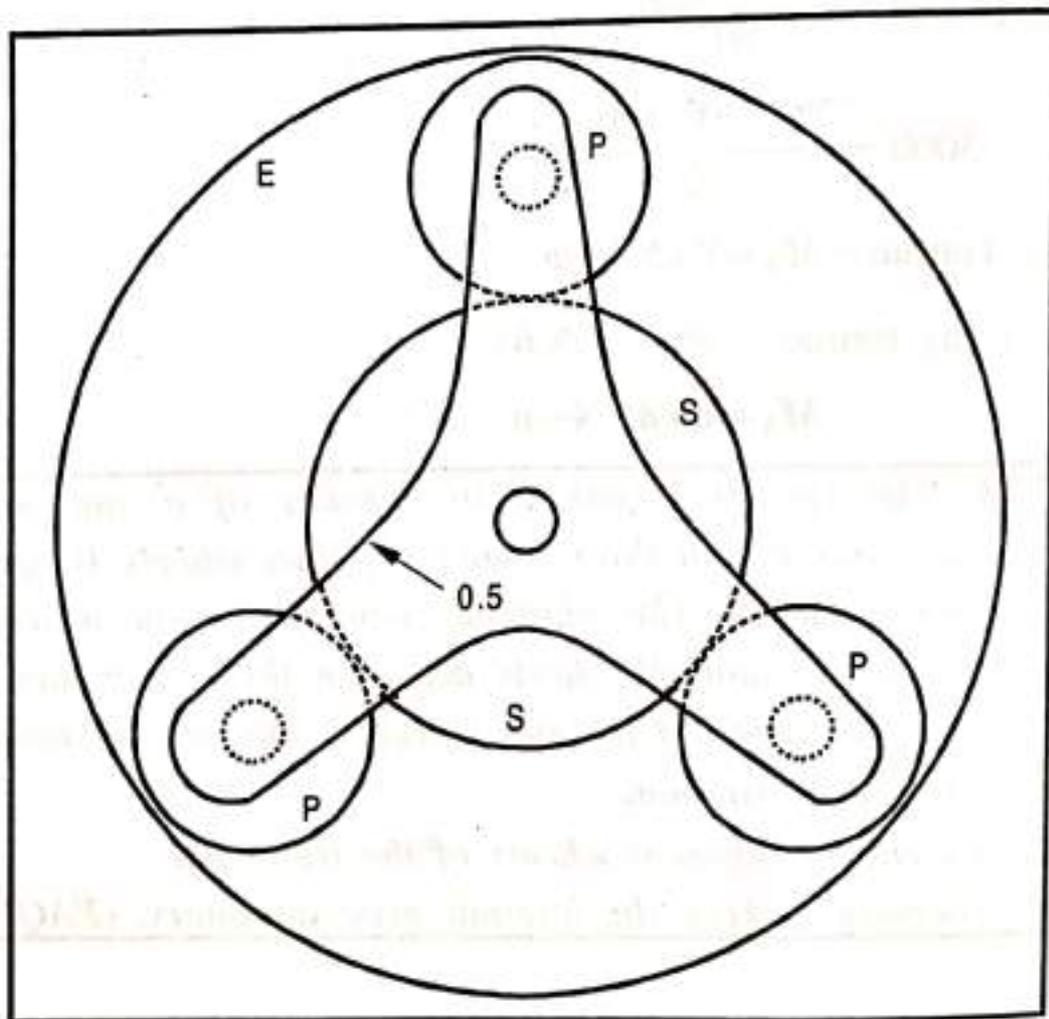
Solution:

$$\text{Speed of planet carrier (arm)} = N_C = \frac{1}{5} N_S$$

where N_S = speed of sun wheel.

By using given data, we can draw the diagram.

Conditions of motion	Revolutions of elements			
	Arm N_C	N_S	Planet wheel N_P	Internal gear N_E
Arm is fixed, rotate sun wheel S through +1 revolution	0	+1	$\left[\frac{N_P}{N_S} = -\frac{T_S}{T_P} \right]$ $N_P = -\frac{T_S}{T_P}$	$\frac{N_E}{N_P} = \frac{T_P}{T_E}$ $N_E = -\frac{T_S}{T_P} \cdot \frac{T_P}{T_E}$ $N_E = -\frac{T_S}{T_E}$
Multiply by x	0	x	$-x \frac{T_S}{T_P}$	$-x \frac{T_S}{T_E}$
Add y	y	x+y	$y - x \frac{T_S}{T_P}$	$y - x \frac{T_S}{T_E}$



Apply the given conditions:

ie $N_C = 1 ; N_S = 5$

From the table,

$$N_C = y = 1 \Rightarrow y = 1$$

$$N_S = x + y = 5$$

$$x + 1 = 5 \Rightarrow x = 4$$

Since the internal gear E is stationary, $N_E = 0$

But from the table

$$N_E = y - x \frac{T_S}{T_E}; \text{ So } y - x \frac{T_S}{T_E} = 0$$

$$1 - 4 \frac{T_S}{T_E} = 0 \quad \dots (i)$$

Since the minimum number of teeth on any wheel is 16, we can take $T_S = 16$. Substitute this in eqn. (i)

$$1 - 4 \frac{T_S}{T_E} = 0$$

$$1 - 4 \frac{16}{T_E} = 0$$

$$1 = \frac{64}{T_E}$$

$$T_E = 64$$

From the geometry of figure.

$$\frac{D_E}{2} = D_P + \frac{D_S}{2}$$

$$\frac{T_E}{2} = T_P + \frac{T_S}{2}$$

$$T_P = \frac{T_E - T_S}{2} = \frac{64 - 16}{2}$$

$$T_P = 24$$

So the number of teeth on different wheels of the train

$$T_S = 16 ; T_E = 64 ; T_P = 24$$

Torque necessary to keep the internal gear stationary

$$\text{Torque on } S \times \omega_S = \text{Torque on } C \times \omega_C$$

(or)

$$M_S \times N_S = M_C \times N_C$$

$$100 \times 5 = M_C \times 1$$

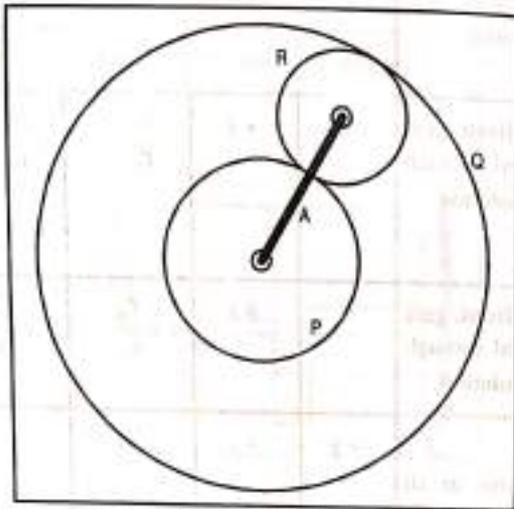
$$M_C = 500 \text{ N-m}$$

Torque necessary to keep the internal gear stationary (ME)

$$= 500 - 100 = 400 \text{ N-m}$$

$$\boxed{M_E = 400 \text{ N-m}}$$

Problem 5.9: The figure shows an epicyclic gear train in which the wheel P having 45 teeth is geared with Q through the intermediate wheel R at the end of the arm A. When P is rotating at 63 rpm in a clockwise direction and A is rotating at 9 rpm also in clockwise direction, Q is required to rotate at 21 rpm in anti clockwise direction. Find the necessary number of teeth in Q and R. (FAQ)



Operation	N_A Arm	N_Q	N_R	N_P
Arm is fixed. Rotate Q through +1 revolution	0	+1	$\frac{N_R}{N_Q} = +\frac{T_Q}{T_R}$	$\frac{N_P}{N_R} = \frac{T_R}{T_P}$ $N_P = -\frac{T_Q}{T_R} \times \frac{T_R}{T_P}$ $N_P = -\frac{T_Q}{T_P}$
Multiply by x	0	x	$+x \frac{T_Q}{T_R}$	$-x \frac{T_Q}{T_P}$
Add y	y	x+y	$y+x \frac{T_Q}{T_R}$	$y-x \frac{T_Q}{T_P}$

Apply the given condition

From table $N_A = y = -9$

$$N_Q = x + y = 21$$

$$x - 9 = 21$$

$$x = 21 + 9 = 30 \text{ rpm}$$

$$x = 30$$

$$N_P = -63 = y - x \cdot \frac{T_Q}{T_P}$$

$$-63 = -9 - \left[30 \times \frac{T_Q}{T_P} \right]$$

$$-54 = -0.666 T_Q$$

$$T_Q = 81$$

From the geometry of the figure,

$$\frac{D_Q}{2} = D_R + \frac{D_P}{2}$$

$$\frac{T_Q}{2} = T_R + \frac{T_P}{2}$$

$$T_R = \frac{T_Q - T_P}{2} = \frac{81 - 45}{2} = 18$$

$$T_R = 18$$

So $T_R = 18$ teeth

$T_Q = 81$ teeth

Problem 5.10: In an epicyclic gear train, annular wheel A having 54 teeth meshes with a planet wheel B which gears with the sun wheel C. The wheels A and C are coaxial. The wheel B is carried on a pin fixed on one end of arm P which rotates about the axis of the wheels A and C. If the wheel A makes 20 rpm in the clockwise direction and the arm P rotates at 100 rpm in anti clockwise direction and the wheel C has 24 teeth, determine the speed and sense of rotation of gear C. (FAQ)

Solution:

From the geometry of the figure,

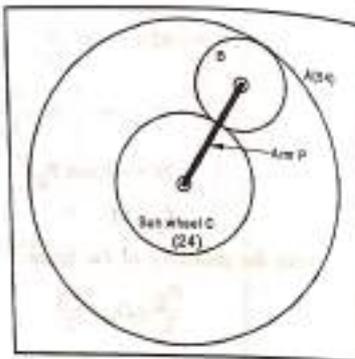
$$\frac{D_A}{2} = D_B + \frac{D_C}{2}$$

$$\frac{T_A}{2} = T_B + \frac{T_C}{2}$$

$$T_B = \frac{T_A - T_C}{2}$$

$$= \frac{54 - 24}{2} = 15$$

$$T_B = 15$$



Apply the given conditions

$$N_P + 100 = y$$

$$y = 100$$

$$N_A = -20 = y - x \frac{T_C}{T_A}$$

$$-20 = 100 - \left[x \times \frac{24}{54} \right]$$

$$-20 = 100 - 0.444x$$

$$0.444x = 120$$

$$x = 270$$

$$N_C = x + y = 100 + 270 = 370 \text{ rpm}$$

$$N_C = 370 \text{ rpm (Anti clockwise)}$$

Problem 5.11: An epicyclic gear consists of a pinion, a wheel of 40 teeth and an annulus with 84 internal teeth concentric with the wheel. The pinion gears with the wheel and the annulus. The arm carries the pinion at one of its end. The arm rotates at 100 rpm. If the annulus is fixed, find the speed of the wheel. Also, find the speed of the annulus, if the wheel is fixed. (FAQ)

Solution:

$$T_S = 40; T_A = 84$$

Condition of motion	Arm N_P	N_C	N_B	N_A
Arm is fixed. Rotate C thro' +1 revolution	0	+1	$\left[\frac{N_B}{N_C} = -\frac{T_C}{T_B} \right]$ $N_B = -\frac{T_C}{T_B}$	$\frac{N_A}{N_B} = \frac{T_B}{T_A}$ $N_A = -\frac{T_C}{T_B} \times \frac{T_B}{T_A}$ $N_A = -\frac{T_C}{T_A}$
Multiply by x	0	+x	$-x \cdot \frac{T_C}{T_B}$	$-x \cdot \frac{T_C}{T_A}$
Add y	y	x+y	$y - x \cdot \frac{T_C}{T_B}$	$y - x \cdot \frac{T_C}{T_A}$

Condition of motion	N_{arm}	N_S	N_P	N_A
Ann. is fixed. Rotate sun wheel S through +1 revolution.	0	+1	$\frac{N_P}{N_S} = -\frac{T_S}{T_P}$ $N_P = -\frac{T_S}{T_P}$	$\frac{N_A}{N_P} = \frac{T_P}{T_A}$ $N_A = -\frac{T_S}{T_P} \times \frac{T_P}{T_A}$ $N_A = -\frac{T_S}{T_A}$
Multiply by x	0	x	$-x \frac{T_S}{T_P}$	$-x \frac{T_S}{T_A}$
Add y	y	x+y	$y - x \frac{T_S}{T_P}$	$y - x \frac{T_S}{T_A}$

$$N_{\text{arm}} = y = 100$$

$$y = 100$$

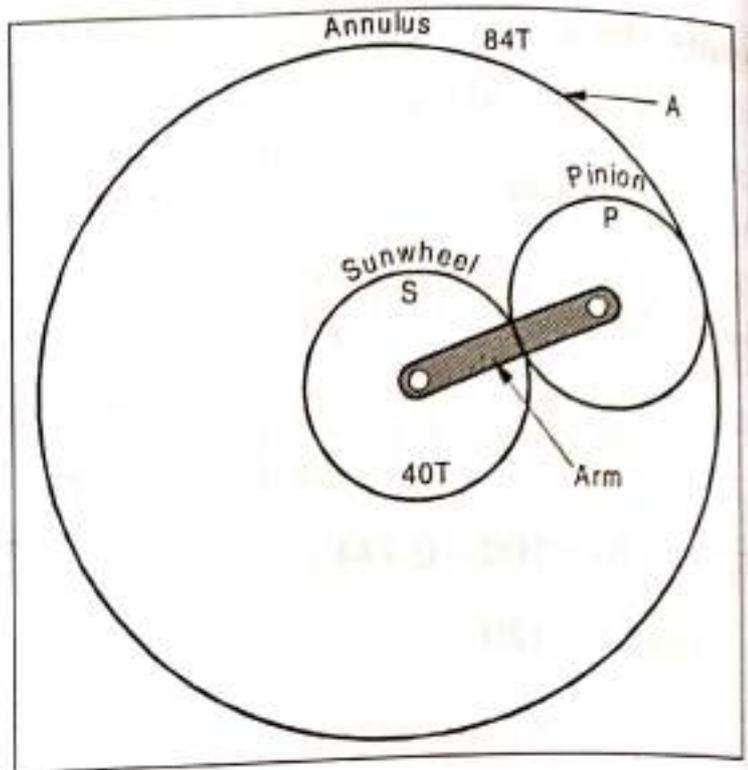
From the geometry of the figure

$$\frac{D_A}{2} = D_P + \frac{D_S}{2}$$

$$\frac{T_A}{2} = T_P + \frac{T_S}{2}$$

$$\frac{84}{2} = T_P + \frac{40}{2}$$

$$T_P = 22$$



Case (i) If the annulus is fixed, then $N_A = 0$: To find N_S

$$N_A = 0 = y - x \frac{T_S}{T_A}$$

$$0 = 100 - x \frac{40}{84}$$

$$x = 100 \times \frac{84}{40} = 210$$

$$x = 210$$

$$N_S = x + y = 210 + 100 = 310 \text{ rpm}$$

$$N_S = 310 \text{ rpm (anticlockwise) (when A is fixed)}$$

Case (ii) If the sun wheel is fixed, then $N_S = 0$; To find N_A

$$N_S = 0 = x + y$$

$$0 = x + 100$$

$$x = -100$$

$$N_A = y - x \frac{T_S}{T_A}$$

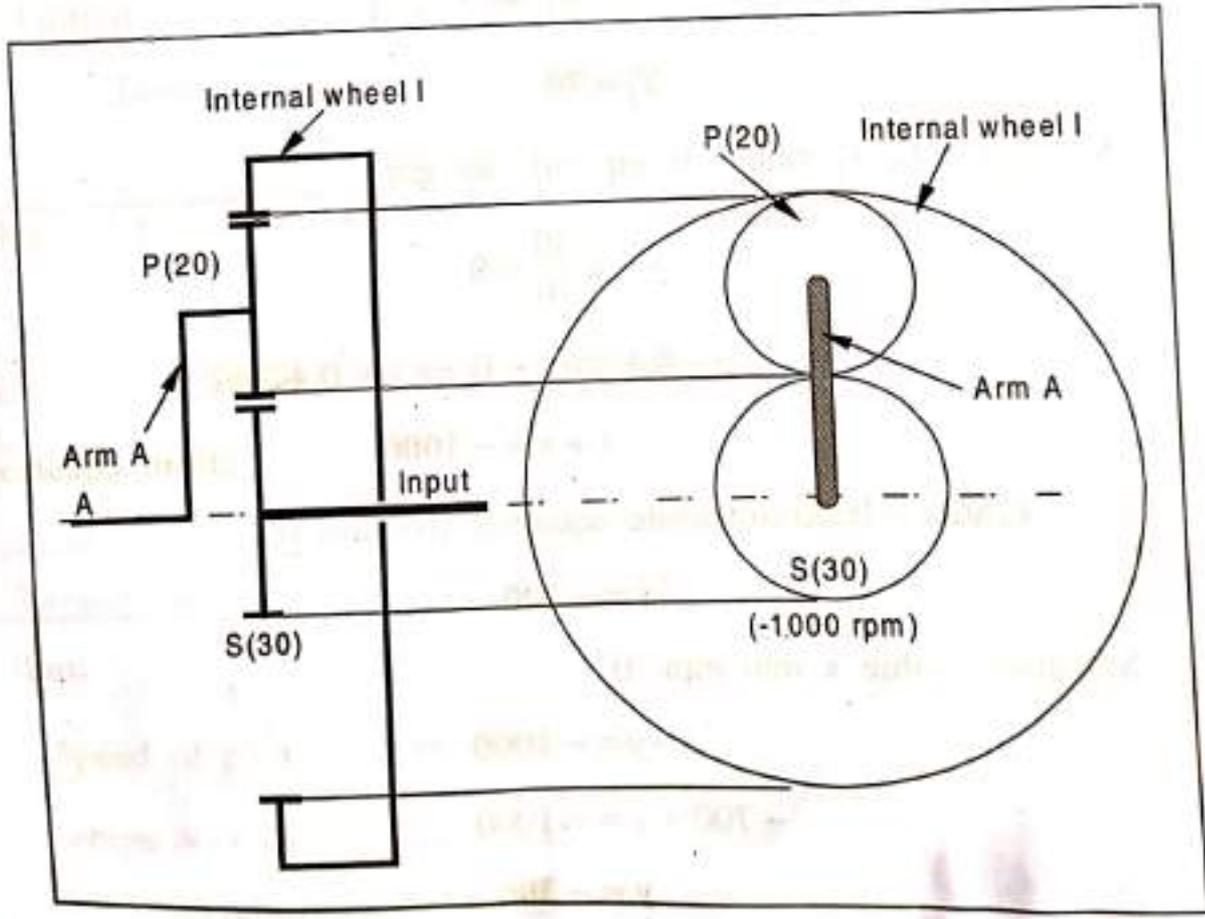
$$= 100 - \left[-100 \times \frac{40}{84} \right] = 147.62$$

$$N_A = 147.62 \text{ rpm (anticlockwise) (when S is fixed)}$$

Problem 5.12: Considering a simple epicyclic gear train consisting of a sun gear S, a planet P, an internal wheel I and an arm A. Explain the method of calculating the speed of the output member A, for an input speed of 1000 rpm (CW) of S, when wheel I is fixed. Number of teeth on S and P are 30 and 20 respectively. For a power input of 3 KW, find the output torque. (FAQ)

Solution

Operation	N_{arm}	N_S	N_P	N_I
Arm is fixed. Rotate sun gear S through +1 revolution	0	+1	$\left[\frac{N_P}{N_S} = -\frac{T_S}{T_P} \right]$ $N_P = -\frac{T_S}{T_P}$	$\frac{N_I}{N_P} = \frac{T_P}{T_I}$ $N_I = -\frac{T_S}{T_P} \times \frac{T_P}{T_I} ; N_I = -\frac{T_S}{T_I}$
Multiply by x	0	x	$-x \frac{T_S}{T_P}$	$-x \frac{T_S}{T_I}$
Add y	y	x+y	$y - x \frac{T_S}{T_P}$	$y - x \frac{T_S}{T_I}$



Apply given condition to respective equations from tables.

$$N_S = -1000 = x + y$$

$$x + y = -1000$$

$$N_I = 0 = y - x \frac{T_S}{T_I}$$

$$y - x \frac{T_S}{T_I} = 0$$

To find T_I

We know, from the geometry of the figure,

$$\frac{D_I}{2} = D_P + \frac{D_S}{2}$$

$$\frac{T_I}{2} = T_P + \frac{T_S}{2}$$

$$T_I = 2 \left[20 + \frac{30}{2} \right]$$

$$T_I = 70$$

Substitute T_S, T_I values in eq. (ii), we get

$$y - x \frac{30}{70} = 0$$

$$y - 0.4286 x = 0 \Rightarrow y = 0.42862 \quad \dots \text{(iii)}$$

$$y + x = -1000 \quad \text{(from equation (i))}$$

$0 - 1.4286 x = 1000$ substitute equation (ii) into (i)

$$x = -700$$

Substitute value x into eqn. (i)

$$x + y = -1000$$

$$-700 + y = -1000$$

$$y = -300$$

So Arm $N_A = y = -300$

Arm $N_A = 300$ rpm (clockwise)

Speed of output member A = 300 rpm (clockwise)

$$\text{Input Power} = \frac{2\pi N_S M_S}{60}$$

$$3000 = \frac{2\pi \times 1000 \times M_S}{60}$$

$$M_S = 28.65 \text{ N-m}$$

Since η is not given, we can take $\eta = 100\%$

So output power = Input power = 3000 W

$$\text{Output power} = \frac{2\pi \times N_A M_A}{60}$$

$$3000 = \frac{2\pi \times 300 \times M_A}{60}$$

$$\text{Output Torque} = M_A = 95.5 \text{ N-m}$$

$$\text{Fixing torque} = 95.5 - 28.65$$

$$M_A = 66.85 \text{ N-m}$$

Problem 5.13: An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star-shaped planet carrier C. The minimum number of teeth on any wheel is 16. The size of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheel S. The driving torque on the sun wheel is 100 N-m. Determine:

- (i) Number of teeth on different wheels of the train and
- (ii) Torque necessary to keep the internal gear stationary. (FAQ)

Solution:

$$\text{Speed of planet carrier (arm)} = N_C = \frac{1}{5} N_S$$

where N_S = speed of sun wheel.

By using given data, we can draw the diagram.