MODULE 2

THEORIES OF FAILURE (STATIC LOADING)

- ✓ When loads are acting on the component, there will be some point in the material, which is critically loaded and this critical condition will lead to failure.
- ✓ Theory of failure gives the condition for the initiation of failure at a point in the material.
- ✓ This theory helps us to find out the load or combination of load that will cause failure of component, thereby helps in design process.

FAILURE ?

A machine component is considered to have failed when it no longer performs its mechanical design function.

- ☐ Yielding or Elastic failure (ductile material)
- ☐ Fracture(brittle material)

In combined loadings, failure theories are needed for representing the material behavior based on yielding and fracture

Utility of theories of failure

- ✓ They help in finding out the load capacity of a component; the load capacity may be single load or combination of loading.
- ✓ When the component is subjected to combined loadings, failure theories are to be utilised for determining the load capacity of machine component based on the observations from uni axial tension test.

1. **Briefly explain the theories of failure ?plot the region of safety for each theory**

❖ THEORIES OF FAILURE

a. Maximum principal stress theory (Rankines Theory)

The failure of machine component subjected to combined action of normal and shear stresses occurs whenever the maximum principal stress reaches the yield (elastic) strength or ultimate strength of the material in uniaxial simple tension test.

Elastic strength = Maximum principal stress

$$\sigma_e = \sigma_1$$

$$\sigma_e = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

... (Eq. 2.16a) **2.8(a)/ Pg 21, DHB**

The dimensions of the component are determined by using a factor of safety.

For design, $$\frac{\sigma_e}{n} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Applicable only for brittle materials. This theory disregards the effects of other principal stresses and the effects of shear stress on other planes and hence it is not used for ductile materials.

b) Maximum shear stress theory (Tresca's theory/Guest's Theory)

The failure of a machine component subjected to combined action of normal and shear stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of tension test at yield point.

$$\tau_{max} = \tau_e$$

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, we have

$$\tau_e = \frac{\sigma_e}{2}$$

$$\therefore \quad \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_e}{2}$$

... (Eq. 2.25a) **2.8(c)/ Pg 22, DHB**

$$\frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \frac{\sigma_e}{2}$$

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \sigma_e$$

... (Eq. 2.25b) **2.8(c)/ Pg 22, DHB**

or $$\sigma_1 - \sigma_2 = \sigma_e$$

... (Eq. 2.25c) **2.8(c)/ Pg 22, DHB**

If factor of safety is considered, then

$$n = \frac{\sigma_e}{\sigma_1 - \sigma_2}$$

For design, $$\frac{\sigma_e}{n} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

or $$\sigma_1 - \sigma_2 = \frac{\sigma_e}{n}$$

For a 3D case: If the principal normal stresses are σ_1, σ_2 and σ_3, where $\sigma_1 \geq \sigma_2 \geq \sigma_3$, then τ_{max} is the largest of:

$$\tau_{1\text{-}2} = \frac{\sigma_1 - \sigma_2}{2};\ \tau_{2\text{-}3} = \frac{\sigma_2 - \sigma_3}{2};\ \tau_{3\text{-}1} = \frac{\sigma_1 - \sigma_3}{2}$$

(Eq. 2.27b) yields...

$$\sigma_1 - \sigma_3 = \frac{\sigma_e}{n}$$

Applicable to ductile materials. The results are on the safer side.

c) Maximum principal strain theory or St Venants Theory

The failure of a machine component subjected to combined action of normal and shear stresses occurs whenever the maximum principal strain of the component becomes equal to the maximum strain of the material in uni axial tension.

or $$\sigma_e = (\sigma_1 - \mu\sigma_2)$$ ... (Eq. 2.20) **2.8(b)/ Pg 22, DHB**

If factor of safety is considered, then

$$n = \frac{\sigma_e}{(\sigma_1 - \mu\sigma_2)}$$

or $$\frac{\sigma_e}{n} = (\sigma_1 - \mu\sigma_2)$$

where μ = Poisson's ratio

This theory is not used in general.

d) Maximum strain energy density theory or Haighs theory

The failure of a machine component subjected to combined action of normal and shear stresses occurs whenever the strain energy density(strain energy per unit volume) at the most critically stressed point in the material becomes equal to strain energy per unit volume of material at yield point.

i.e. $$\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}$$ ... (Eq. 2.34) **2.8(e)/ Pg 22, DHB**

If factor of safety is considered, then

$$n = \frac{\sigma_e}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}}$$

For design, $$\frac{\sigma_e}{n} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}$$

Applicable to ductile materials. This theory yields good approximation.

e)Maximum shear energy theory or Distortion theory or Von mises theory

The failure of a machine component subjected to combined action of normal and shear stress occurs whenever the distortion energy density(shear strain energy per unit volume) of material becomes equal to the distortion energy density (shear strain energy density) of the material in uni axial simple tension test.

i.e. $$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$ ... (Eq. 2.29a) **2.8(d)/ Pg 22, DHB**

or $$\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$ ... (Eq. 2.29b) **2.8(d)/ Pg 22, DHB**

If factor of safety is considered, then

$$n = \frac{\sigma_e}{\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}}$$

For design, $$\frac{\sigma_e}{n} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

or $$\frac{\sigma_e}{n} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

Q N:2

A bolt is designed to take up direct tensile load of 30 kN and a shear load of 16 kN with a factor of safety of 4. The yield strength of the material used is 400 MPa. Calculate the size of the bolt based on various theories of failure. Take $\mu = 0.3$.

Solution: $F = 30$ kN $= 30 \times 10^3$ N, $F_s = 16$ kN $= 16 \times 10^3$ N, factor of safety $n = 4$, yield of elastic stress, $\sigma_e = 400$ MPa, $d = ?$

- Axial stress, $$\sigma_D = \frac{F}{A} = \frac{30 \times 10^3}{\pi d^2/4} = \frac{38.20 \times 10^3}{d^2} \text{ MPa}$$ ... **1.1(a)/ Pg 2, DHB**
- Shear stress, $$\tau = \frac{F_s}{A_s} \; (A_s = A)$$ ... **1.1(c)/ Pg 2, DHB**

$$\tau = \frac{16 \times 10^3}{\pi d^2/4} = \frac{20.37 \times 10^3}{d^2} \text{ MPa}$$

State of stress: Here

$$\sigma_x = \sigma_D = \frac{38.20 \times 10^3}{d^2} \text{ MPa};\ \sigma_y = 0;\ \tau = \tau_{xy} = \frac{20.37 \times 10^3}{d^2} \text{ MPa}$$

According to Maximum normal stress theory

For design $\frac{\sigma_e}{n} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ ... **2.8(a) / Pg 21, DHB**

$$\frac{400}{4} = \left(\frac{38.20 \times 10^3 + 0}{2d^2}\right) + \sqrt{\left(\frac{38.20 \times 10^3 - 0}{2d^2}\right)^2 + \left(\frac{20.37 \times 10^3}{d^2}\right)}$$

$$100 = \left(\frac{19.10 \times 10^3}{d^2}\right) + \left(\frac{27.92 \times 10^3}{d^2}\right)$$

$$100 = \left(\frac{47 \times 10^3}{d^2}\right)$$

$\therefore$ $d = 21.68$ **mm**

According to Maximum shear stress theory

b. **Maximum shear stress theory:**

For design $\frac{\sigma_e}{n} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ ... **2.8(c) / Pg 22, DHB**

$$\frac{400}{4} = \sqrt{\left(\frac{38.20 \times 10^3 - 0}{d^2}\right)^2 + \left[4 \times \left(\frac{20.37 \times 10^3}{d^2}\right)^2\right]}$$

$$100 = \left(\frac{55.84 \times 10^3}{d^2}\right)$$

$d = 23.63$ **mm**

According to Maximum principal strain theory

c. **Maximum strain theory:**

For design $\frac{\sigma_e}{n} = (1-\mu)\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left[(1+\mu)\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]$ ... **2.8(b) / Pg 22, DHB**

$$\frac{400}{4} = \left[(1-0.3) \times \left(\frac{38.20 \times 10^3 + 0}{2d^2}\right)\right] + \left[(1+0.3) \times \sqrt{\left(\frac{38.20 \times 10^3 - 0}{2d^2}\right)^2 + \left(\frac{20.37 \times 10^3 - 0}{2d^2}\right)^2}\right]$$

$$100 = \left(\frac{13.37 \times 10^3}{d^2}\right) + \left(\frac{36.30 \times 10^3}{d^2}\right)$$

$$100 = \left(\frac{49.67 \times 10^3}{d^2}\right)$$

$$d = 22.28 \text{ mm}$$

According to Distortion energy /Von mises Theory

$$\frac{\sigma_e}{n} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \qquad \text{... 2.8(d) / Pg 22, DHB}$$

$$\frac{400}{4} = \sqrt{\left(\frac{38.20 \times 10^3}{d^2}\right)^2 + 0 - \left[\left(\frac{38.20 \times 10^3}{d^2}\right)^2 \times 0\right] + \left[\left(3 \times \frac{20.37 \times 10^3}{d^2}\right)^2\right]}$$

$$100 = \left(\frac{52 \times 10^3}{d^2}\right)$$

$$\therefore \; d = 22.80 \text{ mm}$$

According to strain energy /Haighs Theory

e. Strain energy theory:

For design

$$\frac{\sigma_e}{n} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2} \qquad \text{... 2.8(e) / Pg 22, DHB}$$

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{19.10 \times 10^3}{d^2}\right) \pm \left(\frac{27.92 \times 10^3}{d^2}\right) \qquad \text{... 1.8(c) \& (d) / Pg 5, DHB}$$

$$\therefore \quad \sigma_1 = \frac{47 \times 10^3}{d^2}, \sigma_2 = \left(\frac{-8820}{d^2}\right) \qquad d = 22.44 \text{ mm}$$

QN: 3

A mild steel shaft is subjected to 3500 N·m of bending moment at it critical point and transmits a torque of 2500 N·m. The shaft is made of steel having yield strength of 231 MPa. Estimate the size of the shaft based on various theories of failure and specify the final size. Take FOS = 2 and $\mu = 0.3$.

VTU – Dec. 2013/ Jan. 2014 – 14 Marks

Solution: $M = 3500$ N·m $= 3.5 \times 10^6$ N·mm, $T = 2500$ N·m $= 2.5 \times 10^6$ N·mm, yield stress, $\sigma_e = 231$ MPa, $d = ?$, $n = 2$,

- Bending stress, $\sigma_D = \dfrac{M}{Z} = \dfrac{32M}{\pi d^3} = \dfrac{3.5 \times 10^6}{\pi d^3/32} = \dfrac{35.65 \times 10^6}{d^3}$ ... **1.1(b)/ Pg 2, DHB**

- Shear stress, $\tau = \dfrac{16T}{\pi d^3} = \left(\dfrac{16 \times (2.5 \times 10^6)}{\pi d^3}\right) = \dfrac{12.73 \times 10^6}{d^3}$

... **1.1(d)/ Pg 2, DHB**

State of stress: Here

$$\sigma_x = \sigma_D = \frac{35.65 \times 10^6}{d^3}\ \text{MPa};\ \sigma_y = 0,\ \tau = \tau_{xy} = \frac{12.73 \times 10^6}{d^2}\ \text{MPa}$$

According to maximum normal stress theory

For design $\dfrac{\sigma_e}{n} = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ ... **2.8(a)/ Pg 21, DHB**

$$\frac{231}{2} = \left(\frac{35.65 \times 10^6 + 0}{2d^3}\right) + \sqrt{\left(\frac{35.65 \times 10^6 - 0}{2d^2}\right)^2 + \left(\frac{12.73 \times 10^6}{d^3}\right)^2}$$

$$115.5 = \left(\frac{17.83 \times 10^6}{d^3}\right) + \left(\frac{21.90 \times 10^6}{d^3}\right) \quad \text{... Eq. (a)}$$

$$115.5 = \left(\frac{39.73 \times 10^6}{d^3}\right)$$

$$d = 70.06\ \text{mm}$$

$\therefore$ Standard shaft diameter = 71 mm ... **Tb. 3.5(a)/ Pg 57, DHB**

According to maximum shear stress theory

b. **Maximum shear stress theory:**

For design $\frac{\sigma_e}{n} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ ... **2.8(c)/ Pg 22, DHB**

$$\frac{231}{2} = \sqrt{\left(\frac{35.65 \times 10^6 - 0}{d^3}\right)^2 + \left[4 \times \left(\frac{12.73 \times 10^6}{d^3}\right)^2\right]}$$

$$115.5 = \left(\frac{43.80 \times 10^6}{d^3}\right)$$

$$d = 72.38 \text{ mm}$$

$\therefore$ Standard shaft diameter = 80 mm ... **Tb. 3.5(a)/ Pg 57, DHB**

According to Maximum principal strain theory (St Venants Theory)

For design $\frac{\sigma_e}{n} = (1-\mu)\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left[(1+\mu)\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]$

... **2.8(b)/ Pg 22, DHB**

$$\frac{231}{2} = \left[(1-0.3) \times \left(\frac{35.65 \times 10^6 + 0}{2d^3}\right)\right]$$

$$+ \left[(1-0.3) \times \sqrt{\left(\frac{35.65 \times 10^6 - 0}{2d^3}\right)^2 + \left(\frac{12.73 \times 10^6}{d^3}\right)^2}\right]$$

$$115.5 = \left(\frac{12.48 \times 10^6}{d^3}\right) + \left(\frac{28.47 \times 10^6}{d^3}\right)$$

$$115.5 = \left(\frac{40.95 \times 10^6}{d^3}\right)$$

$\therefore$ $d = 21.52$ mm

$\therefore$ Standard shaft diameter = 71 mm ... **Tb. 3.5(a)/ Pg 57, DHB**

According to Distortion energy theory /Von mises Theory

d. **Distortion energy theory or von Mises theory:**

For design

$$\frac{\sigma_e}{n} = \sqrt{(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2)} \quad \text{... 2.8(d)/ Pg 22, DHB}$$

$$\frac{231}{2} = \sqrt{\left(\frac{35.65\times10^6}{d^3}\right)^2 + 0 - \left[\left(\frac{35.65\times10^6}{d^3}\right)\times 0\right] + \left[3\times\left(\frac{12.73\times10^6}{d^3}\right)^2\right]}$$

$$115.5 = \left(\frac{41.92\times10^6}{d^3}\right)$$

$\therefore \quad d = 71.33$ mm

$\therefore$ Standard shaft diameter = 80 mm ... **Tb. 3.5(a)/ Pg 57, DHB**

According to Maximum strain energy theory

For design

$$\frac{\sigma_e}{n} = \sqrt{(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2} \quad \text{... 2.8(e)/ Pg 22, DHB}$$

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... 1.8(c)\&(d)/ Pg 5, DHB}$$

$$= \left(\frac{17.83\times10^6}{d^3}\right) \pm \left(\frac{21.90\times10^6}{d^3}\right)$$

$$\sigma_1 = \left(\frac{39.73\times10^6}{d^3}\right), \sigma_2 = \left(\frac{-4.07\times10^6}{d^3}\right)$$

$$\frac{231}{2} = \sqrt{\left(\frac{39.73\times10^6}{d^3}\right)^2 + \left(\frac{-4.07\times10^6}{d^3}\right)^2 - \left[2\times0.3\times\left(\frac{39.73\times10^6}{d^3}\right)\times\left(\frac{-4.07\times10^6}{d^3}\right)\right]}$$

$$115.5 = \left(\frac{41.13\times10^6}{d^3}\right)$$

$\therefore \quad d = 71.33$ mm

$\therefore$ Standard shaft diameter = 80 mm ... **Tb. 3.5(a)/ Pg 57, DHB**

Based on above theories, the maximum shaft diameter is $d = 72.38$ mm

$\therefore$ Standard shaft diameter, $d = 80$ mm

QN. 4

A machine member is statically loaded and has a yield strength of 350 MPa. For each of the stress state indicated below, find the factor of safety according to:

a. Maximum normal stress theory

b. Maximum shear stress theory

c. Maximum distortion energy theory.

i. $\sigma_1 = 70$ MPa, $\sigma_2 = 70$ MPa

ii. $\sigma_1 = 70$ MPa, $\sigma_2 = 35$ MPa

iii. $\sigma_1 = 70$ MPa, $\sigma_2 = -70$ MPa

iv. $\sigma_1 = 70$ MPa, $\sigma_2 = 0$ MPa

VTU – Dec. 2011 – 12 Marks

Solution: $\sigma_e = 350$ MPa, $n = ?$

Case i: $\sigma_1 = 70$ MPa, $\sigma_2 = 70$ MPa

a. **Maximum normal stress theory:**

For design $\quad \dfrac{\sigma_e}{n} = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) + \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2} = \sigma_1$

... **2.8(a) / Pg 21, DHB**

i.e. $\quad \dfrac{\sigma_e}{n} = \sigma_1$

$$\frac{350}{n} = 70$$

$\therefore \quad n = 5.0$

b. **Maximum shear stress theory:**

For design $\quad \dfrac{\sigma_e}{n} = \sigma_1 - \sigma_2$ ... **2.8(c) / Pg 22, DHB**

$\therefore \quad \dfrac{350}{n} = (70 - 70)$

$$\frac{350}{n} = 0$$

$n = \infty$ (infinity)

c. **Distortion energy theory:**

For design $\quad \dfrac{\sigma_e}{n} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$ ... **2.8(d) / Pg 22, DHB**

$\therefore \quad \dfrac{350}{n} = \sqrt{70^2 + 70^2 - (70 \times 70)}$

$$\frac{350}{n} = 70$$

$$n = 5.0$$

On similar lines, we have

QN:5(KTU 2018)

The stresses acting at a critical point in a component are the following.

σ_{xx} = 60MPa, σ_{xy} = 30MPa, σ_{xz} = 20MPa, σ_{yy} =40MPa, σ_{zz} = 25MPa and σ_{yz} = 20MPa

The component is made of steel having the following material properties. Ultimate strength in tension =600 MPa, Yield strength in tension =400 MPa, Yield strength in shear =200 MPa and the poisons ratio =0.3.Determine the factor of safety using all the five static failure theories.

Given,

3D State of stress

- σ_{xx} = 60MPa = σ_x
- σ_{yy} = 40MPa = σ_y
- σ_{zz} = 25MPa = σ_z
- σ_{xy} = 30MPa = τ_{xy}
- σ_{xz} = 20MPa = τ_{xz}
- σ_{yz} = 20MPa = τ_{yz}

- Material : Steel
- σ_{ut} = 600 Mpa
- σ_{yt} = 400 Mpa
- τ_{yt} = 200MPa
- Poisson's ratio, μ = 0.3

- FOS ?

Solution,

Step 1: Calculation of Principal Stresses, σ_1, σ_2 & σ_3

$$\begin{vmatrix} (\sigma_x - \sigma) & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & (\sigma_y - \sigma) & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - \sigma) \end{vmatrix} = 0$$

[Ref. PSG Data book Page 7.2

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

Step 1: Calculation of Principal Stresses, σ_1, σ_2 & σ_3

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix}$$

Step 1: Calculation of Principal Stresses, σ_1, σ_2 & σ_3

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$
$$= 60 + 40 + 25$$
$$= 125$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$
$$= 60 \times 40 + 40 \times 25 + 25 \times 60 - 30^2 - 20^2 - 20^2$$
$$= 3200$$

We have

σ_x = 60MPa

σ_y = 40MPa

σ_z = 25MPa

τ_{xy} = 30MPa

τ_{xz} = 20MPa

τ_{yz} = 20MPa

Step 1: Calculation of Principal Stresses, σ_1, σ_2 & σ_3

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$
$$= 60 + 40 + 25$$
$$= 125$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$
$$= 60 \times 40 + 40 \times 25 + 25 \times 60 - 30^2 - 20^2 - 20^2$$
$$= 3200$$

We have

σ_x = 60MPa

σ_y = 40MPa

σ_z = 25MPa

τ_{xy} = 30MPa

τ_{xz} = 20MPa

τ_{yz} = 20MPa

Step 1: Calculation of Principal Stresses, σ_1, σ_2 & σ_3

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

$$\Rightarrow \sigma^3 - 125\,\sigma^2 + 3200\,\sigma - 21500 = 0$$

Solving

$$\Rightarrow \sigma_1 = 93.11 \text{ MPa}$$
$$\sigma_2 = 20.77 \text{ MPa}$$
$$\sigma_3 = 11.12 \text{ MPa}$$

We have

$I_1 = 125$

$I_2 = 3200$

$I_3 = 21500$

Step 2: FOS according to Maximum Principal Stress Theory

Since $\sigma_1 > \sigma_2 > \sigma_3$

According to Maximum Principal Stress Theory [Ref. KM Data book Page 21]

For design $\frac{\sigma_y}{n} = \sigma_1$

$$\Rightarrow n = \frac{400}{93.11} = 4.296$$

We have,

σ_{yt} = 400 MPa

σ_1 = 93.11 MPa

Step 3: FOS according to Maximum Principal Strain Theory

According to Maximum Principal Strain Theory

[Ref. PSG Data book Page 7.3]

For design
$$\frac{\sigma_y}{n} = \begin{cases} \sigma_1 - \mu(\sigma_2 + \sigma_3) \\ \text{or} \\ \sigma_2 - \mu(\sigma_1 + \sigma_3) \\ \text{or} \\ \sigma_3 - \mu(\sigma_1 + \sigma_2) \end{cases}$$

Choose the maximum value

$$\frac{\sigma_y}{n} = \sigma_1 - \mu(\sigma_2 + \sigma_3)$$

$$= 93.11 - 0.3\,(20.77 + 11.12) = 83.543 \text{ MPa}$$

$$\frac{\sigma_y}{n} = 83.543 \text{ MPa}$$

$$n = \frac{400}{83.543} = 4.787$$

Step 4: FOS according to Maximum Shear Stress Theory

According to Maximum Shear Stress Theory [Ref. PSG Data book Page 7.3]

$\sigma_e = (\sigma_1 - \sigma_2)$ or $(\sigma_2 - \sigma_3)$ or $(\sigma_1 - \sigma_3)$

$\sigma_1 - \sigma_2 = 93.11 - 20.77 = 72.34$

$\sigma_2 - \sigma_3 = 20.77 - 11.12 = 9.65$

$\sigma_1 - \sigma_3 = 93.11 - 11.12 = 81.99$

We have,

$\sigma_1 = 93.11$ MPa

$\sigma_2 = 20.77$ Mpa

$\sigma_3 = 11.12$ MPa

Step 4: FOS according to Maximum Shear Stress Theory

Since $(\sigma_1 - \sigma_3) > (\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$

For design $\frac{\sigma_e}{n} = \sigma_1 - \sigma_3 = 81.99$ MPa

$$\Rightarrow n = \frac{400}{81.99} = 4.878$$

We have,

$\sigma_{yt} = 400$ MPa

$\sigma_1 = 93.11$ MPa

$\sigma_2 = 20.77$ Mpa

$\sigma_3 = 11.12$ MPa

$\mu = 0.3$

Step 5: FOS according to Max. Strain Energy Theory

For Safe Design,

$$= \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

$$\frac{\sigma_e}{n} = \sqrt{93.11^2 + 20.77^2 + 11.12^2 - 2 \times 0.3(93.11 \times 20.77 + 20.77 \times 11.12 + 11.12 \times 93.11)}$$

$$= \sqrt{7304.375}$$

$$= 85.465 \text{ MPa}$$

We have,

$\sigma_1 = 93.11$ MPa

$\sigma_2 = 20.77$ Mpa

$\sigma_3 = 11.12$ MPa

$\mu = 0.3$

For design $\frac{\sigma_e}{n} = 85.465$ MPa

$$\Rightarrow n = \frac{400}{85.465} = 4.68$$

We have,

$\sigma_{yt} = 400$ MPa

Step 6: FOS according to Distortion Energy Theory

According to Distortion Energy Theory

For Safe Design,

$$\frac{\sigma_e}{n} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

$$= \sqrt{93.11^2 + 20.77^2 + 11.12^2 - (93.11 \times 20.77 + 20.77 \times 11.12 + 11.12 \times 93.11)}$$

$$= \sqrt{6024.279}$$

$$= 77.616 \text{ MPa}$$

We have,

$\sigma_1 = 93.11$ MPa

$\sigma_2 = 20.77$ Mpa

$\sigma_3 = 11.12$ MPa

Since, $\frac{\sigma_e}{n} = 77.616$ MPa

$$\Rightarrow n = \frac{400}{77.616} = 5.15$$

We have,

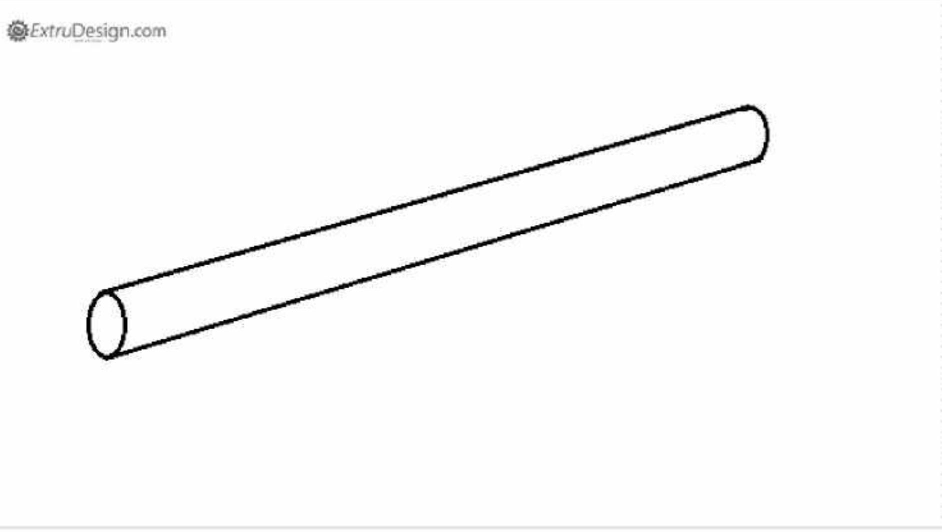
$\sigma_{yt} = 400$ MPa

MODULE 2

FATIGUE LOADING

FATIGUE

- When a material is subjected to repeated stresses (repeated loads/alternating loads), it fails at a stress far below the yield point stress. Such type of failure is referred to as fatigue.
- Fatigue failure is always brittle and catastrophic in nature with no visible warning prior to failure .
- It is observed that about 80 % of failures of mechanical components are due to fatigue failure resulting from fluctuating stresses.
- The decreased resistance of the materials to cyclic stresses is the main characteristics of fatigue failure .
- Fatigue failure is defined as the time delayed fracture under cyclic loading
- Transmission shafts ,connecting rods ,gears ,vehicle suspension springs, ball bearings are subjected to fatigue failure .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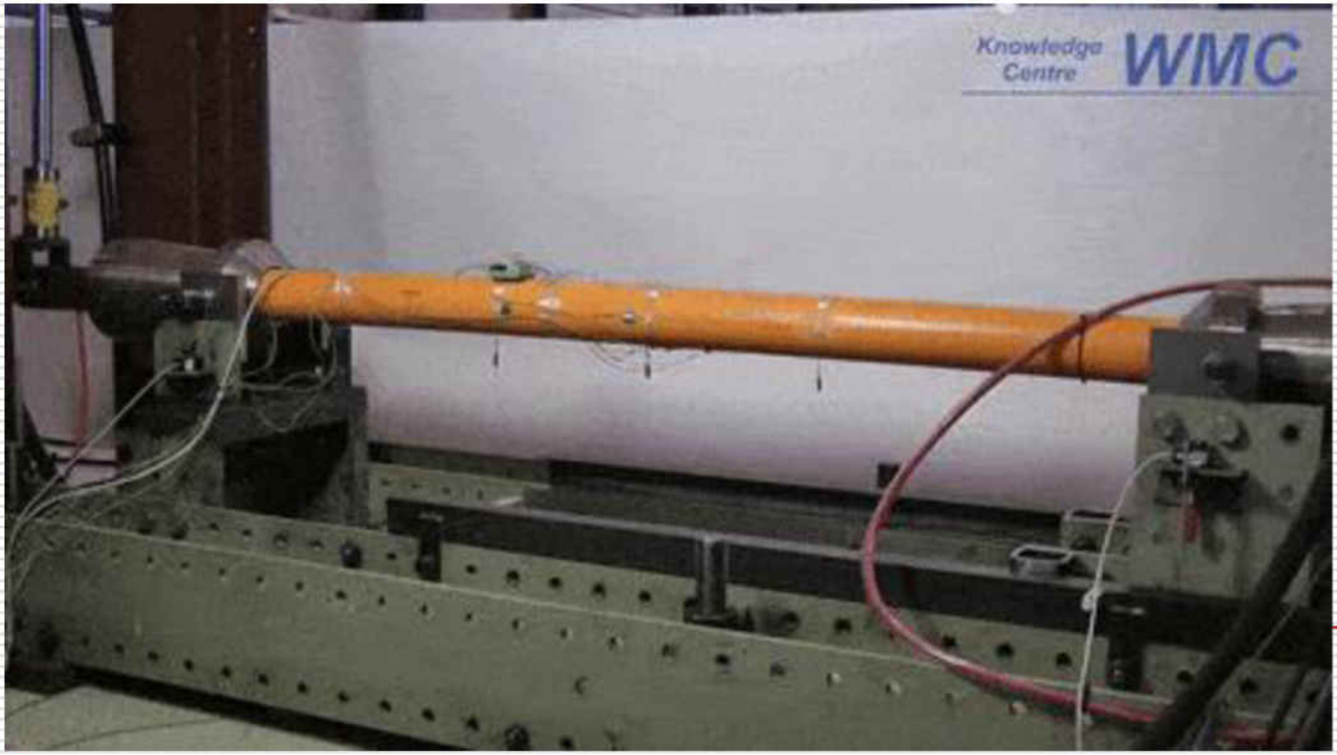
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FATIGUE FAILURE IN VARIOUS FIELD OF ENGINEERING

- ✓ Automobiles in Mechanical Engineering
- ✓ Bridges in Civil Engineering
- ✓ Aircrafts in Aeronautical Enginnering
- ✓ Ship hull in Marine Engineering
- ✓ Pressure vessels in Chemical Engineering
- ✓ Tractors involving Agricultural Engineering

FACTORS TO BE CONSIDERED TO AVOID FATIGUE FAILURE

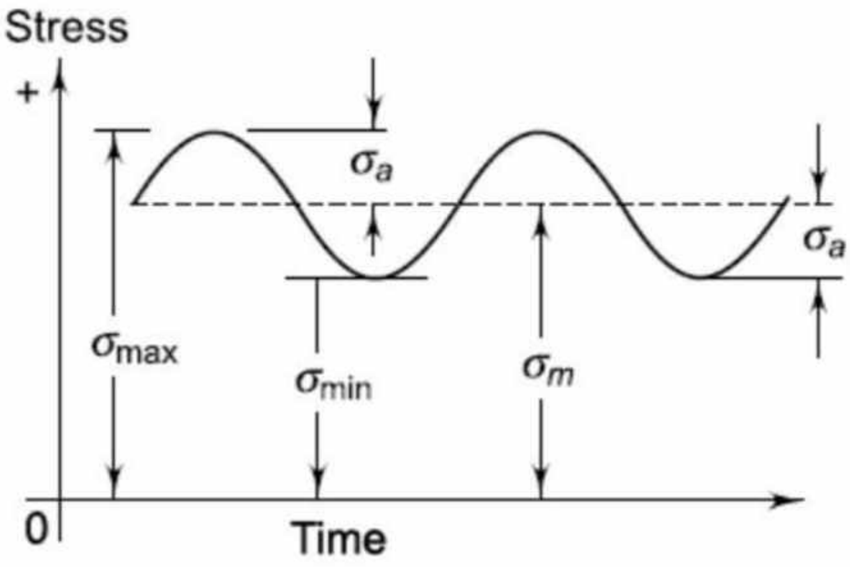
- ✓ Variation in size of the component should be as gradual as possible.
- ✓ Holes , notches and other stress raisers should be avoided.
- ✓ Proper stress deconcentrators such as fillets and notches should be provided wherever necessary.
- ✓ Components should be protected from corrosion.
- ✓ Provide smooth finish on the outer surface of the component ,thereby increasing fatige life.
- ✓ Materials with high fatigue strength should be selected.
- ✓ Residual compressive stresses over the parts surface increases its fatigue strength

Types of models for cyclic stresses

- Fluctuating or alternating stresses
- Repeated stresses
- Reversed stresses

FLUCTUATING STRESS

Fluctuating stress is the stress which varies from a maximum value to a minimum value of the same nature (tensile/compressive).



(a) Fluctuating stresses

$$\sigma_m = \frac{1}{2}(\sigma_{max.} + \sigma_{min})$$

$$\sigma_a = \frac{1}{2}(\sigma_{max.} - \sigma_{min.})$$

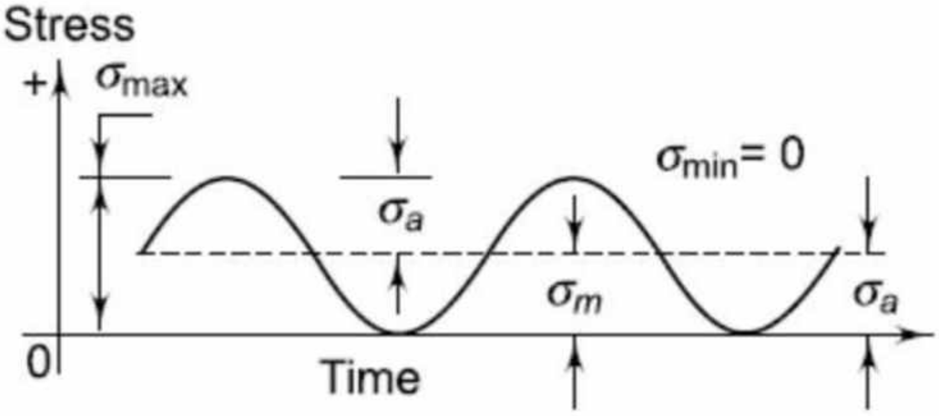
REPEATED STRESS

Repeated/released stress is the stress which varies from zero to a maximum value (tensile/compressive).

Here $\sigma_{min} = 0, R = 0, A = 1$

$$\sigma_m = \sigma_a = \frac{\sigma_{max}}{2}$$

$$\sigma_{min} = 0 \quad R = 0 \quad A = 1$$



(b) Repeated stresses

Stress ratio $R = \dfrac{\sigma_{min}}{\sigma_{max}}$

Amplitude ratio $A = \dfrac{\sigma_a}{\sigma_m} = \dfrac{1-R}{1+R}$