

Module - 3

GRAPHICAL CAM PROFILE SYNTHESIS

- ① Draw the profile of a cam operating a knife edge follower having a lift of 30mm. The cam traces the follower motion SHM for 150° of the rotation followed by a period of dwell for 60°. The follower departs from the next 100° rotation of the cam with uniform velocity again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 30mm. What will maximum velocity and acceleration during lift and return?

Solution

Given

$$L = h = 30 \text{ mm},$$

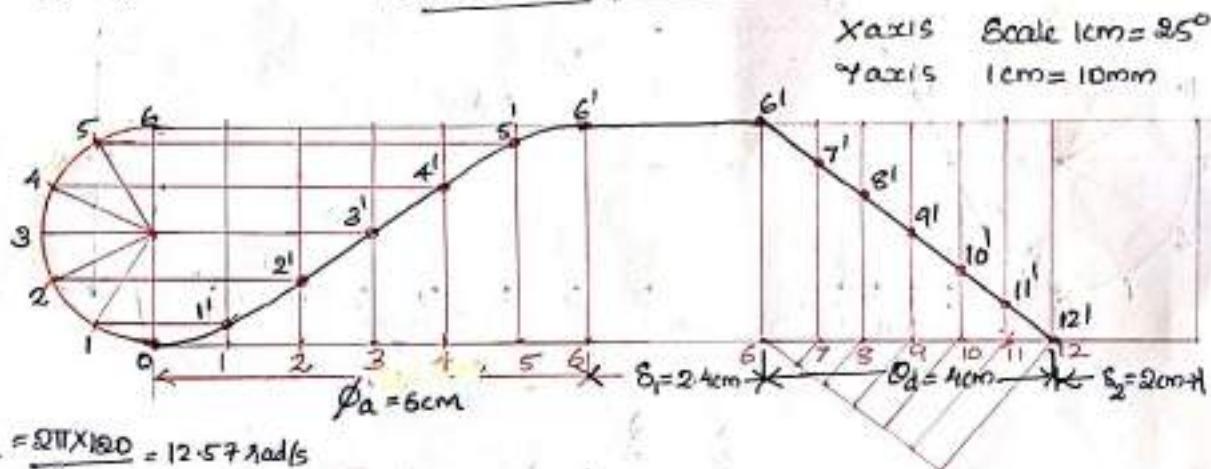
$$\theta_a = \phi_a = 150^\circ$$

$$\theta_d_1 = \delta_1 = 60^\circ$$

$$\theta_a = \phi_d = 100^\circ$$

$$\theta_d_2 = \delta_2 = 50^\circ$$

FOLLOWER DISPLACEMENT DIAGRAMME



$$\omega = \frac{\theta}{t} = \frac{\theta}{60} = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

During ascent (SHM)

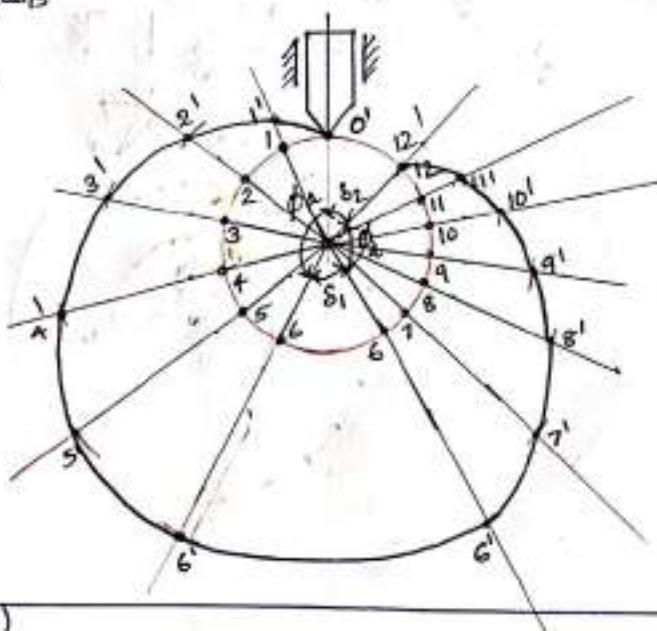
$$V_{max} = \frac{L}{2} \frac{\pi \omega}{\theta_a} = \frac{30}{2} \frac{\pi \times 12.57}{150} = \frac{30 \pi \times 12.57}{300} = 226.3 \text{ mm/s}$$

$$a_{max} = \frac{\omega^2 L}{2} \left(\frac{\pi}{\theta_a} \right)^2$$

$$= 12.57^2 \frac{30}{2} \left(\frac{\pi}{150 \times \frac{180}{180}} \right)^2$$

$$a_{max} = 3413 \text{ mm/s}^2$$

$$= 3.413 \text{ m/s}^2$$



During descent (uniform motion)

$$V_{max} = \frac{L \omega}{\theta_d} = \frac{30 \times 12.57}{100 \times \frac{180}{180}} = 216 \text{ mm/s}$$

- ② A cam with a minimum radius 25mm to be designed for a knife edge follower with following data:
- To raise the follower through 35mm during 60° rotation of the cam
 - Dwell for next 40° of the cam rotation
 - Descending of the follower during the next 90° of the cam rotation
 - Dwell during the rest of the cam rotation.

Draw the profile of the cam. If the ascending and descending of the cam is with SHM and the line of stroke of the follower is offset 10mm from the axis of the cam shaft. What is the maximum velocity and acceleration of the follower during the ascent and descent if the cam rotates at 150 rpm.

Solution

$$L = h = 35\text{ mm} \quad \theta_2 = 170^\circ, N = 150 \text{ rpm}$$

$$\phi_a = 60^\circ$$

$$\delta_1 = 40^\circ$$

$$\phi_d = 90^\circ$$

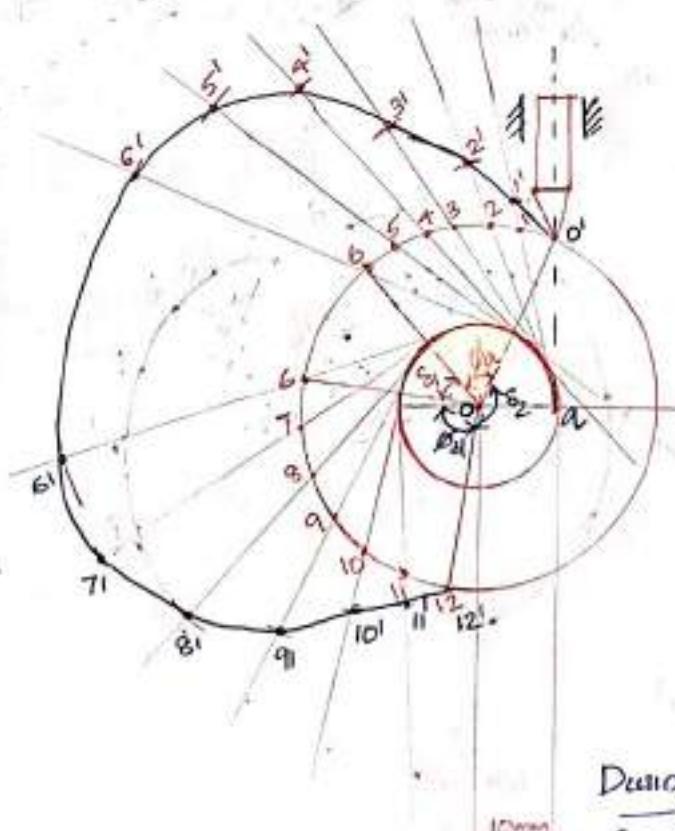
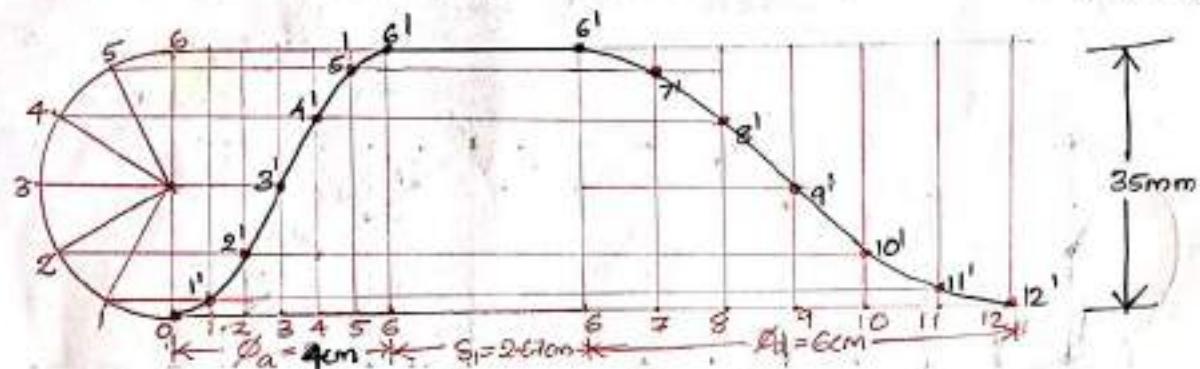
Scale

$$1\text{ cm} = 15^\circ$$

$$\phi_a = 2\text{ cm}$$

$$\delta_1 = 2.67\text{ cm}$$

$$\phi_d = 6\text{ cm}$$



$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 5\pi \text{ rad/sec}$$

During ascent (SHM)

$$V_{max} = \frac{L \cdot \pi \omega}{2 \cdot \phi_a}$$

$$V_{max} = \frac{35 \cdot \pi \times 5\pi}{2 \cdot 60 \times \frac{180}{180}} = 824.7 \text{ mm/s}$$

$$a_{max} = \omega^2 \cdot \frac{L}{2} \left(\frac{\pi}{\phi_a} \right)^2$$

$$a_{max} = (5\pi)^2 \cdot \frac{35}{2} \left(\frac{\pi}{60 \times \frac{180}{180}} \right)^2$$

$$a_{max} = 3888.2 \text{ mm/s}^2$$

During descent

$$V_{max} = \frac{L \cdot \pi \omega}{2 \cdot \phi_d} = \frac{35 \cdot \pi \times 5\pi}{2 \cdot 90 \times \frac{180}{180}} = 599.8 \text{ mm/s}$$

⑤ Draw the profile of a cam operating a roller reciprocating follower, and with following data. Min radius = 25mm, lift = 30mm, roller dia = 150mm, cam lift follower with 180° with uniform acceleration & deceleration, followed by a dwell period of 30° then the followers lowers down during 150° of cam rotation with S.H.M and followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm. Calculate max velocity and acceleration of followers during descend period.

Solution

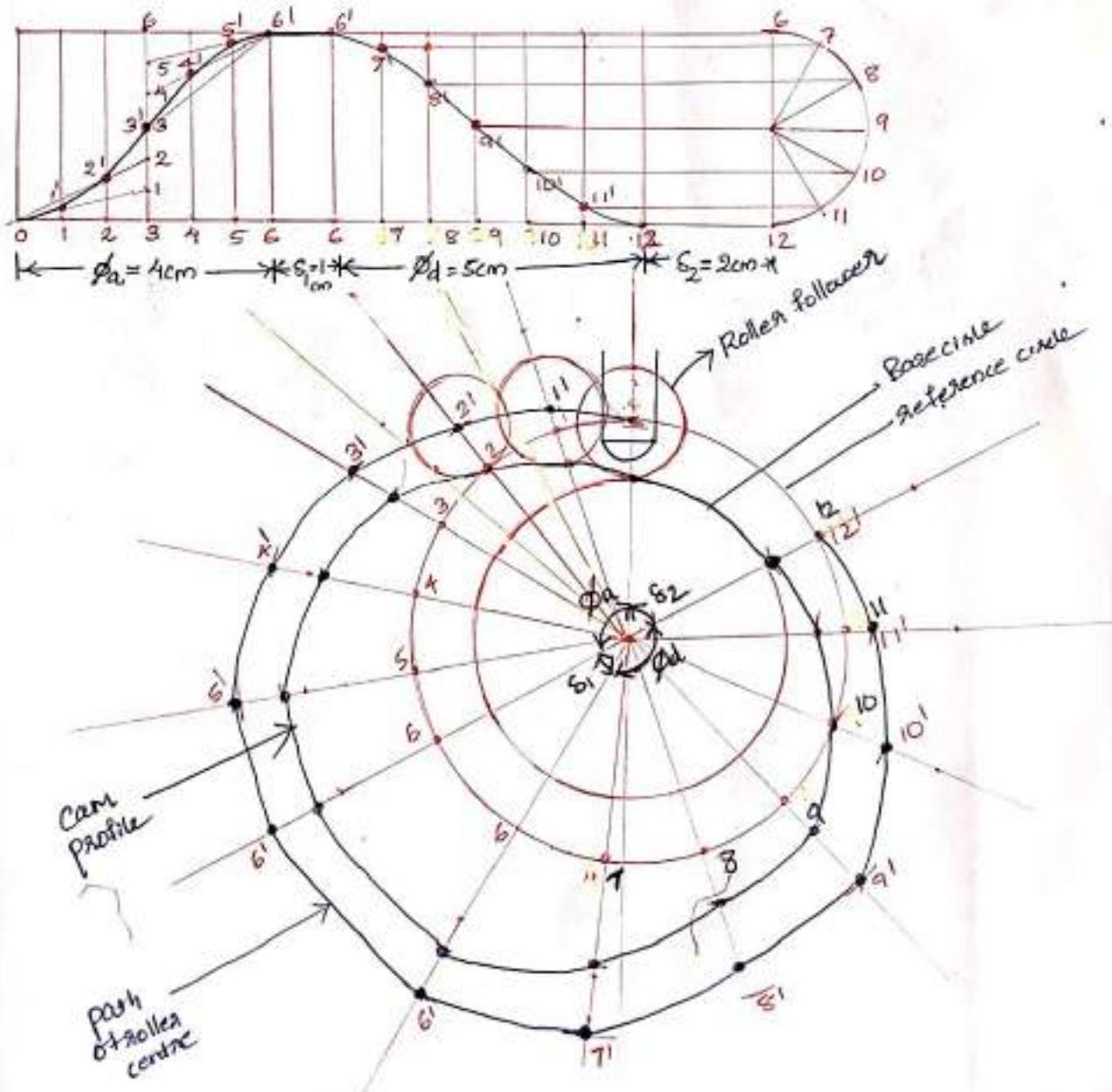
$$\begin{aligned}
 h = h_1 = 30\text{mm} & \quad S_1 = 30^\circ \\
 N = 150\text{rpm} & \quad \phi_d = 150^\circ \\
 r = 25\text{mm} & \quad S_2 = 60^\circ \\
 \phi_a = 120^\circ & \quad R_g = 7.5\text{mm}
 \end{aligned}$$

Note

$$\begin{aligned}
 \text{prime circle radius} &= r + R_g \\
 &= 25 + 7.5 = \underline{\underline{32.5\text{mm}}}
 \end{aligned}$$

Scale

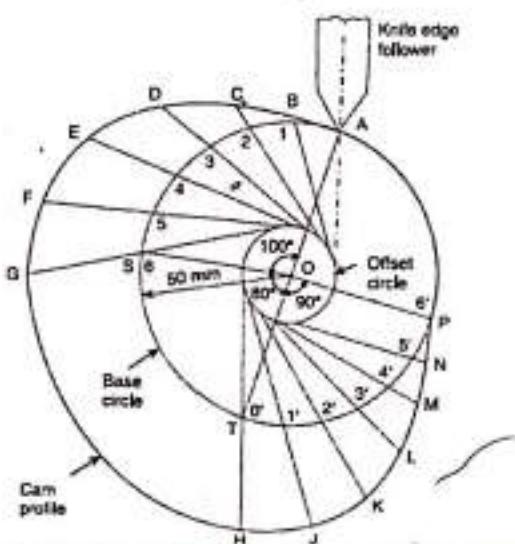
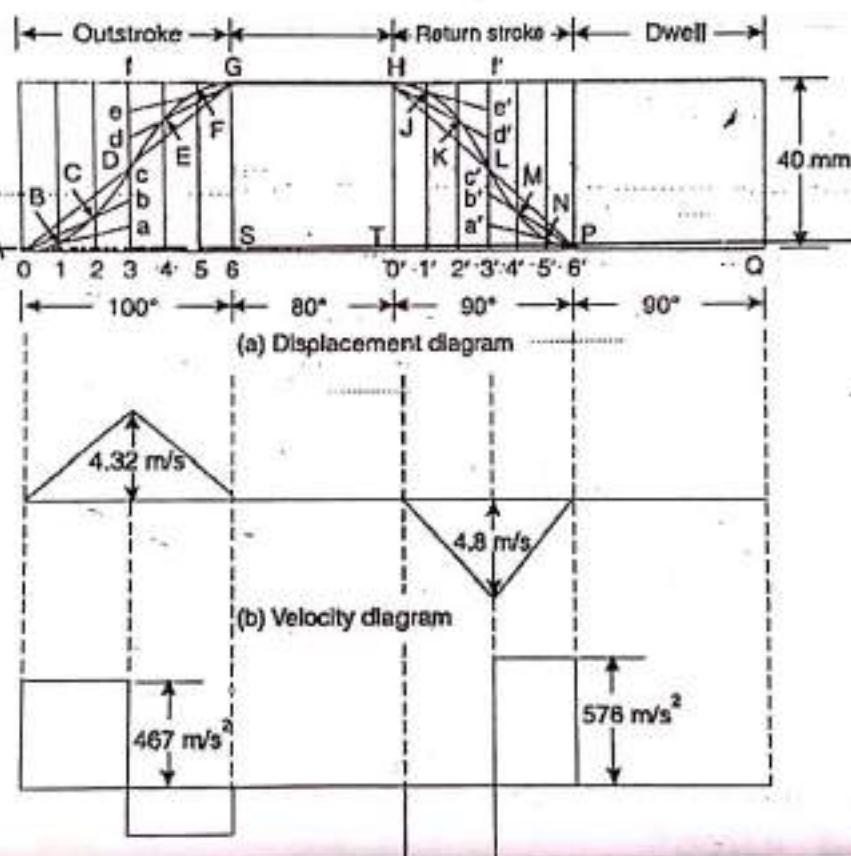
$$\begin{aligned}
 \text{X-axis} \rightarrow 1\text{cm} &= 30^\circ \\
 \text{Y-axis} \rightarrow 1\text{cm} &= 10\text{mm}
 \end{aligned}$$



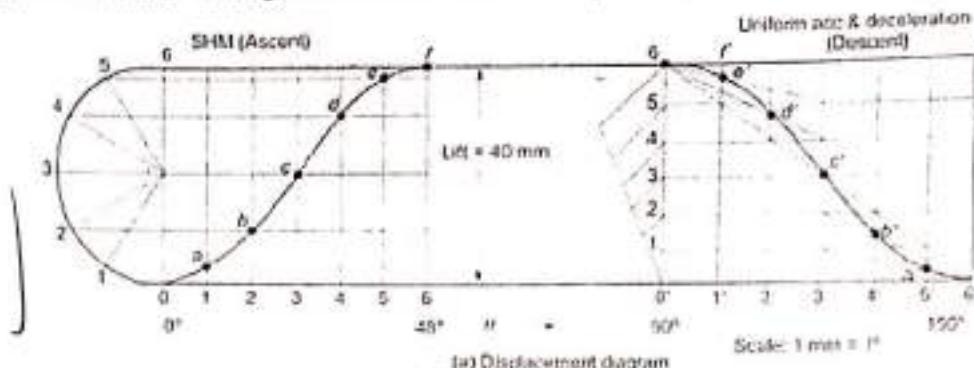
Q3. A cam with a min radius of 50 mm, rotating CW at a uniform speed is required to give a knife edge follower the motion as described below.

- To move outwards through 40 mm during 100 degree rotation of the cam.
- To dwell for next 80 degree.
- To return to its starting position during next 90 degree.
- To dwell for the rest period.

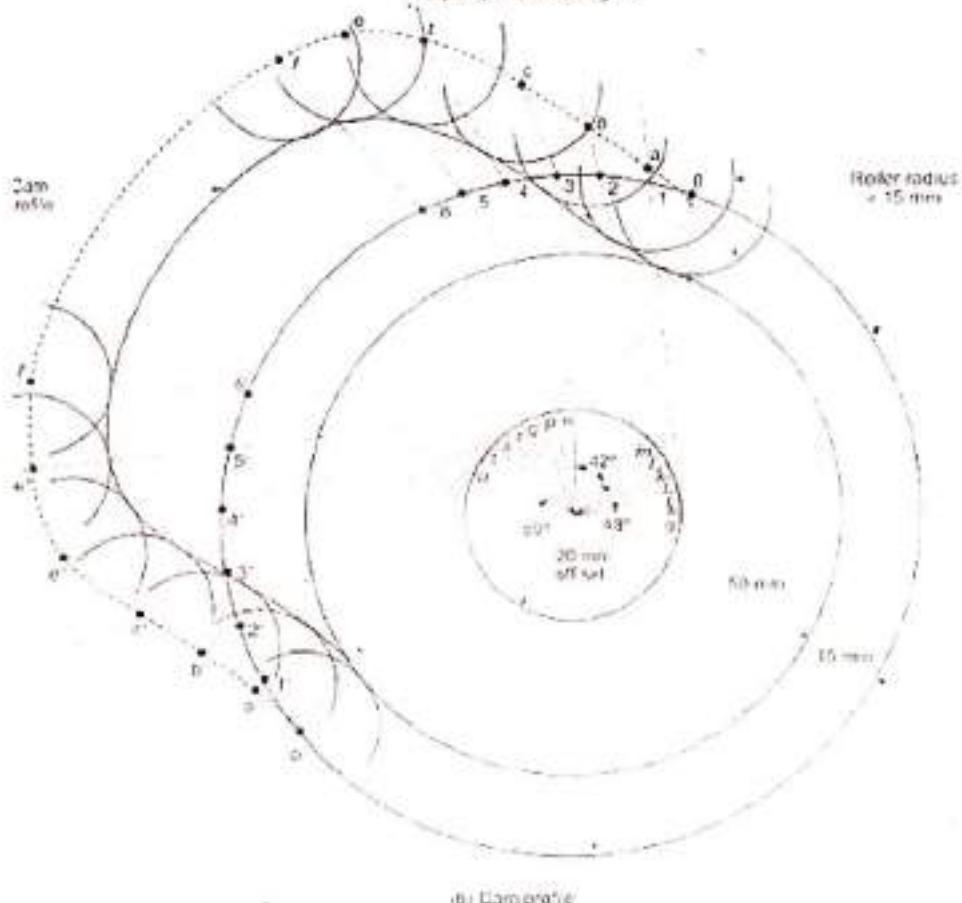
Draw the profile of the cam, when the line of stroke of the follower is offset by 15 mm. The displacement of the follower is to take place with uniform acceleration and retardation. Determine the max velocity and acceleration of the follower when the cam shaft rotates at 900 rpm. draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.



- ④ A cam operate an offset roller follower. The least radius of cam is 30mm. Offset is 20mm. The cam rotates at 360 rpm. The angle of ascent is 48° , angle of dwell is 42° , angle of descent is 60° . The motion is to be SHM during ascent and uniform acceleration and deceleration during descent. draw the cam profile also calculate the maximum velocity and.



(a) Displacement diagram



(b) Cam profile

Cam with offset roller follower having SHM during ascent uniform acceleration and retardation during descent

Soln

$$\omega = \frac{2\pi N}{60} = \underline{\underline{12\pi \text{ rad/s}}}$$

During descent

$$V_{max} = \frac{2 \cdot L \omega}{\theta_d} = \frac{2 \times 40 \times 10^3 \times 12\pi}{\pi/3} = \underline{\underline{2.88 \text{ m/s}}}$$

$$a_{max} = \frac{4 \cdot L \omega^2}{\theta_d^2} = \underline{\underline{207.36 \text{ m/s}^2}}$$

④ The following data relate to a cam profile in which the follower moves with cycloidal during ascent and with constant acceleration and deceleration during descent. The minimum radius of cam = 25mm, roller radius = 7.5mm, lift = 28mm, offset of follower axis = 12mm towards right, angle of ascent = 60° , angle of descent = 90° . Angle of dwell between ascent and descent = 45° . Speed of the cam = 200 rpm. Draw cam profile and determine maximum velocity and acceleration during cut-taking and return stroke?

Ans: Given

$$w = 28 \text{ mm}$$

$$\theta_1 = 45^\circ$$

$$r = 25 \text{ mm}$$

$$\theta_2 = 360 - (60 + 90 + 45) = 160^\circ$$

$$r_t = 7.5 \text{ mm}$$

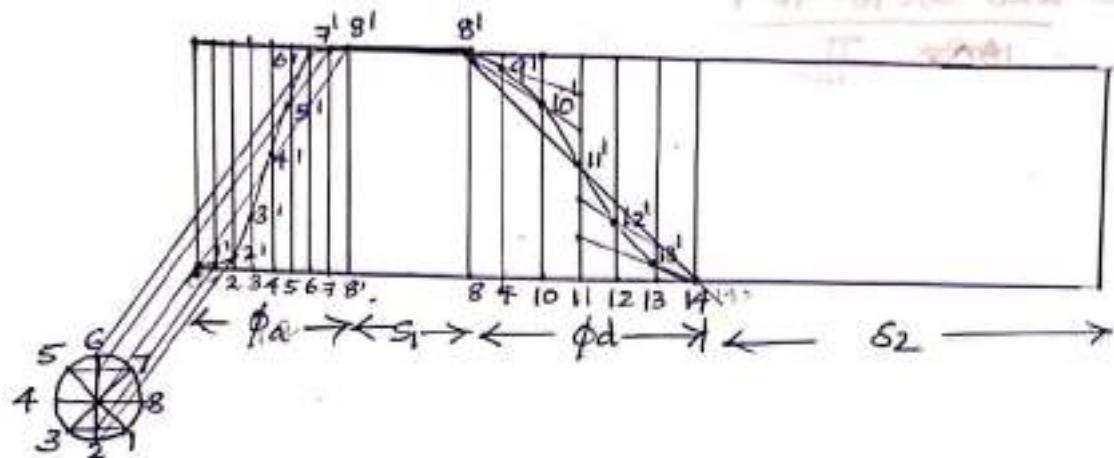
$$N = 200 \text{ rpm}$$

$$z = 12 \text{ mm}$$

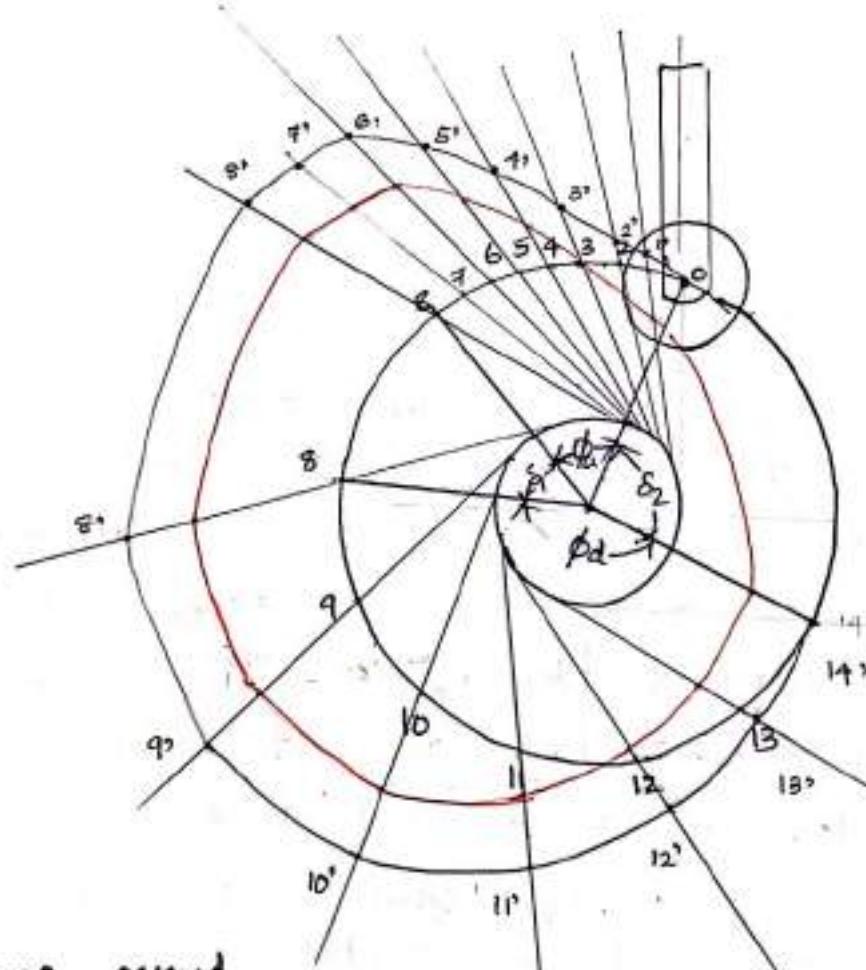
$$r = \frac{hr}{2\pi} = \frac{28}{2\pi} = \underline{\underline{4.45 \text{ mm}}}$$

$$\phi_a = 60^\circ$$

$$\phi_d = 90^\circ$$



⑤



during descent

$$V_{max} = \frac{2hw}{\phi_d} = \cancel{0.28} \times 20 \times 94.3 = \frac{90 \times \pi}{180}$$

$$= 7.66 \text{ m/s}$$

$$f_{max} = \frac{4hw^2}{\phi_d^2} = \frac{4 \times 0.28 \times 20 \times 94.3^2}{(90 \times \pi / 180)^2}$$

$$= 199.093 \text{ m/s}^2$$

during ascend

$$V_{max} = \frac{2hw}{\phi} = \frac{2 \times \pi \times 200}{60} = \underline{\underline{20.993 \text{ m/s}}}$$

$$= \frac{2 \times 0.28 \times 20 \times 94.3}{60 \times \frac{\pi}{180}} = \underline{\underline{11.199 \text{ m/s}}}$$

$$f_{max} = \frac{2hw^2}{\phi_a^2} = \frac{2 \times 0.28 \times \pi \times 20 \times 94.3^2}{(60 \times \frac{\pi}{180})^2} = \underline{\underline{103.65 \text{ m/s}^2}}$$

Q6. Draw the profile of a cam when the roller follower moves with cycloidal motion during outstroke and return stroke as detailed below.

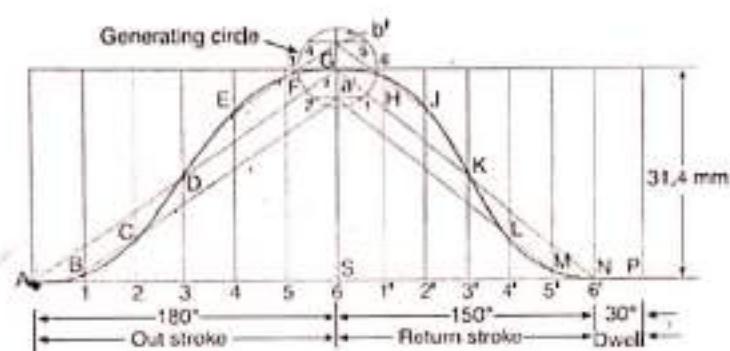
- Outstroke with maximum displacement of 31.4 mm during 180° of cam rotation.
- Return stroke for next 150° of cam rotation .
- Dwell for the remaining 30° of cam rotation .

Minimum radius of cam is 15 mm and roller diameter of follower is 10 mm.the axis of roller follower is offset 10 mm towards right from the axis of cam shaft.

The radius r of circle which rolls along displacement axis to generate cycloid is given by ,

$$2\pi r - \text{stroke length} = \text{lift} = 31.4 \text{ mm}$$

$$r = L/2\pi = 31.4/2\pi = 5 \text{ mm}$$



PROCEDURE FOR DRAWING DISPLACEMENT DIAGRAM

Draw horizontal line ASP ,such that AS= 180° to represent outstroke ,SN = 150° to represent return stroke and NP = 30° to represent dwell period.

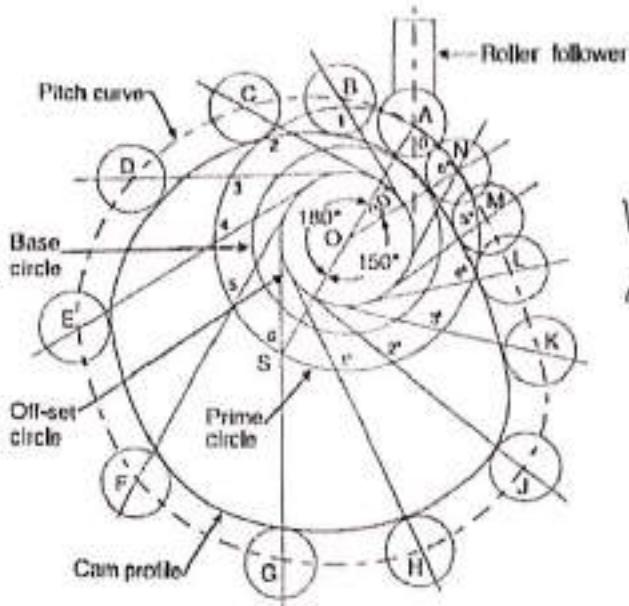
Divide AS and SN into any number of even equal parts (say 6)

From the points 1,2,3 ...etc draw vertical lines and set off equal to the stroke of the follower.

From point G ,draw a generating circle of radius , $r = \text{stroke} / 2\pi = 31.4/2\pi = 5 \text{ mm}$

Divide generating circle in to six equal parts and from the points draw horizontal lines to meet the vertical diameter at a',G and b' .

Join AG and GN .from point a',draw lines parallel to AG and gn to intersect the vertical lines drawn through 1,2 ,4' and 5' at B,C,L and M respectively.similarly draw parallel lines from b' intersecting the vertical lines through 4,5,1' and 2' at E,F,H and J respectively

Outstroke

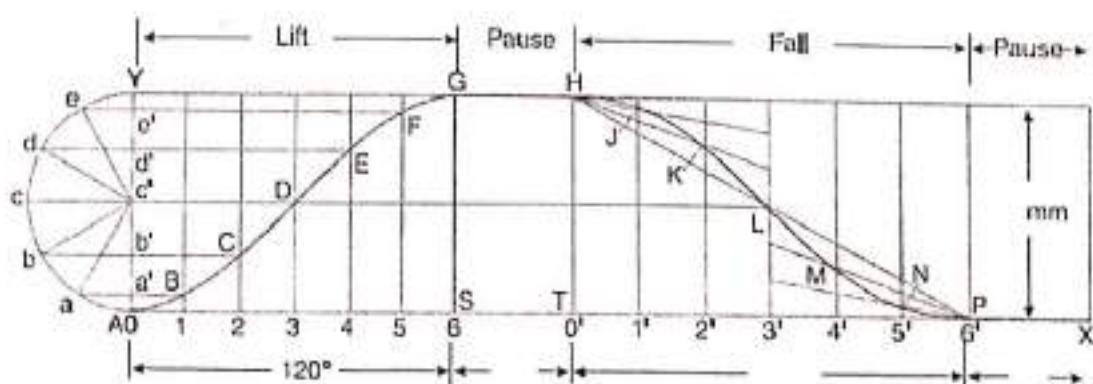
$$V_{FMAX} = \frac{2L\omega}{\theta_0} = 146.67 \text{ cm/s}$$

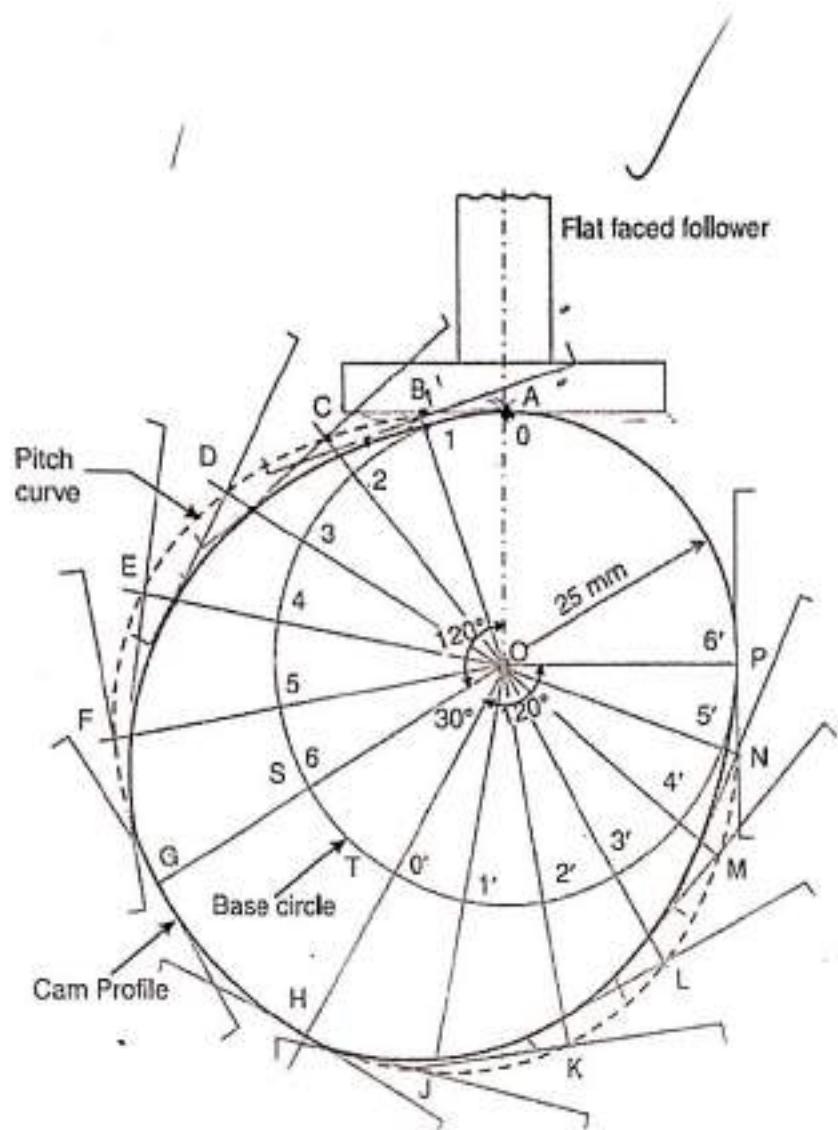
$$a_{MAX} =$$

Q7. It is required to set out the profile of a cam to give the following motion to the reciprocating follower with a flat mushroom contact face.

- Follower to have a stroke of 20 mm during 120 degree of cam rotation.
- Follower to dwell for 30 degree of cam rotation.
- Follower to return to its initial position during 120 degree of cam rotation.
- Follower to dwell for remaining 90 degree of cam rotation.

The minimum radius of the cam is 25 mm. The outstroke of the follower is performed with SHM and return stroke with uniform acceleration and retardation.





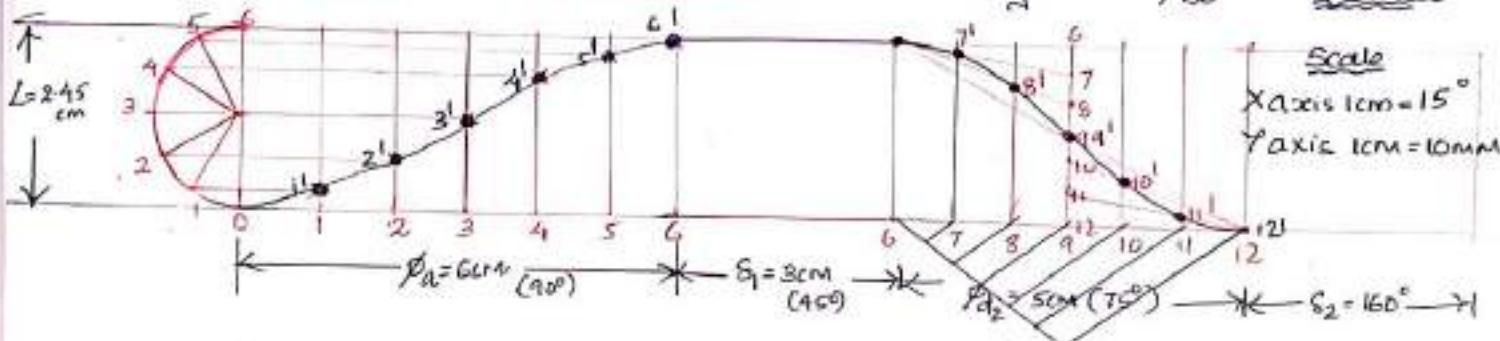
- It is required to set out the profile of cam with Oscillating follower for the following motion.

- Follower to move outward through an angular displacement of 20° during 90° of cam rotation.
- Follower to dwell for 45° of cam rotation.
- Follower to return to its original position of zero displacement in 75° of cam rotation.
- Follower to dwell for remaining period of revolution of the cam.

Distance b/w pivot centre and follower roller centre is 70mm and roller diameter is 20mm. The min radius of cam is 50mm. The location pivot point is 70mm to the left and 60mm above the axis of rotation of the cam. The Motion of follower is to take place with SLM during cutstroke & uniform acceleration and retardation during return stroke.

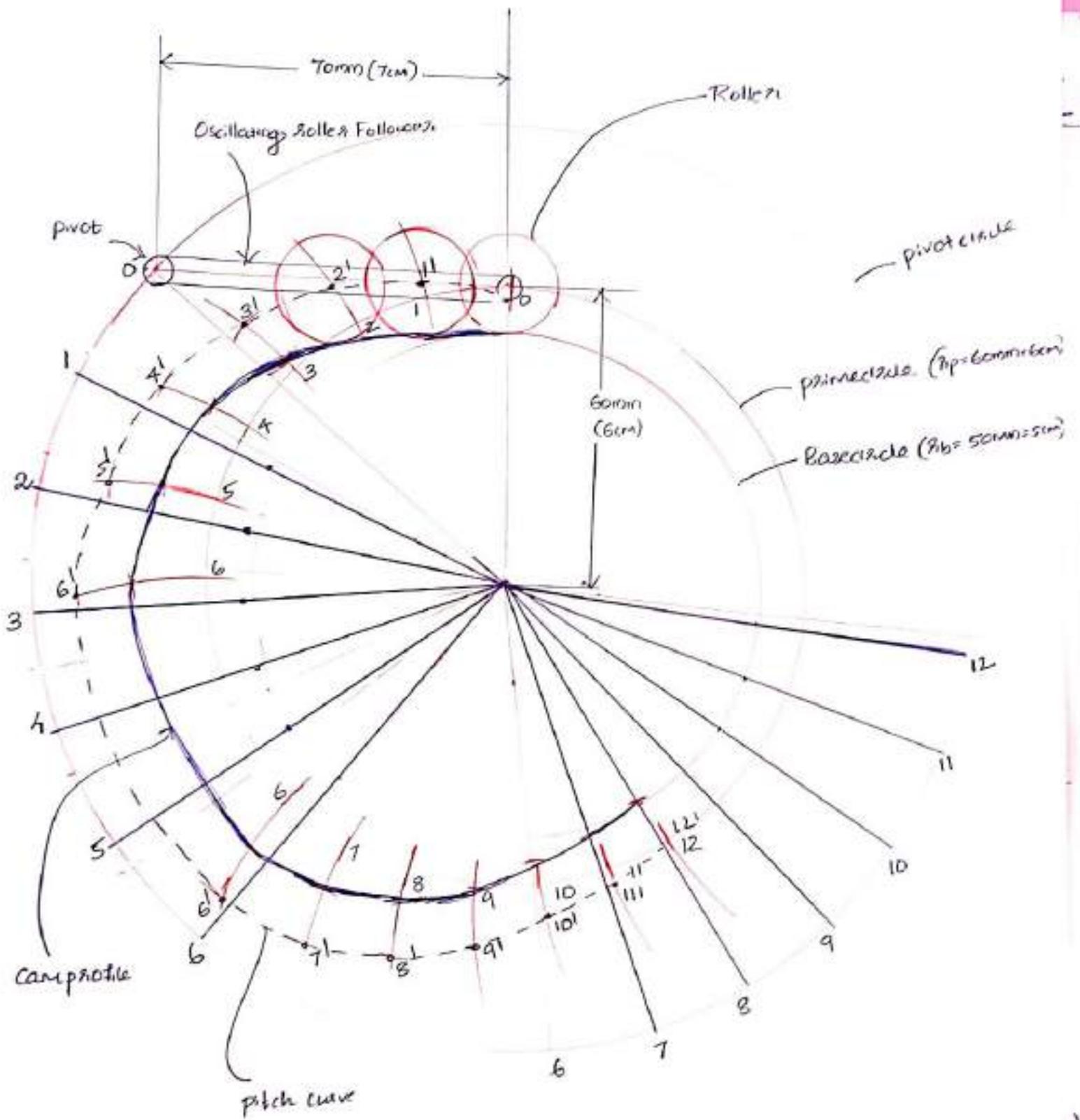
Solution

$$G.D \Rightarrow \phi_a = 90^\circ, S_1 = 45^\circ, \phi_d = 75^\circ, S_2 = 160^\circ, \text{ Lift } L = AA_2 \\ (\text{LHM}) \quad (\text{uniform Acceleration & dwell}) \quad = \text{Arm length} = \frac{20 \times \sqrt{180}}{180} \times 70 = 24.5 \text{ mm}$$



Procedure

- Draw Base circle with centre O & radius = 50mm (Scale - 5cm)
- Draw a prime circle with centre O & radius = 60mm ($r_{ab} + r_{ba}$) (Scale - 6cm)
- Locate pivot centre $A_1 = 70\text{mm}$ left of roller centre & 60mm above axis of rotation [Scale - 7cm] [Scale - 6cm]
- Join OA₁. Draw $\angle A_1OS = 90^\circ$ [ϕ_a], $\angle SOT = 45^\circ$ [S_1], $\angle TOP = 75^\circ$ [ϕ_d]
- Divide $\angle A_1OS$ & $\angle TOP$ in equal six division & mark points as 1, 2, 3, ..., 12 on pivot circle
- Now points on 1, 2, 3, ..., 12 on pivot circle as centre and radius equal to follower arm length = 70mm. Draw circular arcs to intersect the prime circle at points 1', 2', 3', ..., 12'
- Set off the distance 1', 2', 3', ..., along the axis draw equal to distance as measured from the displacement diagram.
- Curve passing through 1', 2', 3', ..., 12' is known as pitch circle
- Now draw circles 1', 2', 3', ..., 12' as centre and radius equal to radius of roller
- Join the bottom of the circles with a smooth curve this is required profile of cam



Q. A cam operating at 150 rpm operates a reciprocating roller follower of radius 2.5 cm. The follower axis is offset by 2.5 cm to the right. The least radius of the cam is 5 cm and the stroke of the follower is 5 cm. Ascent and descent both takes place by uniform acceleration and retardation. Ascent takes place during 75° and descent during 90° of cam rotation. Dwell between ascent and descent is 60° . Draw the cam profile .also sketch velocity and acceleration diagrams and mark salient values.

Based on the flank, cams are classified into two groups,

1. Circular arc cam 2. Straight edged cam or Tangent cam

Circular arc Cam: When the flanks of the cam connecting the nose and base circles are of convex circular arc, such cams are referred as circular arc cams.

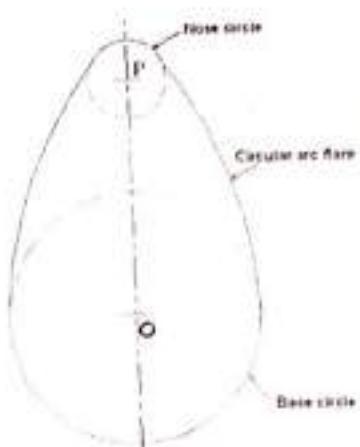
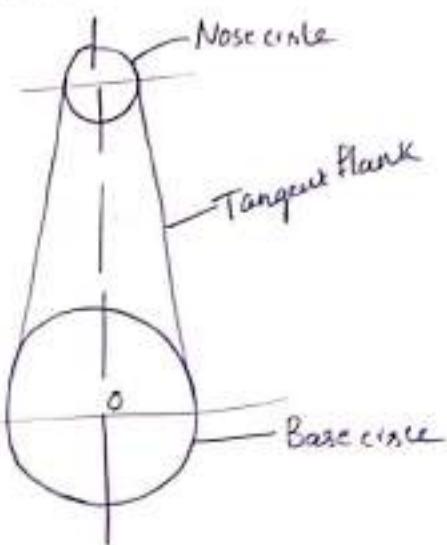


Fig.1

Tangent cam: When the flanks between the nose and base circles are of straight and tangential to both the circles, then, the cams are called tangent cams.



CIRCULAR ARC CAM WITH FLAT FACE FOLLOWER

- (a) Expression for determining the displacement, Velocity/Acceleration of the follower when the flat face of the follower has contact on the circular flank.

Let $R_1 = OB$ = Least Base circle radius

R_2 = Nose circle radius

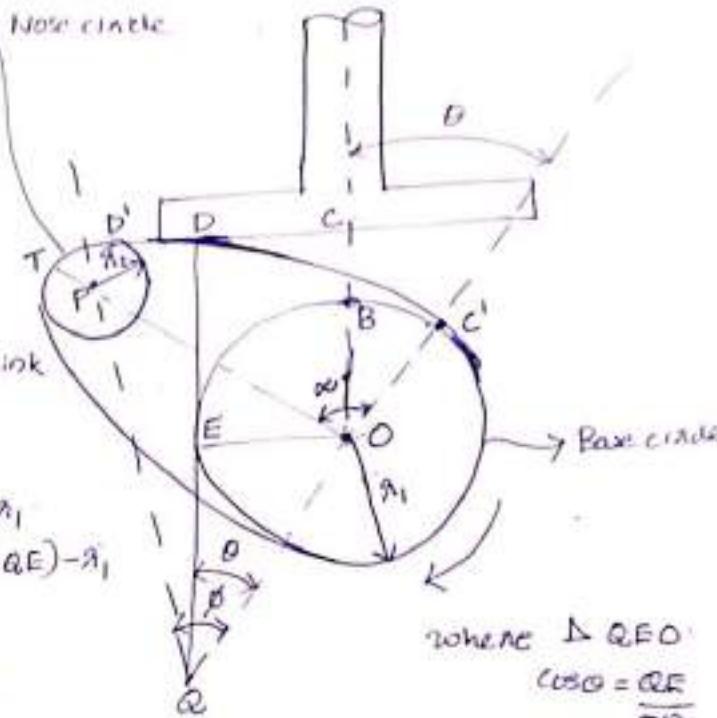
$R = QD$ = Flank circle radius

d = distance b/w centers of cam

β = Nose circle

α = Angle of Ascent

ϕ = Angle of contact on circular link



where $\triangle QEO$

$$\cos \theta = \frac{QE}{QO}$$

$$QE = OB \cos \theta$$

- Displacement

$$x = BC = OC - OB = DE - R_1 \\ = (QD - QE) - R_1$$

$$x = (R - QE \cos \theta) - R_1$$

$$x = (R - (R - R_1) \cos \theta) - R_1$$

$$x = (R - R_1) - (R - R_1) \cos \theta$$

$$\boxed{x = (R - R_1)(1 - \cos \theta)}$$

- Velocity $\Rightarrow v_c = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = (R - R_1) \sin \theta \omega$

$$\boxed{\frac{d\theta}{dt} = \omega}$$

$$\boxed{v = \omega(R - R_1) \sin \theta}$$

Note \Rightarrow At beginning of ascent $v=0$ ($\theta=0$) and it increase with θ

\Rightarrow It will Maximum when the follower is just shift from circular flank to circular nose.

$$\boxed{v_{max} = \omega(R - R_1) \sin \phi}$$

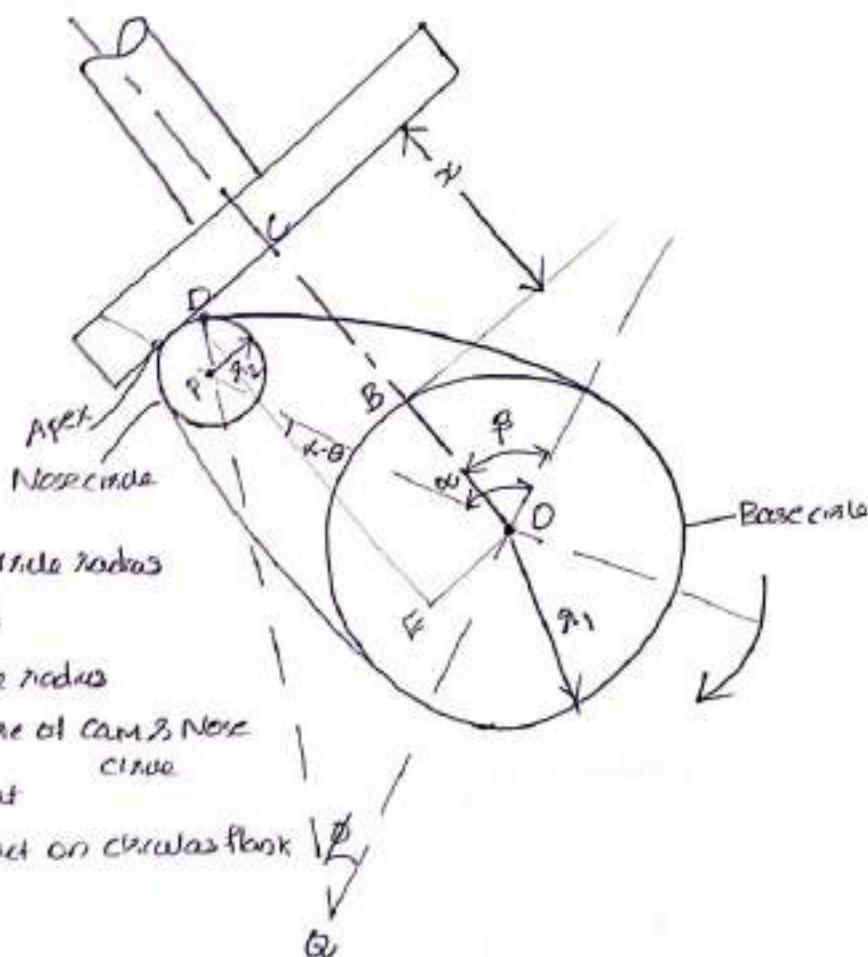
- Acceleration $\Rightarrow a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = \omega(R - R_1) \cos \theta \times \omega = \omega^2(R - R_1) \cos \theta$

Note \Rightarrow At Beginning of Ascent $\theta=0$, Acceleration Max & decrease
towards and Max $\theta=\phi$

$$\boxed{a_{max} = \omega^2(R - R_1)}$$

$$\boxed{a_{min} = \omega^2(R - R_1) \cos \phi}$$

(b) Expression for determining the displacement, Velocity & Acceleration of the follower - when flat face of the follower has contact on base.



Let $r_1 = OB = \text{least base circle radius}$

$r_2 = \text{Nose circle radius}$

$R = OD = \text{Flank circle radius}$

$d = \text{distance b/w centre of cam \& nose circle}$

α : Angle of Ascent

ϕ : Angle of contact on circular flank

$$\begin{aligned} \text{Displacement} \Rightarrow x &= BC = OC - OB = DE - r_1 \\ &= (DP + PE) - r_1 \\ &= (r_2 + OPC \cos(\alpha - \theta)) - r_1 \end{aligned}$$

$$x = r_2 + d \cos(\alpha - \theta) - r_1$$

$$\text{Velocity} \Rightarrow V = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = d \times \sin(\alpha - \theta) \times \omega \times 1$$

$$V = +\omega d \sin(\alpha - \theta)$$

$$\begin{aligned} \Delta PEO \\ \cos(\alpha - \theta) &= \frac{PE}{OP} \\ PE &= OP \cos(\alpha - \theta) \\ OP &= d \end{aligned}$$

Velocity Min when $\alpha = 0$ or $(\alpha - \theta) = 0$ [It happens when follower apex at circular nose]

Velocity Max when $(\alpha - \theta)$ Maximum [It is so when contact changes from circular flank-flank to circular nose $\Rightarrow \alpha - \theta = \phi$]

$$\text{Acceleration} \Rightarrow a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = -\omega^2 d \cos(\alpha - \theta) \times \omega$$

$$a = -\omega^2 d \cos(\alpha - \theta)$$

Negative sign indicate retardation

If Max when $(\alpha - \theta) = 0$ [Follower is at apex of nose]

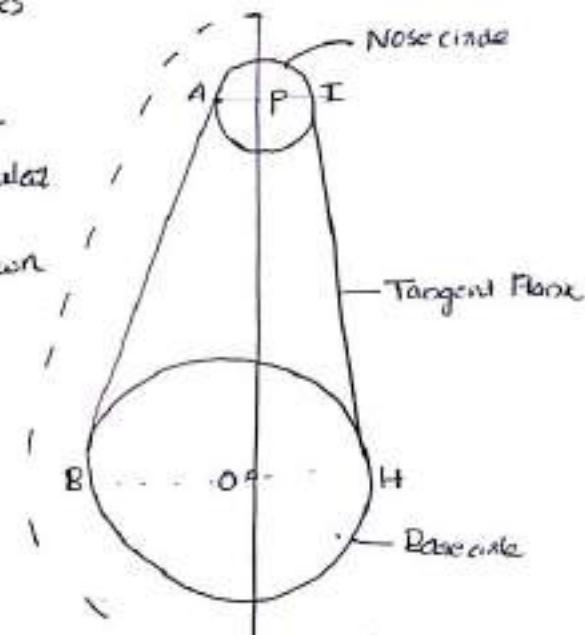
If Minimum when $(\alpha - \theta)$ Maximum [Follower changes contact from circular flank to circular base]

$$a_{\max} = -\omega^2 d$$

$$a_{\min} = -\omega^2 d \cos \phi$$

Tangent cam with Roller reciprocating Follower

- Tangent cam made with straight flanks
- In figure AB & IH are straight lines
- Tangent to base circle at A and I and tangent to nose at B and H. The centre of the circular nose at P
- path of the centre of roller follower shown dotted line



(a)

Expression for determination of displacement, velocity and acceleration of the roller follower when in contact on the straight flank.

Let r_1 = Lead base circle radius

r_2 = Roller radius

r_3 = Nose circle radius

d = distance b/w the cam & nose circles

$$l = (r_1 + r_3)$$

α = Angle of ascent

ϕ = Angle of contact of cam with straight flank

$$\boxed{\text{displacement}} \Rightarrow x = OB - OB = \frac{OB}{\cos \phi} - OB$$

$$x = OB \left(\frac{1}{\cos \phi} - 1 \right)$$

$$\boxed{x = (r_1 + r_3) \left(\frac{1}{\cos \phi} - 1 \right)}$$

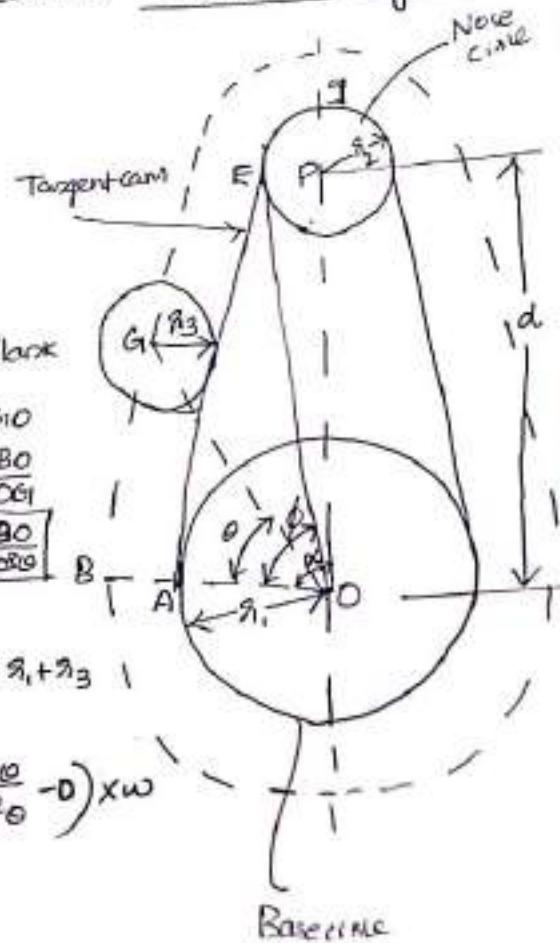
$$\begin{aligned} \triangle BGQ \\ \cos \phi &= BO \\ OG &= \frac{BO}{\cos \phi} \end{aligned}$$

$$OG = \frac{BO}{\cos \phi}$$

Velocity

$$V = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = (r_1 + r_3) \left(\frac{\sin \phi}{r_1^2 \cos^2 \phi} - 0 \right) \times \omega$$

$$\boxed{V = \omega(r_1 + r_3) \left(\frac{\sin \phi}{r_1^2 \cos^2 \phi} \right)}$$



⇒ From eq. of velocity θ increases, Since increase in θ decrease in $\cos \theta$ decrease in velocity increases and velocity will be maximum when θ is maximum. It happens when point of contact just leaving straight flank [when $\theta = \phi$]

$$V_{\max} = \omega(r_1 + r_3) \left(\frac{\sin \theta}{\cos^2 \theta} \right)$$

derivative of $\left(\frac{a}{b} \right)$

$b \times$ derivative of $a - a$ decreases

$$\frac{d}{d\theta} \left(\frac{1}{\cos \theta} \right) = \frac{0 - 1 \times \sin \theta}{\cos^2 \theta}$$

$$= \frac{\sin \theta}{\cos^2 \theta}$$

$$\frac{d}{d\theta} \cos^2 \theta =$$

$$\rightarrow \frac{d}{d\theta} \left(\frac{1 + \cos 2\theta}{2} \right)$$

$$\Rightarrow \frac{1}{2} \times -\sin 2\theta \times 2$$

$$= -\sin 2\theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\text{Acceleration } \Rightarrow a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} =$$

$$a = \omega(r_1 + r_3) \left[\frac{\cos^2 \theta \cdot \cos \theta - 1.5 \sin \theta \cos \theta - \sin 2\theta}{\cos^3 \theta} \right] \times \omega$$

$$a = \omega^2(r_1 + r_3) \left[\frac{\cos^2 \theta \cos \theta + 2 \sin^2 \theta \cos \theta}{\cos^3 \theta} \right]$$

$$a = \omega^2(r_1 + r_3) \left[\frac{\cos^3 \theta + 2(1 - \cos^2 \theta)}{\cos^3 \theta} \right] = \omega^2(r_1 + r_3) \left[\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right]$$

$$a = \omega^2(r_1 + r_3) \left[\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right]$$

$\left[\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right]$ is minimum. This is possible

⇒ Acceleration Min when when $2 - \cos^2 \theta$ is minimum & $\cos^3 \theta$ is maximum. If it is so, when $\theta = 0$ or roller at the beginning of its lift along in the straight

flank.

⇒ Acceleration Maximum when the roller shift from flank to nose side when $\theta = \phi$

$$a_{\min} = \omega^2(r_1 + r_3)$$

$$a_{\max} = \omega^2(r_1 + r_3) \left[\frac{2 - \cos^2 \phi}{\cos^3 \phi} \right]$$

Expression for determination of displacement Velocity & Acceleration of the roller follower when contact with nose

G_1 = Centre of roller
 EP = Lever GIE

$$\text{Displacement} \Rightarrow x = CG_1 - CM$$

$$x = (OE + EG_1) - OB$$

$$x = (OP\cos\phi + G_1P\cos\beta) - OB$$

$$x = OP\cos\phi + G_1P\sqrt{1 - \sin^2\beta} - OB$$

$$x = OP\cos\phi + G_1P\sqrt{1 - \frac{(EP)^2}{(PG_1)^2}} - OB$$

$$x = d\cos(\alpha - \phi) + G_1P\sqrt{1 - \frac{(EP)^2}{(PG_1)^2}} - OB$$

$$\sin\phi = \frac{EP}{\ell} \Rightarrow R\sin\phi = EP$$

$$PG_1 = r_2 + r_3 = l, OB = r_1 + r_3$$

$$x = d\cos(\alpha - \phi) + l\sqrt{1 - \frac{(R\sin\phi)^2}{\ell^2}} - (r_1 + r_3)$$

$$x = d\cos(\alpha - \phi) + \sqrt{\ell^2 - R^2 \sin^2(\alpha - \phi)} - (r_1 + r_3)$$

$$\text{Velocity} \Rightarrow v = \frac{dx}{dt} = \frac{dx}{d\phi} \times \frac{d\phi}{dt} = -ds\in(\alpha - \phi) + \frac{1}{2}(\ell^2 - R^2 \sin^2(\alpha - \phi))^{-\frac{1}{2}} (R^2 \sin 2(\alpha - \phi)) \omega$$

$$v = \omega \left[-ds\in(\alpha - \phi) \cdot x(-1) + \frac{1}{2} (\ell^2 - R^2 \sin^2(\alpha - \phi))^{-\frac{1}{2}} (R^2 \sin 2(\alpha - \phi)) \right]$$

$$v = \omega R \left[\sin(\alpha - \phi) + \frac{R \sin 2(\alpha - \phi)}{2 \sqrt{\ell^2 - R^2 \sin^2(\alpha - \phi)}} \right]$$

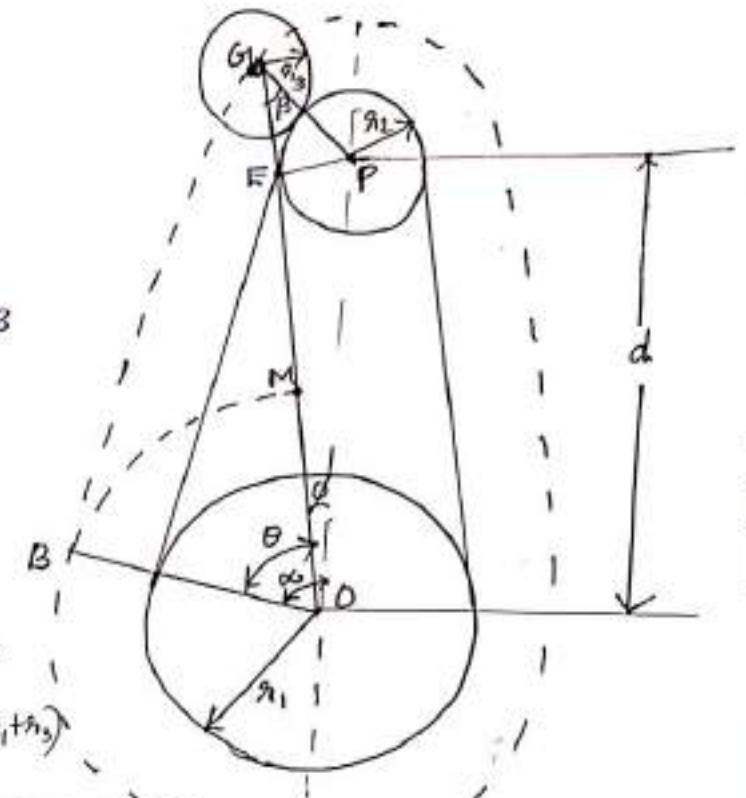
$$\text{Acceleration} \Rightarrow a = \frac{dv}{dt} = \frac{dv}{d\phi} \times \frac{d\phi}{dt}$$

$$a = \omega R \left[\cos(\alpha - \phi)(-1) + \frac{1}{2} \frac{d}{d\phi} \left[R \sin 2(\alpha - \phi) \times (\ell^2 - R^2 \sin^2(\alpha - \phi))^{-\frac{1}{2}} \right] \right] \omega$$

$$= \omega^2 R \left[-\cos(\alpha - \phi) + \frac{1}{2} \left[R \sin 2(\alpha - \phi) \times -\frac{1}{2} \left[\ell^2 - R^2 \sin^2(\alpha - \phi) \right]^{-\frac{1}{2}-1} \right. \right.$$

$$\left. \times (0 - R^2 \sin 2(\alpha - \phi))(-1) \right. \\ \left. + (\ell^2 - R^2 \sin^2(\alpha - \phi))^{-\frac{1}{2}} \times R \cos 2(\alpha - \phi) \times -1 \right]$$

$$a = \omega^2 R \left[-\cos(\alpha - \phi) + \frac{1}{2} \left[-\frac{1}{2} R^3 \sin^2 2(\alpha - \phi) \left[\ell^2 - R^2 \sin^2(\alpha - \phi) \right]^{-\frac{3}{2}} \right. \right. \\ \left. \left. + (-1) \left[\ell^2 - R^2 \sin^2(\alpha - \phi) \right]^{-\frac{1}{2}} \times R \cos 2(\alpha - \phi) \right] \right]$$



Very important problems

①

A symmetrical tangent cam with a least radius of 25mm operates a roller follower of radius 10mm. The angle of ascent is 60° and total lift is 15mm. If speed of cam is 400 rpm, then calculate

(a) principal dimensions of cam

(b) Acceleration of the follower at the beginning of lift, where the roller just touches the nose & apex of circular nose. Assume there is no dwell b/w Ascent & descent.

Solution

Given $r_1 = 25\text{mm}$, $r_3 = 10\text{mm}$, $\alpha = 60^\circ$

$$\text{Total lift} = 15\text{mm}, N = 400 \text{ rpm}, \omega = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = 41.88 \text{ rad/s}$$

(a) Principal dimension of the cam

$$x = d \cos(\alpha - \theta) + \sqrt{l^2 - r_2^2 \sin^2(\alpha - \theta)} - (r_1 + r_3)$$

For total lift (Max displacement) $\theta = \alpha$

$$15 = d + l - (r_1 + r_3)$$

$$15 = d + r_2 + r_3 - r_1 + r_3 \Rightarrow 15 = d + r_2 - 25$$

$$d + r_2 = 40 \quad \boxed{1}$$

From figure $OE = OP + PE \Rightarrow OE = d \cos \alpha + r_2$

$$r_1 = d \cos 60 + r_2$$

Subtract

$$25 = \frac{d}{2} + r_2 \Rightarrow 25 = 0.5d + r_3 \Rightarrow \boxed{25 = 0.5d + r_3} \quad \boxed{2}$$

$$\textcircled{1} - \textcircled{2} \Rightarrow 15 = 0.5d \Rightarrow \boxed{d = 30\text{mm}}$$

But: eq. 1 $\Rightarrow \boxed{r_2 = 10\text{mm}} \Rightarrow$ Nose circle radius

$$\tan \phi = \frac{AC}{AO} \Rightarrow \tan \phi = \frac{25.98}{35} = 0.7423$$

$$\phi = \tan^{-1}(0.7423) = \underline{\underline{36.6^\circ}}$$

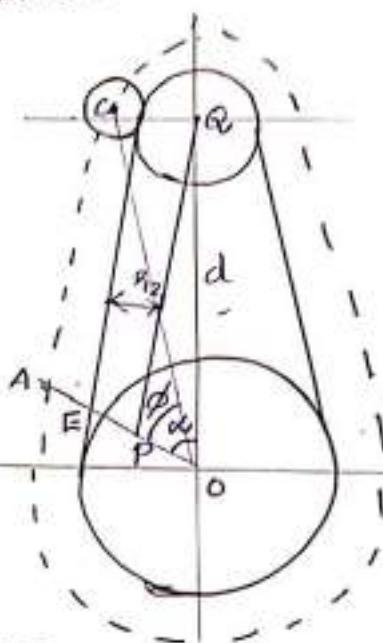
$$AC = PQ$$

$$\sin \alpha = \frac{PQ}{d}$$

$$PQ = 30 \times \sin 60^\circ$$

$$= 25.98\text{mm}$$

$$AO = r_1 + r_3 = 25 + 10 = 35\text{mm}$$



Given:

$$\cos \alpha = \frac{OP}{d}$$

$$OP = r_1 \cos \alpha$$

(ii) Acceleration of follower

(a) At the beginning of lift $\Rightarrow (\theta=0)$

$$a = \omega^2 (r_1 + r_2) \left[\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right] = 41.88^2 (25 + 10) \left(\frac{2-1}{1} \right) = \underline{\underline{61.3877 \text{ m/s}^2}}$$

(b) Acceleration of Follower when roller just touches the nose
when $\theta = \phi = 36.6^\circ$

$$a = \omega^2 (r_1 + r_2) \left[\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right] = 41.88^2 (25 + 10) \left[\frac{2 - \cos^2 36.6}{\cos^3 36.6} \right] = \underline{\underline{160.825 \text{ m/s}^2}}$$

when contact on circular nose

$$\begin{aligned} (l &= r_2 + r_3 \\ &= 20 \text{ mm}) \\ \theta &= \phi = 36.6^\circ \end{aligned}$$

$$\begin{aligned} f &= -\omega^2 d \left[\frac{\cos(\alpha-\theta) + l^2 d \cos 2(\alpha-\theta) + d^3 \sin^4(\alpha-\theta)}{(l^2 - d^2 \sin^2(\alpha-\theta))} \right] \\ &= -41.88^2 \times 30 \left[\frac{\cos(60-36.6) + 20^2 \times 30 \times \cos 2(60-36.6) + 30^3 \sin^4(60-36.6)}{(90^2 - 30^2 \sin^2(60-36.6))} \right] \\ &= \underline{\underline{-161.053 \text{ m/s}^2}} \end{aligned}$$

(c) Acceleration of follower at apex of nose

when $(\theta=\alpha)$

$$a = -\omega^2 d \left[1 + \frac{d}{l} \right] = -41.88^2 \times 30 \left[1 + \frac{30}{90} \right] = \underline{\underline{131.545 \text{ m/s}^2}}$$

Polynomial Cams

The general expression for a polynomial is given by:
 $s = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + \dots + c_n\theta^n$

where

s = displacement of the follower,

θ = cam rotation angle

c_i = constants ($i = 0, \dots, n$)

n = order of the polynomial. For a polynomial of order n we have $n+1$ unknown constant coefficients. These constant can be determined by considering the end conditions. For cam motion we at least want to have continuity in displacement velocity and acceleration which results with the boundary conditions:

for $\theta=0$

Displacement $s=0$

Velocity, $\dot{s}=0$

Acceleration, $\ddot{s}=0$

for $\theta=\beta$

$s=H$

$\dot{s}=0$

$\ddot{s}=0$

Since there are 6 boundary conditions, one can evaluate the value of 6 constants. Hence the polynomial must be of fifth order. The function and its derivatives are:

$$s = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + c_4\theta^4 + c_5\theta^5$$

$$\dot{s}/\omega = c_1 + 2c_2\theta + 3c_3\theta^2 + 4c_4\theta^3 + 5c_5\theta^4$$

$$\ddot{s}/\omega^2 = 2c_2 + 6c_3\theta + 12c_4\theta^2 + 20c_5\theta^3$$