Balancing

Balancing refers to the act of reducing (or even eliminating) the unbalanced forces and couples in a mechanical system. Unbalanced forces prevailing in the system can cause vibrations, noise, ear damage, structural damage, lack of precision and accuracy in the machining process etc.

Static Balancing

In static balancing only the unbalanced forces are required to be balanced as the masses rotate in the same plane.

Example 3.1 *Three masses are attached to a shaft with following properties :*

$m_1 = 3kg$	$r_1 = 30mm$	$\theta_1 = 30°$	plane A
$m_2 = 4kg$	$r_2 = 20mm$	$\theta_2 = 120°$	plane A
$m_3 = 2kg$	$r_3 = 25mm$	$\theta_3 = 270°$	plane A

Find the amount of counter mass at a radial distance of 35 mm for the static balance.

Solution : Let balancing mass, m_b is attached at angular position θ_b

For static balance, conditions are

$$m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_b r_b \sin\theta_b = 0$$

or $3 \times 30 \sin 30° + 4 \times 20 \sin 120° + 2 \times 25 \sin 270° + m_b \times 35 \times \sin\theta_b = 0$

or $\quad 45 + 69.28 - 50 + 35\, m_b \sin\theta_b = 0$

or $\qquad m_b \sin\theta_b = -1.837 \qquad \ldots(i)$

and $\quad m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_b r_b \cos\theta_b = 0$

or $3 \times 30° \cos 30° + 4 \times 20 \cos\theta\, 120° + 2 \times 25 \cos\theta\, 270°$

$\quad + m_b \times 35 \times \cos\theta_b = 0$ or $77.94 - 40 + 0 + 35\, m_b \cos\theta_b = 0$

or $\qquad m_b \cos\theta_b = -1.084 \qquad \ldots(ii)$

Squaring and adding (i) and (ii)

$$mb^2 (\sin^2\theta_b + \cos^2\theta_b) = (-1.837)^2 + (-1.084)^2$$

$$mb^2 = 4.55 \quad \text{or} \quad m_b = 2.13 \text{ kg}$$

Dividing (i) by (ii)

$$\frac{m_b \sin\theta_b}{m_b \cos\theta_b} = \frac{-1.837}{-1.084}$$

or $\qquad \tan\theta_b = 1.695$

or $\qquad \theta_b = 59.4°$

But as values of $\sin\theta_b$ and $\cos\theta_b$ are negative, θ_b lies in third quadrant. Therefore,

$$\theta_b = 180 + 59.4 = 239.4°$$

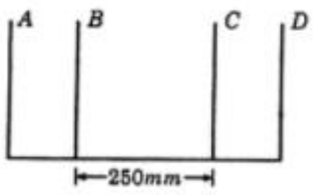
Dynamic Balancing

In dynamic balancing both the unbalanced forces and couples are required to be balanced as the masses rotate in different planes.

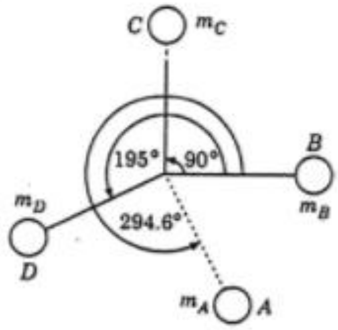
Example 3.3 *A shaft carries four rotating masses A, B C and D which are completely balanced. The masses B, C and D are 50 kg, 80 kg and 70 kg respectively. The masses C and D make angles of 90° and 195° respectively with mass B in the same sense. The masses A, B, C and D are concentrated at radius 75 mm, 100 mm, 50 mm and 90 mm respectively. The plane of rotation of masses B and C are 250 mm apart. Determine :*

(*i*) *the mass A and its angular position*

(*ii*) *the positiion of planes of A and D.*



(b) Position of Planes



(b) Angular Position of Masses

Solution :

Masses, $m_B = 50$ kg ; $m_C = 80$ kg ; $m_D = 70$ kg.

Radius of masses,

$r_A = 75$ mm ; $r_B = 100$ mm ; r_C
$= 50$ mm ; $r_D = 90$ mm.

Angles between masses,

$\angle BOC = 90°$ and $\angle BOD = 195°$ or θ_C
$= 90°$ and $\theta_D = 195°$

Distance between planes B and $C = 250$ mm.

Find : (*i*) m_A and θ_A.

(*ii*) Positions of planes A and D.

Analytical Method :

Fig. 3.6 (*a*) shows the position of planes A, B, C and D whereas Fig. 3.6 (*b*) shows the angular position of masses, B, C and D in which the angular position of mass m_B is assumed in horizontal direction.

Hence, $\theta_B = 0°$

(*i*) The mass A and its angular position.

Let m_A = Mass A

θ_A = Angular position of mass A with respect to mass B.

The four masses are completely balanced, hence, resultant force should be zero.

or $\Sigma\, m \times r \times \cos\theta = 0$ and $\Sigma\, m \times r \times \sin\theta = 0$

Let us first find the product of known mass and corresponding radius.

$\therefore \qquad m_B \times r_B = 50 \times 100 = 5000$

Let us first find the product of known mass and corresponding radius.

$$\therefore \quad m_B \times r_B = 50 \times 100 = 5000$$

$$m_C \times r_C = 80 \times 50 = 4000$$

$$m_D \times r_D = 70 \times 90 = 6300$$

Also we know that $\theta_B = 0$, $\theta_C = 90°$ and $\theta_D = 195°$

Now for $\Sigma\, m \times r \times \cos\theta = 0$ we have

$$m_A \times r_A \times \cos\theta_A + m_B \times r_B \times \cos\theta_B$$
$$+ m_C \times r_C \times \cos\theta_C + m_D \times r_D \times \cos\theta_D = 0$$

or $\quad m_A \times 75 \times \cos\theta_A + 5000 \times \cos 0° + 4000 \times \cos 90° + 6300 \times \cos 195° = 0$

$$75\, m_A \times \cos\theta_A + 5000 + 0 + (-6085.3) = 0$$
$$75\, m_A \times \cos\theta_A = 6085.3 - 5000 = 1085.3 \qquad ...(i)$$

For $\Sigma\, m \times r \times \sin\theta = 0$, we have

$$m_A \times r_A \times \sin\theta_A + m_B \times r_B \times \sin\theta_B + m_C \times r_C \times \sin\theta_C + m_D \times r_D \times \sin\theta_D = 0$$

or $\quad m_A \times 75 \times \sin\theta_A + 5000 \times \sin 0° + 4000 \times \sin 90° + 6300 \times \sin 195° = 0$

or $\quad 75\, m_A \times \sin\theta_A + 0 + 4000 + (-1630.5) = 0$

or $\quad 75\, m_A \times \sin\theta_A = -4000 + 1630.5 = -2369.5 \qquad ...(ii)$

Squaring and adding (i) and (ii), we get

$$75^2 \times m_A^2 \times (\cos^2\theta_A + \sin^2\theta_A) = (1085.3)^2 + (-2369.5)^2$$

or $\quad 5625 \times m_A^2 = 1177876 + 5614530 = 6792406$

$$m_A = \sqrt{\frac{6792406}{5625}} = 34.75 \text{ kg } \textbf{Ans.}$$

Dividing (ii) by (i), we get

$$\frac{75 \times m_A \times \sin\theta_A}{75 \times m_A \times \cos\theta_A} = \frac{-2369.5}{1085.3}$$

or $\quad \dfrac{\sin\theta_A}{\cos\theta_A} = \tan\theta_A = \dfrac{-2369.5}{1085.3}$

In the above equation the numerator (*i.e.*, $\sin\theta_A$) is –ve whereas the denominator (*i.e.*, cos) is +ve. Hence, θ_A lies in fourth quadrant as sine of an angle is –ve and consine of the angle +ve in fourth quadrant.

$$\therefore \quad \theta_A = \tan^{-1}\frac{-2369.5}{1085.3} = \tan^{-1}(-2.183)$$

$$= -65.4° = 360 - 65.4 = 294.6°. \textbf{ Ans.}$$

(ii) Position of planes A and D.

Take plane A as the reference plane.

Then $l_A = 0$

l_B = distance of plane B from plane A

$l_C =$ distance of plane C from plane A $= l_B + 250$

l_D = distance of plane D from plane A

As the four masses are completely balanced, hence the resultant couple about the reference plane should be zero.

or $\quad \Sigma\, m \times r \times l \cos\theta = 0$ and $\Sigma\, m \times r \times l \times \sin\theta = 0$

For $\quad \Sigma\, m \times r \times l \times \cos\theta = 0$ about plane, A, we have

$$m_B \times r_B \times l_B \times \cos\theta_B + m_C \times r_C \times l_C \times \cos\theta_C + m_D \times r_D \times l_D \times \cos\theta_D = 0$$

or $\quad 5000 \times l_B \times \cos 0° + 4000 \times (l_B + 250)\cos 90° + 6300 \times l_D \times \cos 195° = 0$

$(\therefore\ m_B \times r_B = 5000\,;\ m_C \times r_C = 4000\,;\ m_D \times r_D = 6300\,;\ l_C = l_B + 250)$

$5000\, l_B + 0 + 6300 \times l_D \times (-0.966) = 0$

$5000\, l_B - 6085.3\, l_D = 0$

or $\quad l_B = \dfrac{6085.3\, l_D}{5000} = 1.217\, l_D \qquad \ldots(iii)$

$\Sigma\, m \times r \times l \times \sin\theta = \theta$ about plane A, we have

$$m_B \times r_B \times l_B \times \sin\theta_B + m_C \times r_C \times l_C \sin\theta_C + m_D \times r_D \times l_D \times \sin\theta_D = 0$$

or $\quad 5000 \times l_B \times \sin 0° + 4000 \times (l_b + 250)\sin 90° + 6300 \times l_D \times \sin 195° = 0$

$(\therefore\ l_C = l_B + 250)$

or $\ 0 + 4000\,(l_B + 250) + 6300 \times l_D \times (-0.2588) = 0$

or $\ 4000\, l_B + 1000000 - 1630.5\, l_D = 0$

or $\ 4000\,(1.217\, l_D) + 1000000 - 1630.5\, l_D = 0$

$[\because l_B = 1.217\, l_D$ from equation $(iii)]$

or $\ 4868\, l_D + 1000000 - 1630.5\, l_D = 0$

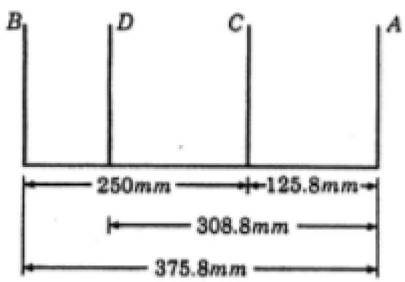
or $\quad 3237.5\, l_D = -1000000$

$\therefore \quad l_D = \dfrac{-1000000}{3237.5} = -308.8$ mm

From equation (iii),

But $\quad l_C = l_B + 250 = -375.8 + 250 = -125.8$ mm

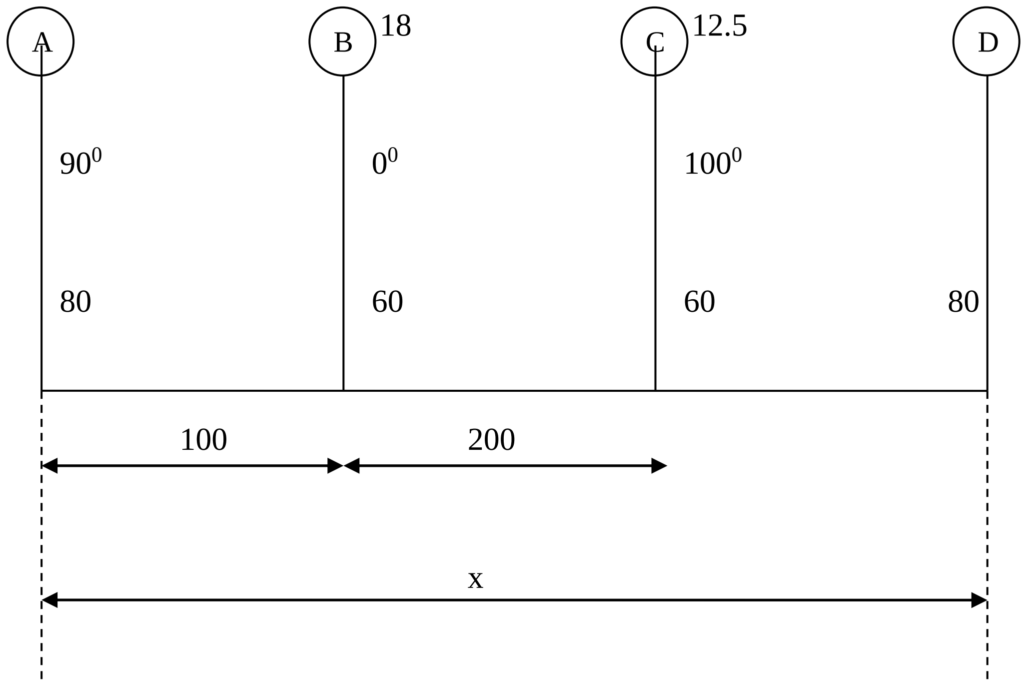
(C) The correct position of planes, A, B, C and D are shown in Fig. 3.7 whereas the angular positiion of mass m_A is shown in Fig. 3.7 (b).



(a) Correct Position of Planes

2. A Shaft carries four masses in parallel planes A, B, C, D. The masses at B & C are 18 Kg & 12.5 Kg respectively and each has an eccentricity of 60mm. The masses at A & D have an eccentricity of 80mm. The angle b/w B & C is 100^0 & that between the masses B & C is 90^0 measured in same direction. The axial distance between the planes A & B is 100mm & that b/w B & C is 200mm. The shaft is in complete balance determine:

1. Masses at planes A & D
2. Distance b/w planer D & A.
3. Ang. Positions of mass D



RP

<u>Balanced Couple Equation.</u>

$M_BR_BL_BSinØ_B + M_CR_CL_CSinØ_C + M_DR_DL_DSinØ_D = 0$

$(18 \times 60 \times 100 \times \text{Sin D}) + (12.5 \times 60 \times 300 \times \text{Sin } 100) + M_DR_DL_DSinØ_D = 0$

$SinØ_D \text{ X } M_DR_D \text{ X } x = -22158174$ – 1

$M_BR_BL_BCosØ_B + M_CR_CL_CCosØ_C + M_DR_DL_DCosØ_D = 0$

$(18 \times 60 \times 100 \times \text{Cos D}) + (12.5 \times 60 \times 300 \times \text{Cos } 100) + M_DR_DL_DCosØ_D = 0$

$CosØ_D \text{ X } M_DR_D \text{ X } x = -68929.16$ - 2

$(1)^2 + (2)^2 = (M_D R_D x)^2 = 5.384 \times 10^{10}$

ie; $M_D R_D \times x = 232055.374$

$M_D \times x = 2900.69$ - (3)

(1) / (2) $\tan Ø_D = -221581.74/-68929.16$; $Ø_D = \underline{\underline{72.72^0}}$ in 3^{rd} quad

ie; $Ø_D = 180 + 72.72 = 252.72$ from$+^{ve}$ X taken CCW

Balanced force eqn;

$M_A R_A \sin Ø_A + M_B R_B \sin Ø_B + M_C R_C \sin Ø_C + M_D R_D \sin Ø_D = 0$

$(M_A \times 80 \times \sin 90) + (18 \times 60 \times \sin D) + (12.5 \times 60 \times \sin 100) + M_D R_D \sin Ø_D = 0$

$80 M_A + M_D R_D \sin Ø_D = 738.6$ - (4)

$M_A R_A \cos Ø_A + M_B R_B \cos Ø_B + M_C R_C \cos Ø_C + M_D R_D \cos Ø_D = 0$

$(M_A R_A \cos 90) + (18 \times 60 \times \cos 0) + (12.5 \times 60 \times \cos 100) + M_D x\ 80 \cos Ø_D = 0$

ie; $M_D \times 80 \times \cos (252.72) = 949.76$

ie; $M_D = -949.76/(\cos(252.72) \times 80) = \underline{\underline{39.96Kg}}$.

Sub M_D in (4);

$80MA + 39.96 \times 80 \times \sin (252.72) = - 738.6$

$80M_A = 2313.91$

$M_A = 2313.91/80 = \underline{\underline{28.92Kg}}$

Sub M_D in (3) ;

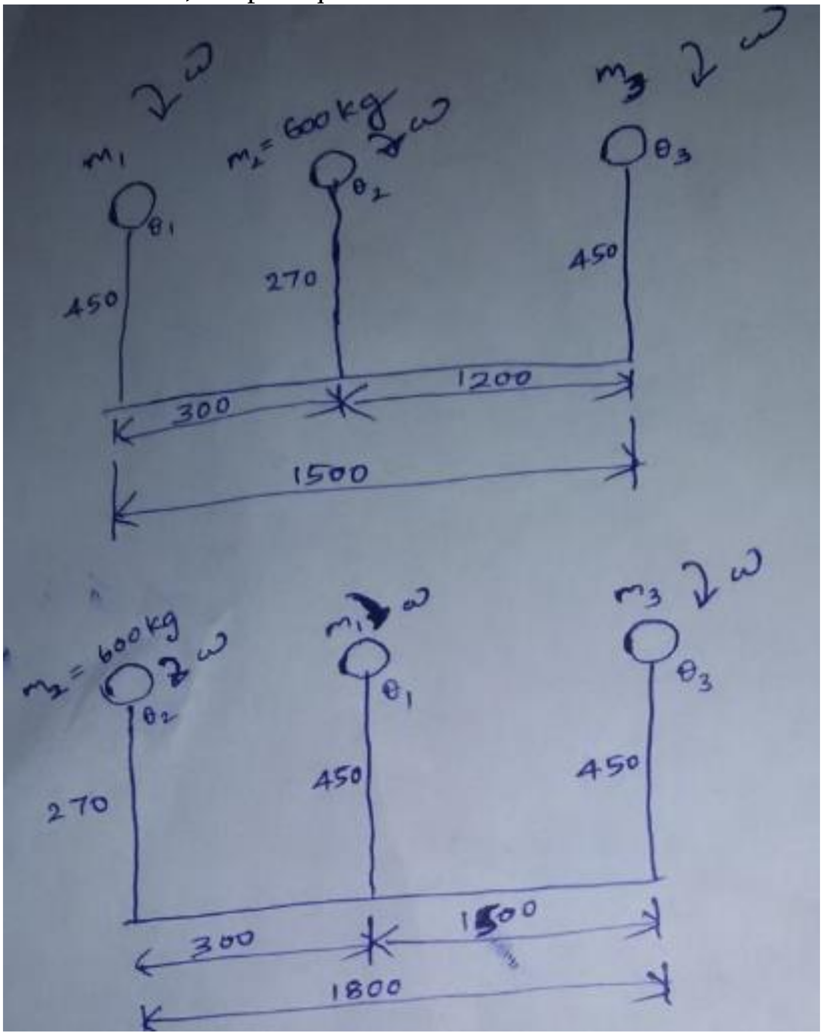
$39.96 \times x = 2900.69$

ie; $x = \underline{\underline{72.58mm}}$

3. A disturbing mass 600 kg is attached to a shaft. The shaft is rotating at a uniform angular velocity of ωrad/sec. and the distance of the C.G. of the disturbing mass from the axis of rotation is 270 mm. The disturbing mass is to be balanced by two masses in two different planes. The distance of the C.G. of the balancing mass from the axis of rotation is 450 mm each. The distance between the two planes of the balancing masses is 1.5 m and the distance between the plane of the disturbing mass and one of the planes of the balancing masses is 300 mm. Determine (a) the distance between the plane of the disturbing mass and the plane of the other balancing mass. (b) the magnitude of the balancing masses when (i) the planes of the balancing masses are on the same side of the plane of the disturbing

mass. (ii) the planes of the balancing masses are on either side of the plane of the disturbing mass.

Case b (i) (Refer phase plane diagram)
Taking the plane of m_3 as RP, couple equations



m_1 x (0.450) x (-1.5) x ω^2 x sin θ_1 + 600 x (0.270) x (-1.2) x ω^2 x sin θ_2 = 0
m_1 x (0.450) x (-1.5) x ω^2 x cos θ_1+ 600 x (0.270) x (-1.2) x ω^2 x cos θ_2 = 0

$-0.675\ m_1\omega^2\ \sin\theta_1 - 194.4\ \omega^2 \sin\theta_2 = 0$
$-0.675\ m_1\omega^2 \cos\theta_1 - 194.4\ \omega^2 \cos\theta_2 = 0$

$-0.675\ m_1 \sin\theta_1 = 194.4 \sin\theta_2$
$-0.675\ m_1 \cos\theta_1 = 194.4 \cos\theta_2$

$(-0.675)^2 (m_1)^2 = (194.4)^2$
$m_1 = 288$ kg

Taking the plane of m_1 as RP, couple equations

600 x (0.270) x (0.3) x ω^2 x sin θ_2 + m_3 x (0.450) x (1.5) x ω^2 x sin θ_3 = 0
600 x (0.270) x (0.3) x ω^2 x cos θ_2 + m_3 x (0.450) x (1.5) x ω^2 x cos θ_3 = 0

0.675 $m_3 \omega^2$ sin θ_3 + 48.6 ω^2sin θ_2 =0
0.675 $m_3 \omega^2$cos θ_3 + 48.6 ω^2cos θ_2 =0

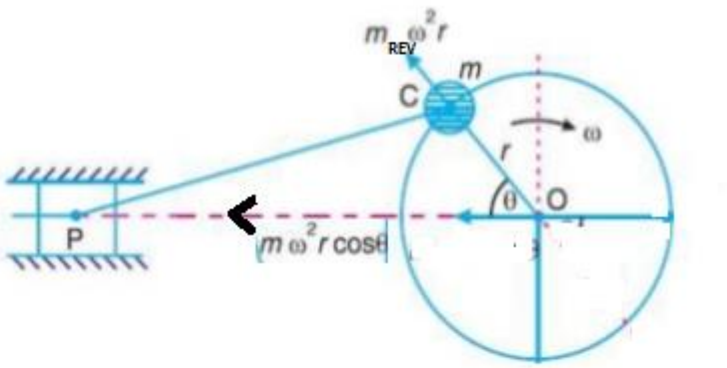
0.675 m_3 sin θ_3 = - 48.6 sin θ_2
0.675 $m_3$cos θ_3 = -48.6 cos θ_2
m_3= 72 kg

Case b (ii) (Refer phase plane diagram)
Taking the plane of m_3 as RP, couple equations

600 x (0.270) x (-1.8) x ω^2 x sin θ_2 + m_1 x (0.450) x (-1.5) x ω^2 x sin θ_1 = 0
600 x (0.270) x (-1.8) x ω^2 x cos θ_2 + m_1 x (0.450) x (-1.5) x ω^2 x cosθ_1 = 0
m_1 = 432 kg
Taking the plane of m_1 as RP, couple equations

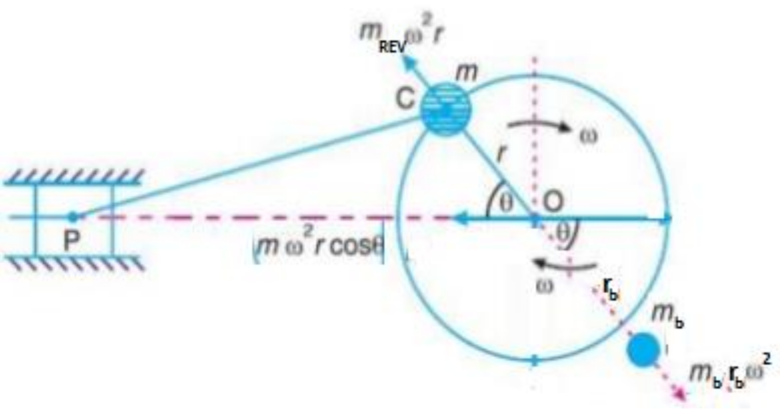
600 x (0.270) x (-0.3) x ω^2 x sin θ_2 + m_3 x (0.450) x (1.5) x ω^2 x sin θ_3 = 0
600 x (0.270) x (-0.3) x ω^2 x cos θ_2 + m_3 x (0.450) x (1.5) x ω^2 x cos θ_3 = 0
m_3= 72 kg

Balancing of Reciprocating mass

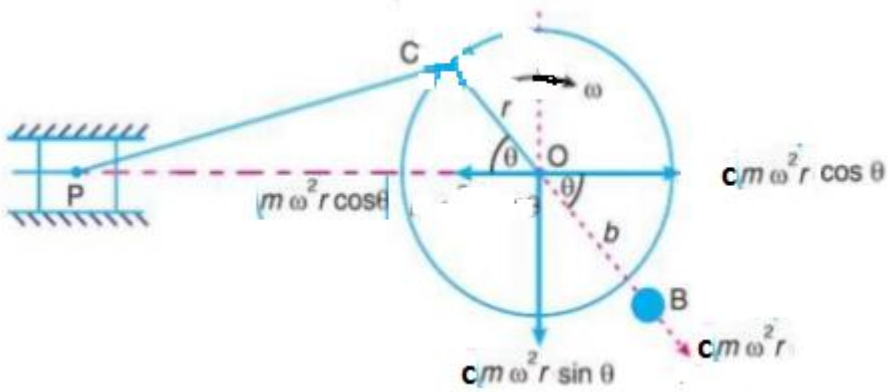


Forces to be balanced are mrω^2cosθ which is the inertial force (due to reciprocating mass m) acting along the L.O.S and centrifugal force due to equivalent revolving mass m_{REV} at crank pin $m_{REV}r\omega^2$. Note that we have excluded the secondary force $\frac{\mathrm{mr\omega 2}}{n} cos2\theta$ from balancing. Therefore the balancing is referred to as primary balancing. N is the length of the connecting rod (l) to crank radius (r) ratio; n=$\frac{l}{r}$

These unbalanced forces can be balanced by placing a radially opposing counterbalancing mass m_b at any suitable radius r_b with speed of rotation ω as shown below:



The value of m_b r_b is selected in such a way that m_b r_b= (cm + m_{REV})r means $m_{REV}r\omega^2$ gets cancelled in the radially opposite directions. The remaining part $cmr\omega^2$ (radially) offers horizontal and vertical balancing components as shown below:



So after balancing has been done the net unbalanced force along X axis (LOS) will be

$mr\omega^2\cos\theta$ - c $mr\omega^2\cos\theta$ = (1-c) $mr\omega^2\cos\theta$ and along vertical direction will be $cmr\omega^2\sin\theta$ and the resultant unbalanced force after partial balancing will be $\sqrt{[(1-c)\ mr\omega^2\cos\theta\ ]^2 + [cmr\omega^2 sin\theta]^2}$

Note that we have effected only a partial balancing of reciprocating mass (m) by a fraction c (c less than 1) and therefore the balancing is referred to as partial primary balancing (secondary force being neglected) and this acts as a compromise between unbalanced forces in the vertical and horizontal directions.

Problem 1 *. A single cylinder reciprocating engine has the following data :*

Speed of engine = 120 r.p.m. ; stroke = 320 mm ; mass of reciprocating parts = 45 kg and mass of revolving parts = 35 kg at crank radius. If 60% of the reciprocating parts and all the revolving parts are to be balanced, then find :

(i) the balance mass required at a radius of 300 mm, and

(ii) the unbalanced force when the crank has rotated 60° from top dead centre.

Sol. Given :

$N = 120$ r.p.m. or $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 120}{60} = 4\pi$ rad/s ;

stroke = 320 mm = 0.32 m or crank radius $r = \dfrac{0.32}{2} = 0.16$ m ; mass of reciprocating parts,

$m = 45$ kg ; mass of revolving parts, $m_{REV} = 35$ kg ; radius of revolving parts, $r = 160$ mm $= 0.16$ m ;

fraction of reciprocating parts to be balanced $= 60\% = \dfrac{60}{100} = 0.6$

or $c = 0.6$; all revolving parts are to be balanced.

(i) Balance mass required at a radius of 300 mm

Let m_b = Balance mass required, and

r_b = Radius of rotation of the balance mass = 300 mm = 0.3 m

Using equation (14.6), we have

$$m_b \times r_b = (m_{REV} + c\,m)\times r$$

or
$$m_b \times 0.3 = (35 + 0.6 \times 45) \times 0.16 = (35 + 27) \times 0.16$$

or
$$m_b = \frac{(35 + 27) \times 0.16}{0.3} = \mathbf{33.06\ kg.}\quad \textbf{Ans.}$$

(ii) Unbalanced force when crank has rotated 60° from top-dead centre

Here $\theta = 60°$ (given)

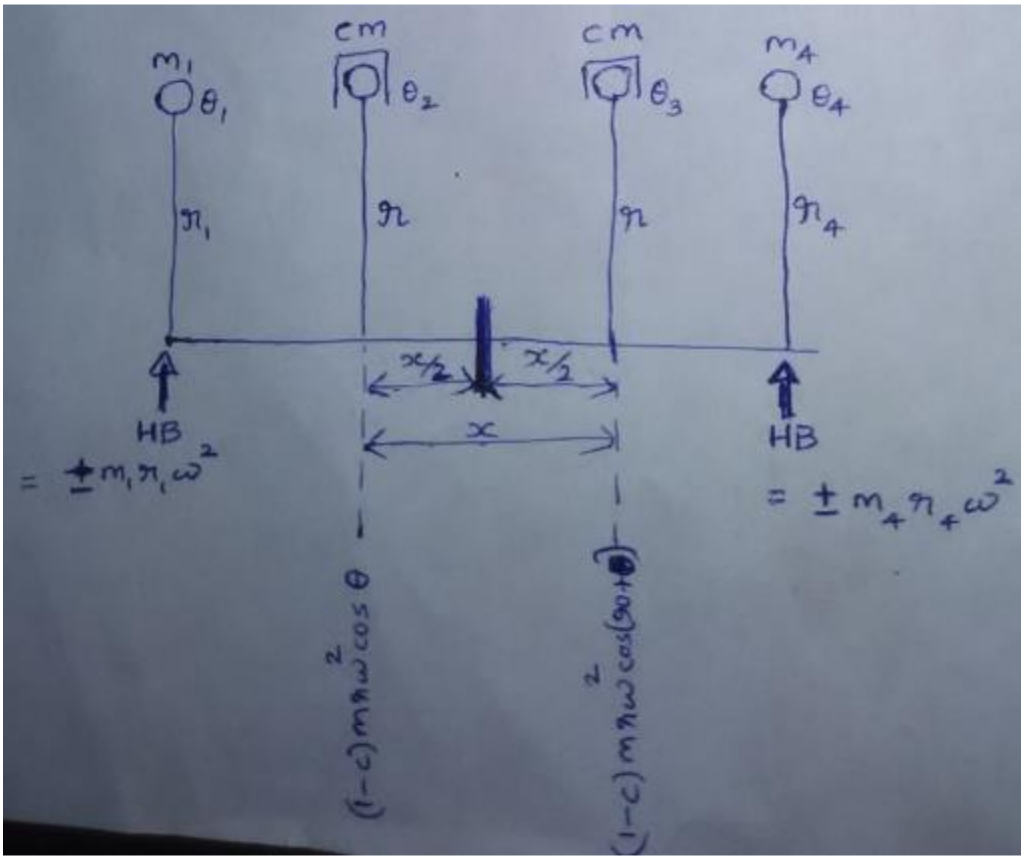
The unbalanced force at any instant is given by equation

$$= m \times \omega^2 \times r\sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

$$= 4.5 \times (4\pi)^2 \times 0.16\sqrt{(1-0.6)^2 \times \cos^2 60° + 0.6 \sin^2 60°}$$

$$= 1136.98\sqrt{.16 \times 0.25 + 0.6 \times 0.75} = 1136.98\sqrt{0.04 + 0.45}$$

$$= 1136.98 \times 0.7 = \mathbf{795.9.}\quad \textbf{Ans.}$$



Effects of Partial balancing in Locomotives

The effects of partial balancing in locomotives are Variation in traction, Swaying Couple and Hammer Blow. (Refer the figure given above)

Variation in Traction

The unbalanced forces along the lines of stroke of the two cylinders causes variation in traction given by

$V = (1-c)mr\omega^2\cos\theta^0 + (1-c)mr\omega^2\cos(90 + \theta)^0$

Max. Value occurs at $\theta=45^0$ and $\theta=225^0$given by

$V_{max} = \pm\sqrt{2}\ (1 - c)\ mr\omega^2$

Swaying Couple

The unbalanced forces along the lines of stroke of the two cylinders constitute a couple about the locomotive central plane called swaying couple which causes the vehicle sway from side to side given by

$S=-\frac{x}{2}(1-c)\ mr\omega^2\cos\theta^0 + \frac{x}{2}(1-c)\ mr\omega^2\cos\ (90 + \theta)^0$

$S_{max}=\pm\frac{1}{\sqrt{2}}(1 - c)\ mr\omega^2 x$

Hammer Blow (HB)

Hammer blow is the maximum vertical unbalanced forces on the plane of wheels caused by the balancing mass placed in the same planes. The tendency is to lift the vehicle of the wheels.

$HB = \pm m_1r_1\omega^2$ or $HB = \pm m_4r_4\omega^2$

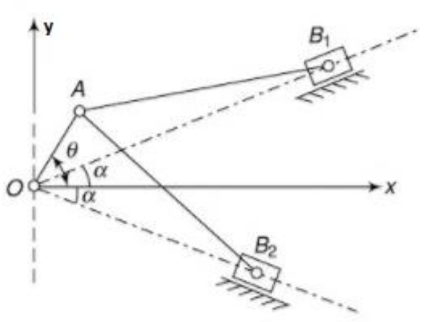
Limiting speed condition of HB is given by HB = W where W is the dead weight on each wheel.

Balancing of V-Engines

Unbalanced forces in V-Engines

In V-engines, a common crank OA is operated by two connecting rods OB_1 and OB_2. Figure shows a symmetrical two cylinder V-cylinder, the centre lines of which are inclined at an angle α to the x-axis.

Let θ be the angle moved by the crank from the x-axis.



Primary force

Primary force of 1 along line of stroke $OB_1 = mr\omega^2 \cos(\theta - \alpha)$

Primary force of 1 along x-axis $= mr\omega^2 \cos(\theta - \alpha) \cos \alpha$

Primary force of 2 along line of stroke $OB_2 = mr\omega^2 \cos(\theta + \alpha)$

Primary force of 2 along the x-axis $= mr\omega^2 \cos(\theta + \alpha) \cos \alpha$

Total primary force along x-axis

$= mr\omega^2 \cos \alpha [\cos(\theta - \alpha) + \cos(\theta + \alpha)]$

$= mr\omega^2 \cos \alpha [(\cos\theta \cos\alpha + \sin\theta \sin\alpha) + (\cos\theta\cos\alpha - \sin\theta\sin\alpha)]$

$= mr\omega^2 \cos\alpha \, 2\cos\theta \cos\alpha$

$= 2mr\omega^2 \cos^2\alpha \cos\theta$

Similarly, total primary force along the y-axis

$mr\omega^2 [\cos(\theta - \alpha)\sin\alpha - \cos(\theta + \alpha)\sin\alpha]$

$= 2mr\omega^2 \sin^2\alpha \sin\theta$

Therefore the resultant primary unbalanced force in V engines is given by :

$$2mr\omega^2\sqrt{\left[(\cos^2\alpha\cos\theta)^2 + (\sin^2\alpha\sin\theta)^2\right]}$$

While evaluating the secondary unbalanced forces the angle made by the crank with the line of stroke gets doubled and the ratio n features in the denominator

Total secondary force along x-axis

$$= \frac{mr\omega^2}{n}\cos\alpha[\cos 2(\theta - \alpha) + \cos 2(\theta + \alpha)]$$

$$= \frac{mr\omega^2}{n}\cos\alpha[(\cos 2\theta\cos 2\alpha + \sin 2\theta \sin 2\alpha) + (\cos 2\theta\cos 2\alpha - \sin 2\theta\sin 2\alpha)]$$

$$= \frac{2mr\omega^2}{n}\cos\alpha\cos 2\theta\cos 2\alpha$$

Similarly, secondary force along z-axis $= \dfrac{2mr\omega^2}{n}\sin\alpha\sin 2\theta\sin 2\alpha$

Resultant secondary force

$$= \frac{2mr\omega^2}{n}\sqrt{(\cos\alpha\cos 2\theta\cos 2\alpha)^2 + (\sin\alpha\sin 2\theta\sin 2\alpha)^2}$$

Balancing Procedure in V-engines

Select the angle of V, $2\alpha = 90^0$. That is $\alpha = 45^0$, the resultant primary unbalanced force becomes equal to $mr\omega^2$. This unbalanced force can be treated as a rotating mass m at crank radius r (rotating with angular speed ω) and therefore can be balanced by placing a radially opposite rotating mass of same speed. Secondary forces are harmonic and smaller in value, therefore usually neglected.

Balancing of Inline engines

In inline engines the cylinders are arranged in such a way that the lines of stroke of all cylinder assembly are made parallel to each other. To balance the unbalanced force in an inline engine, we normally need not add any counter balancing mass externally. But the balancing is done by selecting a suitable configuration (lay out) and firing order. This selection usually depends upon the number of cylinders. This type of balancing is called inherent balancing.See the following example: most of the forces and couples are reduced to zero by selecting proper angular positions , masses and axial distances and not by adding any external counter balancing mass.

EXAMPLE 13.10 In a four cylinder in-line IC engine, the mass of reciprocating parts of cylinder number 1 and 4 are 100 kg and that of cylinder number 2 and 3 are 173 kg. If the crank radius is 150 mm, length of connecting rod is 450 mm and engine speed is 1200 rpm, determine the primary and secondary forces and couples. The cylinders are placed 600 mm apart as shown in Figure 13.26.

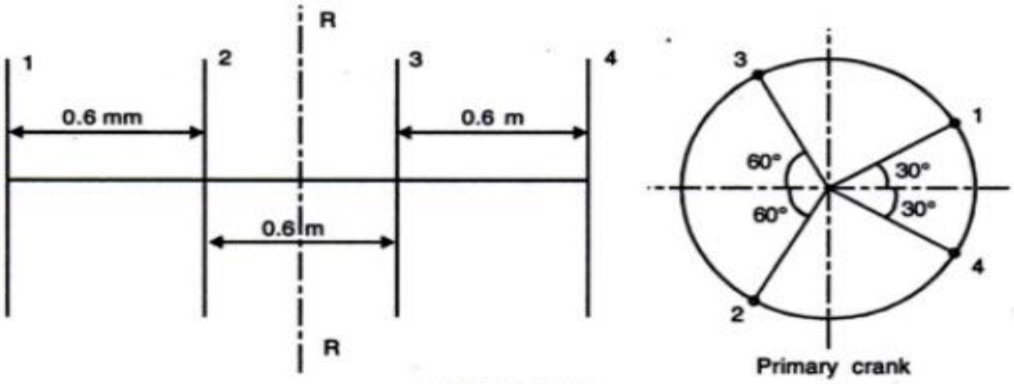


FIGURE 13.26

Solution: Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1200}{60} = 125.66 \text{ rad/s}$$

Ratio:

$$n = \frac{l}{r} = \frac{450}{150} = 3$$

Assuming central plane *R–R* as reference plane, prepare the table as given below:

Plane	*Mass* *m* (kg)	*Radius* *r* (m)	*Primary force* *mr* kg mm	*Secondary force* *mr/n* kg mm	*Distance from* reference plane *l* (m)	*Primary couple* *mrl* kg mm^2	*Secondary couple* *mrl/n* kg mm^2
1	100	0.15	15	5	−0.9	−13.5	−4.5
2	173	0.15	25.95	8.65	−0.3	−7.785	−2.595
3	173	0.15	25.95	8.65	+0.3	7.785	+2.595
4	100	0.15	15	5	+0.9	13.5	+4.5

(i) **Primary forces:**

(a) Horizontal component:

$$F_{PH} = (15 \cos 30° + 25.95 \cos 120° + 25.95 \cos 240° + 15 \cos 330°)\,\omega^2$$
$$= 0$$

(b) Vertical component:

$$F_{PV} = (15 \sin 30° + 25.95 \sin 120° + 25.95 \sin 240° + 15 \sin 330°)\,\omega^2$$
$$= 0$$

Thus, the primary forces are completely balanced.

(ii) **Primary couples:**

(a) Horizontal component:

$$C_{PH} = (-13.5 \cos 30° + 7.785 \cos 120° - 7.785 \cos 240° + 13.5 \cos 330°)\,\omega^2$$
$$= 0$$

(b) Vertical component:

$$C_{PV} = (-13.5 \sin 30° + 7.785 \sin 120° - 7.785 \sin 240° + 13.5 \sin 330°)\,\omega^2$$
$$= 0$$

(iii) **Secondary forces:**

The crank positions for secondary forces are shown in Figure 13.27.

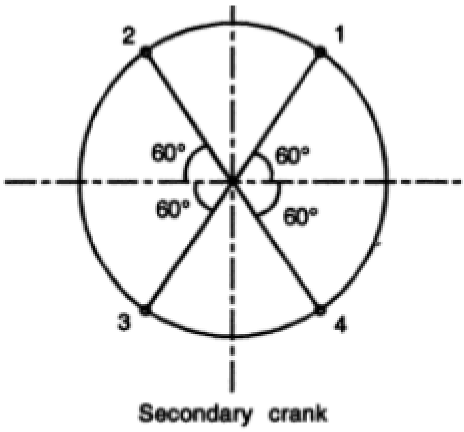


FIGURE 13.27

(a) Horizontal component:

$$F_{SH} = [5 \cos 60° + 8.65 \cos 240° + 8.65 \cos 480° + 5 \cos 660°]\,\omega^2$$
$$= -3.65\,\omega^2$$

(b) Vertical component:

$$F_{SV} = [5 \sin 60° + 8.65 \sin 240° + 8.65 \sin 480° + 5 \sin 660°]\,\omega^2$$
$$= 0$$

So the resultant secondary unbalanced force:

$$F_S = 3.65\ \omega^2 = 3.65 \times 125.66^2 = \mathbf{57635\ N} \qquad \textbf{Ans.}$$

(iv) **Secondary couple:**

(a) Horizontal component:

$$C_{SH} = [-4.5 \cos 60° - 2.595 \cos 480° + 2.595 \cos 240° + 4.5 \cos 660°]\ \omega^2$$
$$= 0$$

(b) Vertical component:

$$C_{SV} = [-4.5 \sin 60° - 2.595 \sin 480° + 2.595 \sin 240° + 4.5 \sin 660°]\ \omega^2$$
$$= -12.2889\ \omega^2$$
$$= -12.2889 \times 125.66^2$$
$$= \mathbf{194047.0\ Nm} \qquad \textbf{Ans.}$$

Balancing Machines

A balancing machine is used to indicate whether a component is in balance or not. If it is out of balance, then machine must be able to measure the magnitude and location of unbalance. Mechanical components whose axial dimensions are small such as gears, pulley, fans and impeller require static balancing. Such balancing is often done through a single plane balancing machine. One typical static balancing machine is shown in Figure.

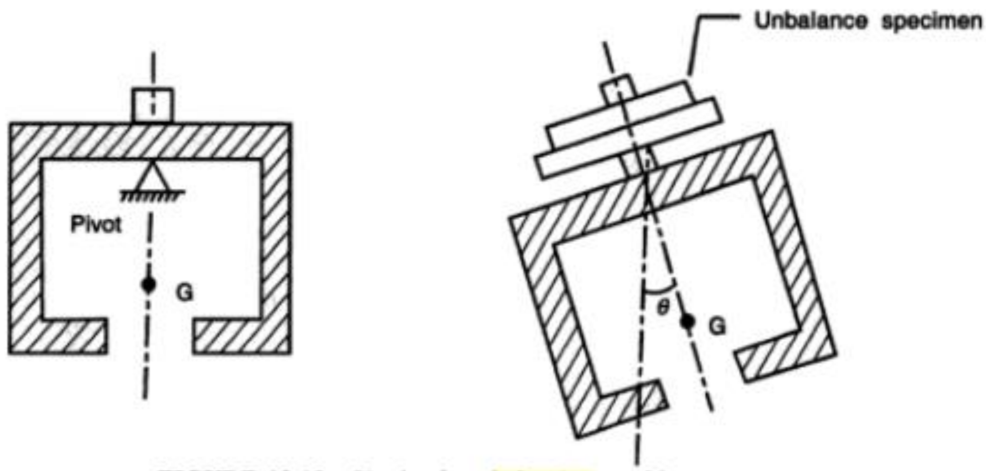
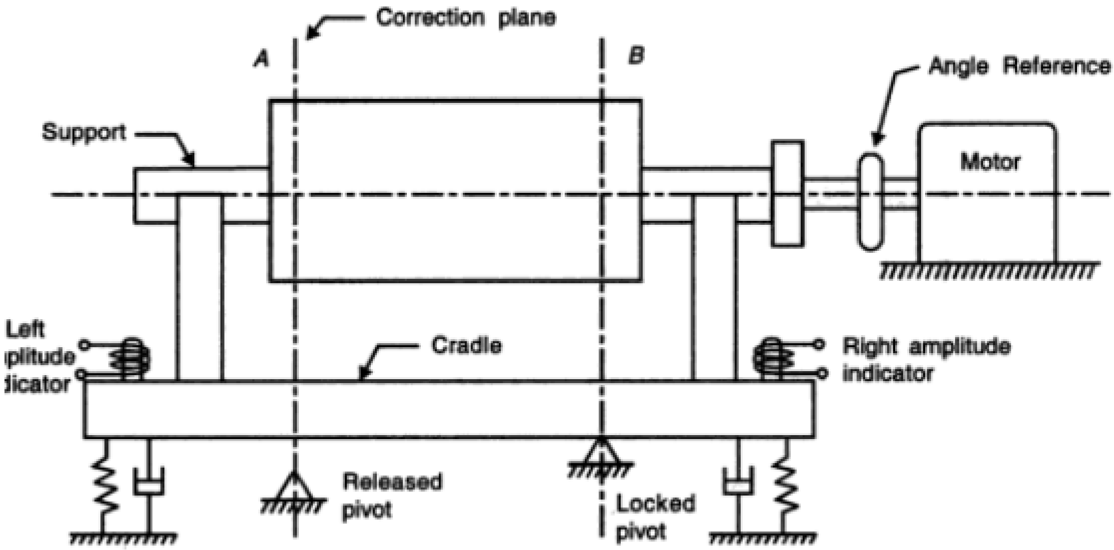


FIGURE 13.12 Single plane balancing machine.

When an unbalanced component is mounted on the platform the pendulum tilts. The direction of tilt gives the location of unbalance and the angle θ indicates the magnitude of unbalance. In the case of axially longer components such as turbine, rotor, armature, etc., the unbalanced centrifugal forces result in couples whose effect is to cause the rotor turn over the end. Thus the purpose of balancing is to measure the unbalanced couple and to add a new couple of same magnitude in the opposite direction. Therefore, balancing of these components require both static and dynamic balancing. The most common types of balancing machines are discussed below:

Pivoted Cradle Balancing Machine

In a pivoted cradle machine the rotor to be corrected is supported on half bearings attached to the cradle and is connected to an electric motor as shown:



The cradle is mounted on spring-dashpots to provide a single degree of freedom vibration system. Often they are made adjustable so that the natural frequency can be tuned to motor speed. Further, the cradle is pivoted about two points which can be adjusted to coincide with the plane of correction. The amplitude of vibration is measured through transducers mounted at each end. In the test run, the pivots are positioned at the plane of correction. One pivot is locked and other is kept free. When the rotor rotates, its amplitude of vibration is measured. The readings obtained will be completely independent of the measurement taken at the other correction plane because an unbalance in the plane of locked pivot will have no moment about that pivot. The measured amplitude of vibration and amount of unbalance are related by the following relation:

$$X = \frac{m_0 e \left(\frac{\omega}{\omega_n}\right)^2}{m\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$X = \frac{m_0 e r^2}{m\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

where m_0e = magnitude of unbalance, X is the amplitude of vibration at correction plane and r = ratio of forcing frequency to the natural frequency ($\omega/\ \omega_n$) The angular position of unbalance is determined by measuring the angular phase difference between a standard sine wave and the wave generated by one of the amplitude transducer. Nowadays, electronic phase meter is attached to measure the phase angle.