

Logic Expressions, Logic Functions & Gates

Logic gates are the fundamental building blocks of digital systems.

The name Logic gate is derived from the ability of such a device to make decisions, in the sense that it produces one output level when some combinations of input levels are present & a different output level when other combinations of input levels are present.

There are just 3 basic types of gates - AND, OR and NOT.

The interconnection of gates to perform a variety of logical operations is called logic design.

Logic gates are electronic circuits because they are made up of a no: of electronic devices & components. Inputs & outputs of logic gates can occur only in 2 levels. These 2 levels are termed HIGH & LOW, or TRUE & FALSE, or ON & OFF, or simply 1 & 0.

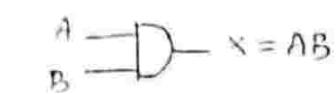
A table which lists all the possible combinations of input variables & the corresponding outputs is called a truthtable.

A logic in which the voltage levels represent logic 1 & logic 0.

Level logic may be true logic or negative logic.

Logic Gates

The AND Gate



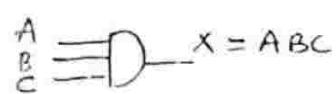
(a) logic symbol.

Inputs Output

A	B	x
0	0	0
0	1	0
1	0	0
1	1	1

(b) Truth table

Fig: A two input AND gate.



(a) logic symbol

Inputs Output

A	B	C	x
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(b) Truth table

Fig: A 3 input AND gate

A device whose output is 1, if & only if all its inputs are 1. Hence the AND gate is also called an all or nothing gate.

The OR Gate

$$A \rightarrow B \quad x = A + B$$

(a) logic symbol

Inputs		Output
A	B	x
0	0	0
0	1	1
1	0	1
1	1	1

(b) Truth table

A device whose output is 1, even if one of its inputs is 1.

Hence an OR gate is also called an any or all gate. It can also be called an inclusive OR gate because it includes the condition both the inputs can be present.

The Not Gate (Inverter)

A not gate, also called an inverter, has only one input & only one output.

It is a device whose output is always -the complement of its input.

$$A \rightarrow \overline{A} \quad x = \overline{A}$$

(a) logic symbol

Input	Output
A	x
0	1
1	0

(b) Truth table

The Universal Gates

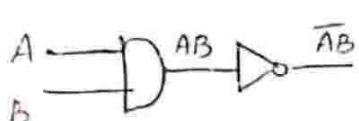
There are 2 universal gates (NAND & NOR) each of which can also realize logic circuits single-handedly.

The NAND & NOR gates are therefore, called universal building blocks.

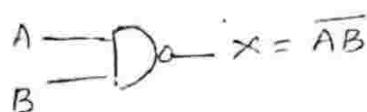
Both NAND & NOR gates can perform all the three basic logic functions (AND, OR & NOT)

∴ AOI logic can be converted to NAND logic or NOR logic.

The NAND gate



(a) AND gate followed
by a NOT gate



(b) A two input
NAND gate
(logic symbol)

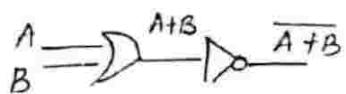
Truth table

Inputs		Output X
A	B	
0	0	1
0	1	1
1	0	1
1	1	0

(c)

NOR Gate

III



Inputs		Outputs
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

NOR means NOT OR. i.e. the OR output is NOTed.

So a NOR gate is a combination of an OR gate & a NOT gate.

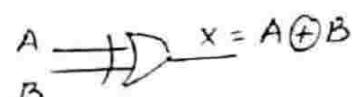
NOR is a contraction of the word NOT-OR.

The Exclusive-OR (x-OR) Gate

An x-OR gate is a two input, one output logic circuit, whose output assumes a logic 1 state when one & only one of its 2 inputs assumes a logic 1 state.

Since an x-OR gate produces an output 1 only when the inputs are not equal it is called an anti-coincidence gate or inequality detector.

The output of an x-OR gate is the modulo sum of its 2 inputs.



② Logic symbol

Inputs		Outputs
A	B	$X = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0



(c) x-OR gate as an inverter

fig: Exclusive-OR gate

The Exclusive-NOR Gate

An X-NOR gate is a combination of an X-OR gate & a NOT gate.

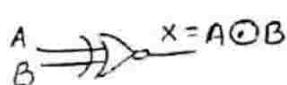
The X-NOR gate is a two input, one output logic circuit, whose output assumes a 1 state only when both the inputs assume a 0 state or when both the inputs assume a 1 state. The output assumes a 0 state, when one of inputs assumes a 0 state and the other a 1 state. It is also called a coincidence gate, because its output is 1 only when its inputs coincide.

It can be used as an equality detector because it outputs a 1 only when its inputs are equal.

The expression for the output of this gate is written as

$$X = A \odot B = AB + \bar{A}\bar{B} = \overline{A \oplus B} = \overline{AB + \bar{A}\bar{B}}$$

and read as 'X' is equal to 'A exnor B'.



(a) Logic symbol.

Inputs Output

A	B	X = A ⊕ B
0	0	1
0	1	0
1	0	0
1	1	1

(b) TruthTable



(c) X-NOR gate as an inverter.

Boolean Laws

Boolean Algebra

Boolean algebra is a system of mathematical logic

It is an algebraic system consisting of the set of elements {0,1}, two binary operators called OR & AND and one unary operator called NOT.

Boolean algebra differs from both the ordinary algebra & the binary number system.

In Boolean algebra, $A+A=A$ & $A \cdot A=A$, because the variable A has only a logical value.

No negative or fractional numbers in Boolean algebra.

In Boolean algebra If $A=1$, then $A \neq 0$
If $A=0$, then $A \neq 1$

AXIOMS AND LAWS OF BOOLEAN ALGEBRA

Axioms or postulates of Boolean algebra are a set of logical expressions that we accept without proofs upon which we can build a set of useful theorems.

Axioms are nothing more than the definitions of the 2 basic logic operations:

AND operation

$$\text{Axiom 1: } 0 \cdot 0 = 0$$

$$\text{Axiom 2: } 0 \cdot 1 = 0$$

$$\text{Axiom 3: } 1 \cdot 0 = 0$$

$$\text{Axiom 4: } 1 \cdot 1 = 1$$

OR operation

$$\text{Axiom 5: } 0 + 0 = 0$$

$$\text{Axiom 6: } 0 + 1 = 1$$

$$\text{Axiom 7: } 1 + 0 = 1$$

$$\text{Axiom 8: } 1 + 1 = 1$$

NOT operation

$$\text{Axiom 9: } T = 0$$

$$\text{Axiom 10: } \overline{0} = 1$$

Complementation Laws

The term complement simply means to invert, ie to change 0s to 1s & 1s to 0s. The five laws of complementation

as follows:

$$\text{Law 1 : } \bar{0} = 1$$

$$2 : \bar{1} = 0$$

$$3 : \text{If } A = 0 \text{ then } \bar{A} = 1$$

$$4 : \text{If } A = 1 \text{ then } \bar{A} = 0$$

$$5 : \bar{\bar{A}} = A \text{ (double complementation law)}$$

AND Laws

The four AND laws are as follows

$$\text{Law 1 : } A \cdot 0 = 0 \text{ (Null law)}$$

$$\text{Law 2 : } A \cdot 1 = A \text{ (Identity law)}$$

$$\text{Law 3 : } A \cdot A = A$$

$$\text{Law 4 : } A \cdot \bar{A} = 0$$

OR Laws

The four OR laws are as follows:

$$\text{Law 1 : } A + 0 = A \text{ (Null law)}$$

$$\text{Law 2 : } A + 1 = 1 \text{ (Identity law)}$$

$$3 : A + A = A$$

$$4 : A + \bar{A} = 1$$

Commutative Laws

commutative law allows change in position of AND or OR variables. There are 2 commutative laws.

$$\text{Law 1 : } A + B = B + A$$

This law can be extended to any no: of variables.

$$\text{For eg } A + B + C = B + C + A = C + A + B = B + A + C$$

$$\text{Law 2 : } A \cdot B = B \cdot A$$

✓
This Law can be extended to any no. of variables.

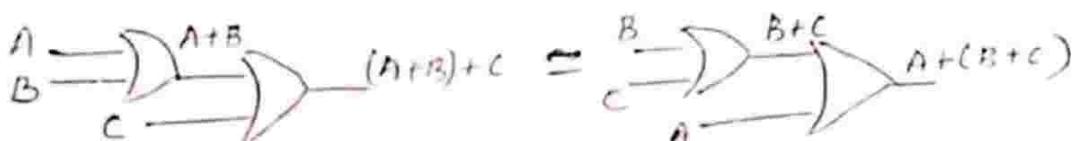
For eg., $A \cdot B \cdot C = B \cdot C \cdot A = C \cdot A \cdot B = B \cdot A \cdot C$

Associative Laws

The associative laws allow grouping of variables.

There are 2 associative laws

Law 1 : $(A+B)+C = A+(B+C)$



Law 2 : $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

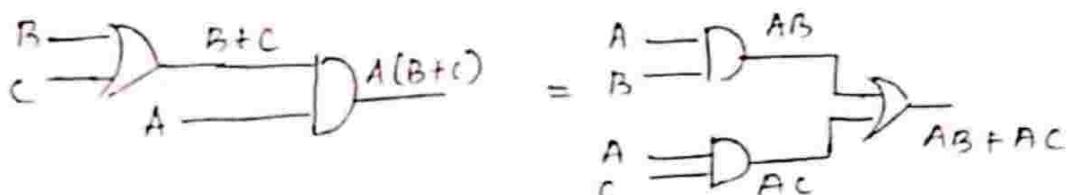


This law can be extended to any number of variables. for eg

$$A(BCD) = (ABC)D = (AB)(CD)$$

Distributive Laws

Law 1 : $A(B+C) = AB + AC$



Law 2 : $A+BC = (A+B)(A+C)$

$$\begin{aligned} \text{RHS} &= (A+B)(A+C) \\ &= AA + BA + AC + BC \\ &= A + AC + AB + BC \end{aligned}$$

$$\begin{aligned}
 &= A(1 + C+B) + BC \quad (\because 1+C+B = 1+B = 1) \\
 &= A \cdot 1 + BC \\
 &= A + BC \\
 &= \text{LHS}
 \end{aligned}$$

Redundant Literal Rule (RLR)

Law 1 : $A + \bar{A}B = A + B$

$$\begin{array}{ccc}
 \overline{A} & \overline{B} & A + \bar{A}B \\
 \overline{A} & B & \overline{A} + B
 \end{array}$$

$$A + \bar{A}B = (A + \bar{A})(A + B)$$

$$= 1 \cdot (A+B)$$

$$= A+B$$

Law 2 : $A(\bar{A} + \bar{B}) = AB$

$$\begin{aligned}
 A(\bar{A} + \bar{B}) &= A\bar{A} + A\bar{B} \\
 &= 0 + AB \\
 &= AB
 \end{aligned}$$

Complement of a term appearing in another term is redundant.

Idempotence Laws

Law 1 : $A \cdot A = A$

If $A=0$, then $A \cdot A = 0 \cdot 0 = 0 = A$

If $A=1$, then $A \cdot A = 1 \cdot 1 = 1 = A$

Law 2 : $A + A = A$

If $A=0$, then $A+A = 0+0 = 0 = A$

If $A=1$, then $A+A = 1+1 = 1 = A$

Absorption Laws

There are two laws

$$\text{Law 1 : } A + A \cdot B = A$$

$$A + A \cdot B = A(1+B) = A \cdot 1 = A$$

Therefore

$$A + A \cdot \text{Any term} = A$$

$$\text{Law 2 : } A(A+B) = A$$

Algebraically we have

$$\begin{aligned} A(A+B) &= A \cdot A + AB = A + AB \\ &= A(1+B) = A \cdot 1 = A \end{aligned}$$

$$A(A + \text{Any term}) = A$$

Consensus Theorem (Included Factor Theorem)

Theorem 1:

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

$$\begin{aligned} LHS &= AB + \bar{A}C + BC \\ &= AB + \bar{A}C + BC(A + \bar{A}) \\ &= AB + \bar{A}C + BCA + BC\bar{A} \\ &= AB(1+C) + \bar{A}C(1+B) \\ &= AB(1) + \bar{A}C(1) \\ &= AB + \bar{A}C \\ &= RHS \end{aligned}$$

Theorem 2:

$$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$$

$$\begin{aligned} LHS &= (A+B)(\bar{A}+C)(B+C) \\ &= (A\bar{A} + AC + B\bar{A} + BC)(B+C) \end{aligned}$$

$$\begin{aligned}
 &= (\bar{A}C + BC + \bar{A}\bar{B}) (B + C) \\
 &= ABC + BBC + \bar{A}BB + \bar{A}C + BC + \bar{A}BC \\
 &= ABC + BC + \bar{A}B + AC + BC + \bar{A}BC \\
 &= AC(1+B) + \bar{A}B(1+C) + AC + BC \\
 &= ABC + \bar{A}B + AC + BC \\
 &= AC + \bar{A}B + BC \\
 &= AC + \bar{A}B + BC
 \end{aligned}$$

$$\begin{aligned}
 \text{RHS} &= (A+B)(\bar{A}C) \\
 &= A\bar{A} + \bar{A}B + A\bar{C} + BC \\
 &= \bar{A}B + A\bar{C} + BC
 \end{aligned}$$

Transposition Theorem

$$AB + \bar{A}C = (A+C)(\bar{A}+B)$$

$$\begin{aligned}
 \text{RHS} &= (A+C)(\bar{A}+B) \\
 &= A\bar{A} + C\bar{A} + AB + CB \\
 &= 0 + \bar{A}C + AB + BC \\
 &= \bar{A}C + AB + BC(A+\bar{A}) \\
 &= \bar{A}C + AB + ABC + \bar{A}BC \\
 &= AB + ABC + \bar{A}C + \bar{A}BC \\
 &= AB + \bar{A}C \\
 &= LHS
 \end{aligned}$$

De Morgan's Theorem

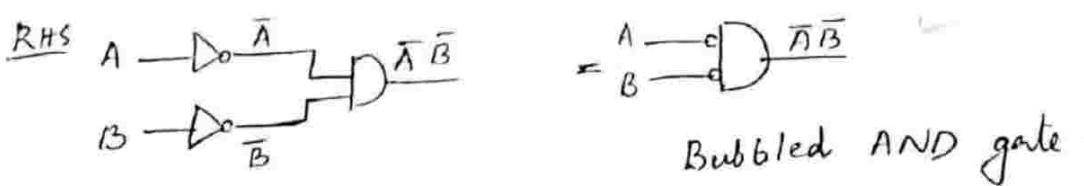
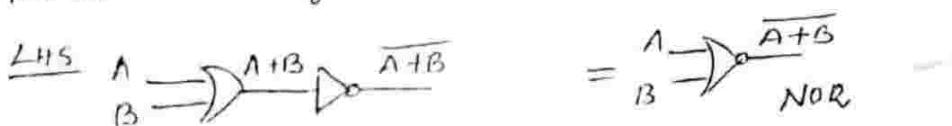
De Morgan's theorem represents two of the most powerful laws in Boolean algebra.

$$\text{Law 1: } \overline{A+B} = \overline{A}\overline{B}$$

This law states that the complement of a sum of variables is equal to the product of their individual complements.

The complement of two or more variables ORed together is the same as the AND of the complements of each of the individual variables.

Each side of this law can be represented as:



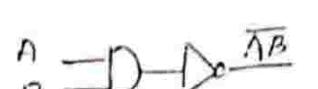
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	?

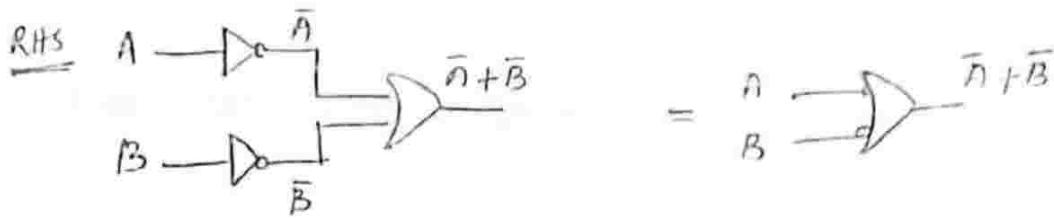
A	B	$\overline{A} \cdot \overline{B}$	$\overline{A+B}$
0	0	1	1
0	1	0	0
1	0	0	1
1	1	0	0

i.e. NOR gate is equivalent to bubbled AND gate

Eg: $A+B+C+D = \overline{\overline{A}\overline{B}\overline{C}\overline{D}}$...

$$\text{Law 2: } \overline{AB} = \overline{A} + \overline{B}$$

LHS  = 



A	B	\overline{AB}	A	B	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
0	0	1	0	0	1	1	1
0	1	1	0	1	1	0	1
1	0	1	1	0	0	1	1
1	1	0	1	1	0	0	0

(ii) shows that the NAND gate is equivalent to a bubbled OR gate.

$$\overline{ABCD} = \bar{A} + \bar{B} + \bar{C} + \bar{D} + \dots$$

It may be seen that the transformations

$$\overline{A+B} = \bar{A}\bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

can be extended to complicated expressions by the following 3 steps:

1. Complement the entire given function

2. change all the ANDs to ORs & all the ORs to ANDs

3. Complement each of the individual variables.

4. change all 0's to 1's & 1's to 0's

This procedure is called demorganization or complementation of switching expressions. It is

$$f(A, B, C, \dots, 0, 1, +, -)_c = f(\bar{A}, \bar{B}, \bar{C}, \dots, 1, 0, +)$$

Apply Demorgan's theorems to the expression

$$f = \overline{AB(CD + EF)(\overline{AB} + \overline{CD})}$$

The given expression is

$$\begin{aligned} f &= \overline{AB(CD + EF)(\overline{AB} + \overline{CD})} \\ &= \overline{\overline{AB}} + \overline{CD + EF} + \overline{\overline{AB} + \overline{CD}} \\ &= \overline{A} \cdot \overline{B} + \overline{CD} + \overline{EF} + \overline{\overline{AB}} \cdot \overline{\overline{CD}} \\ &= AB + \overline{CD} + \overline{EF} + ABCD \\ &= AB + (\overline{C} + \overline{D}) + (\overline{E} + \overline{F}) + ABCD \\ &= AB + (\overline{C} + \overline{D}) + (E + F) + ABCD \end{aligned}$$

✓ Reduce the expression $f = \overline{\overline{AB} + \overline{A} + AB}$

$$\begin{aligned} f &= \overline{AB} \cdot \overline{\overline{A}} \cdot \overline{AB} \\ &= AB \cdot A \cdot \overline{AB} \\ &= AB \cdot \overline{AB} \cdot A \\ &= 0 \cdot A = \underline{\underline{0}} \end{aligned}$$

DUALITY

Given a Boolean identity, we can produce a dual identity by changing all '+' signs to '·' signs, all '·' signs to '+' signs and complementing all 0s & 1s.

The variables are not complemented in this process. The implication of the duality concept is that once a theorem or statement is proved, the dual also thus stands proved. This is called the principle of duality.

$$f(A, B, C, \dots, 0, 1, +, \cdot) = f(A, B, C, \dots, 1, 0, \cdot, +)$$

Relations between complement & dual.

$$f_c(A, B, C, \dots) = \overline{f(\bar{A}, \bar{B}, \bar{C}, \dots)} \\ = f_d(\bar{A}, \bar{B}, \bar{C}, \dots)$$

$$f_d(A, B, C, \dots) = \overline{f(\bar{A}, \bar{B}, \bar{C}, \dots)} \\ = f_c(\bar{A}, \bar{B}, \bar{C}, \dots)$$

The first relation states that the complement of a function $f(A, B, C, \dots)$ can be obtained by complementing all the variables in the dual function $f_d(A, B, C, \dots)$.

The second relation states that the dual can be obtained by complementing all the literals in $\overline{f(A, B, C, \dots)}$

1.1.1 Duals

Given Expression	Dual
1. $A \oplus 1 = 1$	$\bar{A} \oplus 0 = 0$
2. $0 \cdot 1 = 0$	$1 \cdot 0 = 1$
3. $0 \cdot 0 = 0$	$1 \cdot 1 = 1$
4. $1 + 1 = 1$	$0 + 0 = 0$
5. $A \cdot 0 = 0$	$\bar{A} + 1 = 1$
6. $\bar{A} \cdot 1 = A$	$A + 0 = A$
7. $A \cdot A = A$	$\bar{A} + \bar{A} = \bar{A}$
8. $A \cdot \bar{A} = 0$	$A + \bar{A} = 1$
9. $\bar{A} \cdot B = B \cdot A$	$A + \bar{B} = \bar{B} + A$
10. $A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$
11. $A \cdot (B + C) = AB + AC$	$A + BC = (A + B)(A + C)$
12. $A(A + B) = A$	$A + AB = A$
13. $A \cdot (\bar{A} \cdot B) = A \cdot B$	$A + A \cdot B = A + B$
14. $\bar{A}B = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A}B$
15. $(A + B)(A + C)(B + C) = (A + B)(\bar{A} + \bar{C})$	$AB + \bar{A}C + BC = AB + \bar{AC}$
16. $(A + C)(\bar{A} + B) = AB + \bar{AC}$	$AC + \bar{AB} = (A + B)(\bar{A} + C)$
17. $A + \bar{B}C = (A + \bar{B})(A + C)$	$A(\bar{B} + C) = (\bar{A}\bar{B} + AC)$
18. $(A + B)(C + D) = AC + AD + BC + BD$	$(AB + CD) = (A + C)(A + D)(B + C)(B + D)$
19. $A \cdot B = AB + A\bar{B}$	$AB = (A + B)(\bar{A} + B)(A + \bar{B})$
20. $A + B \overline{(C + DE)} = A + BCDE$	$A[B + \overline{(C + D + E)}] = A \cdot (B + \bar{C} + D + E)$
21. $\overline{AB + A + AB} = 0$	$\overline{\overline{A + B} \cdot \bar{A}} \cdot (A + B) = 1$
22. $AB + \overline{AC} + A\bar{B}C (AB + C) = 1$	$(A + B)(\overline{A + C}) \cdot [(A + \bar{B} + C) + (A + B)C] = 0$
23. $ABD + ABCD = ABD$	$(A + B + D)(A + B + C + D) = (A + B + D)$
24. $\overline{AB + ABC} + A(B + A\bar{B}) = 0$	$(A + B) \cdot (A + B + C) - (A + [B(A + \bar{B})]) = 1$
25. $A + \bar{B}C(A + \bar{B}C) = A + \bar{B}C$	$A \cdot [(\bar{B} + C) + A \cdot (\bar{B} + C)] = A \cdot (\bar{B} + C)$

Reducing Boolean Expressions

Reduce the expression $f = A[B + \bar{C}(\bar{A}B + A\bar{C})]$

$$\begin{aligned}
 f &= A[B + \bar{C}(\bar{A}B + A\bar{C})] \\
 &= A[B + \bar{C}(\bar{A} + \bar{B}) \cdot (\bar{A} \cdot C)] \\
 &= A[B + \bar{C}(\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}C + \bar{B}C)] \\
 &= A[B + \bar{C}(\bar{B} + \bar{A}\bar{B} + \bar{A}C + \bar{B}C)] \\
 &= A[B + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}C\bar{C} + \bar{B}C\bar{C}] \\
 &= A[B + \bar{A}\bar{C} + \bar{B}\bar{C}] \\
 &= AB + A\bar{A}\bar{C} + A\bar{B}\bar{B}\bar{C} \\
 &= \underline{\underline{AB}}
 \end{aligned}$$

Reduce the expression $f = A+B[AC + (B+\bar{C})D]$

$$\begin{aligned}
 f &= A + B[AC + BD + \bar{C}D] \\
 &= A + ABC + BBD + B\bar{C}D \\
 &= A(1+BC) + BD(1+\bar{C}) \\
 &= A + BD \\
 &= \underline{\underline{A+BD}}
 \end{aligned}$$

Reduce the expression $f = (\overline{A+\bar{B}C})(A\bar{B} + ABC)$

$$\begin{aligned}
 f &= (\overline{A+\bar{B}C})(A\bar{B} + ABC) \\
 &= \overline{A}\overline{\bar{B}C}(A\bar{B} + ABC) \\
 &= \overline{ABC}A\bar{B} + \overline{ABC}ABC \\
 &= A\bar{A}B\bar{B}C + A\bar{A}BCBC \\
 &= 0 + 0 = \underline{\underline{0}}
 \end{aligned}$$

Reduce the expression $f = (B+BC)(B+\bar{B}C)(B+D)$

$$\begin{aligned}
 f &= (BB + B\bar{B}C + BBC + BC\bar{B}C)(B+D) \\
 &= BB + BBC(B+D)
 \end{aligned}$$

$$\begin{aligned}
 &= BB(1+C)(B+D) \\
 &= B(B+D) \\
 &= BB + BBD \\
 &= BB(1+D) = \underline{\underline{B}}
 \end{aligned}$$

Boolean Functions And their Representation

A function of 'n' Boolean variables denoted by $f(x_1, x_2, \dots, x_n)$ is another variable of algebra & takes one of 2 possible values, 0 & 1.

The various ways of representing a given function are given in this section.

Sum - of - products (SOP) form: This form is also called the Disjunctive Normal Form (DNF)

$$\text{For example } f(A, B, C) = \bar{A}B + \bar{B}C$$

Product - of - sums (POS) form: This form is also called Conjunctive Normal Form (CNF).

$$f(A, B, C) = (\bar{A} + \bar{B})(B + C)$$

Standard Sum - of - Products Form:

(In this form, the function is the sum of a no: of product terms where each product term contains all the variables of the function either in complemented or uncomplemented forms.)

The following steps are followed for the expansion of a Boolean expression in SOP form to the standard SOP form:

- (i) Write down all the terms
- (ii) If one or more variables are missing in any term, expand

that term by multiplying it with the sum of each one of the missing variable & its complement.

3) Drop out the redundant terms. \rightarrow

Also the given expression can be directly written in terms of its minterms by using the following procedure:

- 1) Write down all the terms
- 2) Put 1s in terms where variables must be inserted to form a minterm.
- 3) Replace the non-complemented variables by 1s & the complemented variables by 0s & use all combinations of 1s in terms of 0s & 1s to generate minterms.
- 4) Drop out all the redundant terms.

(A product term which contains all the variables of the function either in complemented or uncomplemented form is called a minterm. \rightarrow)

\checkmark If $f(A, B, C) = \bar{A}B + \bar{B}C = \bar{A}B(C + \bar{C}) + \bar{B}C(A + \bar{A})$
 $= \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

(The minterm are denoted as $m_0, m_1, m_2 \dots$

For a 3 variable function $m_0 = \bar{A}\bar{B}\bar{C}$, $m_1 = \bar{A}\bar{B}C$, \dots

$m_2 = \bar{A}B\bar{C}$, $m_3 = \bar{A}BC$, $m_4 = A\bar{B}\bar{C}$, $m_5 = A\bar{B}C$, $m_6 = AB\bar{C}$,

$m_7 = ABC$ \rightarrow

$$f(A, B, C) = m_3 + m_2 + m_5 + m_1$$

$$f(A, B, C) = \sum m(1, 2, 3, 5) \quad \checkmark$$

Standard Product of sums form:

(Each sum is a sum of all the variables. A variable appears in uncomplemented form if it has a value of 0 in the combination and appears in complemented form if it has a value of 1 in the combination)

(A sum term which contains each of the n variables in either complemented or uncomplemented forms is called a maxterm.)

Maxterms are often represented as $M_0, M_1, M_2 \dots$ where the suffixes denote their decimal code

$$f(A, B, C) = M_0 \cdot M_1 \cdot M_2 \cdot M_3 \text{ or simply as}$$

$$f(A, B, C) = \pi M(0, 4, 6, 7)$$

where π represents the product of all maxterms.

function $f(A, B, C) = (\bar{A} + \bar{B})(A + B)$ is given by the product of sums

$$\begin{aligned} f(A, B, C) &= (\bar{A} + \bar{B} + C\bar{C})(A + B + C\bar{C}) \\ &= (\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})(A + B + C)(A + B + \bar{C}) \end{aligned}$$

The expansion of a Boolean expression to the standard POS form is conducted as follows:

- 1) If one or more variables are missing in any sum term, expand that term by adding the products of each of the missing terms & its complement.
- 2) Drop out the redundant terms.

XII

Expand $\bar{A} + \bar{B}$ to maxterms & minterms.

$$\begin{aligned}\bar{A} + \bar{B} &= \bar{A}(B + \bar{B}) + \bar{B}(A + \bar{A}) \\&= \bar{A}B + \bar{A}\bar{B} + \bar{B}A + \bar{B}\bar{A} \\&= \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B} \\&= \bar{A}B + \bar{A}\bar{B} + AB \\&= 01 + 00 + 10 \\&= M_1 + m_0 + m_2 \\&= \sum m(0, 1, 2)\end{aligned}$$

Hence the POS form is $\bar{A}M_3$

Expand $A(B+A)B$ to maxterms & minterms

$$\begin{aligned}A &= A + B\bar{B} = (A+B)(A+\bar{B}) \\B &= B + A\bar{A} = (B+A)(B+\bar{A}) \\A(\bar{B}+A)B &= (A+B)(A+\bar{B})(\bar{B}+A)(B+A)(B+\bar{A}) \\&= (A+B)(A+\bar{B})(A+\bar{B}) (A+B)(\bar{A}+B) \\&= (A+B)(A+\bar{B})(\bar{A}+B) \\&= (00)(01)(10) \\&= M_0 \cdot M_1 \cdot M_2 \\&= \bar{A}M(0, 1, 2)\end{aligned}$$

The maxterm M_3 is missing in the POS form. So, the SOP form will contain only the minterm m_3 . ($\sum m(3)$)

Karnaugh Maps (2, 3, 4)

(a) 2 variable K Map

A \ B	0	1
0	0	1
1	2	3

A \ B	0	1
0	$\bar{A}\bar{B}$	$\bar{A}B$
1	$A\bar{B}$	AB

The mapping of the expression $\sum m(0, 2, 3)$

A \ B	0	1
0	1	0
1	1	1

Mapping the expressions $\bar{A}B + A\bar{B}$

$$\begin{aligned}\bar{A}B + A\bar{B} &= 01 + 10 \\ &= \sum m(1, 2)\end{aligned}$$

K Map

A \ B	0	1
0	0	1
1	1	0

Minimization of SOP Expressions

$$\bar{A}B + A\bar{B} = \bar{A}$$

$$\begin{aligned}f_1 &= \bar{A}\bar{B} + \bar{A}B = 00 + 01 \\ &= \sum m(0, 1)\end{aligned}$$

A \ B	0	1
0	1	1
1	0	0

$$f_1 = \bar{A}$$

$m_0 + m_2$

A	B	0	1
0	1	1	0
1	1	0	1

$f_2 = \bar{B}$

 $m_1 + m_3$

A	B	0	1
0	0	1	1
1	0	1	1

$f_3 = B$

 $m_2 + m_3$

A	B	0	1
0	0	0	0
1	1	1	1

$f_4 = D$

 $m_0 + m_1 + m_2 + m_3$

A	B	0	1
0	1	1	1
1	1	1	1

$f_5 = I$

3 variable k Map

A	BC	00	01	11	10
0	0	1	3	2	4
1	4	5	7	6	0

A	BC	00	01	11	10
0	AB̄C	ĀB̄C	ĀBC	ĀB̄C	AB̄C
1	ĀB̄C	ĀB̄C	ABC	ĀB̄C	ABC

Map the expression, $\bar{A}\bar{B}C + A\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$
 $\leq_m (1, 5, 2, 6, 7)$

A	BC	00	01	11	10
0	0	1	0	1	0
1	0	1	1	1	0

 $Eg: f = \sum_m (1, 2, 4, 6, 7)$

A	BC	00	01	11	10
0	0	1	0	1	0
1	0	1	1	1	1

$f = A\bar{C} + AB + B\bar{C} + \bar{A}B\bar{C}$

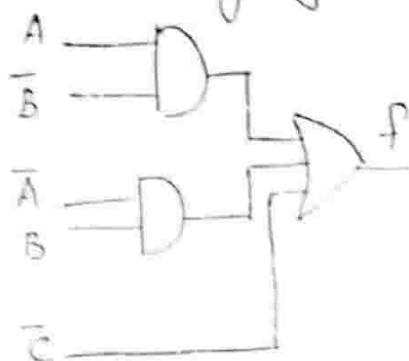
$$\textcircled{1} \quad f = \sum m(0, 2, 3, 4, 5, 6)$$

$A \setminus BC$

	00	01	11	10
00	1		1	
01	1	1		
11				
10				1

$$= \bar{C} + A\bar{B} + \bar{A}B$$

Implementation using gates



Four Variable K Map

$AB \setminus CD$

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$AB \setminus CD$

	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}B\bar{C}\bar{D}$
11	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$ABC\bar{D}$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

$$\textcircled{1} \quad f = \sum m(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$$

$AB \setminus CD$

	00	01	11	10
00	1	1		1
01		1	1	
11	1	1		
10	1	1		1

$$\bar{A}D + A\bar{C} + \bar{B}\bar{D}$$

② $f = \Sigma m(0, 1, 3, 4, 5, 6, 7, 13, 15)$

AB	CD	00	01	11	10
00	Q	1	1	1	
01	Q	1	1	1	1
11		1	1	1	
10					

$$f = \bar{A}\bar{C} + BD + \bar{A}B + \bar{A}D$$

POS = $\pi M(2, 8, 9, 10, 11, 12, 14)$

AB	CD	00	01	11	10
00					0
01					0
11	Q	0	0	0	0
10	Q	0	0	0	0

$$(\bar{A}+B)(\bar{A}+D)(B+\bar{C}+D)$$

③ POS $f = \pi M(4, 6, 11, 14, 15)$

AB	CD	00	01	11	10
00					
01	0				0
11		0	0	0	0
10		0			

$$f = (\bar{A} + \bar{C} + \bar{D})(\bar{C} + \bar{A} + \bar{B})(A + \bar{B} + D)$$

Don't Care Combinations

It often occurs that for certain input combinations, the value of the output is unspecified.

The ^{input} combinations for which the values of the expression are not specified are called don't care combinations or optional.

combinations

The output is a don't care for these invalid combinations.

✓ Reduce the expression $f = \sum m(1, 5, 6, 12, 13, 14) + d(2, 4)$ and implement the real minimal expression in universal logic

AB \ CD	00	01	11	10
00		1 ⁽¹⁾		X
01	X ⁽²⁾	1		1 ⁽³⁾
11	1	1		
10				

SOP minimal

$$= BC + \overline{C}D\overline{A} + \overline{B}\overline{D}D$$

$$= BC + \overline{A}\overline{C}D + B\overline{D}$$

AB \ CD	00	01	11	10
00	0 ⁽¹⁾		0 ⁽²⁾	(X)
01	X		0	
11			0	
10	0	0 ⁽³⁾	0	10

POS = $\sum m(0, 3, 7, 8, 9, 10, 11, 15)$

$\times d(2, 4)$.

POS minimal is

$$f_{min} = (\overline{C} + \overline{D})(\overline{A} + B)(B + D)$$

NOR logic implementation

$$f_{min} = \overline{(B + D)} + \overline{(\overline{A} + B)} + \overline{(\overline{C} + \overline{D})}$$

