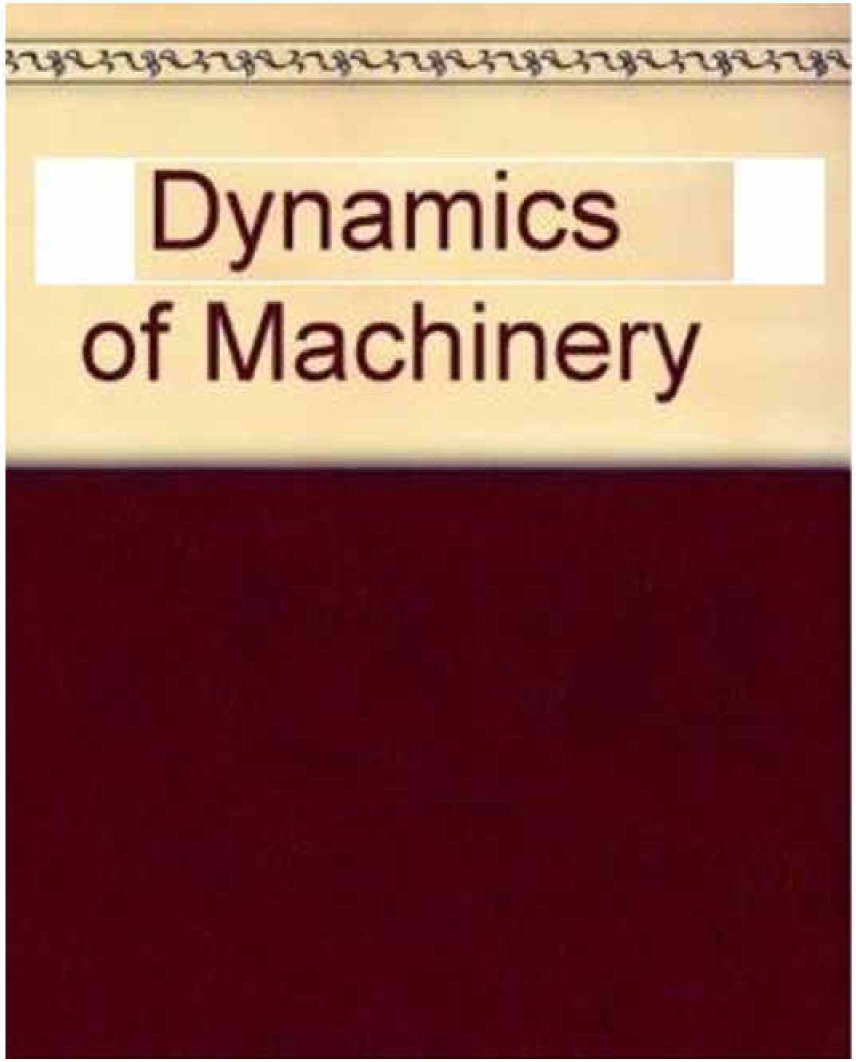


MODULE 4: GYROSCOPE

KTU MECHANICAL ENGINEERING

GYROSCOPE

- The word gyre is a Greek word means circular motion.
- A **gyroscope** is a spinning body which is free to move in other directions under the action of external forces.
- A **gyroscope** is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum.

EXAMPLES OF GYROSCOPE

- Earth
- Spinning top
- Boomerang
- yo-yos
- Frisbees

BASICS

- Linear momentum (mv)
- Conservation of linear momentum
- Angular momentum (Iω)
- Conservation of angular momentum
- Force
- Torque

PROPERTIES OF GYROSCOPE

- **RIGIDITY** :The axis of rotation (spin axis) of the gyro wheel tends to remain in a fixed direction in space if no external force is applied to it.
- **PRECESSION** : The axis of rotation has a tendency to turn at a right angle to the direction of an applied force.

APPLICATIONS OF GYROSCOPE

- Inertial guidance : gyro compass (utilises rigidity property of gyroscope)
- Rate gyro: it will measure angular velocity .
- Stabiliser –antiroll stabiliser

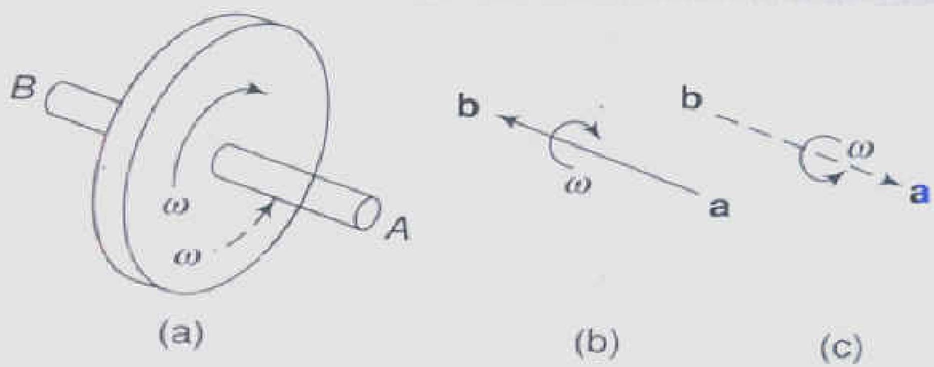
ANGULAR MOTION AND CONVENTIONAL VECTOR REPRESENTATION

- In problems involving gyroscopic effects, it is very important to determine correctly the sense of gyro couples and gyro reaction couples.
- This is possible only if one understands the convention used in representing angular motion characteristics like displacement, velocity and acceleration.

Angular velocity is a vector quantity and requires following parameters to be specified

- magnitude of angular velocity
- direction of axis of spin(being normal to plane of spin)
- the sense of angular velocity ie clockwise or counterclockwise

ILLUSTRATION OF RIGHT HAND THUMB RULE



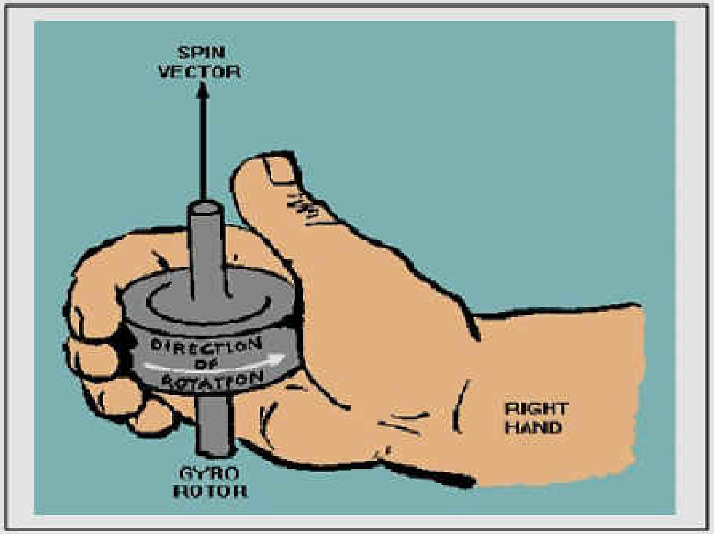


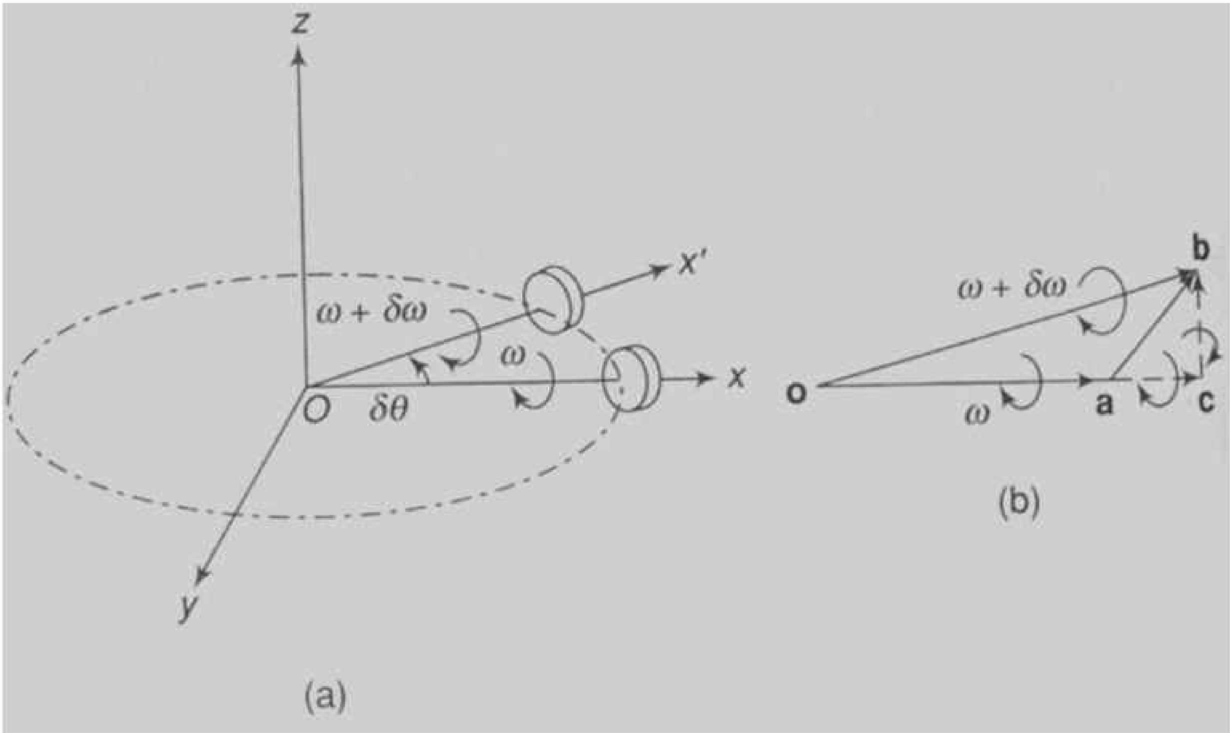
ILLUSTRATION OF RIGHT HAND THUMB RULE

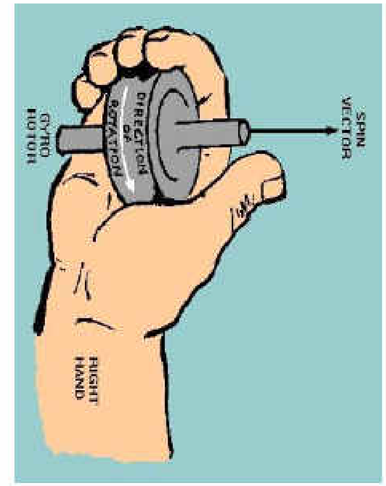
Why this subject is important?

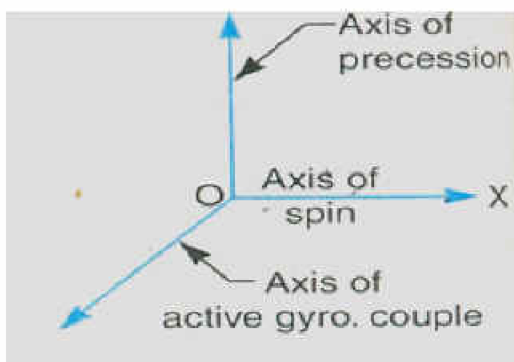
- While designing machines and machine components, forces and couples due to gyroscopic effects must be taken in to account.
- A designer must account of these forces and couples in selection of bearings and rotating parts.
- With the present trend of increasing machine speeds and decreasing factor of safety, designers must consider gyroscopic forces and couples in machine design calculations.

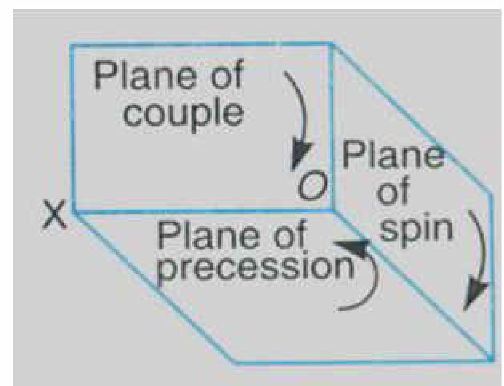
DERIVATION OF ANGULAR ACCELERATION OF A SPINNING DISC UNDERGOING PRECESSION

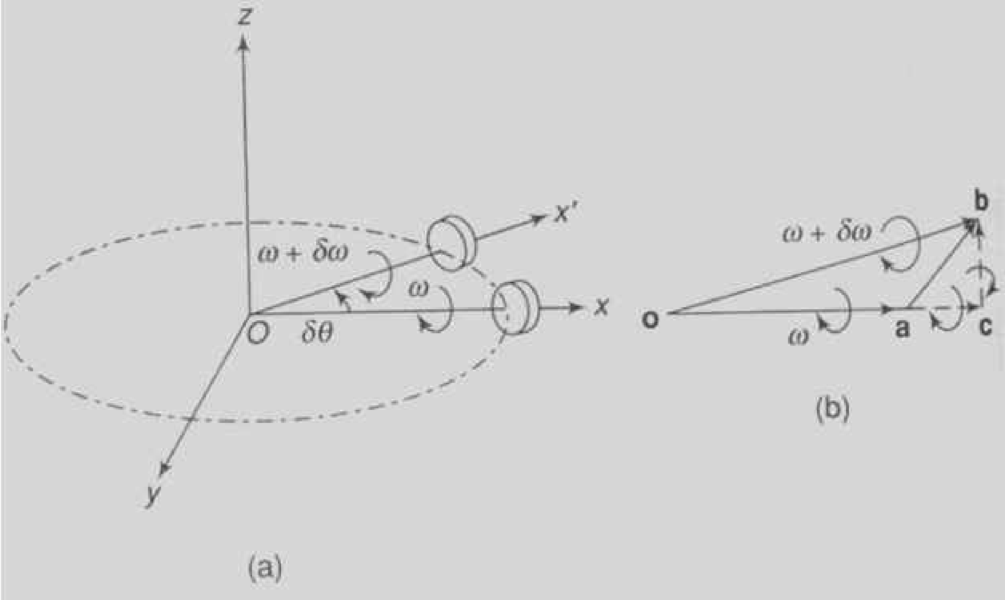
PRECESSIONAL MOTION AND ANGULAR ACCELERATION











$\overrightarrow{oa}$ → initial angular velocity vector

$\overrightarrow{ob}$ → angular velocity vector in new position

$\overrightarrow{ab}$ → change in angular velocity vector

The vector ab can be resolved in to two components.

- ac representing angular velocity change in a plane normal to x axis .
- cb representing angular velocity change in a plane normal to y axis.

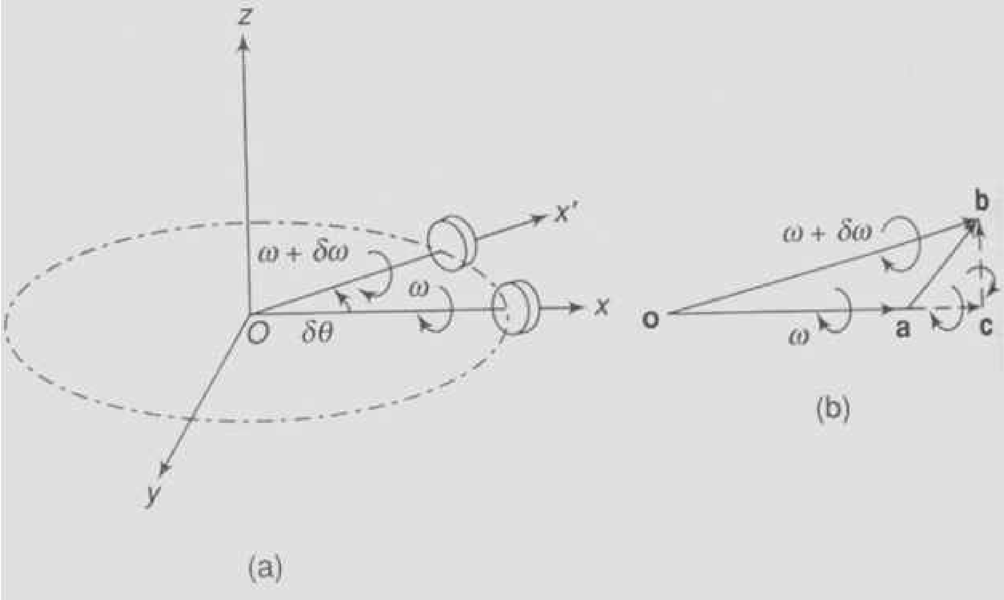
Change of angular velocity, ac = $(\omega + \delta\omega)\cos\delta\theta - \omega$

Rate of change of angular velocity= $\lim_{\delta t \to 0} \frac{((\omega + \delta\omega)\cos\delta\theta - \omega)}{\delta t}$

Therefore angular acceleration= $\lim_{\delta t \to 0} \frac{((\omega + \delta\omega)\cos\delta\theta - \omega)}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$, $\cos\delta\theta \to 1$, $\sin\delta\theta \to \delta\theta$

Therefore angular acceleration due to change in magnitude of angular velocity of rotor = $\lim_{\delta t \to 0} \frac{((\omega + \delta\omega) - \omega)}{\delta t} = \lim_{\delta t \to 0} \frac{(\delta\omega)}{\delta t} = d\omega/dt$



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The vector ab can be resolved in to two components.

- ac representing angular velocity change in a plane normal to x axis .
- cb representing angular velocity change in a plane normal to y axis.

Change of angular velocity, cb= $(\omega+ \delta\omega) \sin \delta\theta$

Rate of change of angular velocity= $\lim_{\delta t\to 0}\frac{((\omega+ \delta\omega)\sin \delta\theta)}{\delta t}$

Therefore angular acceleration=$\lim_{\delta t\to 0}\frac{((\omega+ \delta\omega)\sin \delta\theta)}{\delta t}$

As $\delta t \to 0 \to 0$, $\delta\theta \to 0$, $\sin \delta\theta \to \delta\theta$

Therefore angular acceleration due to change in direction of angular velocity of rotor (axis of spin)

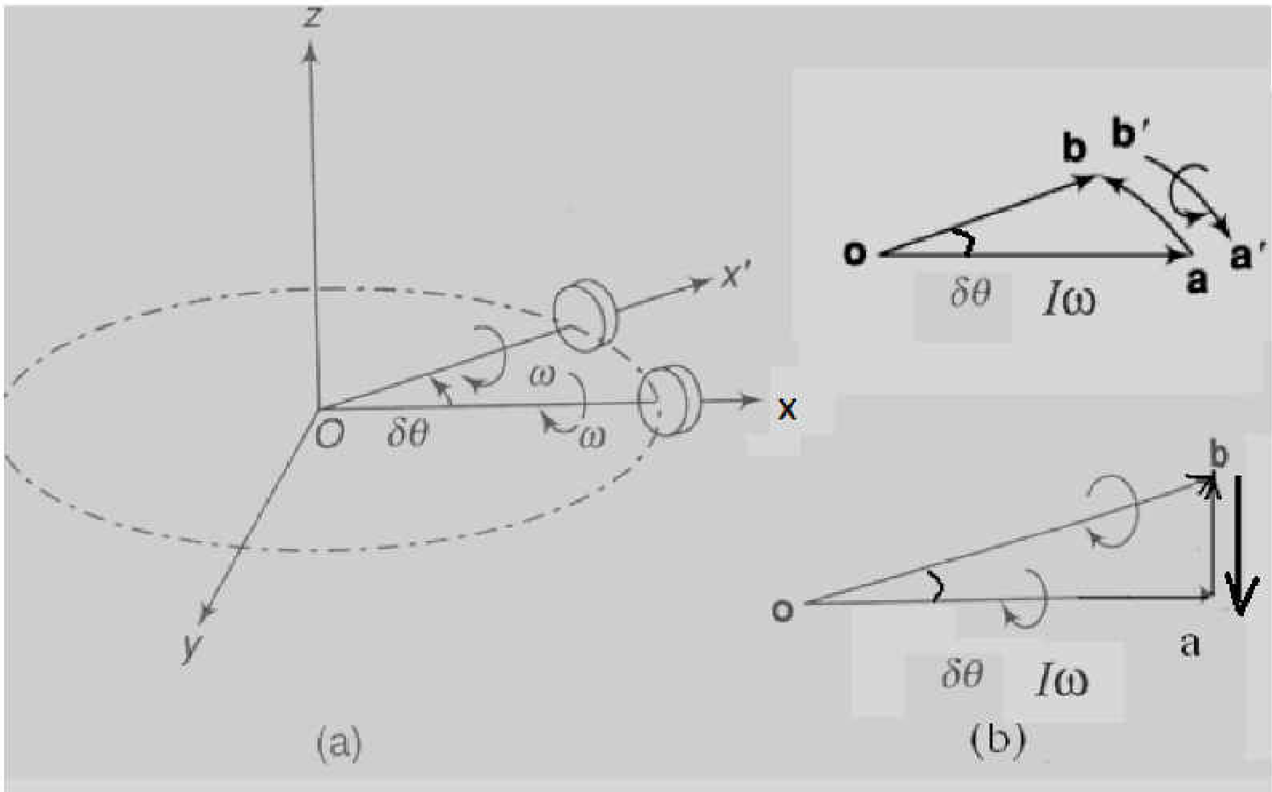
$$= \lim_{\delta t\to 0}\frac{((\omega+ \delta\omega)\delta\theta)}{\delta t} = \lim_{\delta t\to 0}\left[\frac{\omega\delta\theta}{\delta t} + \frac{\delta\omega\delta\theta}{\delta t}\right] = \omega\, d\theta/dt = \omega\, \omega_p$$

TOTAL ANGULAR ACCELERATION OF A SPINNING BODY UNDERGOING PRECESSIONAL MOTION

- Total angular acceleration, $\alpha = d\omega/dt + \omega\omega_p$
- This shows that the total angular acceleration of the rotor is the sum of
- $d\omega/dt$,representing change in the magnitude of angular velocity of rotor.
- $\omega.\omega_p$,representing change in the direction of axis of spin, direction of cb is from c to b in the vector diagram (being a component of ab), the acceleration acts clockwise in the vertical plane XZ (when viewed from front along the y axis).
- Note: $\omega_p = d\theta/dt$,angular velocity of precession

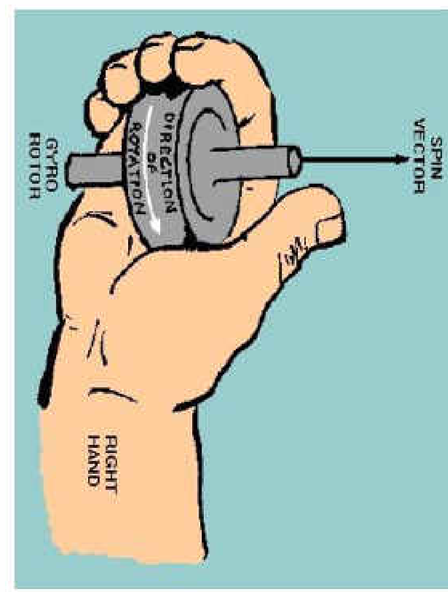
DERIVATION OF GYROSCOPIC COUPLE

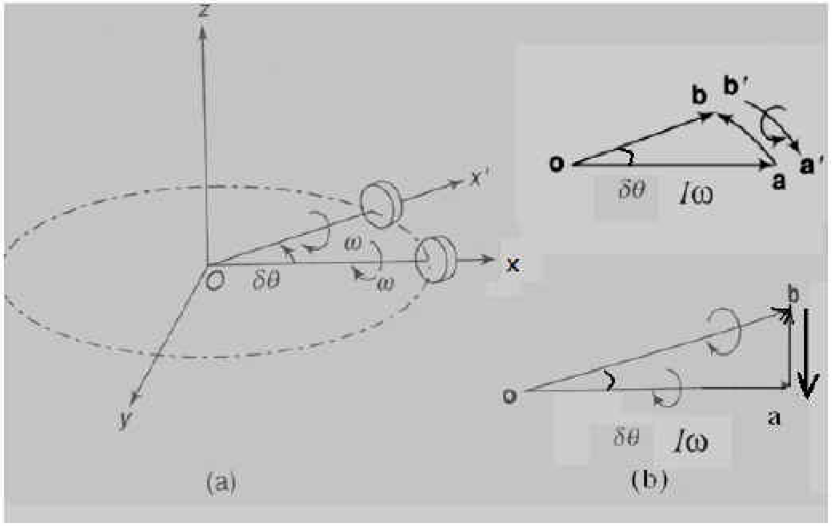
GYROSCOPIC COUPLE



ab → Applied couple

b′a′ → Reaction couple





We know angular momentum of spinning disc, $L = I\omega$
Thus vector oa = initial angular momentum
Similarly vector ob = angular momentum of spinning disc after time δt
A change in angular momentum of the disc occurs because of the change in the direction of momentum.
Then vector ab = change of angular momentum vector in time $\delta t(\Delta L)$

Rate of change of angular momentum = $\lim_{\delta t \to 0} (\Delta L)/\delta t$

$= \lim_{\delta t \to 0} (I\,\omega\,\delta\theta)/\delta t$

Rate of change of angular momentum = $I\,\omega\, d\theta/dt$

$= I\,\omega\,\omega_p$

- Rate of change of angular momentum is a couple and in this case it is gyroscopic couple.
- So gyroscopic couple = $I\,\omega\,\omega_p$

Active Gyroscopic couple and Reactive Gyroscopic couple

- Gyrocouple , which is also known as active couple ,must be applied to a rotating disc for obtaining desirable precessional motion of the axis of the spin. This couple is usually applied to the shaft.
- The shaft inturn exerts an equal and opposite (reaction) couple on the bearings. This is perfectly in accordance with newtons third law of motion. Thus the precessional motion of the axis of spin causes a gyroscopic reaction couple to act on the frame to which bearings are fixed. Being equal and opposite to gyrocouple,magnitude of gyroreaction couple is given by $T = I \omega \omega_p$.
- The effect produced by the reactive gyroscopic couple on a rotating body is called gyroscopic effect.

Example 1 *A disc with radius of gyration of 60 mm and a mass of 4 kg is mounted centrally on a horizontal axle of 80 mm length between the bearings. It spins about the axle at 800 rpm counterclockwise when viewed from the right-hand side bearing. The axle precesses about a ver axis at 50 rpm in the clockwise direction when viewed from above. Determine the resultant reaction at each bearing due to the mass and the gyroscopic effect.*

$m = 4$ kg $\quad N = 800$ rpm $\quad k = 0.06$ m $\quad N_p = 50$ rpm $\quad I = mk^2 = 4 \times (0.06)^2 = 0.0144$ kg.m^2

$l = 80$ mm $= 0.08$ m

$$\omega = \frac{2\pi \times 800}{60} = 83.78 \text{ rad/s}$$

$$\omega_p = \frac{2\pi \times 50}{60} = 5.24 \text{ rad/s}$$

$$\therefore \quad C = I\,\omega\omega_p = 0.0144 \times 83.78 \times 5.24 = 6.32 \text{ N.m}$$

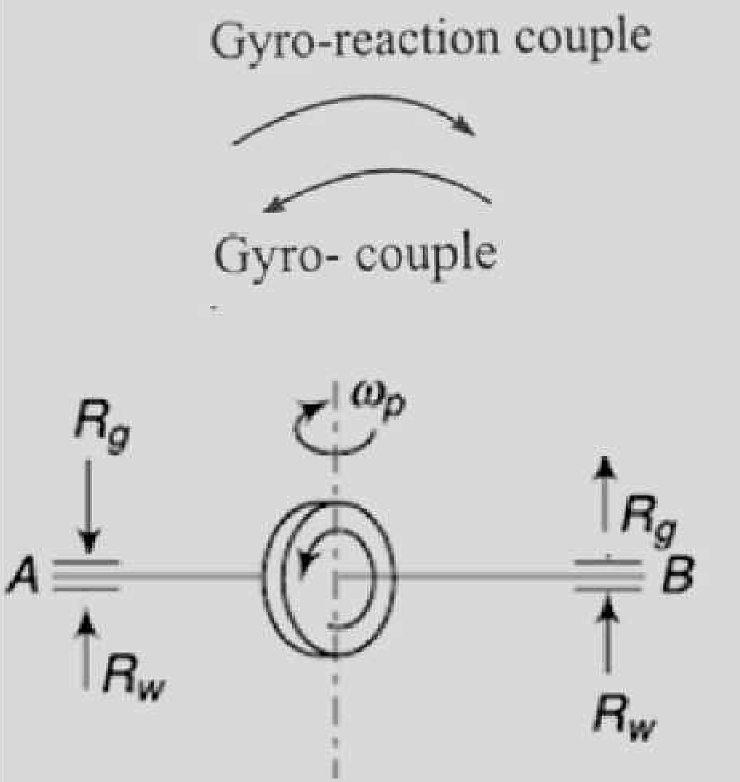
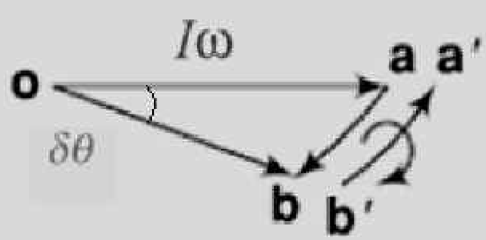


FIG (a)



FIG(b)