

STATIC FORCE ANALYSIS

The general function of any machine is to transmit motion and forces from one actuator to the components that perform the desired task. The forces are transmitted between machine members through mating surfaces ie one gear through meshing teeth to another gear, from a connecting rod through a bearing to a lever ,from a v- belt to a pulley ,from a cam to a follower or from brake drum to a brake shoe etc. In the design of machine elements it is necessary to know the magnitudes as well as directions of the forces transmitted from input to output. The analysis helps in selecting proper sizes of the machine components to withstand the stresses developed in them. If proper sizes are not selected, the components may fail during the machine operations. On the other hand, if the members are designed to have more strength than required, the machine may not be able to compete with others due to more cost, weight, size etc.

Forces in mechanism arise from various sources ,for example forces of gravity, forces of assembly, forces from applied loads, forces from energy transmission, frictional force, inertia forces, spring forces, impact forces and forces due to change of temperature .All these forces must be considered in the final design of a machine for its successful operation.

These forces may be classified in to two categories- static force and dynamic force.

Static forces are those forces acting on the members whose magnitude doesn't depend on acceleration and mass of the component.

Dynamic forces are forces produced due to dynamic action of machine element or inertia.

If the components of a machine accelerate, inertia forces are produced due to their masses. However if the magnitudes of inertia force produced due to accelerating masses are small compared to the externally applied loads, they can be neglected while analyzing the mechanism such an analysis is known as static force analysis.

In static force analysis, for a known value of force acting on one element of a mechanism, the forces in other elements are determined without any consideration for motion of other elements. **FORCE**

A force is a vector quantity that represents pulling or pushing action of a part.

MOMENT OR TORQUE

It is a twisting action produced by a force.

$$\text{Moment}_A = F \cdot d$$

F – Force applied, d – Perpendicular distance between reference point A and line of action of force.

A – Reference point designation.

NEWTONS LAWS OF MOTION

LAW1: Everybody preserves its state of rest or uniform motion in a straight line, except in so far as it is compelled to change the state by impressed force.

LAW 3: Reaction is always equal and opposite to action.

LAW1: If all the forces acting on the body are balanced, ie resultant force is zero, and then the body will either remain at rest or will continue to move in a straight line at uniform velocity.

LAW 3: when two bodies interact, a pair of reaction forces come in to existence, these forces have the same magnitude and opposite sense and they act along the straight line common to two bodies.

FREE BODY DIAGRAMS

The diagram so obtained is called free because the machine parts or portion of the body has been isolated from the remaining machine elements, and their effects have been replaced by forces and moments.

The free body diagram of various members ie input link 2, coupler link 3 and output link 4 are shown in fig above. Various forces acting on each member are also shown. As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.

Advantages of free body diagram

1. **It is a medium to convert ideas in to physical model.**
2. They assists in seeing and understanding all facets of the problem.
3. It helps to build up a suitable mathematical relation between various forces acted upon a link.

STATIC EQUILIBRIUM

A body is in static equilibrium, if it remains in its state of rest or motion. If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion .The state of equilibrium can be changed by the application of forces and moments.

A body to be in static equilibrium, the following two necessary and sufficient conditions must be met.

- The vector sum of all the forces acting on the body is zero.(force polygon must be closed)
- The vector sum of all the moments about any arbitrary point is zero. (couple polygon must be closed)

Mathematically,

$$\left. \begin{array}{l} \Sigma F = 0 \\ \Sigma M = 0 \end{array} \right\}$$

- In a planar system forces can be described by two dimensional vectors and therefore

$$\left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{array} \right\}$$

Static equilibrium is applicable in many machines where movement is relatively slow.

Eg 1: in lifting cranes, bucket load and the static weight loads may be quite high relative to any dynamic loads due to accelerating masses.

Eg 2: static equilibrium is applicable in many machines where movement is relatively slow. These include clamps, latches, support linkages and many hand operated tools such as pliers and cutters.

Note: force convention

The force applied by link i on link j is denoted by F_{ij} .it means force applied by link 2 to link 1 is represented as F_{21} . Force applied by link 3 to link 4 is represented as F_{34} and so on.

ANALYSIS OF A TWO FORCE MEMBER

A special case of equilibrium which is of considerable interest is that of a member that is subjected to only two forces. This type of part is termed as a two force member. Many mechanism links, particularly couplers and connecting rods are two force members.

In order for a two force member to be in equilibrium, the two forces must

- a) Have the same magnitude.
- b) Act along the same line.
- c) Be opposite in sense.

Since the two forces must act along the same line, the only line that can satisfy this constraint is the line that extends between the points, where the forces are applied. Thus a link with only two forces simply exhibits tension or compression.

This fact can be extremely useful in force analysis. When the locations of the forces are known, directions of forces are defined. When the magnitude and sense of a single force are known, other forces magnitude and sense can be immediately determined. Thus the analysis of a two force member is simple.

Equilibrium of three force and four force members

1. Three force system

A member under the action of three forces will be in equilibrium, if

- a. The resultant of all the forces is zero.
- b. The lines of action of all the forces intersect at a point known as point of concurrency.

2. Four force systems

Normally in most of the cases, the above conditions for equilibrium of a member are found to be sufficient. However in some problems it may be found that the number of forces on a member is four or even more than that. In such cases first look for all the forces completely known and combine them in to a single force representing the sum of the known forces. This may reduce the number of forces acting on a body to two or three. However in planar mechanisms, a four force system is also solvable if only one force is known completely along with line of action of others.

Member with two forces and torque

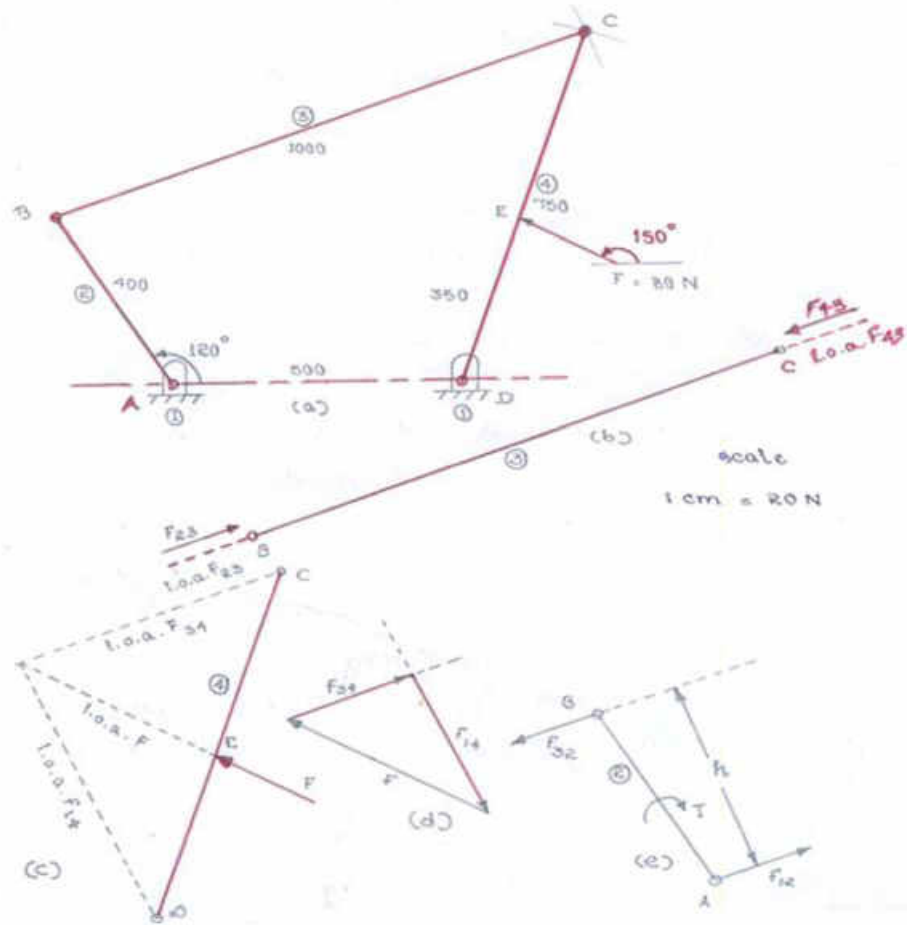
A member under the action of two forces and an applied torque will be in equilibrium if

- a. Forces are equal in magnitude, parallel in direction and opposite in sense.
- b. The forces form a couple which is equal and opposite to the applied torque.

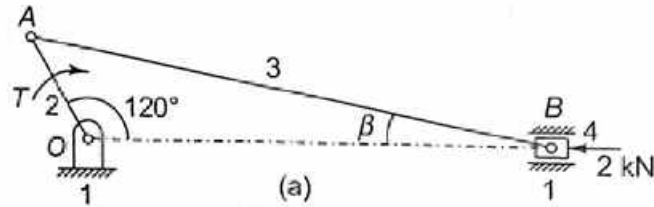
1. A four link mechanism with the following dimensions is acted upon by a force $80 < 150^\circ$ N on the link DC. AD= 500mm, AB= 400 mm, BC=1000mm, DC=750 mm, DE= 350 mm.

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration. ?

Answer :- Let us take the scale $10 \text{ mm} = 100 \text{ mm}$ or $1 : 10$



Q. A slider crank mechanism with the following dimensions is acted upon by a force $F=2\text{KN}$ at B as shown in figure below. $OA=100\text{ mm}$, $AB=450\text{ mm}$. Determine the input torque T on the link OA for the static equilibrium of the mechanism for the given configuration.



From force triangle, $F_{34} = 2.04\text{KN}$

Also $F_{34} = F_{43} = F_{23} = F_{32}$

Member 2 will be in equilibrium, if

F_{12} is equal parallel and opposite to F_{32} and

$T = F_{32} \cdot h = 2.04 \times 75 = 153\text{ KNmm (CCW)}$

($h = 75\text{mm}$ on measurement)

The input torque has to be equal and opposite to this couple

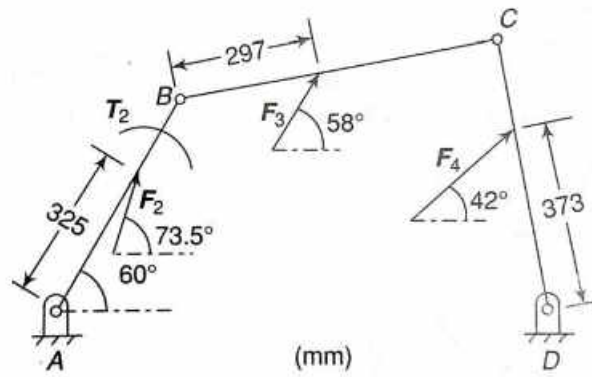
$T = 153\text{ KNmm} = 153\text{ Nm (CW)}$

PRINCIPLE OF SUPERPOSITION

In linear systems, if a number of loads act on a mechanical system, the net effect is equal to the superposition of the effects of the individual loads taken one at a time. A linear system is one in which the output force is directly proportional to the input force, i.e. in mechanisms where coulomb or dry friction is neglected.

Q. A four link mechanism is subjected to the following external forces as shown in figure & table below. determine the shaft torque T_2 on the input link AB for the static equilibrium of the mechanism. also find the forces on the bearings A, B, C and D

Link	Length	Force	Magnitude	Point of application force (r)
AB (2)	500 mm	F_2	$80 \angle 73.5^\circ \text{N}$	325 mm from A
AB (3)	660 mm	F_3	$144 \angle 58^\circ \text{N}$	297 mm from B
AB (4)	560 mm	F_4	$60 \angle 42^\circ \text{N}$	373 mm from D
AB (1)	1000 mm	-	(Fixed link)	



Subproblem (a) → neglecting F_3 and F_4

	<p>Link 3 and 4 are both two force members.</p> <p>therefore F_{41} can be along BC and F_{43} along DC. As F_{43} is to be equal and opposite of F_{34}, both must be zero.</p> <p>Also $F_{43} = F_{23} = F_{32} = 0$.</p> <p>Hence link 2 is in equilibrium under the action of two forces and torque T_{2a}.</p> <p>$T_{2a} = F_2 \times h_{2a} = 6 \text{ Nm (clockwise)}$</p>
--	--

Subproblem (b) → neglecting F_2 and F_4

	<p>Link 4 is a two force member.</p> <p>Therefore $F_{34} = F_{14}$, magnitudes unknown, directions parallel to DC.</p> <p>Link 3 is a 3 force member in which F_3 is completely known, line of action of F_{43} is known (parallel to DC) and F_{23} is completely unknown.</p> <p>From force triangle, $F_{43} = 50 \text{ N}$</p> <p>Also $F_{43} = F_{34} = F_{14} = 50 \text{ N}$</p> <p>$F_{23} = 113 \text{ N}; F_{23} = F_{32} = 113 \text{ N}$</p> <p>Link 2 is subjected to two force and torque T_{2b}.</p> <p>For equilibrium, F_{12} is equal, parallel and opposite to F_{32}.</p> <p>$T_{2b} = F_{32} \times h_{2b} = 113 \times 0.16 = 18.1 \text{ Nm (CCW)}$</p>
--	--

Subproblem (c) → neglecting F_2 and F_3

	<p>Link 4 is a 3 force member in which one force F_4 is known.</p> <p>However line of action of F_{34} can be obtained from the equilibrium of the link 3, which is a two force member.</p> <p>F_{34} will be equal and opposite to F_{43} which is along BC.</p> <p>From force triangle, $F_{14} = 34.8$ N</p> <p>Also $F_{34} = F_{43} = F_{23} = F_{32} = 34$ N</p> <p>Link 2 is subjected to two forces and torque T_{2c}.</p> <p>$T_{2c} = F_{32} \times h_{2c} = 34 \times 0.38 = 12.9$ Nm (CCW)</p>
--	--

Net crankshaft torque = $T_{2a} + T_{2b} + T_{2c} = -6 + 18.1 + 12.9 = 25$ Nm (CCW)

To find the magnitude of forces on the bearings, the result obtained in a, b and c have to be added vectorially ($F_{14} = 80$ N; $F_{34} = 60$ N; $F_{23} = 137$ N; $F_{14} = 204$ N)

PRINCIPLE OF VIRTUAL WORK

The principle of virtual (imaginary) work can be stated as “the work done during a virtual displacement from the equilibrium is equal to zero”. Virtual displacement may be defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it in to free bodies.

Consider a slider crank mechanism as shown in fig below.

<p>It is acted upon by external piston force F, external crankshaft torque T and the force at the bearings. As the crank rotates through a small angular displacement $\delta\theta$, the corresponding displacement of the piston is δx.</p>	
---	--

The various forces acting on the system are

- Bearing reactions at O (performs no work)
- Force of cylinder on piston, perpendicular to piston displacement (produces no work).
- Bearing forces at A and B, being equal and opposite (AB is a two force member), no work is done.
- Work done by torque, $T = T \cdot \delta\theta$
- Work done by force, $F = F \cdot \delta x$

Work done is positive if a force acts in the direction of the displacement and negative if it acts in the opposite direction .According to the principle of virtual work, $W= T \cdot \delta\theta + F \cdot \delta x=0$.

As virtual displacement must take place during the same interval δt ,

$$T \frac{d\theta}{dt} + F \frac{dx}{dt} = 0.$$

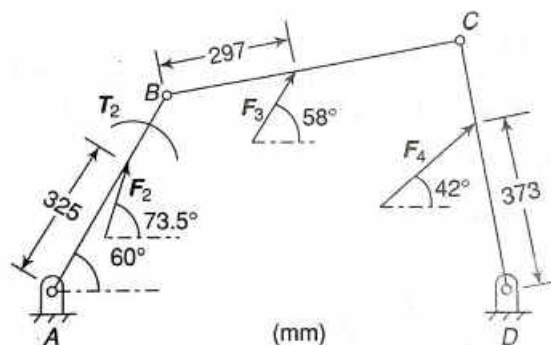
$$T\omega + Fv = 0$$

Where ω is the angular velocity of the crank and v , the linear velocity of the piston. $T = \frac{-F}{\omega} v$

The negative sign indicates that for equilibrium, T must be applied in the opposite direction to the angular displacement.

Q.A four link mechanism is subjected to the following external forces as shown in figure &table below.determine the shaft torque T_2 on the input link AB for the static equilibrium of the mechanism.also find the forces on the bearings A,B,C and D

Link	Length	Force	Magnitude	Point of application force (r)
AB (2)	500 mm	F_2	$80 \angle 73.5^\circ \text{N}$	325 mm from A
AB (3)	660 mm	F_3	$144 \angle 58^\circ \text{N}$	297 mm from B
AB (4)	560 mm	F_4	$60 \angle 42^\circ \text{N}$	373 mm from D
AB (1)	1000 mm	-	(Fixed link)	

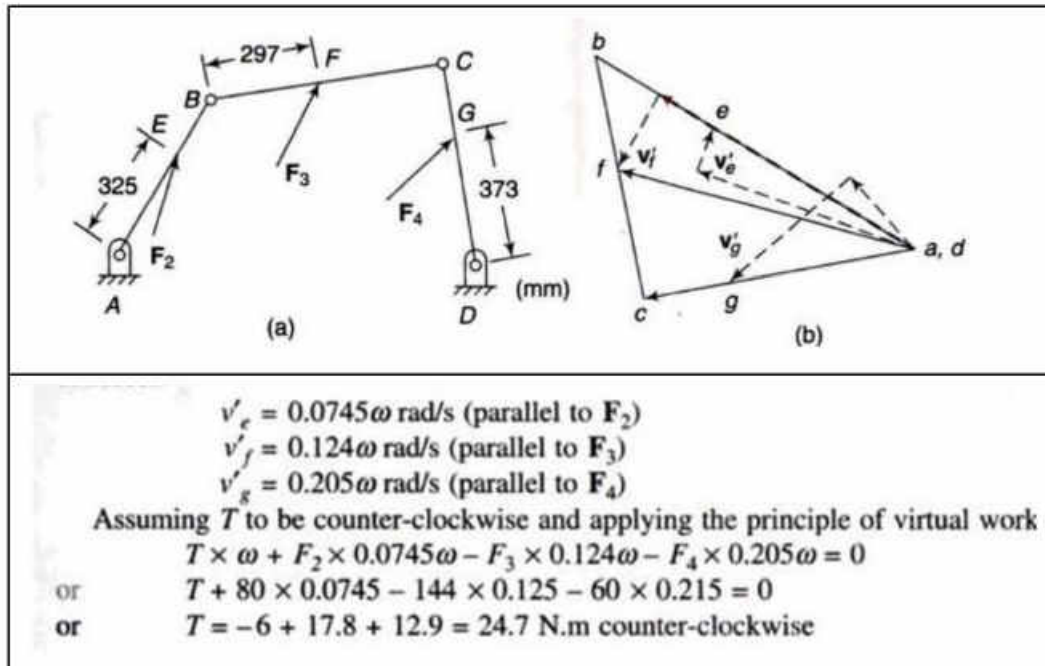


Solution

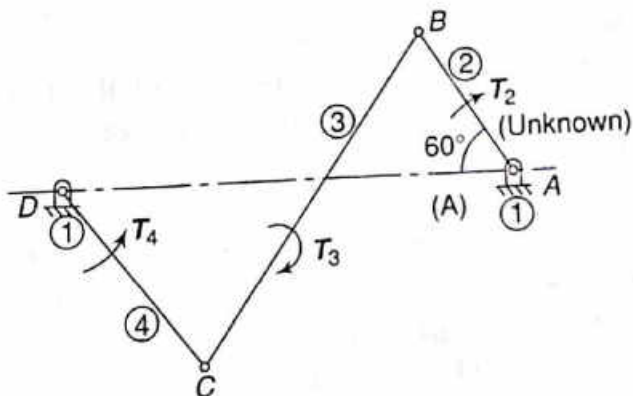
Assume that the line AB has an instantaneous angular velocity of ω rad/s counter clockwise.

then $v_b = 0.5$ m/s.

from the configuration diagram ,draw the velocity diagram.locate the points E,F and gG on the velocity diagram and locate the velocity vectors for the same .take their components parallel and perpendicular to the direction of forces.



Q. In a four link mechanism as shown, torque T_3 and T_4 have magnitudes of 30 Nm and 20 Nm respectively. The link lengths are $AD = 800 \text{ mm}$, $AB = 300 \text{ mm}$, $BC = 700 \text{ mm}$ and $CD = 400 \text{ mm}$. For the static equilibrium of the mechanism, determine the required input torque T_2 ? (solution using superposition method)



Subproblem a (neglecting T_3)

(a) Free body diagram of link 4 showing forces F_{14} , F_{34} and torque T_4 . (b) Free body diagram of link 3 showing forces F_{32} , F_{43} and torque T_{23} .

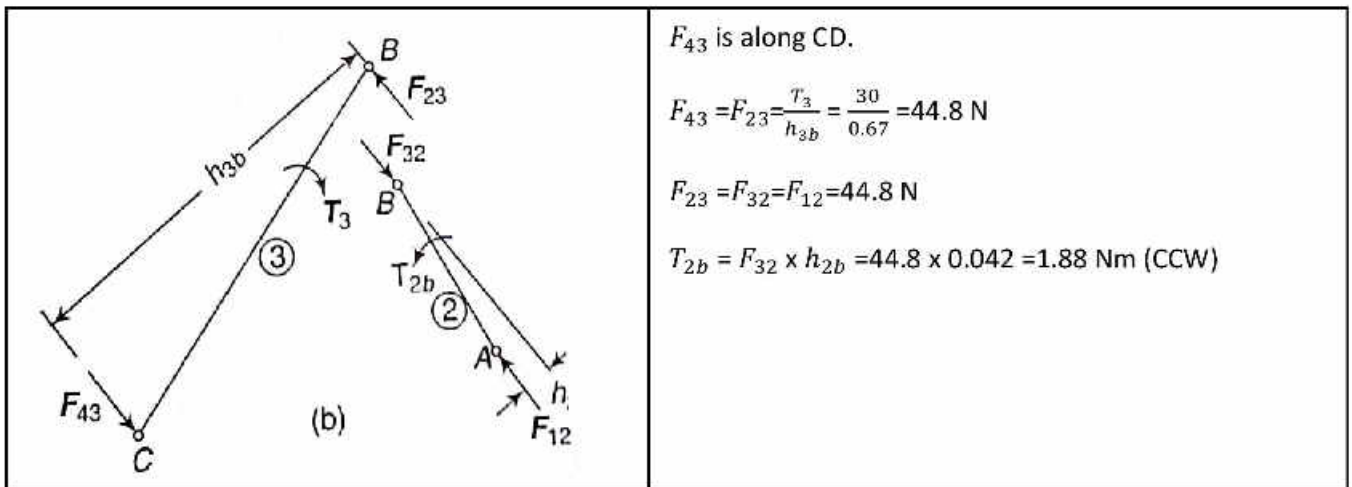
Torque T_4 on the link 4 is balanced by a couple having two equal, parallel and opposite forces at C and D.

as the link 3 is a two force member, F_{43} and F_{23} will be parallel to BC (same line of action).

$$F_{34} = F_{14} = \frac{T_4}{h_{4a}} = \frac{20}{0.383} = 52.2 \text{ N}$$

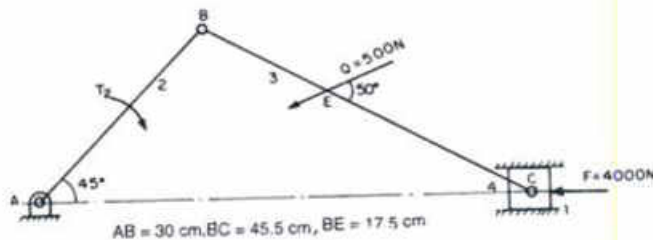
Also $F_{34} = F_{43} = F_{23} = F_{32} = F_{12} = 52.2 \text{ N}$

Subproblem b (neglecting torque T_4)



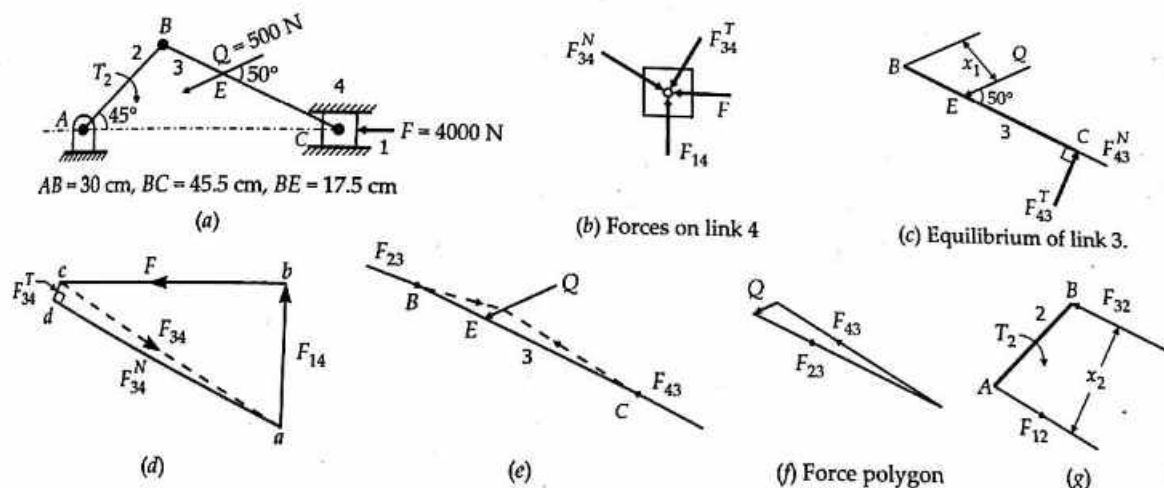
Ans : $T_2 = T_{2a} + T_{2b} = 14.3 + 1.88 = 16.18 \text{ N (CCW)}$

Q. Determine the couple T_2 as applied in the figure below.



Solution

The forces of link 4 are shown in fig (b). the link 3 is a three force member. So the force F_{43} is not acting along BC. Let the force F_{43} be broken in to two components one along BC and other perpendicular to BC. let these components be denoted by F_{43}^N (normal) and F_{43}^T (tangential)



Procedure for finding various terms is as under

1. draw link 3 and find force F_{43}^T by the equation of moments.

$$Q \cdot x_1 = F_{43}^T \cdot BC$$

$$F_{43}^T = \frac{Q \cdot x_1}{BC}$$

Consider equilibrium of link 4 ;

- Direction of F_{14} is perpendicular to the sliding surface. F_{43}^T is completely known. Refer to Fig. (d). Draw $bc = F = 4000$ N along the sliding surface with some suitable scale. From c draw $cd = F_{43}^T = 153.85$ perpendicular to link BC . From d draw a line perpendicular to cd representing F_{34}^N . This line is known in direction only. From point b draw a line perpendicular to bc representing F_{14} . The line of action of force F_{14} and line of action of force F_{34}^N intersect at point a . Join a to c . Then ac represents the force F_{34} . By measurement $F_{34} = 4590$ N; $F_{14} = 2250$ N.
- Coming back to the equilibrium of link 3, it is a three force member. The forces are F_{43} (both magnitude and direction known) Q for which magnitude and direction are given and F_{23} magnitude and direction both are unknown. These are shown in Fig. (e). With the help of force polygon as shown in Fig. (f), force F_{23} can be determined in magnitude and direction. By measurement $F_{23} = F_{32} = 4900$ N.
- Couple on link 2 can be calculated as shown in Fig. (g)

$$T_2 = F_{32} \cdot x_2 = 4900 \times 27.5 = 1347.5 \text{ N-m} \text{ It is in clockwise sense.}$$

EXAMPLE 12.1 Determine the torque required to be applied at the crank shaft of a slider-crank mechanism to bring it in equilibrium. The slider is subjected to a horizontal force of 5000 N and a force of magnitude 1000 N is applied on the connecting rod as shown in Figure 12.10. The dimensions of various links are as under:

$$OA = 250 \text{ mm, } AB = 750 \text{ mm and } AC = 250 \text{ mm, } \angle BOA = 40^\circ$$

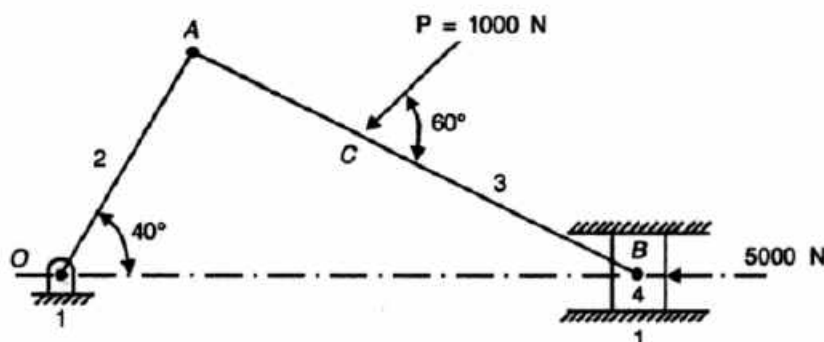


FIGURE 12.10 Slider-crank mechanism.

Solution The graphical solution procedure is given as under:

1. Draw configuration diagram of the mechanism as shown in Figure 12.11(a).
2. Draw free body diagram of link 3 as shown in Figure 12.11(b). The force F_{43} is broken into two components—one along the link F'_{43} and another normal to the link F''_{43} .
3. Taking moments about point A,

$$P \times x = F''_{43} \times AB$$

where $x = AC \cos 30^\circ = 250 \times \cos 30^\circ = 216.5 \text{ mm}$

Force: $F''_{43} = \frac{1000 \times 216.5}{750} = 288.67 \text{ N}$

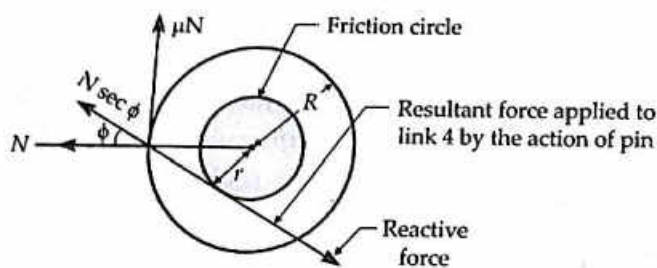
FRICITION IN MECHANISMS

When two members of a mechanism move relative to each other, friction occurs at the joints. The presence of friction increases the energy requirements of a machine.

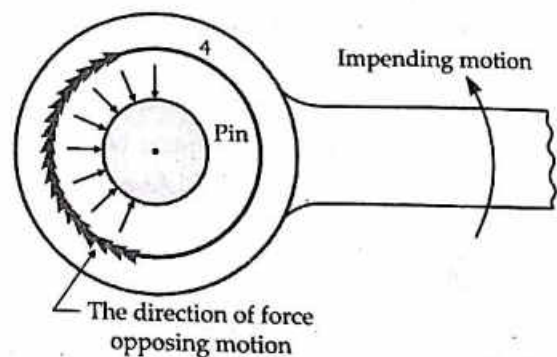
Friction at the bearing is taken in to account by drawing friction circles and at the sliding pairs by considering the angle of friction.

Friction in pin joints

Resultant force on a pin passes through its centre, if friction is neglected. If friction is taken into account it does not pass through its centre. For the precise magnitude and direction of the force for design purpose it is necessary to consider the friction in pin joints. A pin fitted with link 4 is shown below. pin is rigidly held and the impending motion of the link is anticlockwise. The friction force opposes the impending motion.



(a) Pin is assumed to be held rigidly



(b)

The net resultant of forces N and μN is calculated as

$$\begin{aligned} & \sqrt{N^2 + (\mu N)^2} \\ &= N\sqrt{1 + \mu^2} = N\sqrt{1 + \tan^2 \phi} \quad (\because \mu = \tan \phi) \\ &= N \sec \phi \end{aligned}$$

where μ = coefficient of friction and ϕ = friction angle

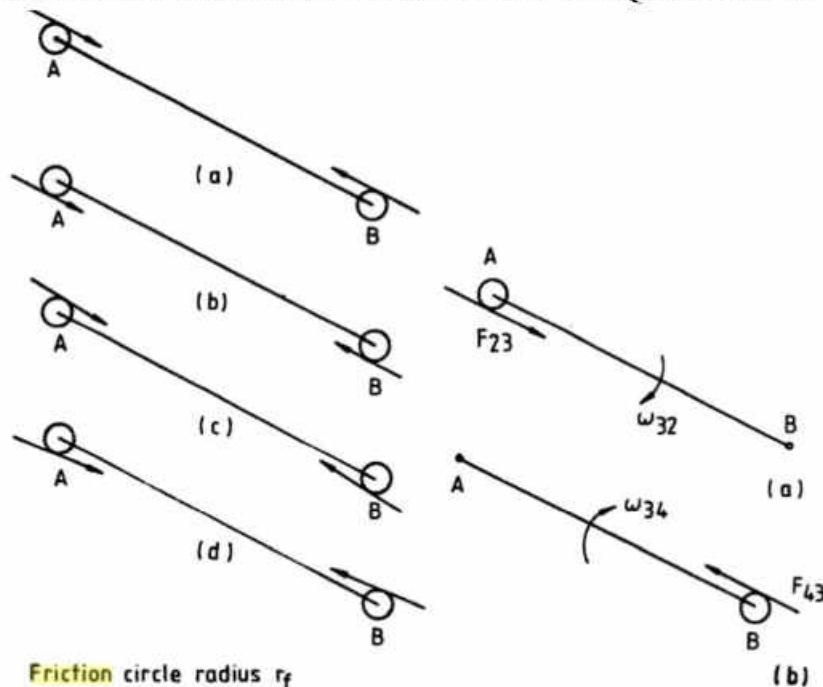
The moment of the friction force should be equal to the moment of resultant force about the centre of the pin :

$$\begin{aligned} (N \sec \phi)(r) &= (\mu N)(R) \\ \sec \phi \cdot r &= \mu \cdot R = \tan \phi \cdot R \\ r &= R \cdot \sin \phi \end{aligned} \quad (\because \mu = \tan \phi)$$

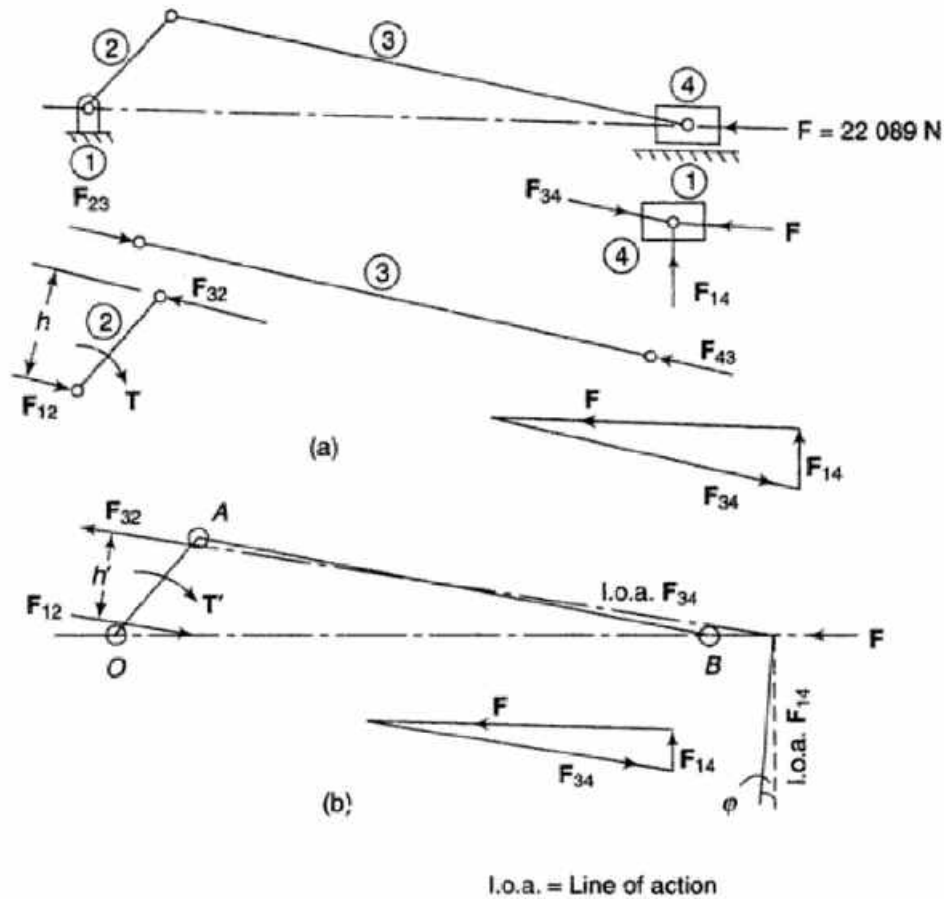
Where R = radius of pin

r = the distance from the centre of the pin to the resultant force($N \sec \phi$)

DIFFERENT POSSIBILITIES OF LINK 3 IN EQUILIBRIUM



Q. The length of crank of a slider crank mechanism is 300 mm and of connecting rod is 1.25 m. the diameters of the journals at the crosshead, crankpin, and crankshaft are 80 mm, 120 mm and 140 mm respectively. the steam pressure on the piston is 450 kN/m^2 which has a diameter of 250 mm. coefficient of friction between the cross head and the guides is 0.07 and for journals it is 0.05. Solve graphically to determine the magnitude and sense of torque T applied on input link for the static equilibrium of the linkage. take coefficient of friction for the journals as 0.4 instead of 0.05 (a fictitious high value of coefficient of friction is taken so that friction circles of reasonable diameter may be drawn on a smaller scale).



$$F_{34} = 22\,500 \text{ N}$$

Now,

$$F_{34} = -F_{43} = F_{23} = F_{32}$$

$$T = F_{23} \times h = 22\,500 \times 0.261 = 5872.5 \text{ N.m clockwise}$$

When friction is considered [Fig. 12.26(b)],

$$\text{Radius of friction circle at } O = 0.4 \times \frac{140}{2} = 28 \text{ mm}$$

$$\text{Radius of friction circle at } A = 0.4 \times \frac{120}{2} = 24 \text{ mm}$$

$$\text{Radius of friction circle at } B = 0.4 \times \frac{80}{2} = 16 \text{ mm}$$

As the crank moves counter-clockwise, $\angle OAB$ decreases. AB rotates clockwise relative to OA . Thus, tangent at A is to be such that a counter-clockwise friction couple is obtained.

At B , $\angle OBA$ is increasing. Therefore, BA rotates clockwise relative to the piston. Thus the tangent to the friction circle is to be such that it gives a counter-clockwise friction couple.

$$\text{For the sliding pair, } \phi = \tan^{-1} 0.07 = 4^\circ$$

The point of intersection of F_{34} and F gives the point of concurrency for the forces on the slider. Force F_{14} , i.e. the reaction of the guide, is inclined to the perpendicular to the slider path, and passes through the point of concurrency.

By drawing a force triangle for the forces acting on the slider, F_{34} is obtained.

The force at A is equal, parallel and opposite to F_{32} and tangent to the friction circle such that a clockwise friction couple is obtained.

$$T' = F_{32} \times h' = 22\,200 \times 0.202 = 4484 \text{ N.m clockwise}$$

The applied load on the piston of an offset slider-crank linkage shown in Fig.1 is 100 N, and the coefficient of friction between the slider and the guide is 0.27, using graphical method determine the magnitude and sense of torque τ_2 applied on OA for the static equilibrium of the linkage.

