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## Course Code: ECT204

 Course Name: SIGNALS AND SYSTEMS1 Determine energy of the signal $x(t)=e^{-2 t} u(t) \quad 3$
2 Plot the waveform of the following signal

$$
x(t)=u(t+1)-2 u(t)+u(t-1)
$$

3 Perform linear convolution of signals $\mathrm{x}_{1}[n]=[2,2,2,2]$ and $\mathrm{x}_{2}[n]=[1,1,1,1]$
4 Find Laplace Transform and sketch ROC for the signal $x(t)=e^{2 t} u(t)+e-{ }_{-}^{3 t} u(t)$
5 State sampling theorem of a band limited Continuous time signal.
6 Find the Nyquist rate and Nyquist interval of the following signal
$x(t)=3 \sin 100 \pi t+2 \cos 200 \pi t$
7
Find DTFT of the signal $x[n]=\frac{1}{2}\left[\left(\frac{1}{2}\right)^{n}+\left(\frac{1}{4}\right)^{n}\right] u[n]$
8 State and prove differentiation property of DTFT
9 Derive the relation between DTFT and Z transform
10 Evaluate the transfer function $\mathrm{H}(\mathrm{z})$ of an LTI system described by
$y[n]-\frac{1}{2} y[n-1]=2 x[n]$

## PART B

(Answer one full question from each module, each question carries 14 marks)

## Module - 1

11 a) Test whether the following signals are periodic or not. If periodic, determine the fundamental period and frequency.

1) $x(t)=3 \cos (5 t+\pi / 6)$
2) $x(t)=e^{(j \pi-2) t}$
b) Evaluate the discrete-time convolution sum with required plots for the following signal $y[n]=3^{n} u[-n+3] * u[n-2]$

12 a) Evaluate the autocorrelation of the signal $x(t)=e^{-t} u(t)$
b) Evaluate the continuous time convolution integral for the following with proper plots.
$\mathrm{y}(\mathrm{t})=\{\mathrm{u}(\mathrm{t})-\mathrm{u}(\mathrm{t}-2)\} * \mathrm{u}(\mathrm{t})$

## Module -2

13 a) Find the trigonometric Fourier Series of the given continuous time square wave $\mathrm{x}(\mathrm{t})$. Plot the magnitude and phase spectra.

b) Using the standard transforms and properties find Fourier Transforms of the following signals
i. $\quad x(t)=t e^{-2 t} u(t)$
ii. $\quad x(t)=\sin (2 \pi t) e^{-t} u(t)$

14 a) A periodic signal has the Fourier series representation


Without determining $x(t)$, find the Fourier series $Y(k)$ and $\omega_{0}^{\prime}$ for
i. $y(t)=x(3 t)$
ii. $\mathrm{y}(\mathrm{t})=\mathrm{dx}(\mathrm{t}) / \mathrm{dt}$
iii. $y(t)=x(t-1)$
b) Find time domain signal represented by the Fourier Series coefficients

$$
\begin{gathered}
\mathrm{X}(\mathrm{k})=\mathrm{j} \delta(\mathrm{k}-1)-\mathrm{j} \delta(\mathrm{k}+1)+\delta(\mathrm{k}-3)+\delta(\mathrm{k}+3), \omega_{0}=2 \pi \\
\text { Module -3 }
\end{gathered}
$$

15 a) A second order LTI system is described by the given differential equation. Use Laplace Transform to determine the transfer function the system
$\frac{d^{2}}{d t^{2}} \mathrm{y}(\mathrm{t})+4 \frac{d}{d t} \mathrm{y}(\mathrm{t})+3 \mathrm{y}(\mathrm{t})=4 \mathrm{x}(\mathrm{t})+2 \frac{d}{d t} \mathrm{x}(\mathrm{t})$
Also find the output $y(t)$ of the system for a given input $x(t)=e^{-2 t} u(t)$.
b) An arbitrary band-limited continuous time signal $x(t)$ is sampled with an impulse train. With spectral details, explain the following conditions
(i)
Oversampling
(ii) Critical Rate
(iii) Aliasing

16 a) Determine a differential equation description for a system with the following transfer function
$\mathrm{H}(\mathrm{s})=\frac{2(s-2)}{(s+1)^{2}(s+3)}$
b) Determine whether the system described by the following system is
i. Both causal and stable
ii. Whether a causal and stable inverse systems exist or not?
$\mathrm{H}(\mathrm{s})=\frac{(s+1)(s+2)}{(s+1)\left(s^{2}+2 s+10\right)}$

## Module -4

17 a) i. Find convolution of the following two sequences using DTFT

$$
\begin{aligned}
& \mathrm{x}_{1}[\mathrm{n}]=[1,2,3,1] \\
& \mathrm{x}_{2}[\mathrm{n}]=[1,2,1,-1]
\end{aligned}
$$

ii. Find Inverse DTFT of

$$
\begin{aligned}
|\mathrm{H}(\omega)|=1 & -\omega_{0} \leq \omega \leq \omega_{0} \\
0 & \text { otherwise }
\end{aligned}
$$

b) Compute DTFS coefficients of the given discrete time signal. Plot its magnitude and frequency spectrum.

$$
x[n]=\cos \left(\frac{6 \pi}{13} n+\frac{\pi}{6}\right)
$$

18 a) Use the defining equation for the DTFS to determine the time domain signal represented by the following DTFS coefficients by inspection

$$
X[k]=2 j \sin \left(\frac{4 \pi}{19} k\right)+\cos \left(\frac{10 \pi}{19} k\right)
$$

b) Given DTFT of $\mathrm{x}[\mathrm{n}]=\mathrm{n}(3 / 4)^{\mathrm{n} \mid} \longleftrightarrow \mathrm{X}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)$.

Using properties of DTFT, find $\mathrm{y}[\mathrm{n}]$ for the following $\mathrm{Y}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)$
i. $\mathrm{Y}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)=\frac{d}{d \Omega} X(e j \Omega)$
ii. $\mathrm{Y}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)=\mathrm{X}\left(\mathrm{e}^{\mathrm{j} \Omega}\right) * \mathrm{X}\left(\mathrm{e}^{\mathrm{j}(\Omega-\pi / 2)}\right)$

## Module -5

19 a) Determine the Z Transform and ROC for the following signal. Sketch the ROC, poles and zeroes in the Z-plane.
$x[n]=(2 / 3)^{|n|}$

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b) Write the impulse response of the system function whose algebraic expression is given below. Also check and justify the causality and Stability.

$$
H(z)=\frac{1}{\left(1-\frac{1}{2} z^{-1}\right)}+\frac{1}{\left(1-2 z^{-1}\right)}, \quad \frac{1}{2}<|z|<2
$$

20 a) Evaluate the inverse Z-Transform by partial fraction method for the given $\mathrm{X}(\mathrm{z})$.

$$
X(z)=\frac{3-\frac{5}{6} z^{-1}}{\left(1-\frac{1}{4} z^{-1}\right)\left(1-\frac{1}{3} z^{-1}\right)}, \quad|z|>\frac{1}{3}
$$

b) Evaluate Z-Transform of the following.
i. $\quad x[n]=\left[r^{n} \cos \omega_{0} n\right] u[n]$
ii. $\quad x[n]=n\left(\frac{1}{3}\right)^{n} u[n]$

