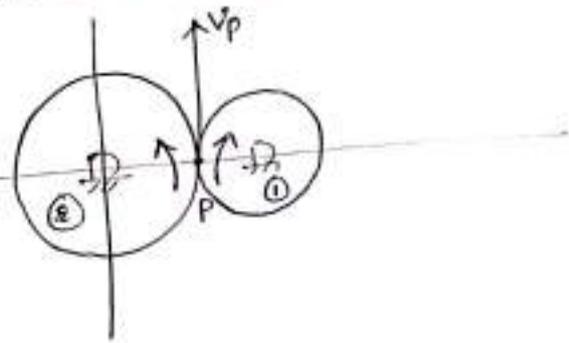


Module-4

Gears

Gears are used to transmit motion from one shaft to another or b/w a shaft and a slide.

If the power transmitted b/w 2 shafts is small, motion b/w these may be obtained by using 2 plane cylinders ① and ② as shown in figure. Such wheels are termed as friction wheels.



$$v_p = \omega_1 r_1 = \omega_2 r_2$$
$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$$
$$\frac{N_1}{N_2} = \frac{r_2}{r_1}$$

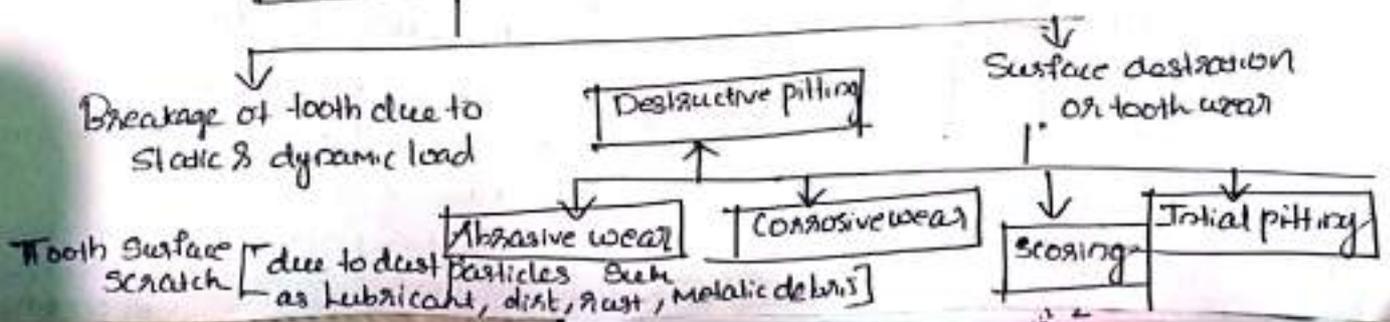
Advantage of Gear drives

- ① It is positive drive and velocity ratio remains constant
- ② The drive is compact, since centre distance b/w shafts are relatively small.
- ③ It can transmit more power
- ④ The efficiency of gear drive is very high (99% of case of spur gear)
- ⑤ In the gear box, shifting of gear is possible. However velocity ratio can be changed over a wide range.

Disadvantages

- ① Gear drives are costly
- ② Maintenance cost is high
- ③ Manufacturing process of gear is complicated
- ④ Gear drive requires careful attention
- ⑤ Accurate alignment of shaft is required

Gear tooth Failures



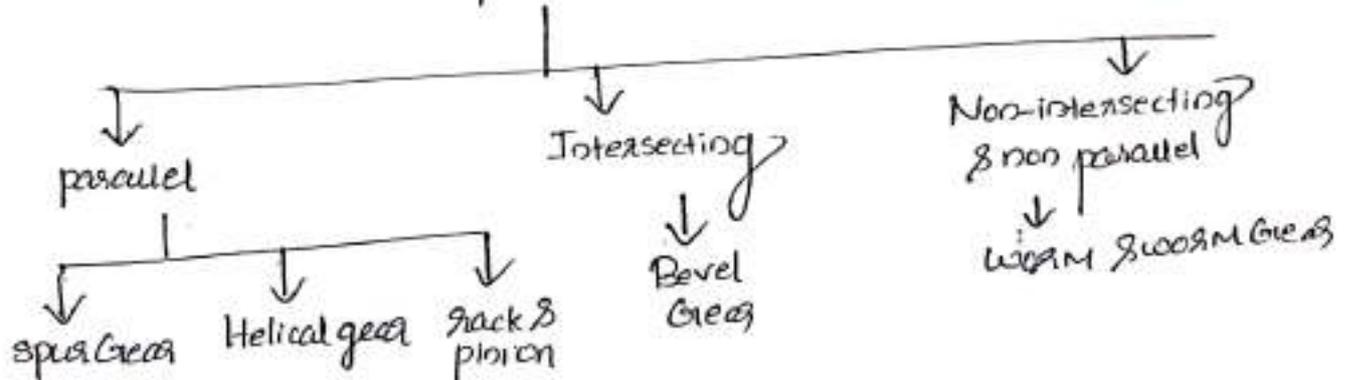
Material For Gears

- ① Metallic Material Eg: CI, Steel, Bronze
- ② Non-metallic material Eg: wood, Rawhide, Compressed paper

Lubrication of Gears

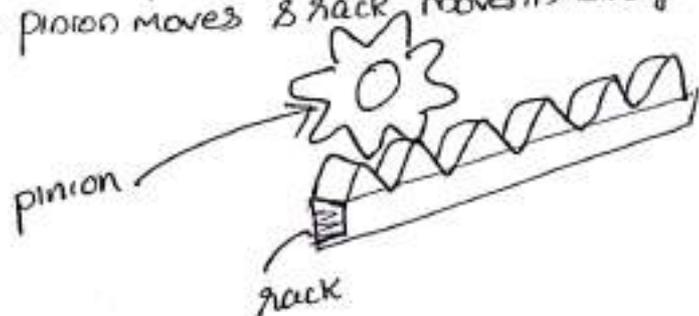
- ① Grease lubrication
- ② Splash lubrication
- ③ Spray lubrication

Classification Gears According to the position of shaft



Rack & pinion

Rack is a special use of spur gear having infinitely large diameter. The teeth are load flat. Torque converted to linear force by meshing rack with pinion. pinion moves & rack moves in straight line.



Classification of Gears

[slow-speed Gears]

① Spur Gear

- used to connect 2 parallel & coplanar shafts.
- Teeth parallel to axis of rotation
- Transmit power from one shaft to another (1-to-shaft)
- used to transmit torque & angular velocity.
- More efficient, No slip



Large gear = Gear
Small gear = Pinion

It used in toys, clocks, household applications. Application
There are internal & External Gears.

Advantages

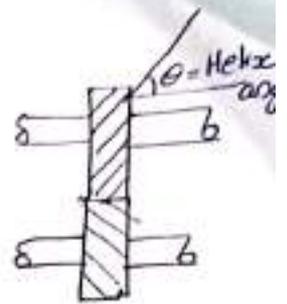
- ① Simplicity in design
- ② Economy of Manufacture & maintenance
- ③ Absence of end-thrust

Disadvantages

- ① Noisy operation, vibration
- ② H mesh along entire width at once sudden impact of tooth on cause noise

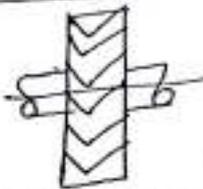
② Helical Gear

- It transmit power b/w parallel shaft
- The teeth of the gear are inclined to the axis of shaft in the form of helix. Hence the name called Helical Gear
- These are usually high speed Gear. Helical Gear take higher load than Spur gear.

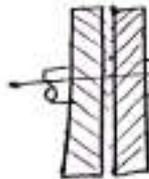


Helix angle = $14.5^\circ - 20^\circ$

Herringbone Gear



Double Helical Gear



They are formed by joining two helical gears of identical pitch and diameter, but of opposite hand.

* In Herringbone & double helical gears, chance of slip less.

- Disadvantages
- Expensive

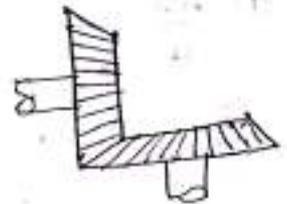
- Application
- High power application such as ship drives

Advantages

- chance of slip less
- It take higher load than spur gear.
- No need of thrust bearing

③ Bevel Gear

- Bevel gears are used for shaft whose axes intersect each other at right angles.
- angle not be 90° . Slightly less than or greater than 90°
- Teeth of bevel gear can be cut straight or spiral
- It impose both radial & thrust load on the shaft.



Advantages

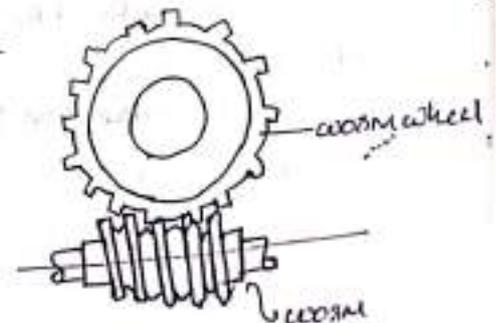
- operating angle can be changed. \therefore It is flexible in operation.
- Efficiency of bevel gear high compared to worm gear.
- Due to rolling action of bevel gear, sliding friction lower.

Disadvantages

- It has limited gear ratio
- Not suitable for high speed reduction
- It produce noise at high speed.

④ Worm and Worm wheel

- power transmitted b/w non intersecting shaft at right angle each other.
- worm-threaded screw and toothed wheel
- used for high speed reduction ratio
- worm in the form of threaded screw
It mesh with a wheel
- worm impose high thrust load & worm wheel impose high radial load.



Disadvantages

- Efficiency low
- ① gear gear material expensive
- ② High power losses
- ③ Generate Heat

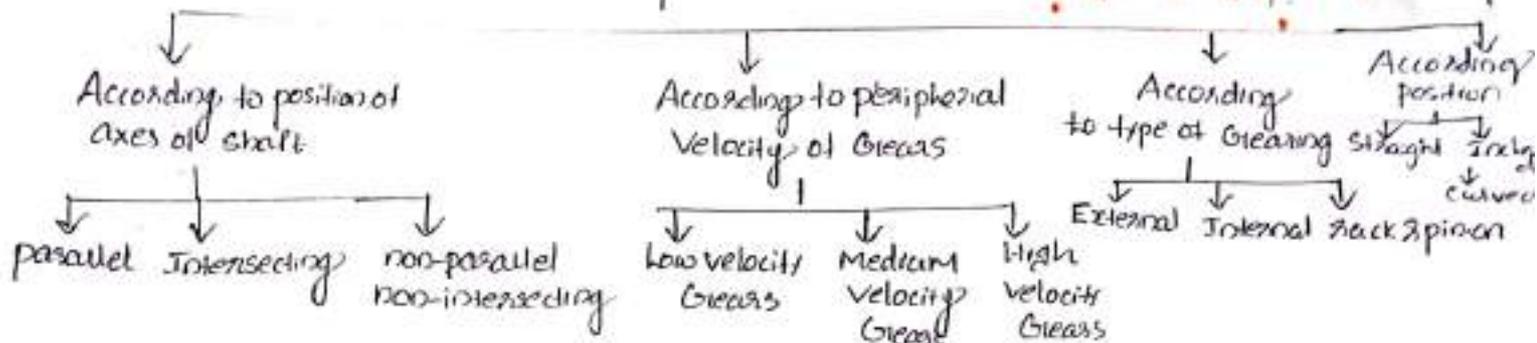
Application
elevators, lifting m/c

① silent operation

② High speed reduction ratio 300:1

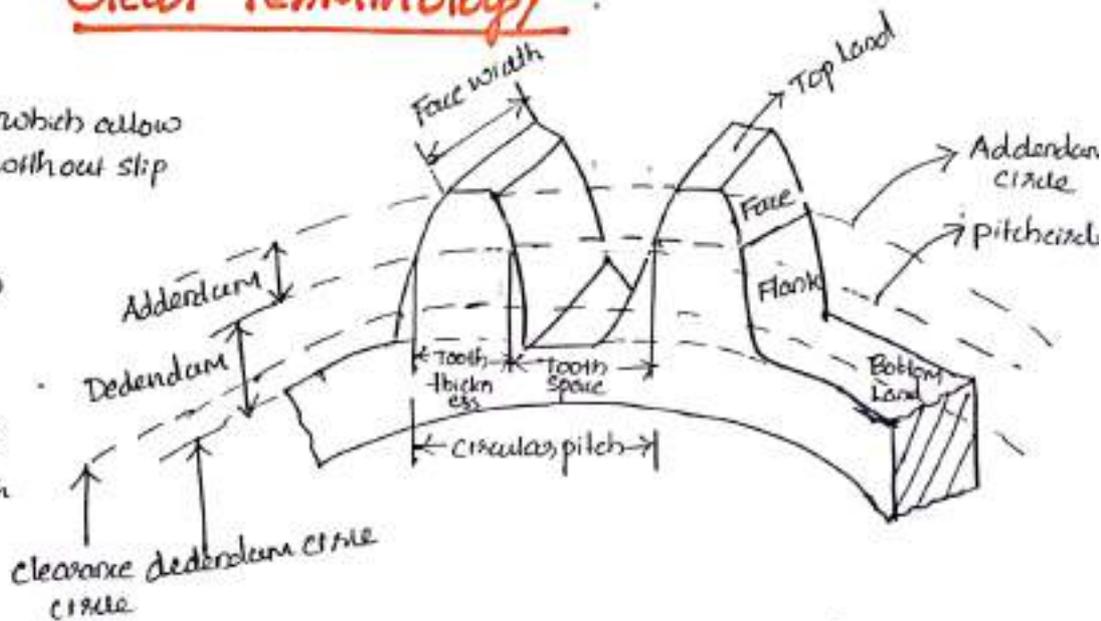
③ worm & worm wheel selflocking? characteristics: Friction reverse motion occupy lesser space

General classification



Gear Terminology

- ① pitch circle
• It is imaginary circle which allow pure rolling action without slip
- ② pitch point
point of contact of two pitch circle.
- ③ Addendum circle
It is the circle passing through the tips of teeth



- ④ Addendum
It is radial height of tooth above pitch circle. [Addendum = 1 Module]
- ⑤ dedendum or root circle
It is circle passing through the root of the teeth
- ⑥ Dedendum
It is the radial depth of a tooth below pitch circle. [Dedendum = 1.57 Module]
- ⑦ clearance
Radial difference b/w addendum and dedendum of a tool.
- ⑧ Face
Tooth surface b/w pitch circle and top line.
- ⑨ Flank
Tooth surface b/w pitch circle and bottom land including fillet.
- ⑩ Tooth thickness
It is the thickness of tooth measured along a pitch circle.
- ⑪ Backlash
Difference b/w space width & tooth thickness along pitch circle.
- ⑫ Circular pitch (P_c) ⇒ It is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.

$$P_c = \frac{\pi d}{T}$$

$d \rightarrow$ pitch diameter
 $T \rightarrow$ Number of teeth

(13) Diametral pitch (P)

It is the number of teeth per unit length of pitch circle diameters

$$P = \frac{T}{d}$$

(14) Module (m)

It is the ratio of the pitch diameter in mm to the number of teeth.

$$m = \frac{d}{T} \quad \therefore P_c = \pi m$$

(15) Gear ratio:

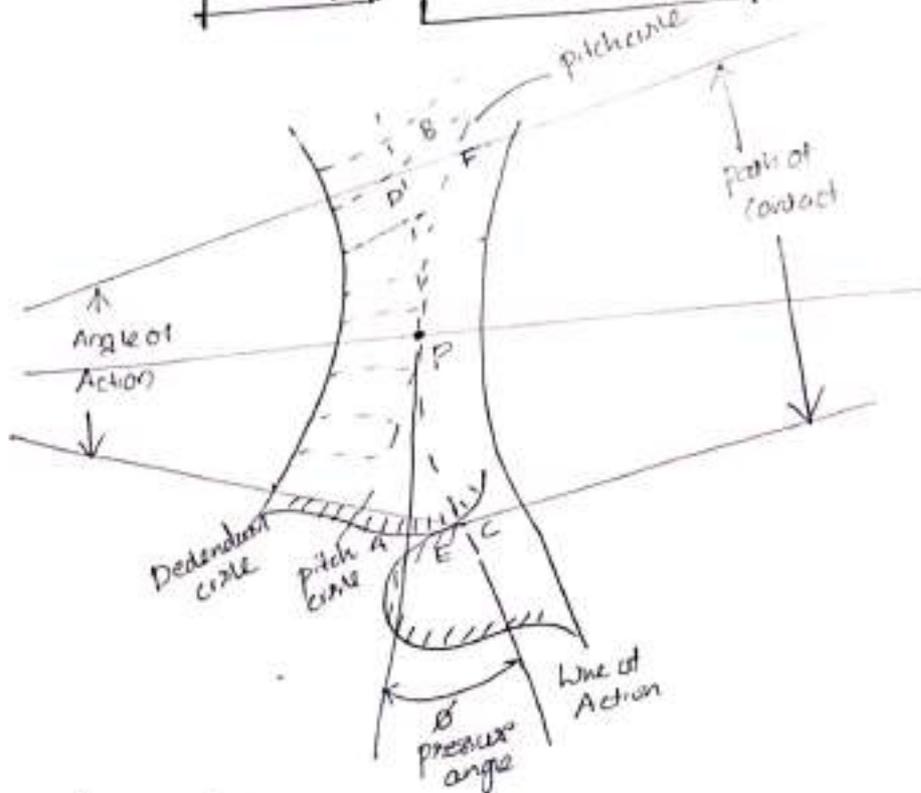
It is the ratio of the number of teeth on gear to that on the pinion.

$$G_1 = \frac{T_2}{T_1} \quad \text{or} \quad G_1 = \frac{T}{t}$$

$T = T_2 \rightarrow$ No. of teeth on the wheel
 $t = T_1 \rightarrow$ No. of teeth on pinion

(16) Velocity ratio (V_R) - Ratio of angular velocity of the follower to the angular velocity of driving gear.

$$V_R = \frac{\omega_2}{\omega_1} \quad V_R = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$



(a) Line of Action / pressure line

- The force, which driving tooth exerts on the driven tooth is along a line from the pitch point to the point of contact of the two teeth.

This line is also the common normal at the point of contact of the mating gears and is known as line of action or the pressure line.

(b) pressure angle or Angle of obliquity (ϕ)

The angle b/w the pressure line and the common tangent to the pitch circles is known as pressure angle.

For more power transmission and lesser pressure on the bearings the pressure angle must be kept small.

Standard pressure angle = $20^\circ - 25^\circ$

- ① Two spur gears have a velocity ratio of $\frac{1}{3}$. The drive gear has 72 teeth of 8mm module & rotates 800rpm. Calculate no. of teeth and speed of the driven. What will be the pitch line velocity?

Solution

Given $\Rightarrow VR = \frac{1}{3} \Rightarrow T_2 = 72 \text{ mm}, m = 8 \text{ mm}, N_2 = 800 \text{ rpm}$

$VR = \frac{T_1}{T_2} \Rightarrow T_1 = 72 \times \frac{1}{3} = 24$

$VR = \frac{N_2}{N_1} \Rightarrow N_1 = \frac{N_2}{VR} = \frac{800}{\frac{1}{3}} = 2400 \text{ rpm}$

$V = r_1 \omega_1 = r_2 \omega_2 \Rightarrow \omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2400}{60} = 251.33 \text{ rad/s}$

$m = \frac{d_1}{T_1} \Rightarrow d_1 = 24 \times 8 = 192 \text{ mm}$

$V = r \omega = \frac{192}{2} \times 251.33 = 24133 \text{ mm/s} = 24.133 \text{ m/s}$

Fundamental Law of Gearing

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio b/w two gears.

\Rightarrow Figure shows two bodies 1 & 2 representing a portion of two gears in mesh.

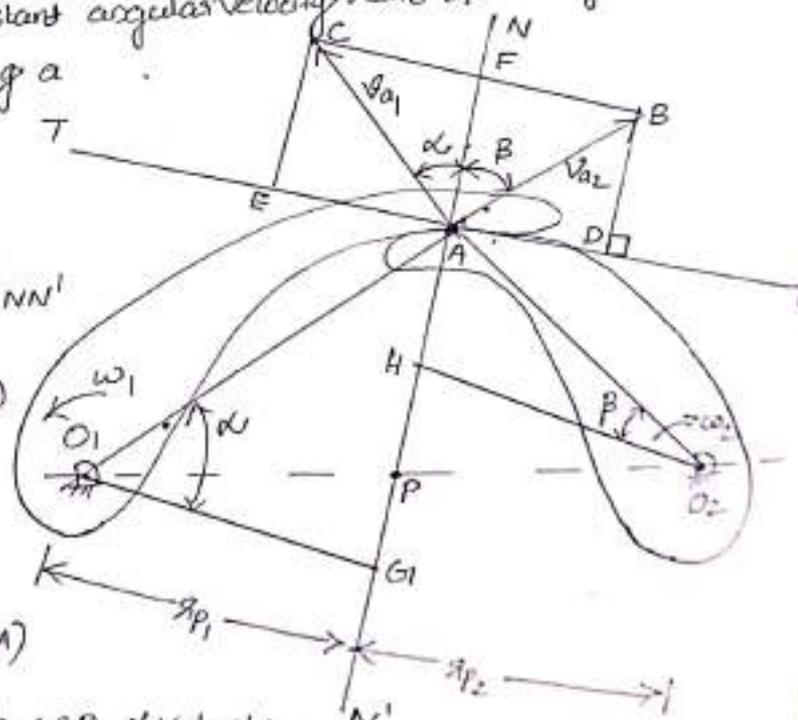
\Rightarrow Two bodies 1 & 2 pivoted at O_1 & O_2 and contacting point A.

\Rightarrow Common tangent & Normal are TT' & NN' at point A.

$\Rightarrow \omega_1$ & ω_2 angular velocity of body 1 & 2.

V_{A1} = Velocity of point A considering to be a point of body 1 (radius O_1A)

V_{A2} = Velocity of point A considering to be a point on body 2 (radius O_2A)



Component velocity $V_{A1} \cos \alpha$ & $V_{A2} \cos \beta$ of velocity vectors V_{A1} & V_{A2} respectively along the common normal NN' must be equal. If they are not equal, either body 1 would dig into body 2.

$$V_{a1} \cos \alpha = V_{a2} \cos \beta \quad \text{--- (1)}$$

$$\left. \begin{aligned} V_{a1} &= (O_1 A) \omega_1 \\ V_{a2} &= (O_2 A) \omega_2 \end{aligned} \right\} \text{(2)}$$

$$\text{Sub eq (2) in eq (1)} \Rightarrow O_1 A \omega_1 \cos \alpha = O_2 A \omega_2 \cos \beta \quad \text{--- (3)}$$

Consider right angles $\triangle O_1 G_1 A$ & $\triangle O_2 H A$

$$\left. \begin{aligned} O_1 A \cos \alpha &= O_1 G_1 \\ O_2 A \cos \beta &= O_2 H \end{aligned} \right\} \text{(4)}$$

$$\text{Sub eq (4) in (3)} \Rightarrow O_1 G_1 \omega_1 = O_2 H \omega_2 \Rightarrow \boxed{\frac{\omega_2}{\omega_1} = \frac{O_2 H}{O_1 G_1}}$$

$\triangle O_1 G_1 P$ & $\triangle O_2 H P$ are similar

$$\frac{O_1 G_1}{O_2 H} = \frac{O_1 P}{O_2 P} = \frac{P G_1}{P H} \Rightarrow \frac{O_2 H}{O_1 G_1} = \frac{O_2 P}{O_1 P} = \frac{P H}{P G_1}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}} \quad \boxed{\frac{\omega_1}{\omega_2} = \frac{P G_2}{P G_1}}$$

- Law of gearing states that for transmitting constant angular velocity ratio common normal to the contacting surfaces of mating teeth, at every instantaneous point of contact, must pass through a fixed point on the line of centres of two gears. This fixed point is called pitch point
- which divides the line of centres in the inverse ratio of angular velocities of mating gears.

Velocity of sliding

$$\text{Velocity of sliding} = V_{a2} \sin \beta + V_{a1} \sin \alpha$$

$$V_{sl} = \omega_2 O_2 A \sin \beta + \omega_1 O_1 A \sin \alpha$$

$$V_{sl} = \omega_2 \times AH + \omega_1 \times AG_1$$

$$V_{sl} = \omega_1 AG_1 + \omega_2 AH$$

$$V_{sl} = \omega_1 (AP + PG_1) + \omega_2 (AP - PH)$$

$$V_{sl} = (\omega_1 + \omega_2) AP + \omega_1 PG_1 - \omega_2 PH$$

Consider similar triangles $\triangle O_1 G_1 P$ & $\triangle O_2 H P$

$$\frac{O_2 H}{O_1 G_1} = \frac{O_2 P}{O_1 P} = \frac{P H}{P G_1} = \frac{\omega_1}{\omega_2} = \frac{P G_2}{P G_1} \quad \left(\omega_1 P G_1 = \omega_2 P H \right)$$

$$\boxed{V_{sl} = (\omega_1 + \omega_2) AP}$$

V_{sl} = Sum of angular velocities \times distance of point of contact from pitch point

Relation b/w Base circle radii, pitch circle radii & central distance

Terms { where O_1, O_2 - centre of gear 1 & 2, R_{b1} & R_{b2} = Base circle dia of Gear 1 & 2
 ω_1, ω_2 - Angular Velocities of gear 1 & 2, R_{p1} & R_{p2} }

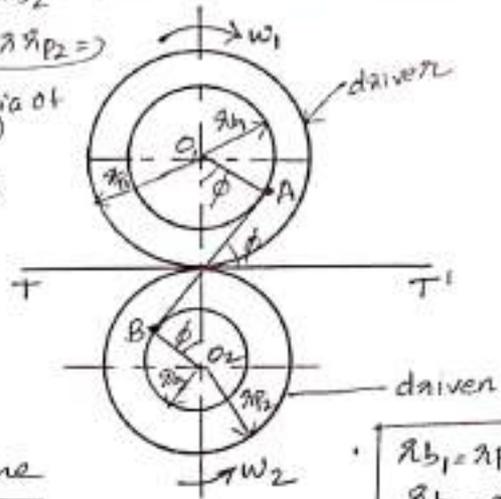
[AB] Line of Action (pressure line)

pitch circle dia of Gears 1 & 2

- It represents BA in Figure. The force which driving tooth exert on the driven tooth is along a line from pitch point to the pitch point of contact of the two teeth.

This line also the Common normal at the point of contact of the mating gears.

It is known as line of Action or pressure line



$$R_{b1} = R_{p1} \cos \phi$$

$$R_{b2} = R_{p2} \cos \phi$$

pressure Angle (Angle of obliquity) ϕ

- It is the angle b/w pressure line and common tangent to the pitch circle
- For more power transmission and lesser pressure on bearings, pressure angle (ϕ) must be kept small

Standard pressure angle = $20^\circ - 25^\circ$

$\phi = 14.5^\circ$ Absolute

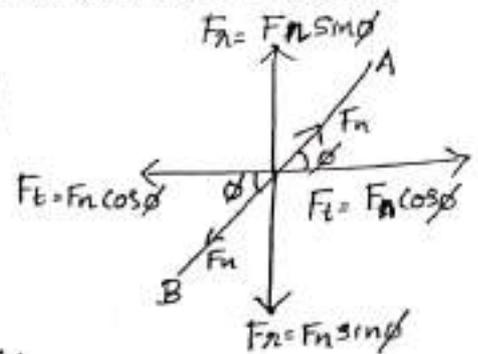
Significance of pressure angle

Let F_n be force transmitted from driver (pinion) to (Gear) driven along AB

Resolving force along tangent TT' & common line of centres

$$F_t = F_n \cos \phi, F_r = F_n \sin \phi$$

- Here F_t produce driving torque ($F_t R_{p2}$) and useful component.
- while component F_r tends to separate the shafts at O_1, O_2 apart. The component F_r is undesirable as it produce bending in the shaft and increases bearing loads.



From purely load transmission point of view, it is desirable to have pressure angle ϕ as small as possible.

Effect of increasing centre distance & velocity ratio

- consider pair of meshing Gears having involute teeth
- centre closer will lead to jamming or deforming teeth.
- If centre distance increase \rightarrow it lead to clearance & backlash b/w mating teeth
- If centre distance increased we have new pitch circles of larger radii for each gear, keeping the ratio of pitch circle radii same.

r_p & R_p be pitch circle radius $\frac{r_p}{R_p}$ same

$$r_{b1} = r_{p1} \cos \phi, \quad R_{b2} = R_{p2} \cos \phi$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{r_{p2}}{r_{p1}} = \frac{r_{b2}}{r_{b1}}}$$

Velocity ratio remain unchanged
If increase centre distance
 ϕ increases, centre distance increase

- ① Two spur gears having velocity ratio $\frac{1}{3}$. The driven gear has 72 teeth and 8mm module and rotates at 300rpm. Calculate
- No: of teeth & speed of driver
 - What will be the pitch line velocities.

GID $\rightarrow T_2 = 72, v.R = \frac{1}{3}, N_2 = 300 \text{ rpm}, m = 8 \text{ mm}$

(a) $v.R = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{3} \Rightarrow \frac{N_2}{N_1} = \frac{1}{3} \Rightarrow \frac{300}{N_1} = \frac{1}{3} \Rightarrow N_1 = \frac{3N_2}{1} = \frac{3 \times 300}{1} = 900 \text{ rpm}$

$$\frac{T_1}{T_2} = \frac{1}{3} \Rightarrow T_1 = \frac{1}{3} \times T_2 = \frac{1}{3} \times 72 = \underline{24}$$

(b) pitch line velocity, $V_p = \omega_1 r_{p1} = \omega_2 r_{p2}$

$$m = \frac{d_{p1}}{T_1} = \frac{d_{p2}}{T_2} \Rightarrow d_{p1} = m T_1 = 8 \times 24 = 192 \text{ mm}, r_{p1} = \underline{96 \text{ mm}}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 900}{60} = \underline{94.24 \text{ rad/sec}}$$

pitch line velocity, $V_p = \omega_1 r_{p1} = 94.24 \times 96 \times 10^{-3} = \underline{9.047 \text{ m/s}}$

- ② The number of teeth on a spur gear is 30 and it rotates at 200rpm. What will be its circular pitch and pitch line velocities if it has a module of 2mm.

GID $T = 30, m = 2 \text{ mm}, N = 200 \text{ rpm}$

$$\text{Circular pitch} = \frac{\pi d_p}{T} = \pi m = \pi \times 2 = \underline{6.28 \text{ mm}}$$

$$V_p = \omega r = \frac{2\pi N}{60} \frac{d_p}{2} = \frac{2\pi N \times m T}{60 \times 2} = \frac{628.3 \text{ mm/s}}{2} = \underline{0.6283 \text{ m/s}}$$

- ③ The following data refer to two meshing gear velocity ratio = $\frac{1}{3}$. Module = 4mm, pressure angle = 20° , centre distance = 200mm. Determine the number of teeth and base circle radius of gear wheel

GID

$$v.R = \frac{1}{3}, \phi = 20^\circ, m = 4 \text{ mm}, C = 200 \text{ mm}$$

$$v.R = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{3}$$

$$\boxed{T_2 = 3T_1}$$

(a) $C = \frac{d_{p1} + d_{p2}}{2} = \frac{m(T_1 + T_2)}{2} \Rightarrow 200 = \frac{4(T_1 + 3T_1)}{2} \Rightarrow T_1 = \underline{25}$

$T_2 = 3 \times 25 = \underline{75}$ (No: of teeth on Gear wheel = 75)

(b) $d_{p2} = m T_2 = 4 \times 75 = 300 \text{ mm}, r_{p2} = \frac{d_{p2}}{2} = \frac{300}{2} = 150 \text{ mm}$

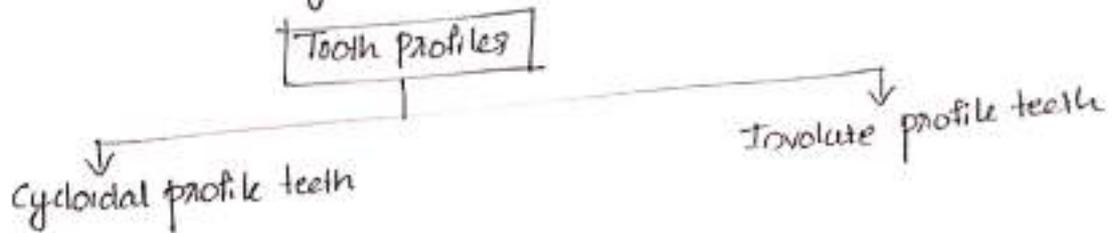
$$r_{b2} = r_{p2} \cos \phi = 150 \times \cos 20^\circ = \underline{141.2 \text{ mm}}$$

Forms of Teeth

* Conjugate Teeth

- When mating tooth profiles are so shaped that they produce a constant angular velocity ratio during mesh (Apply law of gearing). The tooth surface are said to be conjugate teeth
- Even though a large No. of conjugate curves are possible, it is in random profile and difficult to manufacture, high cost of production, wear etc. So conjugate teeth not used in normal use.

In Actual practice two types of teeth commonly used which satisfy law of Gearing.



① Cycloidal Profile teeth

A cycloid is a locus of a point on circumference of a circle wheel roll without slipping on a fixed straight line.

It has two variants (a) Epicycloid (b) Hypocycloid

(a) Epicycloid

It is the locus of a point on circumference of a circle, which roll without slipping on outside circumference of another circle of finite radius.

(b) Hypocycloid

It is a locus of a point on the circumference of circle, which roll without slipping on inside circumference of another circle.

Formation of cycloidal teeth of a Gear

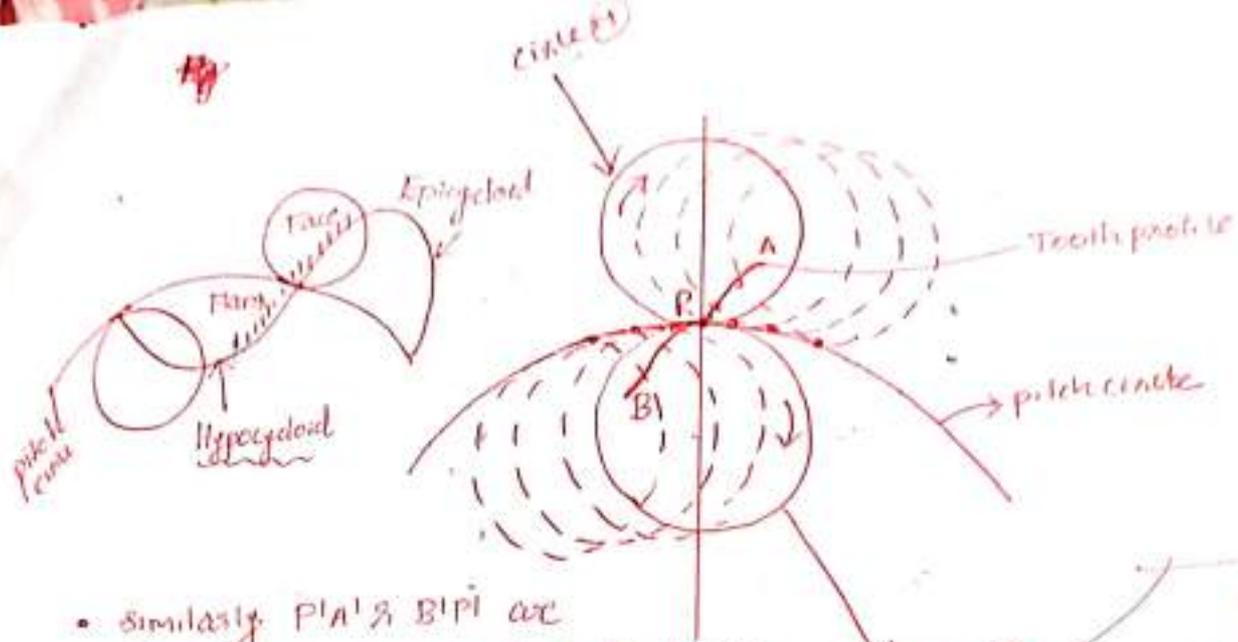
- Rolling circle M, roll on the out side pitch circle to the right without slipping and point P on the circumference of circle traces an epicycloid.

$PA = \text{position}$ = Face position

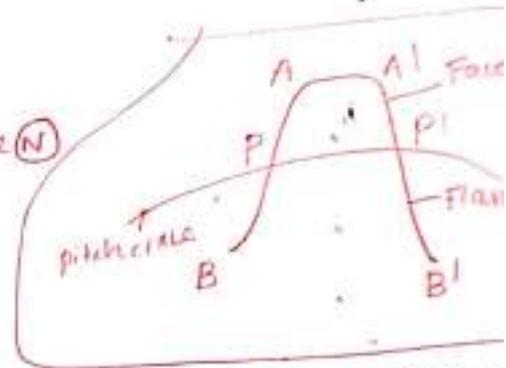
- Rolling circle N, without slipping on the pitch circle to the left on the inside circle. The point P on the circumference of circle traces hypocycloid.

$PB = \text{Flank position}$

profile BPA = one side of cycloidal tooth

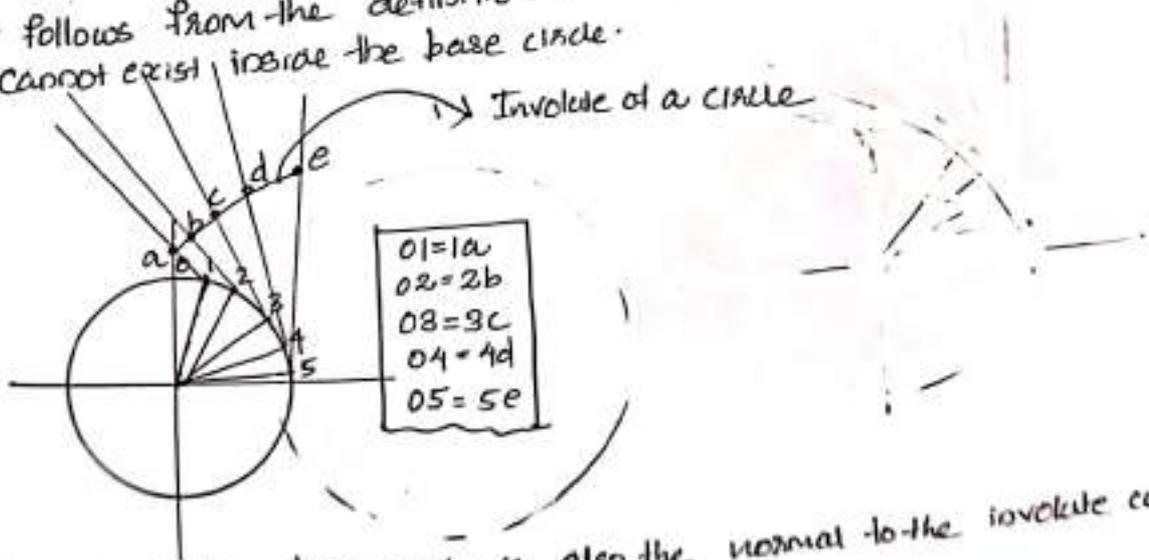


- Similarly, $P'A'$ & $B'P'$ are formed opposite side of tooth profile traced by the point P' where circle M & N rolls opposite direction.
- In cycloidal tooth profile pressure angle (ϕ) varies constantly and zero at pitch point and increase during faces.
- Variation in ϕ lead to additional wear & wear. Also change bearing reaction at the shaft supports.



Involute Tooth Profile

- It is the locus of a point on a tight string as the string is unwrapped from the circumference of a circle.
- The circle from which the tight string is unwrapped or over which the line rolls without slipping is called Base circle.
- It follows from the definition of involute curve that the involute curve cannot exist inside the base circle.



- Tangent to the base circle is also the normal to the involute curve at corresponding point.

Comparison b/w Involute & Cycloidal Profiles

Characteristic	Involute Gears	Cycloidal Gears
① Pressure angle	constant throughout the engagement	Varies from commencement to end
② Ease of Manufacture	Easy to Manufacture	Difficult to Manufacture
③ Centre distance	Do not require exact centre distance	Require exact centre distance
④ Interference	May occur	No interference
⑤ Strength	Less	More
⑥ wear	More	Less
⑦ Running	Smooth	Less smooth

* Advantages of Involute Profile

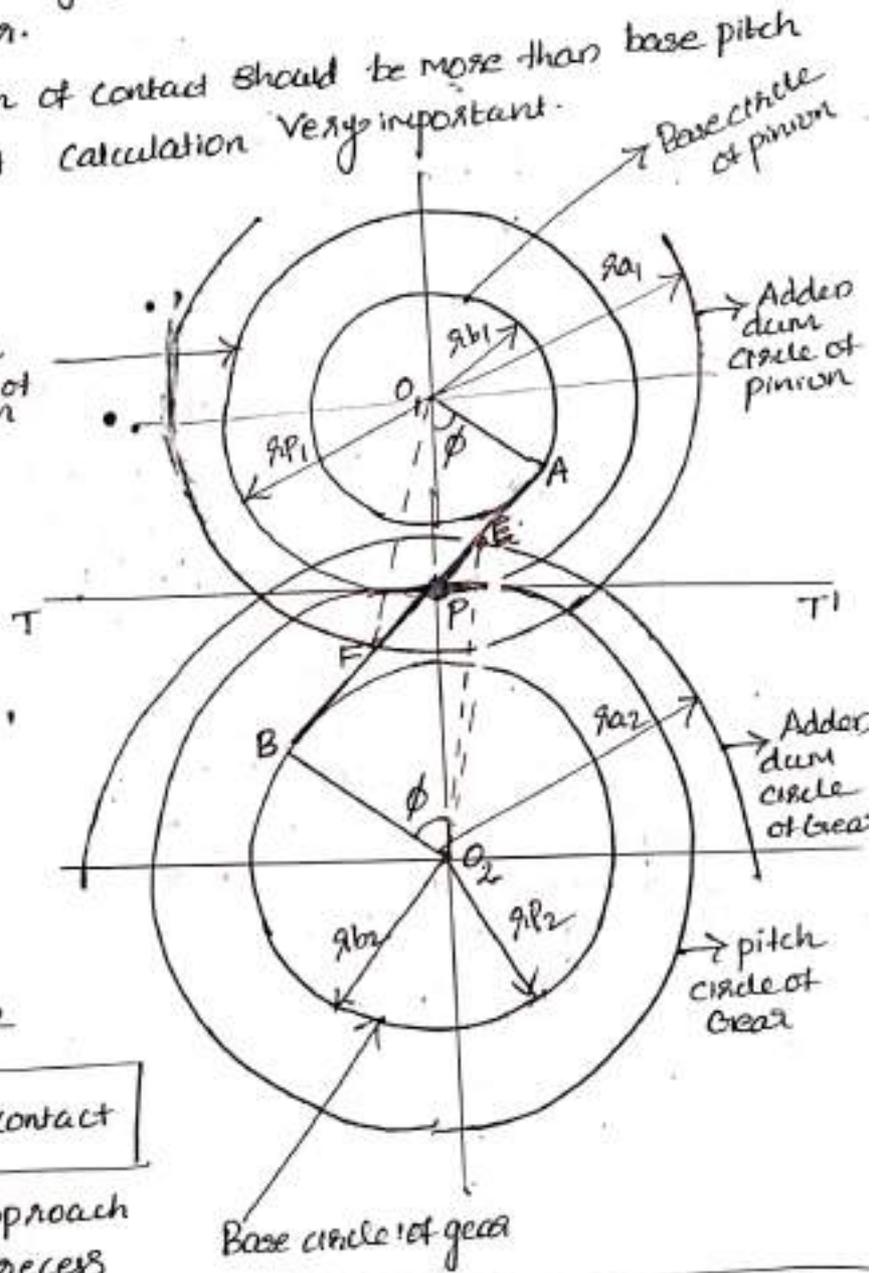
- ① Maintain conjugate action
- ② Operating pressure angle (ϕ) is constant throughout engagement
- ③ Maintain conjugate action even if centre distance b/w Gears varies.
- ④ Easy to Manufacture
- ⑤ Smooth operation.

Length of path of Contact

- For a smoother transmission of power, it is necessary that length of arc of contact between a pair of teeth must at least be equal to circular pitch of teeth.
- when length of arc of contacts equals circular pitch, a second pair of gear teeth begins to engage before the engagement b/w the preceeding pair is over.
- It seen the length of path of contact should be more than base pitch
- length of path of contact calculation very important.

Figure

- consider pinion (1) drives Gear (2) shown figure
- pinion rotates C.W direction
- Flank portion near the base circle of pinion comes in contact with addendum circle of mating gear tooth at point E
- Gear pair rotates further point of contact along common tangent AB to the base circle. Contact continue upto point F. Addendum of pinion contact with flank portion of tooth of Gear 2



Length EF = Length of path of contact

Length EP = Length of path of approach
 Length FP = Length of path of recess

Length of path of contact (EF) = Length of path of Approach (EP) + Length of path of Recess (FP)

Vimp

Length of path of Approach (EP)

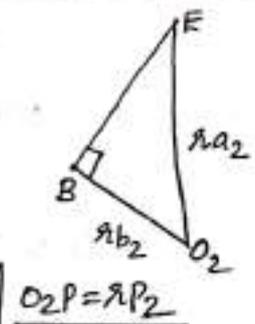
$EP = BE - BP$ — (1)

Sub: $BE \propto BP$ in eq (1)

Length of path of Approach
 = $(r_{a2}^2 - r_{b2}^2)^{1/2} - r_{p2} \sin \phi$

where $\Delta BO_2E \Rightarrow BE = (r_{a2}^2 - r_{b2}^2)^{1/2}$

where $\Delta BO_2P \Rightarrow \sin \phi = \frac{BP}{O_2P}$
 (A) $BP = (r_{p2} \sin \phi)$



Length of path of recess (FP)

$$FP = AF - AP \quad \text{--- (2)}$$

$$FP = (\frac{r_{a1}^2 - r_{b1}^2}{2})^{1/2} - r_{p1} \sin \phi \quad \text{--- (B)}$$

where $\Delta O_1 AF$

$$AF = (\frac{r_{a1}^2 - r_{b1}^2}{2})^{1/2}$$

$\Delta O_1 AP$

$$\sin \phi = \frac{AP}{PO_1} \quad | \quad O_1P = r_{p1}$$

$$AP = r_{p1} \sin \phi$$

Length of path of contact = EP + FP

$$L.P.C = (\frac{r_{a2}^2 - r_{b2}^2}{2})^{1/2} + (\frac{r_{a1}^2 - r_{b1}^2}{2})^{1/2} - (r_{p1} + r_{p2}) \sin \phi$$

$$r_{a2} \rightarrow r_{p2} + a$$

$$r_{a1} \rightarrow r_{p1} + a$$

$$r_{b1} = r_{p1} \cos \phi$$

$$r_{b2} = r_{p2} \cos \phi$$

Length of Arc of Contact

It is the length of the arc of pitch circle described by a point of a tooth on pitch circle from beginning to the end of engagement between a pair of teeth in Mesh.

- Consider tooth surface CDE revolves C-C-W to the position GHI through angle θ at point D on the pitch circle moves to position H & simultaneously a point C moves to the position EH on base circle.

$$\angle CO_2G = \angle DO_2H$$

$$\text{arc } CG = r_{b2} \theta \quad \text{--- (1)}$$

$$\text{arc } DH = r_{p2} \theta \quad \text{--- (2)}$$

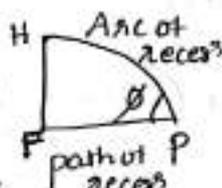
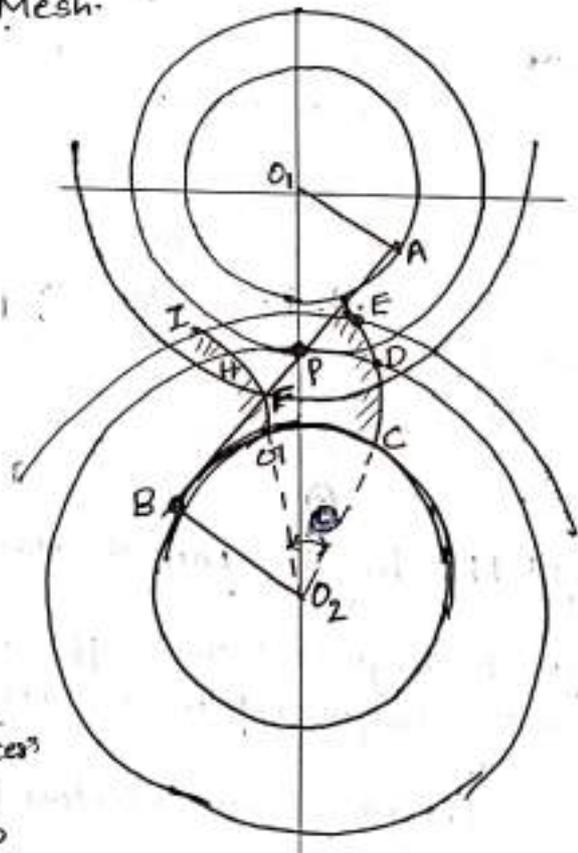
From (1) $\Rightarrow \theta = \frac{\text{arc } CG}{r_{b2}}$

$$\text{arc } PH = \frac{PF}{\cos \phi}$$

$$\text{arc of recess} = \frac{\text{path of recess}}{\cos \phi} \quad \text{--- (1)}$$

$$\text{arc } PD = \frac{\text{path of approach}}{\cos \phi} \quad \text{--- (2)}$$

$$\text{Arc of contact} = \text{Arc of Approach} + \text{Arc of recess} = \frac{\text{path of Approach} + \text{path of recess}}{\cos \phi}$$



$$\text{Arc of contact} = \frac{\text{Length of path of contact}}{\cos \phi}$$

Contact Ratio

- Contact Ratio is one of the measurable quantity of Gear drive.
- It is defined as average number of pairs of teeth which are in contact.
- Length of arc of contact of a pair of mating teeth and circular pitch are both measured along pitch circle.

$$\text{Contact Ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

Its values lies b/w 1 & 2 (Not integer No.)

Contact Ratio = 1

- One tooth together with adjacent tooth space will occupy the entire arc of contact.
- Other words when tooth just enter into engagement, the preceding tooth is just to go out of contact from its mating tooth. Thus all through the motion of arc of contact, exactly one pair of teeth remains constant.

Contact Ratio 1.4

Eg: Contact Ratio = 1.4 \Rightarrow when a pair of teeth is just entering into contact (engagement), the preceding tooth pair continues to remain in contact. Thus for a short spell of time (40% total spell time) two pairs of teeth will be in mesh & remaining time, there will be contact for one pair of teeth only.

① A pinion having 30 teeth drives a gear having 80 teeth. The profile of gear is involute with 20° pressure angle, 12mm Module and 10mm Addendum. Find length of path of contact, arc of contact & contact ratio.

Solution

G.D $\rightarrow T_1 = 30, T_2 = 80, \phi = 20^\circ, m = 12\text{mm}, a = 10\text{mm}$

(a) $L.P.C = (\lambda a_2^2 - \lambda b_2^2)^{1/2} + (\lambda a_1^2 - \lambda b_1^2)^{1/2} - (\lambda P_1 + \lambda P_2) \sin \phi$

$\lambda a_2 = \lambda P_2 + a$

$\lambda P = \frac{mT}{2} \Rightarrow \lambda P_1 = \frac{mT_1}{2} = \frac{12 \times 30}{2} = 180\text{mm}, \lambda P_2 = \frac{mT_2}{2} = \frac{12 \times 80}{2} = 480\text{mm}$

$\lambda a_2 = \lambda P_2 + a = 480 + 10 = 490\text{mm}, \lambda a_1 = \lambda P_1 + a = 180 + 10 = 190\text{mm}$

$L.P.C = (\lambda a_2^2 - \lambda b_2^2)^{1/2} + (\lambda a_1^2 - \lambda b_1^2)^{1/2} - (\lambda P_1 + \lambda P_2) \sin \phi$
 $= (\lambda a_2^2 - \lambda P_2^2 \cos^2 \phi)^{1/2} + (\lambda a_1^2 - \lambda P_1^2 \cos^2 \phi)^{1/2} - (\lambda P_1 + \lambda P_2) \sin \phi$
 $L.P.C = (490^2 - 480^2 \cos^2 20^\circ)^{1/2} + (190^2 - 180^2 \cos^2 20^\circ)^{1/2} - (180 + 480) \sin 20^\circ$
 $= 52.39\text{mm}$

(b) Length of arc of contact = $\frac{\text{length of path of contact}}{\cos \phi} = \frac{52.39}{\cos 20^\circ} = 55.66\text{mm}$

(c) contact ratio = $\frac{\text{length of arc of contact}}{\text{Circular Pitch } (P_c)} = \frac{55.66}{37.7} = 1.5$

$P_c = \pi m$
 $= \pi \times 12$
 $= 37.7\text{mm}$

② Two involute gears in mesh have 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6mm. The pitch line velocity is 1.5m/s & addendum equal to one module.

Determine (a) Angle of action of pinion [Angle measured through by pinion action on part of teeth in mesh]
 (b) Maximum velocity of sliding

G.D $\phi = 20^\circ, T_1 = 24, m = 6\text{mm}, \text{Gear ratio} = \frac{T_2}{T_1} = 3, T_2 = 3T_1 = 3 \times 24 = 72$
 $a = 1 \text{ module}$

(a) Angle of Action = $\frac{\text{arc of contact}}{\lambda P}$

$\lambda a_1 = \lambda P_1 + a = 72 + 6 = 78\text{mm}$
 $\lambda a_2 = \lambda P_2 + a = 216 + 6 = 222\text{mm}$

$\lambda P_1 = \frac{mT_1}{2} = \frac{6 \times 24}{2} = 72\text{mm}$
 $\lambda P_2 = \frac{mT_2}{2} = \frac{6 \times 72}{2} = 216\text{mm}$

$L.P.C = (\lambda a_2^2 - \lambda P_2^2 \cos^2 \phi)^{1/2} + (\lambda a_1^2 - \lambda P_1^2 \cos^2 \phi)^{1/2} - (\lambda P_1 + \lambda P_2) \cos \phi$
 $= (222^2 - 216^2 \cos^2 20^\circ)^{1/2} + (78^2 - 72^2 \cos^2 20^\circ)^{1/2} - (72 + 216) \sin 20^\circ = 30.22\text{mm}$

Arc of contact = $\frac{L.P.C}{\cos \phi} = \frac{30.22}{\cos 20^\circ} = 32.16\text{mm}$

Angle of action = $\frac{\text{arc of contact}}{\lambda P_1} = \frac{32.16}{72} = 0.4467 \text{ radians}$

(b) Max velocity of sliding = $(\omega_1 + \omega_2) \times \text{max path}$
 path of Approach = $(r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} - r_{p2} \sin \phi$
 $= (222^2 - 216^2 \cos^2 20)^{1/2} - 216 \sin 20 = 16.04 \text{ mm}$

path of recess = $(r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - r_{p1} \sin \phi$
 $= (78^2 - 72^2 \cos^2 20)^{1/2} - 72 \sin 20 = 14.18 \text{ mm}$

path of Approach > path of recess
 Max path = 16.04 mm

Max Velocity of sliding = $\left(\frac{V_P}{r_{p1}} + \frac{V_P}{r_{p2}}\right) \times 16.04 = \left(\frac{1500}{72} + \frac{1500}{216}\right) \times 16.04$

$V_{\text{max}} = 445.6 \text{ mm/s} = 4.456 \text{ m/s}$

(3) The Number of teeth on each of the two equal spur gears in mesh is 40. The teeth have 20° involute profile and the module is 6mm. If the arc of contact is 1.75 times of circular pitch. Find the Addendum

Solution

GID $\rightarrow \phi = 20^\circ$, $m = 6 \text{ mm}$, Arc of contact = $1.75 P_c$, $T_1 = T_2 = 40$

$P_c = \pi m = \pi \times 6 = 18.85 \text{ mm}$, $r_{p1} = r_{p2} = \frac{mT}{2} = \frac{6 \times 40}{2} = 120 \text{ mm}$

Length of arc of contact = $1.75 P_c = 1.75 \times 18.85 = 33 \text{ mm}$

Arc of contact = $\frac{L.P.C}{\cos \phi} \Rightarrow L.P.C = 33 \times \cos 20 = 31 \text{ mm}$

$L.P.C = 31 \Rightarrow (r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} + (r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - (r_{p1} + r_{p2}) \sin \phi = 31$

Assume $r_{p1} = r_{p2}$, $r_{a1} = r_{a2}$

$2(r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} = 31 \Rightarrow 2(r_{a2}^2 - 120^2 \cos^2 20)^{1/2} = 31$

$r_{a2} = 126.14 \text{ mm}$

Addendum = $r_{a2} - r_{p2}$ or $r_{a1} - r_{p1}$

$= 126.14 - 120 = 6.14 \text{ mm}$

(4) A pair of 20° Full depth involute spur gear having 30 & 50 teeth & P.D of module 4mm are in mesh, a small gear rotates 1000 rpm

(a) Determine sliding velocities at engagement & disengagement of a pair of teeth.

(b) Contact ratio

Addendum = 1 module

G10

$\phi = 20^\circ, T_1 = 30, T_2 = 50, m = 4\text{mm}$

$N_1 = 1000\text{rpm}$

$r_{p1} = \frac{mT_1}{2} = \frac{4 \times 30}{2} = 60\text{mm}, r_{p2} = \frac{mT_2}{2} = \frac{4 \times 50}{2} = 100\text{mm}$

$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 1000}{60} = 104.72\text{rad/sec}$

$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{30}{50} = \frac{3}{5} \Rightarrow \omega_2 = \frac{\omega_1 \times 3}{5} = \frac{104.72 \times 3}{5} = 62.832\text{rad/s}$

Addendum = 1 module = 4mm

$r_{a1} = r_{p1} + a = 60 + 4 = \underline{64\text{mm}}, r_{a2} = r_{p2} + a = 100 + 4 = \underline{104\text{mm}}$

path of approach = $(r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} - r_{p2} \sin \phi$
 $= (104^2 - 100^2 \cos^2 20) ^{1/2} - 100 \sin 20 = 10.36\text{mm}$

path of recess = $(r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - r_{p1} \sin \phi$
 $= (64^2 - 60^2 \cos^2 20) ^{1/2} - 60 \sin 20 = 9.76\text{mm}$

Sliding velocity during path of approach = $(\omega_1 + \omega_2)$ path of approach
 $= (104.72 + 62.832) 10.36 = \underline{1375.84 \text{ mm/s}}$

Sliding velocity during path of recess = $(\omega_1 + \omega_2)$ path of recess
 $= (104.72 + 62.832) 9.76 = \underline{1635.3 \text{ mm/s}}$

(b)

contact ratio = $\frac{\text{Arc of contact}}{P_c}$
 $= \frac{21.41}{\pi \times 4} = \underline{1.704}$

Length of arc of contact
~~ratio~~ = $\frac{\text{path of contact}}{\cos \phi}$
 $= \frac{10.36 + 9.76}{\cos 20} = \underline{21.41\text{mm}}$
 $P_c = \pi m = \underline{\pi \times 4}$

(5)

Two involute gears in a mesh have module of 8mm & a pressure angle of 20° . The larger gear has 57 & pinion has 23 teeth. If the addendum of pinion & gear wheels are equal to one module

Find (a) Contact ratio

(b) Angle of action of pinion & gear wheel

(c) Ratio of sliding to rolling velocity

* beginning of contact

* pitch point

* End of contact

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} \Rightarrow \omega_2 = \omega_1 \times \frac{23}{57}$$

Gr: D

$$\phi = 20^\circ, T_1 = 23, T_2 = 57, m = 8 \text{ mm}, a = 8 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}} \quad \text{--- (1)}$$

$$(a) \cdot \text{Arc of contact} = \frac{L.P.C}{\cos \phi}$$

$$r_{p1} = \frac{m T_1}{2} = \frac{8 \times 23}{2} = 92 \text{ mm}$$

$$r_{p2} = \frac{m T_2}{2} = \frac{8 \times 57}{2} = 228 \text{ mm}$$

$$r_{a1} = r_{p1} + a = 92 + 8 = 100 \text{ mm}$$

$$r_{a2} = r_{p2} + a = 228 + 8 = 236 \text{ mm}$$

$$L.P.C = (r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} + (r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - (r_{p1} + r_{p2}) \sin \phi$$

$$L.P.C = (236^2 - 228^2 \cos^2 20^\circ)^{1/2} + (100^2 - 92^2 \cos^2 20^\circ)^{1/2} - (92 + 228) \sin 20^\circ = 39.76 \text{ mm}$$

$$\text{Arc of contact} = \frac{39.76}{\cos 20^\circ} = 42.31 \text{ mm}$$

$$P_c = \pi m = \pi \times 8 \text{ mm}$$

$$\text{Sub: eq (1)} \Rightarrow \text{Contact ratio} = \frac{\text{Arc of contact}}{P_c} = \frac{42.31}{8\pi} = 1.68$$

(b) Angle of Action (θ)

$$\theta_p = \frac{\text{Arc of contact}}{r_{p1}} = \frac{42.31}{92} = 0.46 \text{ radians}, \quad 0.46 \times \frac{180}{\pi} = 26.3^\circ$$

$$\theta_g = \frac{\text{Arc of contact}}{r_{p2}} = \frac{42.31}{228} = 0.1856 \text{ radians}, \quad 0.1856 \times \frac{180}{\pi} = 10.63^\circ$$

(c)

$$\begin{aligned} \frac{\text{Sliding velocity}}{\text{Rolling velocity}} \Big|_{\text{engagement}} &= \frac{(\omega_1 + \omega_2) \text{ path of approach}}{\text{pitch line velocity}} = \frac{(\omega_1 + \frac{23}{57} \omega_1) LPA}{\omega_1 r_{p1}} \\ &= \omega_1 \left(1 + \frac{23}{57}\right) \times \left[(r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} - r_{p2} \sin \phi \right] \\ &= \frac{\omega_1 \times 92}{\left(1 + \frac{23}{57}\right) \left[(236^2 - 228^2 \cos^2 20^\circ)^{1/2} - 228 \sin 20^\circ \right]} \\ &= 0.32 \end{aligned}$$

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} \Big|_{\text{pitch point}} = \frac{(\omega_1 + \omega_2) 0}{\text{pitch line velocity}} = 0$$

$$\begin{aligned} \frac{\text{Sliding velocity}}{\text{Rolling velocity}} \Big|_{\text{End of contact}} &= \frac{(\omega_1 + \omega_2) \text{ path of recess}}{\omega_1 r_{p1}} \\ &= \left(\omega_1 + \frac{23}{57} \omega_1 \right) \left[(r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - r_{p1} \sin \phi \right] \\ &= \left(1 + \frac{23}{57}\right) \frac{\omega_1 r_{p1}}{(100^2 - 92^2 \cos^2 20^\circ)^{1/2} - 92 \sin 20^\circ} = 0.287 \end{aligned}$$

Interference in Involute Gears

Introduction

- one of the major attributes of gear drive is that it provides a constant velocity ratio.
- power transmission through a pair of teeth is along the line of action or common normal to the two involutes at the point of contact and this common normal is also common tangent to the two base circles and pass through the pitch point.
- At any instant, the portion of tooth profiles which are in contact must be involute. So line of action does not deviate.
- Involute starts at base circle & generate outward.
- It is impossible to have an involute inside base circle.

If any of two surface of tooth profile is not an involute, the two surfaces would not touch each other & transmission of power would not be proper. Mating of two non-conjugate (non-involute) teeth is known as interference

- Because of two ~~types~~ teeth does not slide properly and rough action & binding occur.
- It will not maintain constant angular velocity ratio which can lock the two gears.

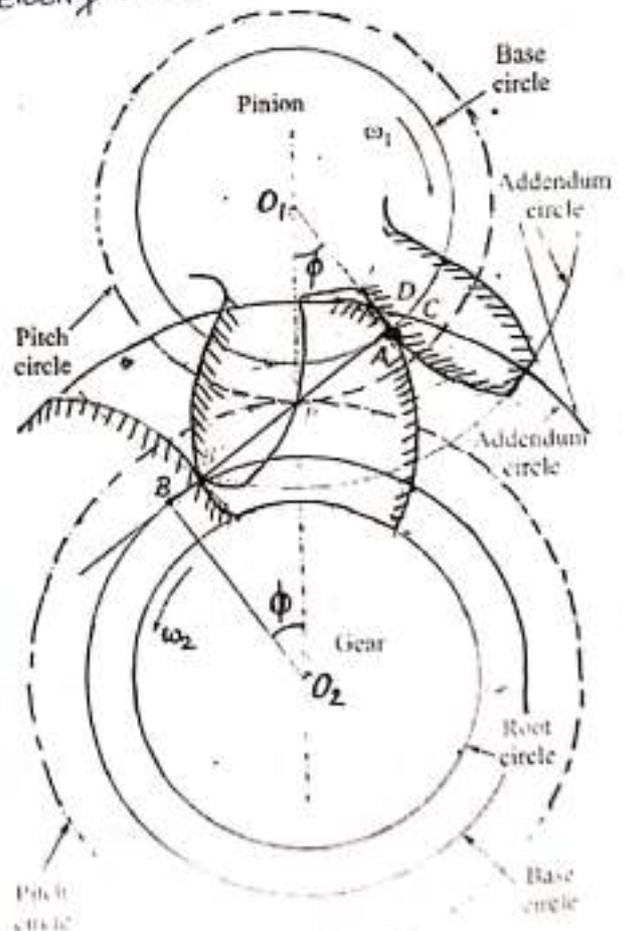
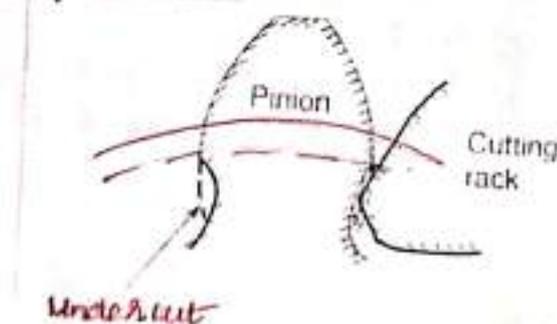
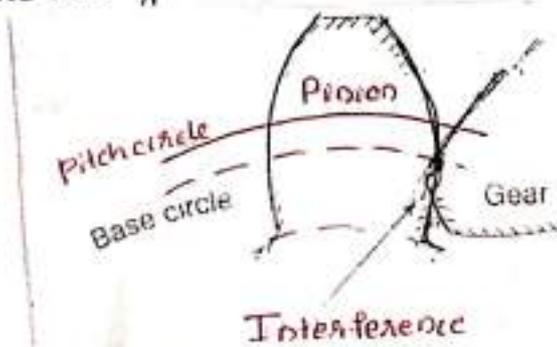


Figure illustrating condition of interference

The line AB = line of action of two point A & B
 ⇒ Here AB is the extreme limit of length of action (contact). These points are called interference points.
 ⇒ Beginning of contact occurs before the interference point is met, then the involute position of driven gear will mate with non-involute position of driver pinion and involute interference is said to occur.

Conclusions

- ① Max possible length of path of contact avoiding interference = AB. It is the distance between point of tangency to base circle.
- ② Interference possible only when diameter of base circle larger than diameter of addendum circle [Necessary condition]

*** Minimum Number of Teeth to avoid interference**
Maintaining constant angular velocity ratio $\lambda = \frac{\omega_2}{\omega_1} \leq 1$

From fig ⇒ length of path of approach in limiting condition = AP = $r_{p1} \sin \phi$
 length of path of recess in limiting condition = PB = $r_{p2} \sin \phi$

The limiting condition for avoiding interference is obtained when the addendum circle of gear cuts the line of action at the point of tangency to base circle.
 From fig ⇒ O_1P = Pitch circle radius of pinion (r_{p1})
 O_2P = pitch circle radius of gear (r_{p2})
 O_2A = Addendum circle radius of gear (r_{a2})
 or

In order to avoid interference in a pair of involute gears in mesh, the addendum circle of either must not intersect line of Action (Contact) outside the point of tangency to the two base circles.

x From fig, interference occurs. It will occur on pinion tooth flank position.
 x Limiting condition indicated in point A where slight increase in addendum circle radius will lead to interference.

Δ in O_2PA , Apply cosine law
 $O_2A^2 = O_2P^2 + AP^2 - 2O_2P \cdot AP \cos(90 + \phi)$
 $r_{a2}^2 = r_{p2}^2 + r_{p1}^2 \sin^2 \phi + 2r_{p2} r_{p1} \sin^2 \phi$
 $r_{a2} = r_{p2} \sqrt{1 + \frac{r_{p1}^2 \sin^2 \phi}{r_{p2}^2} + 2 \frac{r_{p2} r_{p1} \sin^2 \phi}{r_{p2}^2}}$

where $\frac{r_{p1}}{r_{p2}} = \frac{\omega_2}{\omega_1} = \lambda$

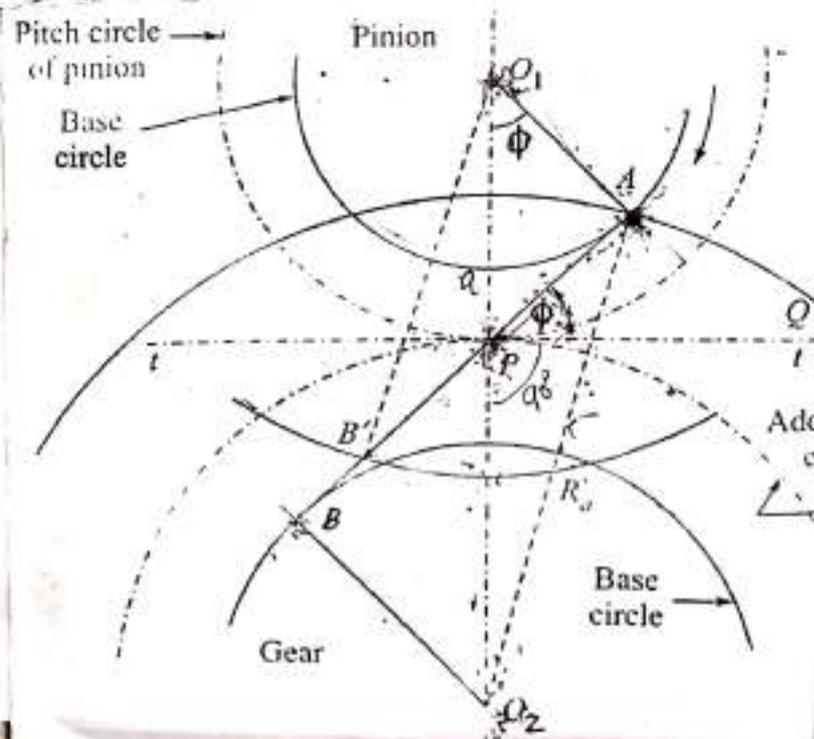


Figure illustrating limiting length of path of contact to

$$r_{a2} = r_{p2} (1 + \lambda^2 \sin^2 \phi + 2\lambda \sin^2 \phi)$$

$$r_{a2} \leq r_{p2} (1 + \lambda^2 \sin^2 \phi + 2\lambda \sin^2 \phi)^{1/2}$$

$$r_{a2} \leq r_{p2} (1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2}$$

where

r_{a2} = Addendum radius of gear

r_{p2} = pitch circle radius of gear

Addendum $a = O_2A - O_2P = r_{a2} - r_{p2} = m \cdot a_w$

Addendum = Factor \times Module

$$a = r_{a2} - r_{p2} = r_{p2} [(1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2} - 1]$$

$$a = m a_w = r_{p2} (1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2} - 1$$

$$a_w \frac{2 r_{p2}}{T_2} = r_{p2} (1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2} - 1$$

Module

$$m = \frac{d_{p2}}{T_2} = \frac{2 r_{p2}}{T_2}$$

Min: no. of teeth on gear to avoid interference

$$T_{2min} = \frac{2 a_w}{[(1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2} - 1]}$$

$$T_{2min} = \frac{2 a_w}{[(1 + \frac{1}{G_1} (\frac{1}{G_1} + 2) \sin^2 \phi)^{1/2} - 1]}$$

\therefore No. of teeth on gear to avoid interference $\geq T_{2min}$

$$\lambda = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} \Rightarrow T_1 = \lambda T_2$$

Min: No of teeth on pinion to avoid interference

$$T_{1min} = \frac{2 a_w \lambda}{[(1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2} - 1]}$$

Q1 Two 20° involute spur gears have a module of 10mm. The addendum equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth? Will gear interfere with the pinion?

Solution

G.D $\rightarrow \phi = 20^\circ, m = 10\text{mm}, a = 1m = 10\text{mm}, T_2 = 40, T_1 = 20, r_{p1} = \frac{m T_1}{2} = \frac{10 \times 20}{2} = 100\text{mm}$

$r_{p2} = \frac{m T_2}{2} = \frac{10 \times 40}{2} = 200\text{mm}$, Actual Addendum radius of gear $r_{a2} = r_{p2} + a = 200 + 10 = 210\text{mm}$

To avoid interference, max value of addendum radius of gear

where $\lambda = \frac{T_1}{T_2} = \frac{20}{40} = 0.5$

$$r_{a2max} = r_{p2} (1 + \lambda^2 \sin^2 \phi + 2\lambda \sin^2 \phi)^{1/2}$$

$$r_{a2max} = 200 (1 + 0.5^2 \sin^2 20 + 2 \times 0.5 \times \sin^2 20)^{1/2} = 214.1\text{mm}$$

Here $r_{a2} \leq r_{p2} (1 + \lambda(\lambda+2) \sin^2 \phi)^{1/2}$

$210 \leq 214.1\text{mm}$

Actual Addendum radius of gear \leq Max. value of Addendum radius

no interference occur

② Two 20° involute spur gears mesh externally and give a velocity ratio of $\frac{1}{3}$. The module is 3mm and addendum is equal to 1.1 module. If the pinion rotates at 120 rpm.

9.14

Determine (a) M2: No. of teeth on each wheel - to avoid interference
(b) Contact ratio

Solution

(a) Velocity ratio, $\lambda = \frac{1}{3}$, $\phi = 20^\circ$, $N_p = 120$ rpm, $a = a_w \times m$, $a = 1.1m$, $G_1 = 3$, $m = 3$ mm, $a_w = 1.1 \times 3 = 3.3$ mm

$$(T_{min}) = \frac{2a_w}{(1 + \frac{1}{G_1}(\frac{1}{G_1} + 2) \sin^2 \phi)^{1/2} - 1} = \frac{2 \times 3.3}{(1 + \frac{1}{3}(\frac{1}{3} + 2) \sin^2 20^\circ)^{1/2} - 1} = 49.44$$

Take higher whole No. divisible by velocity ratio. ($T_2 = 51$)

$$\frac{T_1}{T_2} = \frac{1}{3} \Rightarrow T_1 = \frac{1}{3} \times 51 = 17$$

(b) Contact ratio = $\frac{\text{Length of Arc of Contact}}{\text{Circular pitch}}$

where $L.A.C = \frac{L.P.C}{\cos \phi} \Rightarrow L.P.C = (r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} + (r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - (r_{p1} + r_{p2}) \sin \phi$

where $r_{a2} = r_{p2} + a = 76.5 + 3.3 = 79.8$ mm
 $r_{a1} = r_{p1} + a = 25.5 + 3.3 = 28.8$ mm

$$\begin{aligned} r_{p1} &= \frac{m T_1}{2} = \frac{3 \times 17}{2} = 25.5 \text{ mm} \\ r_{p2} &= \frac{m T_2}{2} = \frac{3 \times 51}{2} = 76.5 \text{ mm} \end{aligned} \quad \left| \begin{aligned} a &= 1.1 \times m \\ &= 1.1 \times 3 \\ &= 3.3 \text{ mm} \end{aligned} \right.$$

$$L.P.C = (79.8^2 - 76.5^2 \cos^2 20^\circ)^{1/2} + (28.8^2 - 25.5^2 \cos^2 20^\circ)^{1/2} - (25.5 + 76.5) \sin 20^\circ$$

$$L.P.C = 15.738 \text{ mm}$$

$$L.A.C = \frac{15.738}{\cos 20^\circ} = 16.748 \text{ mm} \Rightarrow \text{Contact ratio} = \frac{L.A.C}{P_c} = \frac{16.748}{\pi \times 3} = 1.78$$

Thus, 1 pair of teeth will always remain in contact whenever for 78% of the time, 2 pairs of teeth will be in contact

③ Two 20° involute spur gears have a module of 10mm. The Addendum is one module. The larger gear has 50 teeth & pinion has 13 teeth. Does interference occur? If occur to what value should the pressure angle be changed to eliminate interference?

Solution $\phi = 20^\circ$, $T_2 = 50$, $m = 10$ mm, $a = 10$ mm, $T_1 = 13$

$$r_{p2} = \frac{m T_2}{2} = \frac{10 \times 50}{2} = 250 \text{ mm}, \quad r_{p1} = \frac{m T_1}{2} = \frac{10 \times 13}{2} = 65 \text{ mm}$$

$$\begin{aligned} r_{a2} &= r_{p2} + a \\ &= 250 + 10 = 260 \text{ mm} \\ r_{a1} &= r_{p1} + a \\ &= 65 + 10 = 75 \text{ mm} \end{aligned}$$

$$r_{a2max} = r_{p2} (1 + \lambda (\lambda + 2) \sin^2 \phi)^{1/2}$$

$$r_{a2max} = 250 \left(1 + \frac{13}{50} \left(\frac{13}{50} + 2 \right) \sin^2 20^\circ \right)^{1/2} = 258.45 \text{ mm}$$

$$\lambda = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{13}{50}$$

$$\boxed{260 \geq 258.45} \Rightarrow \text{Actual Addendum radius} \geq \text{Max Value } (r_{a2max})$$

\Rightarrow Interference occur

New value of pressure angle ϕ to avoid interference

$$r_{a2max} = r_{p2} (1 + \lambda (\lambda + 2) \sin^2 \phi)^{1/2}$$

$$260 = 250 \left(1 + \frac{13}{50} \left(\frac{13}{50} + 2 \right) \sin^2 \phi \right)^{1/2} \Rightarrow \phi = 21.88^\circ$$

- ④ Two involute gears in mesh have a velocity ratio of $\frac{1}{3}$. The arc of approach is not to be less than the circular pitch. When the pinion is the driver. The pressure angle of involute teeth is 20° . Determine
- Least No. of teeth in each gear
 - Addendum of wheel in terms of Module

Solution

$$\phi = 20^\circ, \lambda = \frac{1}{3}, \text{ Arc of approach} = \text{Circular pitch} = \pi m$$

$$\text{path of Approach} = \pi m \cos 20 = 2.452m$$

$$\text{Max length of approach} = r_{p1} \sin \phi = \frac{m T_1}{2} \sin \phi = \frac{m T_1}{2} \sin 20 = 0.171 m T_1$$

$$0.171 m T_1 = 2.452 m$$

$$T_1 = 17.26 \approx 18 \text{ teeth} \quad \& \quad T_2 = 18 \times 3 = 54$$

$$r_{a2 \text{ max}} = r_{p2} (1 + \lambda (\lambda + 2) \sin^2 \phi)^{1/2} = \frac{m T_2}{2} (1 + \frac{1}{3} (\frac{1}{3} + 2) \sin^2 20)^{1/2}$$

$$= \frac{m \cdot 54}{2} (1 + \frac{1}{3} (\frac{1}{3} + 2) \sin^2 20)^{1/2} = \underline{28.2 \text{ mm}}$$

$$a = r_{a2} - r_{p2} = 28.2 \text{ mm} - 27 \text{ mm} = \underline{1.2 \text{ mm}}$$

- ⑤ The following data relate to two meshing involute gears.
 No. of teeth on gear wheel = 60, pressure angle = 20° , Gear ratio = 1.5
 Speed of gear wheel = 1000 rpm, Module = 8 mm

The addendum on each wheel is such that the path of approach & path of recess on each side are 40% of Max. possible length each. Determine addendum for the pinion and gear length of arc of contact?

$$G_1 = 1.5$$

$$V.R = \frac{1}{1.5} = \frac{T_1}{T_2}$$

$$T_2 = 60$$

Solution

$$r_{p2} = \frac{m T_2}{2} = \frac{8 \times 60}{2} = \underline{240 \text{ mm}}, \quad r_{p1} = \frac{m T_1}{2} = \frac{8 (60/1.5)}{2} = \underline{160 \text{ mm}}$$

$$\text{Max. possible length of path of approach} = r_{p1} \sin \phi$$

$$\text{Actual length of path of approach} = 0.4 r_{p1} \sin \phi$$

$$\text{Actual length of path of recess} = 0.4 r_{p2} \sin \phi$$

$$0.4 r_{p1} \sin \phi = (r_{a2}^2 - r_{p2}^2 \cos^2 \phi)^{1/2} - r_{p2} \sin \phi$$

$$0.4 \times 160 \times \sin 20 = (r_{a2}^2 - 240^2 \cos^2 20)^{1/2} - 240 \sin 20 \Rightarrow r_{a2} = \underline{248.3 \text{ mm}}$$

$$\text{Addendum of (wheel) gear} = 248.3 - 240 = \underline{8.3 \text{ mm}}$$

$$0.4 r_{p2} \sin \phi = (r_{a1}^2 - r_{p1}^2 \cos^2 \phi)^{1/2} - r_{p1} \sin \phi$$

$$\Rightarrow r_{a1} = \underline{174 \text{ mm}}$$

Addendum of pinion

$$= r_{a1} - r_{p1}$$

$$= 174 - 160 = \underline{14 \text{ mm}}$$

$$\text{Arc of contact} = \frac{\text{path of contact}}{\cos \phi}$$

$$= 0.4 (r_{p1} \sin \phi + r_{p2} \sin \phi)$$

$$\cos \phi$$

$$= \underline{58.2 \text{ mm}}$$

4. Worm Gears		
Merits	(i)	Higher speed reduction upto 300:1 is possible in case of worm gears.
	(ii)	Worm gears are silent in operation.
	(iii)	Worm and worm gears have self-locking characteristics, which restricts their reverse motion. This characteristic makes them suitable for applications like elevators, hoisting machines, etc.
	(iv)	Worm gears occupy lesser space.
	(v)	These gears can be used for reducing speed and increasing torque.
Demerits	(i)	Worm gear materials are expensive.
	(ii)	Worm gears have high power losses
	(iii)	Worm gears generate a lot of heat during their operation.
	(iv)	Efficiency of worm gears are very low when compared to other gears.

3.15 MERITS AND DEMERITS OF EACH TYPE OF GEARS

1. Spur gears	
Merits	(i) Spur gears are simple in design, hence it is easy to manufacture and install them.
	(ii) It is highly suitable for compact structures.
	(iii) Spur gears are more efficient when compared to the other gears of same size.
	(iv) Spur gear teeth are parallel to its axis. Hence, spur gear train does not produce axial thrust. Therefore, the gear shafts can be mounted easily using ball bearings.
	(v) Spur gears are less expensive when compared to other gears.
Demerits	(i) Spur gears produce significant noise at high speeds.
	(ii) Tooth engagements in spur gears are not gradual when compared to other gears.
	(iii) The loads in spur gears are transmitted over fewer teeth only. This makes their design weaker when compared to other gears.
	(iv) It is not suitable for applications requiring heavy load. This is because, spur gears take significant stresses during operation, which makes them vulnerable to wear and tear.
	(v) Spur gears are not suitable for transmitting power between non-parallel shafts.
2. Helical Gears	
Merits	(i) Helical gears can be used for transferring power between non parallel shafts.
	(ii) At any given time their load is distributed over several teeth i.e., minimum two or three teeth of each gear are always in contact with other gears. This results in lesser wear and makes them suitable for higher load applications.
	(iii) During engagement, the teeth engage a little at a time, instead of the entire face of each tooth at once. This allows for a smoother and quieter operation.

	(iv) For same tooth size (module) and equivalent width, helical gears can handle more load than spur gears because the helical gear tooth is positioned diagonally.
Demerits	(i) Designing and Manufacturing costs of helical gears are more when compared to the spur gears.
	(ii) Design of helical gears is too complicated when compared to the spur gears.
	(iii) While meshing, helical gear makes a sliding-type contact between two gears. This leads to the generation of more heat, which will eventually results in the power loss and decrease in efficiency. Therefore, for reducing the sliding friction, additives are added to the lubricants.
	(iv) Also, during meshing, helical gear develops an unwanted thrust in the axial direction. This leads to a decrease in gear efficiency. Therefore, special thrust bearings need to be used for reducing the axial thrust in helical gears.
3. Bevel Gears	
Merits	(i) The operating angles of bevel gears can be changed. This characteristic of bevel gears makes them flexible in their operation.
	(ii) Efficiency of bevel gears is quite high when compared to the worm gears.
	(iii) Due to the rolling action of bevel gears, the sliding friction in bevel gearing mechanism is lower.
Demerits	(i) To get the maximum efficiency, the bevel gears should be precisely positioned and assembled with the respective shaft.
	(ii) It has a limited gear ratio, so more gear are required in a gear train to achieve a high total gear ratio.
	(iii) These gears are not suitable for high-speed reduction.
	(iv) Bevel gears produce significant noise at high speeds.

value, it interferes with the addendum of the pinion and two gears are blocked.

However if a cutting rack having similar teeth is used to cut the teeth in the pinion, it will remove that portion of the pinion tooth which would have interfered with gear as shown in Fig. A gear having its material removed in this manner is said to be undercut and the process undercutting. Undercutting ^{not only} weakens the pinion tooth, but may also remove a small portion of the ~~involute~~ ^{involute} adjacent to the base circle, which may cause a serious reduction in the length of action.

Standard proportions of Interchangeable gear.
(Gear Standardisation)

A set of gears is interchangeable when any two gears selected from the set will mesh and satisfy the fundamental law of gearing. For interchangeability, all gears of the set have same circular pitch, module, pressure angle, addendum and dedendum. Besides the above conditions, the tooth thickness must be $(\pi/2)$ times the module.

Advantage of interchangeable gears.

- a) easy availability of production tools
- b) can be produced quickly + economically.

Full depth tooth.

addendum = 1 module.

> provides larger working depth.

Stub tooth.

> working depth smaller than full depth tooth.

This is obtained by cutting short addenda

and dedendum

Pressure angle.

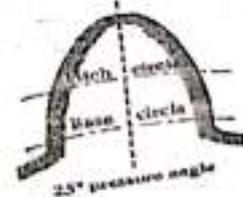
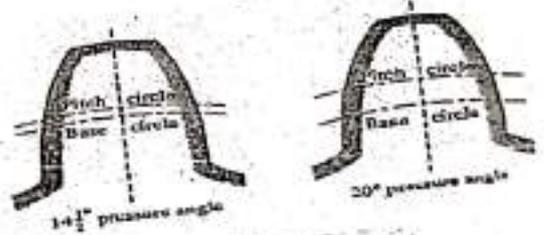
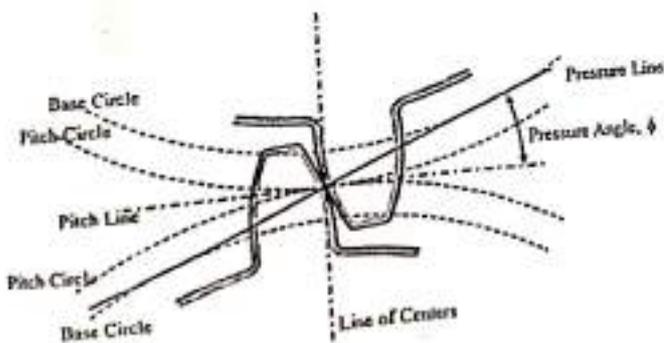
Standard pressure angle used in practice

are $14\frac{1}{2}^\circ$ & 20° .

pressure angles of $17\frac{1}{2}^\circ$, $22\frac{1}{2}^\circ$ & 25° are used sometimes.

The $14\frac{1}{2}^\circ$ pressure angle used in early designs is the oldest tool systems designs.

Pressure Angle (ϕ)



$\phi = 14\frac{1}{2}^\circ, 20^\circ, 25^\circ$

Effect of module on size of teeth



Relative tooth sizes (approximate) for different modules m .

Classification of gears according to position of axis of shaft.

a. Parallel

1. Spur Gear.
2. Helical Gear.
3. Rack & pinion.

b. Intersecting

Bevel Gear.

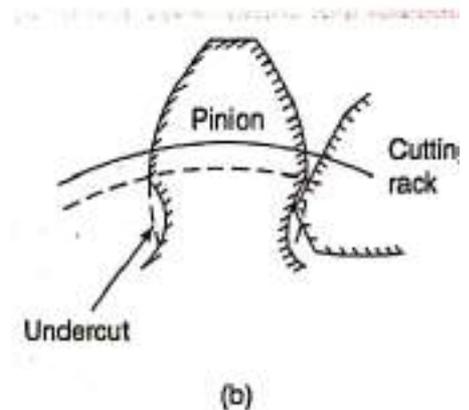
c. Non-intersecting & non parallel.

Worm & worm gears.

Methods of reducing or eliminating interference

1. Undercutting

- ❑ Cutting rack having similar teeth is used to cut the teeth in the pinion
- ❑ It will remove that portion of pinion tooth which would interfere with the gear
- ❑ Gear having its material removed in the manner is said to be undercut and process of undercutting
- ❑ In pinion with small number of teeth, this can seriously weaken tooth. Actual gear meshes with undercut pinion. No interference occur



2. **Increasing slightly the center distance** between the meshing gears would also eliminate interference.
3. **Increasing the number of teeth** on the gear can also eliminate the chances of interference. Increasing number of teeth, keeping the diameter same makes teeth smaller and reduces the tendency to interfere
4. **Elimination of interference is possible by tooth stubbing.** In this process a portion of the tip of the teeth is removed, thus preventing that portion of the tip of tooth in contacting the non-involute portion of the other meshing tooth. In this case also, the teeth are weakened.
5. **Use of a larger pressure angle can eliminate interference.** Having a larger pressure angle results in a smaller base circle. As a result, more of the tooth profiles become involute. In this case, the tip of the tooth of one gear will not have a chance to contact the flank of the other gear on its non-involute portion.

6. **Tooth thinning.** Depending on the class of gear there are recommended and there AGMA has charts to follow. In very high application such as Gas turbines operating at high speed. load and temperature further thinning may be required as the bending of teeth can cause interference. These information's available with very few manufacturers. Further tooth bending and whiplash at the elevated load, temperature, speed and pattern of contact makes it highly complicated.

Non-standard gears

- ☐ The term non-standard gears apply to such gears as are modified by changing some standard parameters like pressure angle, addendum, tooth depth or centre distance
- ☐ these changes are made to improve the performance of the gear operation or from the economical point of view.
- ☐ The recent trend these days is to make the design of machines as compact as possible to reduce their size and weight which also results in reduction in the costs.
- ☐ Consider a gear set to have a velocity ratio of 4:1. If a pinion of 80 mm pitch diameter is selected for purpose, the pitch diameter of the gear is 320mm, Thus space requirement of the gear is 400 mm

pitch diameter of the pinion is reduced by 10 mm, the pitch diameter of the gear is reduced by 40 mm, and the overall reduction in Space is 50 mm,
- ☐ Also, the sizes of other components associated with the gear set such as shafts, casings and bearings are also reduced.
- ☐ The only way to have a smaller size of gears is to reduce the number of teeth.
- ☐ However, for a typical type of teeth, it is observed that if the number of teeth is reduced from a certain number, the problems of interference, undercutting and contact ratio hamper the smooth running of the gears.
- ☐ Therefore, the main reason to employ non-standard gears is to prevent interference and undercutting and to maintain a reasonable contact ratio.
- ☐ It should be remembered that as an involute is generated, its radius of curvature goes on becoming larger and larger, being zero at the base circle.
- ☐ As far as possible, the curve near the base should be avoided because high stresses are developed in the region of sharp curvature.

Centre-distance Modifications

- ❑ The number teeth on a pinion can be reduced from the minimum allowable number by increasing the centre distance marginally and by changing the tooth proportions and the pressure angle of the gears.
- ❑ A reduction in the interference and improvement of contact ratio brought this way.
- ❑ The teeth can be generated with rack cutters of standard pressure angles by displacing the pitch line of the rack from the pitch circle of the gear. T

his action produces teeth which are thicker than before. As the teeth are cut with a displaced or offset cutter, they will engage at a new pressure angle and at a new centre distance.

Clearance Modifications

- ❑ If the clearance between mating teeth is increased to 0.3 m or 0.4 m instead of the usual value of 0.25 m to have a larger fillet at the root of the tooth, the fatigue strength of the tooth is increased.
- ❑ This way some extra depth is available to smoothen the tooth profile. Interchangeability is not lost this way.

Addendum Modifications

- ❑ In cases where it is not possible to change the centre distances, modifications can be made to the addendum. In such cases.
- ❑ there has to be no change in the pitch circles and the pressure angles.
- ❑ However, the contact region is shifted away from the pinion centre towards the gear centre, decreasing the Womb action and increasing the recess action.

Internal gears

- ❑ Internal gears are the ones with the teeth formed on the inner surface of a cylinder or cone. Internal gears mesh with spur gears.
- ❑ There are two types of tooth shape, one being parallel and the other one with a helix in respect to the axis. However gears with teeth parallel to the axis have higher demand
- ❑ **Planetary gear mechanism:** The most popular application for internal gears is usage for planetary gear trains or gear reducers composed of a carrier, several spur gears, called planet gears and a sun gear in the center, rotating inside an internal gear.



Backlash

- ❑ It is the difference between space width and the tooth thickness along pitch circle
- ❑ **Methods to Minimize Gear Backlash**
- ❑ For these applications, there are three basic ways to reduce or eliminate backlash: **precision gears, modified gears, and special designs that use components other than gears.**

TERMINOLOGY OF HELICAL GEAR

10.19 TERMINOLOGY OF HELICAL GEARS

Refer Fig. 10.32.

Helix Angle (ψ) It is the angle at which the teeth are inclined to the axis of a gear. It is also known as *spiral angle*.

Circular Pitch (p) It is the distance between the corresponding points on adjacent teeth measured on the pitch circle. It is also known as *transverse circular pitch*.

Normal Circular Pitch (p_n) Normal circular pitch or simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth. The normal circular pitch of two mating gears must be same.

$$P_n = p \cos \psi$$

Also, we have, $p = \pi m$ as for spur gears

$$P_n = \pi m_n$$

and $m_n = m \cos \psi$

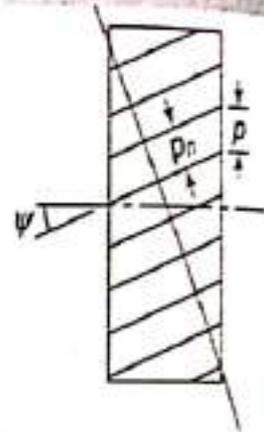


Fig. 10.32

According to type of gears: Gears can be classified as external gears, internal gears, and rack and pinion.

1. External gears mesh externally - the bigger one is called "gear" and the smaller one is called "pinion"
2. Internal gears mesh internally - the larger one is called "annular" gear and the smaller one is called "pinion".
3. Rack and pinion type – converts rotary to linear motion or vice versa. There is a straight line gear called "rack" on which a small rotary gear called "pinion" moves.

10.1 CLASSIFICATION OF GEARS

Gears can be classified according to the relative positions of their shaft axes as follows:

1. Parallel Shafts

Regardless of the manner of contact, uniform rotary motion between two parallel shafts is equivalent to the rolling of two cylinders, assuming no slipping. Depending upon the teeth of the equivalent cylinders, i.e., straight or helical, the following are the main types of gears to join parallel shafts:

Spur Gears They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load [Fig. 10.2(a)].

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axes of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

Further, if the gears have external teeth on the outer surface of the cylinders, the shafts rotate in the opposite direction [Fig. 10.2(a)]. In an internal spur gear, the teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction as shown in [Fig. 10.2(b)].

Spur Rack and Pinion Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is a plane (Fig. 10.3). The spur rack and pinion combination converts rotary motion into translatory motion, or vice-versa

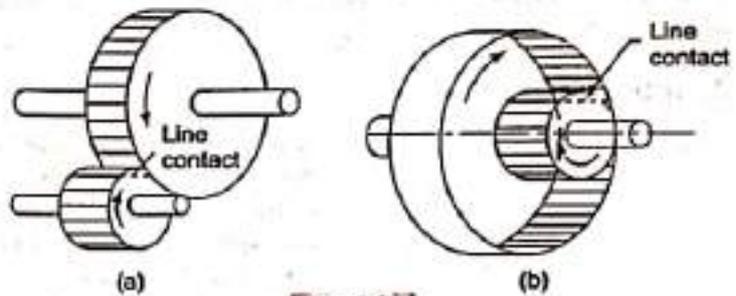


Fig. 10.2

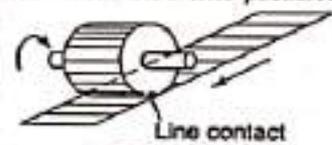


Fig. 10.3



CS 10.23 TERMINOLOGY OF WORM GEARS

Refer Fig. 10.35.

(i) **Axial Pitch (p_a)** It is the distance between corresponding points on adjacent teeth measured along the direction of the axis.

(ii) **Lead (L)** The distance by which a helix advances along the axis of the gear for one turn around is known as lead.

In a single helix, the axial pitch is equal to lead. In a double helix, this is one-half the lead, in a triple helix, one third of lead, and so on.

(iii) **Lead Angle (λ)** It is the angle at which the teeth are inclined to the normal to the axis of rotation. Obviously, the lead angle is the complement of the helix angle.

i.e., $\psi + \lambda = 90^\circ$

In case of worms, the lead angle is very small and the shaft axes of worm and worm gear are at 90°

$$\psi_1 + \psi_2 = 90^\circ$$

$$(90^\circ - \lambda_1) + \psi_2 = 90^\circ$$

or

i.e., lead angle of worm = helix angle of the gear w/

Also, p_n of worm = p_n of wheel

$$p_{a1} \cos \lambda_1 = p_2 \cos \psi_2$$

but

$$\lambda_1 = \psi_2$$

i.e., axial pitch of worm = circular pitch of wheel

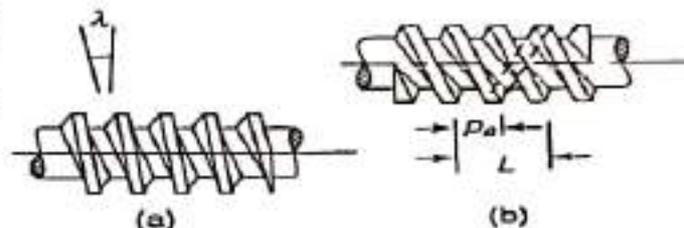


Fig. 10.35

10.26 BEVEL GEARS

To have a gear drive between two intersecting shafts, bevel gears are used. Kinematically, bevel gears are equivalent to rolling cones. Some of the common terms used in bevel gears are illustrated in Fig. 10.36(a).

Let γ_g, γ_p = pitch angles of gear and pinion respectively

r_g, r_p = pitch radii of gear and pinion respectively.

The pitch cones for two mating external bevel gears are shown in Fig. 10.36(b).

We have,

$$\sin \gamma_g = \frac{r_g}{OP} = \frac{r_g}{r_p / \sin \gamma_p} = \frac{r_g}{r_p} \sin (\theta - \gamma_p)$$

$$\text{or } \sin \gamma_g = \frac{r_g}{r_p} (\sin \theta \cos \gamma_p - \cos \theta \sin \gamma_p)$$

Dividing both sides by $\cos \gamma_g$,

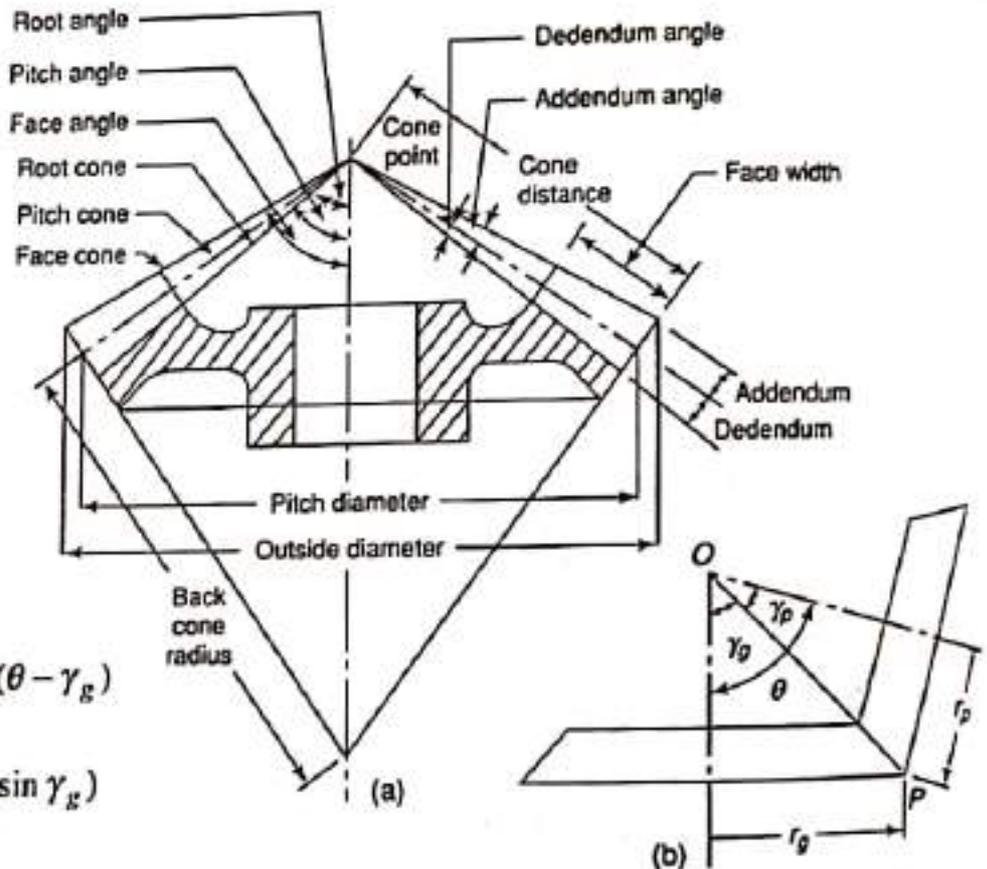


Fig. 10.36