

## MODULE 1

### PLAIN AND REINFORCED CONCRETE

#### Plain Concrete

Concrete may be defined as any solid mass made by the use of a cementing medium; the ingredients generally comprise sand, gravel, cement and water. That the mixing together of such disparate and discrete materials can result in a solid mass (of any desired shape), with well-defined properties, is a wonder in itself. Concrete has been in use as a building material for more than a hundred and fifty years. Its success and popularity may be largely attributed to

- (1) Durability under hostile environments (including resistance to water),
- (2) Ease with which it can be cast into a variety of shapes and sizes, and
- (3) Its relative economy and easy availability.

The main strength of concrete lies in its compressive-bearing ability, which surpasses that of traditional materials like brick and stone masonry. Advances in concrete technology, during the past four decades in particular, have now made it possible to produce a wide range of concrete grades, varying in mass density ( $1200 - 2500 \text{ kg/m}^3$ ) and compressive strength ( $10 - 100 \text{ MPa}$ ).

Concrete may be remarkably strong in compression, but it is equally remarkably weak in tension. Its tensile strength is approximately one-tenth of its compressive strength. Hence, the use of plain concrete as a structural material is limited to situations where significant tensile stresses and strains do not develop, as in hollow (or solid) block wall construction, small pedestals and mass concrete applications (in dams, etc.).

#### Reinforced Concrete

Concrete would not have gained its present status as a principal building material, but for the invention of reinforced concrete, which is concrete with steel bars embedded in it. The idea of reinforcing concrete with steel bars resulted in a new composite material, having the potential of resisting significant tensile stresses, which was impossible. Thus, the construction of load-bearing flexural members, such as beams and slabs, became viable with this new material. The steel bars (embedded in the tension zone of the concrete) compensate for the concrete's incapacity for tensile resistance, effectively taking up all the tension, without separating from the concrete. The bond between steel and the surrounding concrete ensures strain compatibility, i.e., the strain at any point in the steel is equal to that in the adjoining concrete. Moreover, the

reinforcing steel imparts ductility to a material that is otherwise brittle. In practical terms, this implies that if a properly reinforced beam were to fail in tension, then such a failure would, fortunately, be preceded by large deflections caused by the yielding of steel, thereby giving ample warning of the impending collapse.

Tensile stresses occur either directly or indirectly. Temperature and shrinkage effects may also induce tensile stresses. In all such cases, reinforcing steel is essential, and should be appropriately located. If insufficient steel is provided, cracks would develop and propagate, and could possibly lead to failure.

Reinforcing steel can also supplement concrete in bearing compressive forces, as in columns provided with longitudinal bars. These bars need to be confined by transverse steel ties, in order to maintain their positions and to prevent their lateral buckling. The lateral ties also serve to confine the concrete, thereby enhancing its compression load-bearing capacity.

## CHARACTERISTICS OF STRUCTURAL DESIGN

The design of a structure must satisfy three basic requirements:

- 1) Stability to prevent overturning, shifting or buckling of the structure, or parts of it, under the action of loads;
- 2) Strength to resist safely the stresses induced by the loads in the various structural members; and
- 3) Serviceability to ensure satisfactory performance under service load conditions – which implies providing adequate stiffness and reinforcements to constrain deflections, crack-widths and vibrations within acceptable limits, and also providing impermeability and durability (including corrosion-resistance), etc.

There are two other considerations that a sensible designer ought to bear in mind, viz., economy and aesthetics. One can always design a massive structure, which has more-than-adequate stability, strength and serviceability, but the ensuing cost of the structure may be exorbitant, and the end product, far from aesthetic.

## DESIGN CODES AND HANDBOOKS

### Purpose of Codes

National building codes have been formulated in different countries to lay down guidelines for the design and construction of structures. The codes have evolved from the

collective wisdom of expert structural engineers, gained over the years. These codes are periodically revised to bring them in line with current research, and also, current trends.

The codes serve at least four distinct functions:

- Firstly, they ensure adequate structural safety, by specifying certain essential minimum requirements for design.
- Secondly, they render the task of the designer relatively simple; often, the results of sophisticated analyses are made available in the form of a simple formula or chart.
- Thirdly, the codes ensure a measure of consistency among different designers.
- Finally, they have some legal validity, in that they protect the structural designer from any liability due to structural failures that are caused by inadequate supervision and/or faulty material and construction.

### **Basic Code for Design**

The design procedures, described in this book, conform to the following Indian code for reinforced concrete design, published by the Bureau of Indian Standards, New Delhi:

**IS 456 : 2000 — Plain and reinforced concrete — Code of practice (fourth revision)**

### **Loading Standards**

The loads to be considered for structural design are specified in the following loading standards:

**IS 875 (Parts 1-5) : 1987 — Code of practice for design loads (other than earthquake) for buildings and structures (second revision)**

**Part 1 : Dead loads**

**Part 2 : Imposed (live) loads**

**Part 3 : Wind loads**

**Part 4 : Snow loads**

**Part 5 : Special loads and load combinations**

**IS 1893 : 2002 — Criteria for earthquake resistant design of structures (fifth revision).**

### **Design Handbooks**

The Bureau of Indian Standards has also published the following handbooks, which serve as useful supplements to the 1978 version of the Code. Although the handbooks need to be updated to bring them in line with the recently revised (2004 version) of the Code, many of the provisions continue to be valid (especially with regard to structural design provisions).

**SP 16 : 1980 — Design Aids (for Reinforced Concrete) by IS 456 : 1978**

**SP 24 : 1983 - Exploratory Handbook on IS 456 : 1978**

**SP 34 : 1987 - Handbook on Concrete Reinforcement and Detailing**

**SP 23 : 1982 — Design of Concrete Mixes**

### **Design Philosophies**

Over the years, various design philosophies have evolved in different parts of the world, with regard to reinforced concrete design.

1. Working Stress Method
2. Load factor method
3. Limit state Method

#### **Working Stress Method**

This method of design was the oldest one. It is based on the elastic theory and assumes that both steel and concrete are elastic and obey Hooke's law. It means that the stress is directly proportional to strain up to the point of collapse. Based on the elastic theory, and assuming that the bond between steel and concrete is perfect, permissible stresses of the materials are obtained. The basis of this method is that the permissible stresses are not exceeded anywhere in the structure when it is subjected to worst combination of working loads.

In this method, the ultimate strength of concrete and yield stress of steel are divided by factors of safety to obtain permissible stresses. These factors of safety take into account the uncertainties in manufacturing of these materials. As per IS 456, a factor of safety of 3 is to be used for bending compressive stresses in concrete and 1.78 for yield strength of steel.

The main drawbacks of the working stress method of design are as follows:

- (i) It assumes that concrete is elastic which is not true as the concrete behaves in elastically even at low level of stresses.
- (ii) It uses factors of safety for stresses only and not for loads. Hence, this method does not give true margin of safety with respect to loads because we do not know the failure load.
- (iii) It does not use any factor of safety with respect to loads. It means, there is no provision for the uncertainties associated with the estimation of loads.
- (iv) It does not account for shrinkage and creep which are time dependent and plastic in nature.

- (v) This method gives uneconomical sections.
- (vi) It pays no attention to the conditions that arise at the time of collapse.

#### Limit state method

The object of limit state design is based on the concept of achieving an acceptable probability that a structure will not become unservable in its lifetime for the use for which it is intended. It should be able to withstand safely all the loads that are going to act on it throughout its life along with satisfying the serviceability requirements. To ensure proper degree of safety and serviceability, the design must include all relevant limit states.

The aim of this method is that the structure should be able to withstand safely all the load that are liable to act on it throughout its life and it should also satisfy the serviceability requirements of limiting deflection and cracking. Limit state is defined as the acceptable limit of safety and serviceability requirements before failure.

This method is based on the actual stress-strain curves of steel and concrete. For concrete, the stress-strain curve is nonlinear. In this method, partial safety factors are applied to get design values of stresses. Design loads are obtained by multiplying partial safety factors of load to the working loads. This method is more economical as it gives thinner sections.

The two major limit states which are usually considered are the following:

1. **The ultimate strength limit state, or the limit state of collapse,** which deals with the strength and stability of the structure under the maximum overload it is expected to carry. This implies that no part or whole of the structure should fail apart under any combination of expected overload.
2. **The serviceability limit state** which deals with conditions such as deflection, cracking of the structure under service loads, durability (under a given environment in which the structure has been placed), overall stability (i.e. resistance to collapse of the structure due to an accident such as a gas explosion), excessive vibration, fire resistance, fatigue, etc.

Sl. No.	Working Stress Method	Limit State Method
(1)	This method is based on the plastic theory which assumes that concrete and steel are elastic and the stress-strain curve is linear for both.	This method is based on the actual stress-strain curves of steel and concrete. For concrete, the stress-strain curve is non-linear.
(2)	In this method the factor of safety etc. are applied to the yield stresses or the permissible stresses.	In this method partial safety factors are applied to get design values of stresses.
(3)	No factor of safety is used for loads.	Design loads are obtained by multiplying partial safety factors of load to the working loads.
(4)	Exact margin of safety is not known.	Exact margin of safety is known.
(5)	This method gives thicker sections, so less economical.	This method is more economical as it gives thinner sections.
(6)	This method accounts for the actual loads, permissible stresses and factors of safety are known. So it is called as deterministic method.	This method is based upon the probabilistic approach which depends upon the actual data or experience, hence it is called as non-deterministic method.

## Working load

### CHARACTERISTIC LOAD AND CHARACTERISTIC STRENGTHS

#### Characteristic strength

The strengths that one can safely assume for the materials (steel and concrete) are called their characteristic strengths.

#### Characteristic strength of concrete

The term 'characteristic strength' means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall. It is denoted by  $f_{ck}$  in N/mm<sup>2</sup>. The value of  $f_{ck}$  for different grades of concrete are specified in IS 456-2000, Table 2.

#### Characteristic strength of steel

The characteristic strength of steel is taken as the minimum yield stress/0.2 percent proof stress specified in the relevant Indian Standard Specifications. In case of mild steel it is taken as minimum yield strength and in case of IISYSD bars it is taken as 0.2 percent proof stress.

#### Characteristic load

The term 'characteristic load' means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. The maximum working load that the structure has to withstand and for which it is to be designed is called the characteristic load. There are characteristic dead loads and characteristic live loads. In absence of any data, loads given in IS codes are taken as characteristic load.

Characteristic design load = Mean load + KS

Characteristic strength = Mean strength - KS

K is taken as 1.65 in Indian Standards

S = Standard Deviation (IS456-2000 table 8)

DESIGN VALUES

IS 456 : 2000 Pg No : 68

#### Design strength of materials

The strength to be used for design should be the reduced value of the characteristic strength by the factor denoted by the partial safety factor for the materials.

The design strength of the materials, is given by

$$f_d = \frac{f_c}{\gamma_m}$$

$f_c$  = characteristic strength of the material

$\gamma_m$  = partial safety factor appropriate to the material and the limit state being considered.

Partial safety factor for concrete  $\gamma_{mc} = 1.5$

Partial safety factor for steel  $\gamma_{ms} = 1.15$

#### Design Loads

The design loads are obtained by multiplying Characteristic loads and the appropriate partial safety factors

The design load,  $P_d$  is given by

$$P_d = \gamma \cdot P_c$$

$P_c$  = characteristic load

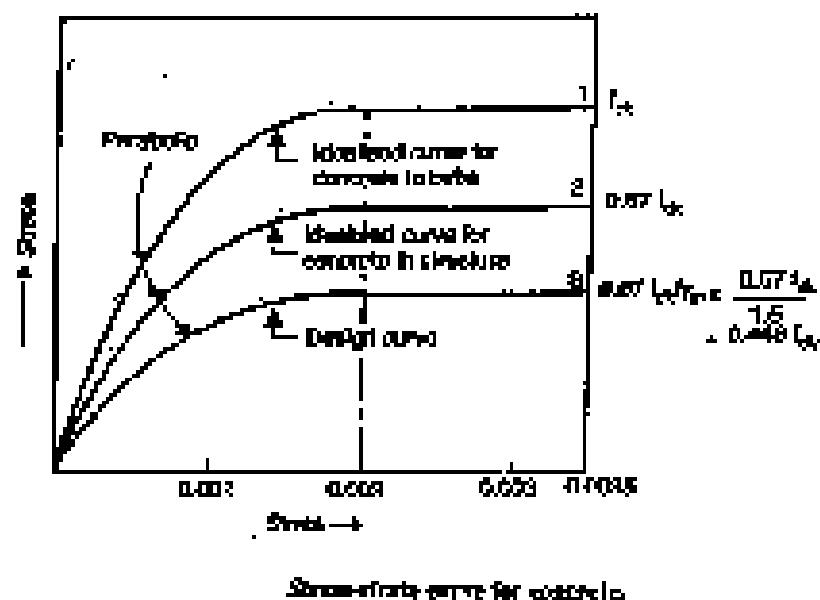
$\gamma$  = partial safety factor appropriate to the nature of loading and the limit state being considered.

#### STRESS - STRAIN RELATIONSHIP FOR CONCRETE

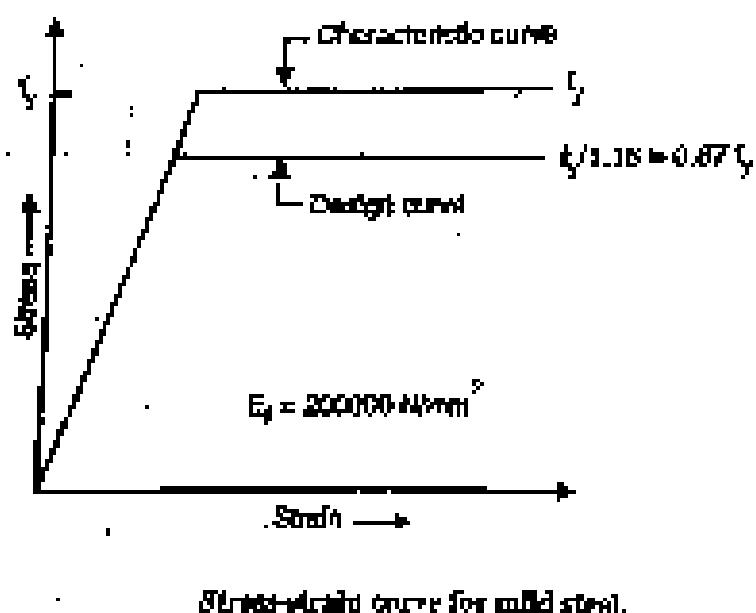
The experimental or actual stress-strain curve for concrete is very difficult to use in design. Therefore, IS code 456(2000) has simplified or idealized it as shown in Fig.

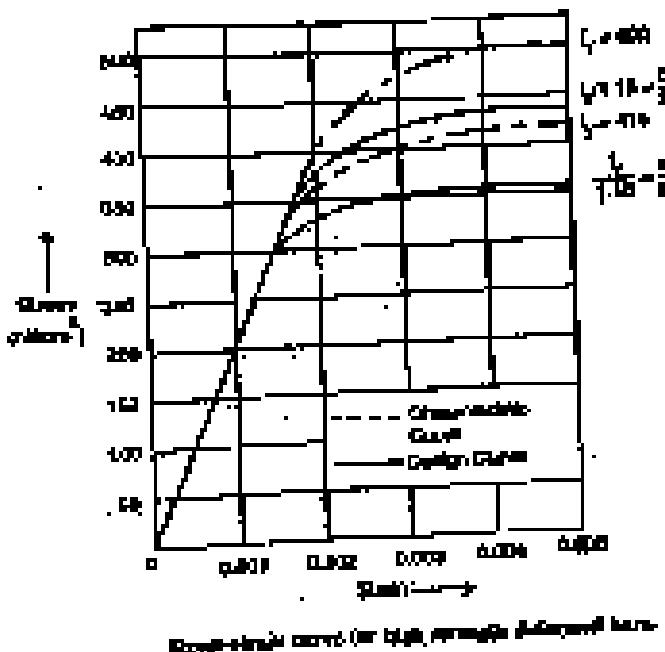
- For design purposes, the compressive strength of concrete in the structure is taken as 0.67 times the characteristic strength. The 0.67 factor is introduced to account for the difference in the strength indicated by a cube test and the strength of concrete in actual structures.
- The partial safety factor ( $\gamma_{mc}$ ) equal to 1.5 is applied in addition to this 0.67 factor.

- The initial portion of the curve is parabolic. After a strain of 0.002 (0.2%), the stress becomes constant with increasing load, until a strain of 0.0095 is reached and hence the concrete is assumed to have failed.



### STRESS - STRAIN RELATIONSHIP FOR STEEL





## ✓ LIMIT STATE OF COLLAPSE : PLIUXURE OR BENDING

### Assumptions

Plastic sections normal to the axis remain plane after bending.

1. Plastic sections normal to the axis remain plane after bending.
2. The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 to bending.
3. The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress strain curve is given in Fig. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor  $\gamma_c = 1.5$  shall be applied in addition to this.
4. The tensile strength of the concrete is ignored.
5. The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in Fig. 23. For design purposes the partial safety factor  $\gamma_s$  equal to 1.15 shall be applied.
6. The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_r}{E} + 0.002$$

$f_r$  = characteristic strength of steel

$E$  = modulus of elasticity of steel

### TYPES OF R.C.C BEAMS

R.C.C. beams are of following three types:

#### (i) Simply Reinforced Beams:

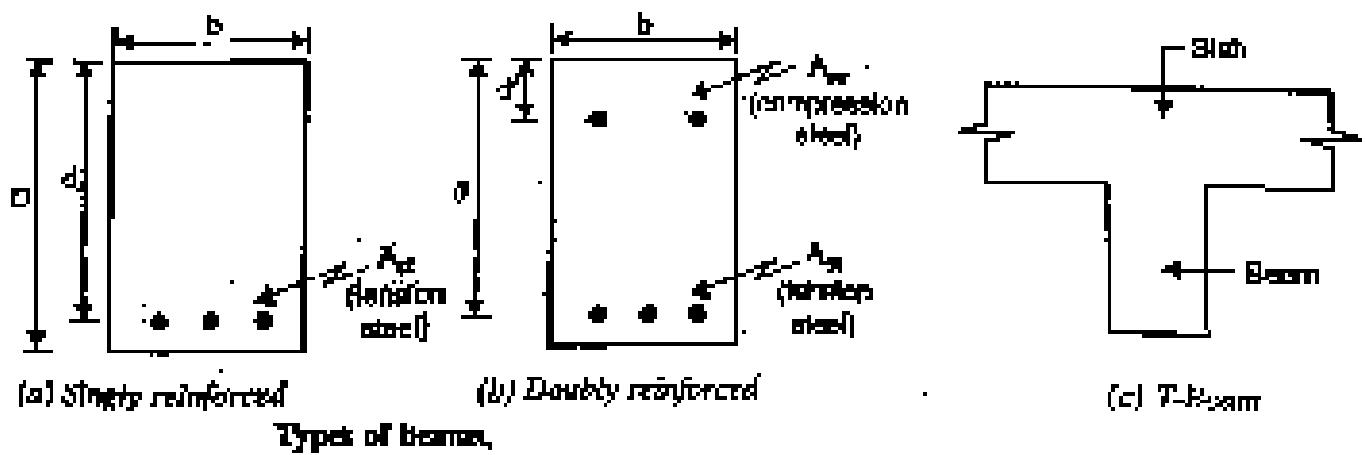
The beams in which steel reinforcement is placed in the tensile zone only are called as singly reinforced beams.

#### (ii) Doubly Reinforced Beams:

The beams in which reinforcement is placed in the bustle as well as compression zone are called as doubly reinforced beams.

#### (iii) Flanged Beams (T beams and L beams)

In most reinforced concrete structures, the slab and beams act as monolithic. Thus, the beam forms a part of the floor system. When the beam bends, a part of the slab also bends along with the beam. So, the individual beams in a floor system act as T beams and the end beams as L beams. The beams in which a portion of the slab acts together with the beam for resisting compressive stresses are called as flanged beams. Figure shows singly reinforced, doubly reinforced and T-beam sections.

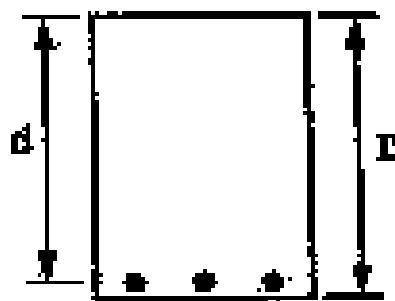


### Lever arm

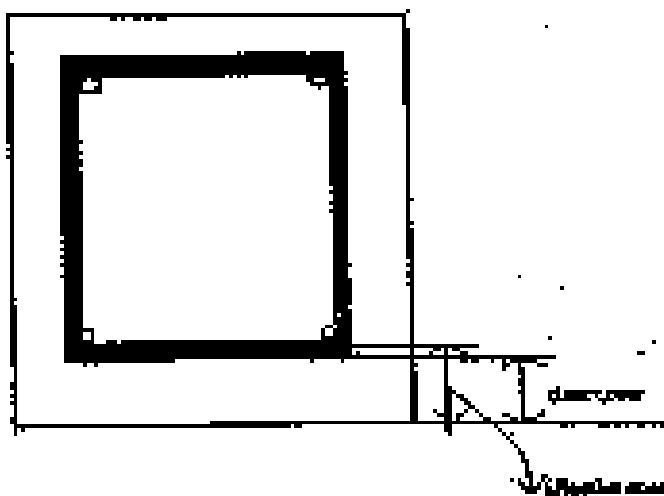
It is the distance between the resultant compressive force and the resultant tensile force.

## **Effective depth**

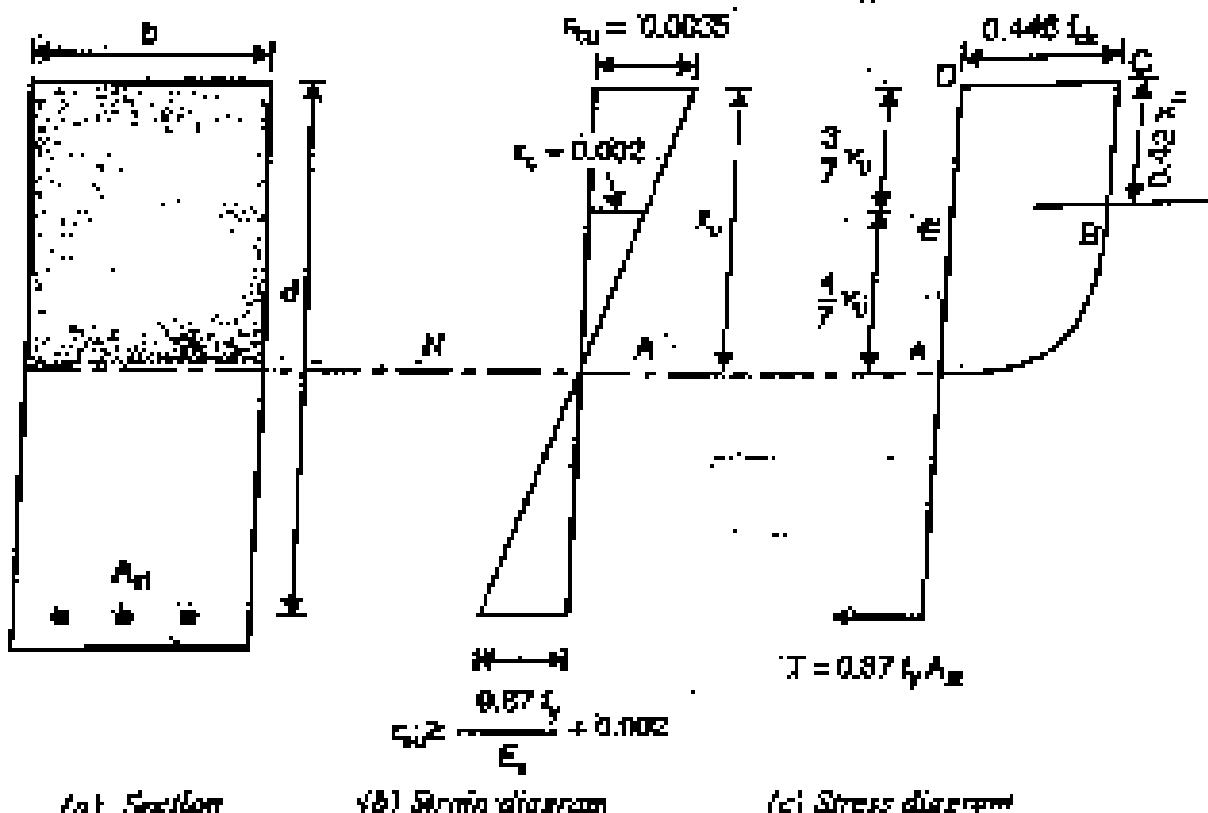
The effective depth of the beam is the distance from the tension steel to the edge of the compression fiber.



## **Clear cover and effective cover**



ANALYSIS OF A SINGLY REINFORCED BEAM SP 10 pg 19



Stress and strain distribution for a singly reinforced beam as per IS 456.

#### Strain Distribution

The assumption (i) of the limit state theory gives a linear strain distribution across the cross section as shown in Fig. (b). It varies as zero at the neutral axis and maximum at the extreme fiber. The various salient points of the strain diagram are:

- Strain at neutral axis = 0,
- Maximum or ultimate strain in concrete at extreme fiber,  $\epsilon_u = 0.0035$
- Strain at constant stress of  $0.67 f_y = 0.002$ ,
- Ultimate strain in steel corresponding to maximum stress at failure,

$$\epsilon_u = \frac{0.87 f_y}{E_y} + 0.002. \quad \frac{1}{1.15} \approx 0.87$$

#### Stress Distribution

The stress diagram is shown in Fig. (c). It has a parabolic shape from A to B and then linear from B to C above the neutral axis. The various salient points of the stress diagram are:

(i) Stress at neutral axis (Point A) = 0.

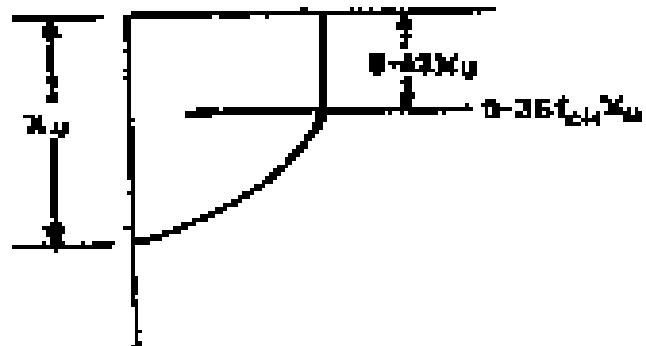
(ii) Stress at 0.002 strain (Point B) =  $\frac{0.67f_y}{1.5} = 0.446 f_y \approx 0.45 f_y$

(iii) Stress at extreme fibre (Pt. C) =  $0.446 f_y = 0.15 f_y$

(iv) Below the neutral axis, the concrete is assumed to be cracked and maximum stress in steel

$$= \frac{f_y}{1.15} + 0.27 f_y$$

Stress Block Parameters IS 456 Pg 6<sup>th</sup>. Fig 22.



#### Stress Block Parameters

For the stress-strain curve of concrete the design stress block parameters are taken as following as per IS 456: 2000.

Area of stress block =  $0.36 f_c k_x$ .

Depth of centre of compressive force from the extreme fibre in compression =  $0.42 x_n$

#### Neutral Axis Depth ( $x_n$ )

Neutral axis is the axis at which the stresses are zero and it is situated at the centre of gravity of the section. The depth of neutral axis for a singly reinforced beam is calculated by taking equilibrium of tensile and compressive forces.

Fig. 4.5. Concrete stress-block parameters in compression.

b = Width of section

d = Effective depth of beam

$A_s$  = Area of steel reinforcement

$x_n$  = Depth of neutral axis

C = Total compression

T = Total tension

Total Tension ( $T$ ) =  $0.87 f_y A_s$   
 Total compression ( $C$ ) =  $0.36 f_{ck} A_c + 0.42 x_e$  from the top extreme fibre (as per IS 456 : 2000)

For equilibrium of forces

Total tension = Total compression

$$0.87 f_y A_s = 0.36 f_{ck} A_c +$$

$$x_e = \frac{0.87 f_y A_s}{0.36 f_{ck} A_c}$$

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Q-I-I Reaction

which is given in Clause 31.1 of IS code.

$$\frac{x_e}{d} = \frac{0.87 f_y A_s}{0.36 f_{ck} A_c}$$

**Limiting Depth of Neutral Axis ( $x_{n, max}$ )**

The strain distribution for the singly reinforced beam is shown in Fig. 4.7. The maximum strain in concrete is 0.0035. As per code, the strain in steel at failure should not be less than  $\frac{0.87 f_y}{E_y} + 0.002$ . This limits the depth of neutral axis to its maximum or limiting value.

From the Fig. -

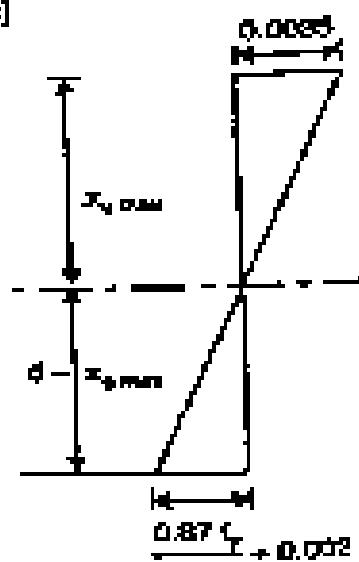
$$\frac{0.0035}{x_{n, max}} = \frac{\frac{0.87 f_y}{E_y} + 0.002}{d - x_{n, max}} \quad (\text{From similar triangles})$$

$$\frac{x_{n, max}}{d - x_{n, max}} = \frac{0.0035}{\frac{0.87 f_y}{E_y} + 0.002}$$

$$x_{n, max} \left( \frac{0.87 f_y}{E_y} + 0.002 + 0.0035 \right) = 0.0035 d$$

$$\frac{x_{n, max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_y} + 0.0035}$$

The above equation gives the limiting or maximum values of depth of neutral axis for different grades of steel.



Strain diagram

Values of  $x_{n, max}/d$

Grade of Steel	$f_y (\text{N/mm}^2)$	$\frac{x_{n, max}}{d}$
M20 steel (Fy 335)	250	0.59
Fy 415	415	0.48
Fy 500	500	0.46

The value of  $E_y$ , i.e. modulus of elasticity of steel is taken as  $2 \times 10^5 \text{ N/mm}^2$ .

# BALANCED, UNDER-REINFORCED AND OVER-REINFORCED SECTION

## Balanced Section

$$\frac{0.87 f_y}{E_s} + 0.002$$

In balanced section the steel reinforcement reaches its yield strain i.e.  $\frac{0.87 f_y}{E_s} + 0.002$  at the same time as the concrete reaches the ultimate strain value of 0.0035.

$$\text{i.e. } \frac{x_0}{d} = \frac{x_{\text{max}}}{d}$$

$$(ii) P_u = P_{\text{bal}}$$

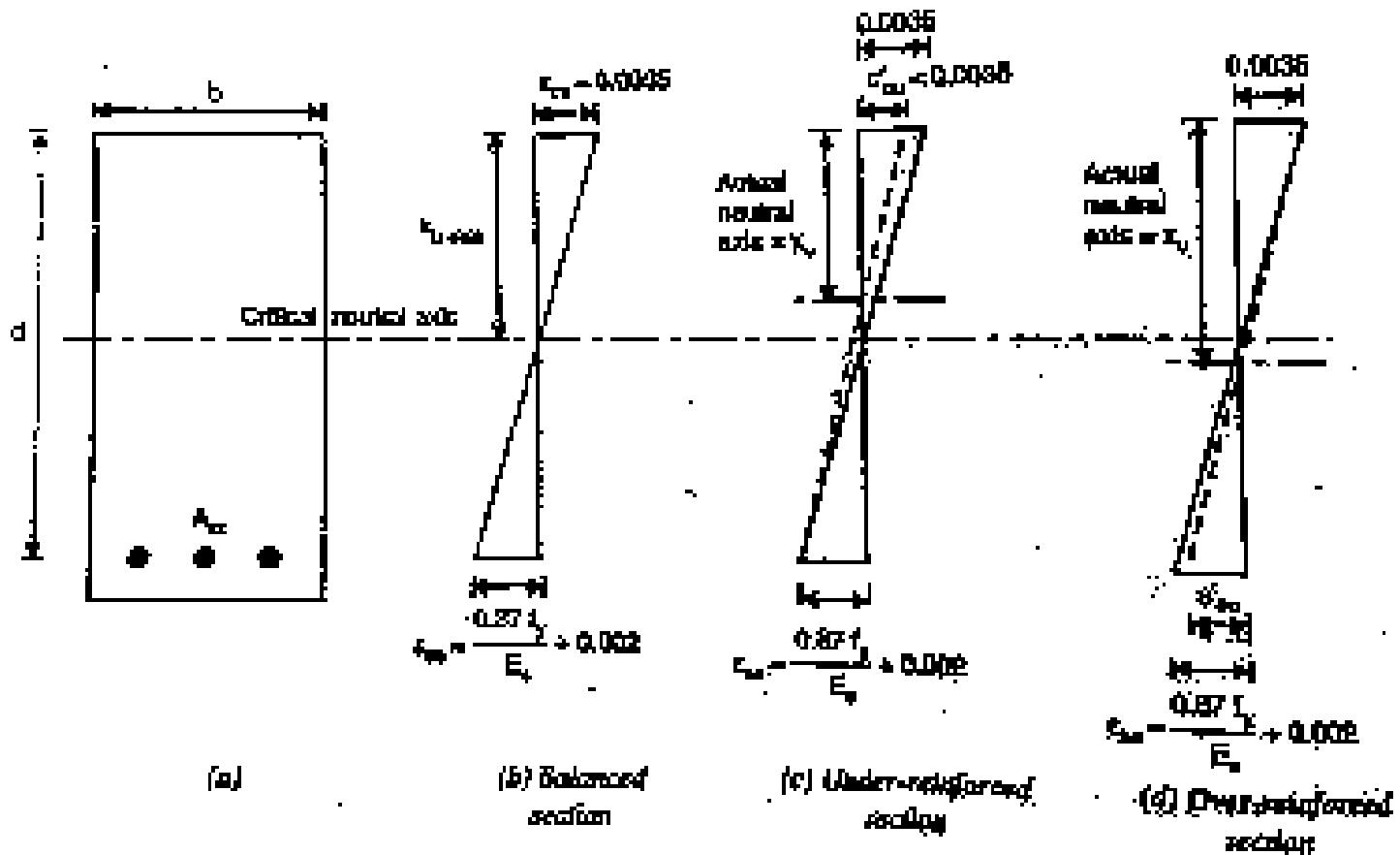
(iii) The yield strain in steel and ultimate strain in concrete reaches their maximum values at the same time.

(iv) The moment of the resistance is equal to its limiting value  $M_{u, \text{lim}}$  and can be calculated as:

$$M_{u, \text{lim}} = 0.36 f_{ct} \frac{x_{\text{max}}}{d} \left( 1 - \frac{0.42 x_{\text{max}}}{d} \right) bd^2$$

OR

$$M_{u, \text{lim}} = 0.87 f_y A_s d \left( 1 - \frac{0.42 x_{\text{max}}}{d} \right)$$



## Under-Reinforced Section

In this case, the steel fails first by reaching its yield strain value, although in concrete the ultimate strain has not reached. Steel is ductile material and it gives sufficient warning before

failure, hence the under-reinforced sections are preferred by designers. In under-reinforced section the percentage of steel is less than its maximum or limiting value.

- (i) The strain in steel reaches its yield value first i.e.,  $\frac{0.87 f_y}{E_y}$  but at that time the strain in concrete is less than 0.0035.
- (ii)  $\frac{x_e}{d} < \frac{x_{max}}{d}$  i.e., depth of neutral axis is less than the limiting value.
- (iii)  $P < P_{ult}$  i.e., the percentage of steel is less than the maximum or limiting value of percentage of steel required for balanced section. Hence the under-reinforced sections are economical.
- (iv) The section fails in a ductile manner.
- (v) The moment of resistance is calculated as follows:

$$M_r = 0.87 f_y A_s d \left( 1 - \frac{0.42 x_e}{d} \right)$$

### Over-reinforced Section

The over-reinforced section is that in which strain in concrete reaches its ultimate value earlier than the steel. This means that over-reinforced beam fails by crushing failure of concrete. Concrete being brittle, fails suddenly without warning. Therefore code IS 456: 2000 recommends that over-reinforced sections should be redesigned.

- (i) Strain in concrete reaches its ultimate value i.e., 0.0035 first and the strain in steel at that time is less than  $\frac{0.87 f_y}{E_y} + 0.0035$ .
- (ii)  $\frac{x_e}{d} > \frac{x_{max}}{d}$  i.e., depth of neutral axis is greater than its limiting or maximum value.
- (iii)  $P > P_{ult}$  the percentage of steel is greater than that in the balanced section. Thus, this section is un-economical.
- (iv) Failure is sudden, without warning.
- (v) The moment of resistance of over-reinforced section is calculated as follows:

By putting  $x_p = x_{max}$

$$M_{r,unr} = 0.87 f_y A_s \cdot \frac{x_{max}}{d} \left( 1 - 0.42 \frac{x_{max}}{d} \right) b d^2$$

### FINDING MOMENT OF RESISTANCE OF SINGLY REINFORCED BEAM

The moment of resistance of a singly reinforced beam can be obtained as follows (IS 456: 2000, Annex G):

1. For the given grades of concrete and steel ( $f_c$  and  $f_y$  known), find the depth of neutral axis of the given section:

$$\frac{x_e}{d} = \frac{0.87 f_y A_s}{0.36 f_c b d}$$

## Moment of Resistance

The moment of resistance is equal to the product of the tensile force by two equal and opposite forces i.e., total compression and total tension ( $C$  and  $T$ ).

Ultimate moment of resistance =  $M_u$

$$M_u = C \times lever arm = T \times lever arm$$

$$C = 0.46 f_{ck} b \cdot x_c \text{ which acts at } 0.42 x_c \text{ from the top most fiber.}$$

$$T = 0.87 f_y A_s$$

$$\text{Lever arm} = d - 0.42 x_c$$

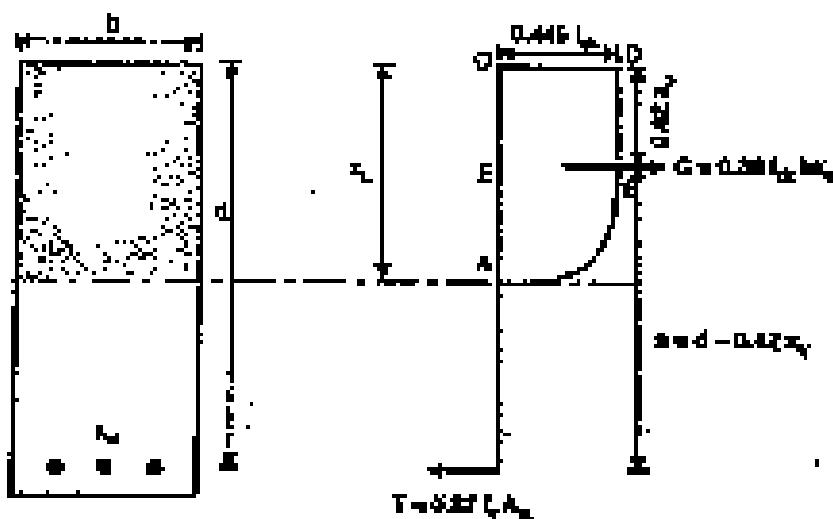
$$M_u = 0.46 f_{ck} b \cdot x_c (d - 0.42 x_c)$$

$$M_u = 0.46 f_{ck} b d A_s \left( 1 - \frac{0.42 x_c}{d} \right)$$

$$M_u = 0.46 f_{ck} \cdot \frac{b}{2} \left( 1 - \frac{0.42 x_c}{d} \right) d^2 \quad ... (ii)$$

## Moment of Resistance (Clause G 1.1, IS 456)

The moment of resistance of singly reinforced beam under first mode of collapse is calculated as follows:



The above equation gives the value of ultimate moment of resistance of the singly reinforced beam. The moment of resistance of this beam can also be obtained from tension side as follows:

$$\begin{aligned}
 M_u &= T \times lever arm \\
 &= 0.87 f_y A_s (d - 0.42 x_c) \\
 &= 0.87 f_y A_s d \left( 1 - \frac{0.42 x_c}{d} \right) \quad ... (iii)
 \end{aligned}$$

By putting value of  $x_c$  in Eq. (ii), we get

$$M_u = 0.87 f_y A_s d \left( 1 - \frac{A_s f_y}{bd f_{ck}} \right)$$

### Limiting Value of Moment of Resistance

The depth of neutral axis is limited to  $x_{\text{max}}$ . The maximum value of neutral axis depth gives the maximum or limiting value of moment of resistance.

The Limiting or maximum value of moment of resistance is given by  $M_{\text{max}}$  which we get by substituting  $x_{\text{max}}$  for  $x$ , in eqn. (7)

$$M_{\text{max}} = 0.87 f_y A_s \cdot \frac{x_{\text{max}}}{d} \left( 1 - \frac{0.42 x_{\text{max}}}{d} \right) bd^2 \quad (\text{Part C.1.1 (c) of IS 456})$$

For example:  $M_{\text{max}}$  for mild steel is obtained as follows:

$$\frac{x_{\text{max}}}{d} \approx 0.53 \quad (\text{for mild steel by pulling } f_y = 250 \text{ N/mm}^2 \text{ in the expression for } \frac{x_{\text{max}}}{d})$$

$$M_{\text{max}} = 0.87 f_y A_s \times 0.53 \left( 1 - 0.42 \times 0.53 \right) bd^2 = 0.148 f_y A_s bd^2$$

The following Table gives various values of  $M_{\text{max}}$  for various grades of concrete and steel.

Limiting Values of Moment of Resistance (Nm/mm)

Concrete Grade	Mild Steel (Fe 250)	High Strength Steel (Fe 415)	High Strength Steel (Fe 500)
M15	$M_{\text{max}} = 0.148 f_y A_s bd^2$ 2.22 $bd^2$	$M_{\text{max}} = 0.138 f_y A_s bd^2$ 2.07 $bd^2$	$M_{\text{max}} = 0.133 f_y A_s bd^2$ 2.00 $bd^2$
M20	2.96 $bd^2$	2.76 $bd^2$	2.66 $bd^2$
M25	3.70 $bd^2$	3.43 $bd^2$	3.31 $bd^2$
M30	4.48 $bd^2$	4.14 $bd^2$	3.99 $bd^2$
M35	5.21 $bd^2$	4.83 $bd^2$	4.65 $bd^2$

### Percentage of Steel

The percentage of tensile reinforcement can be found by equating the tensile and compressive forces

$$T = C$$

$$0.87 f_y A_s = 0.36 f_c b x_c$$

$$\frac{A_s}{b} = \frac{0.36 f_c x_c}{0.87 f_y}$$

$$\frac{A_s}{b d} = \frac{0.36 f_c x_c}{0.87 f_y d}$$

$$P_t = \frac{A_s}{b d} = \frac{0.36 f_c x_c}{0.87 f_y d}$$

As the value of  $\frac{x_c}{d}$  is limited, its maximum value gives the maximum percentage of steel.

$$P_{s, \text{max}} (\%) = \frac{0.36 f_c}{0.87 f_y} \cdot \frac{x_{\text{max}}}{d} \times 100$$

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$$x_n = \frac{(0.97 f_y A_s)}{0.35 f_{ct} b}$$

(A<sub>s</sub> is given)

### 3. Calculate x<sub>n</sub> and x<sub>n, min</sub>:

- (i) If  $\frac{x_n}{d} < \frac{x_{n,\min}}{d}$ , the beam is under reinforced and moment of resistance is calculated by the following equation:

$$M_n = 0.87 f_y A_s A \left( 1 - \frac{0.42 x_n}{d} \right)$$

- (ii) If  $\frac{x_n}{d} = \frac{x_{n,\min}}{d}$ , the beam is designed as balanced section and moment of resistance of the section is given by following expression:

$$M_{n, \text{bal}} = 0.36 f_{ct} \frac{x_{n,\min}}{d} \left( 1 - \frac{0.42 x_{n,\min}}{d} \right) b d^3$$

- (iii) If  $\frac{x_n}{d} > \frac{x_{n,\min}}{d}$ , the moment of the resistance of the section is equal to M<sub>n, min</sub> but the code recommends that the section is to be redesigned as it is a case of over reinforced section.

Introduction Plain and Reinforced concrete - Properties of concrete and reinforcing steel - Objectives of design - Different design philosophies working stress and limit state methods - Limit state method of design - Introduction to BIS code - Types of limit states - characteristic and design values - partial safety factors - type of loads and their factors

Limit state of collapse in bending - assumptions - strain - strain relationship of steel and concrete - analysis of singly reinforced rectangular beams - balanced - under reinforced - over reinforced sections - moment of resistance - initial deflections

## Plain and Reinforced Concrete

### Plain Concrete (PC)

- Concrete may be defined as any solid mass made by the use of cementitious materials, the ingredients generally comprising cement, aggregate, cement and water.
- Its uses and popularity may be due to:
  - Durability under hostile environments
  - Ease with which it can be cast into various shapes and sizes
  - It is cheap, strong and very available
- Plain concrete compares with malleable or hardens under hydration
- It reacts well with water and does not chemically bond to chemicals

\* Plain cement concrete has good compressive strength.  
But very little tensile strength thus limiting its uses in  
construction (weak in bending, shear and tension)

### Reinforced Cement Concrete (RCC)

- \* It is a composite material
- \* To improve the tensile strength of PCC, reinforcements  
are embedded in concrete such type of concrete is known  
as RCC.
- \* Concrete is RCC resist compression
- \* Reinforcement / steel is concrete resist tension.
- \* Twisted steel bars are generally used for concrete bond,  
prevent slipping
- \* Stirrups are provided to resist shear force.
- \* Main bars are provided to resist BM.
- \* Distribution bars are provided in slabs to resist shearing  
seepage etc.

### Uses of RCC.

- \* Used in building
- \* Flyovers
- \* Water tanks
- \* Road & Railway
- \* Chimney & towers
- \* Retaining walls
- \* Bunkers & Silos.

## Advantages & Disadvantages of RCC

- \* **Strength**  
RCC has very good strength in tension and compression.
- \* **Durability**  
RCC structures are durable if designed and built properly. They can last upto 100 yrs.
- \* **Moldability**  
RCC sections can be given any shape easily by proper designing of the formwork.
- \* **Economy**  
RCC is cheaper as compared to steel & prestressed concrete. The maintenance cost is less.
- \* **Transportation**  
The raw materials required for RCC such as cement, aggregate, water and steel are easily available and can be easily transported.
- \* **Fire Resistance**  
RCC structures are more fire resistant than commonly used materials.
- \* **Seismic Resistance**  
Properly designed RCC structure are extremely resistant to earthquake.

## Disadvantages of RCC

- \* RCC structures are heavier structures.
- \* RCC needs lots of formworks & skilled labours.
- \* Concrete takes time to attain its full strength thus RCC structures can be used in construction after immediate purpose.

## Grades of Concrete (IS 456 Part 1 page 16)

Ordinary Concrete - M<sub>10</sub>, M<sub>15</sub>, M<sub>20</sub>.

Standard Concrete - M<sub>25</sub>, M<sub>30</sub>, M<sub>35</sub>, M<sub>40</sub>, M<sub>45</sub>, M<sub>50</sub>, M<sub>55</sub>.

High strength Concrete - M<sub>60</sub>, M<sub>65</sub>... M<sub>90</sub>.

For RCC work - not lower than M<sub>20</sub>.

For post-tensioning - M<sub>25</sub> and above.

For prestressed & posttressed concrete - M<sub>40</sub> & above.

## Properties of Concrete (IS 456 page 15 & 16)

The properties of concrete depends upon the properties & proportions of its ingredients. Following are the important properties:

- Compressive strength
- Workability
- Durability
- Tensile strength
- Modulus of elasticity
- Poisson's Ratio
- Creep
- Shrinkage.

## Compressive Strength (IS 456 Cl. 6.1 pg 15)

- \* Primarily depends on age, cement content and w/c ratio.
- \* Compressive strength of concrete is determined by cube test.
- \* Compressive strength of concrete is given in terms of Characteristic compressive strength.

- \* Compressive strength of concrete is defined as the strength of 15cm cube at 28 days in N/mm<sup>2</sup>
- \* Compressive strength of concrete is simply the last results obtained during actual practice in laboratory conditions.
- \* Characteristic compressive strength is defined as the strength of concrete below which not more than 5% of the test results are expected to fail.

It is represented by 'fc'.

- \* Characteristic compressive strength is obtained by multiplying compressive strength of concrete with factor of safety

$$C.S. = 1.5 \times C.B.$$

where 1.5 is the factor for concrete.

### Workability (IS 456 Ch 4 page 11)

Flow with which concrete can be batched, mixed, transported, placed, compacted and cured.

The factors affecting workability are:

- \* W/C ratio
- \* Size and shape of aggregates
- \* CA to FA ratio
- \* Grading of aggregates
- \* Surface texture of aggregate
- \* Cement Content
- \* Use of admixtures etc.

### Durability (IS 456 Ch 4 page 11)

Concrete should be durable to environment during its life. The various types of exposure conditions are listed in IS 456 Table 3 page 18.

## Tensile Strength (f<sub>t</sub>) (IS 456 Ch 2 Pg 16)

Tensile strength of concrete is correlated with characteristic compressive strength of concrete.

Tensile strength,  $f_t = 0.01 f_{ck}$  N/mm<sup>2</sup>.

where  $f_{ck}$  is the characteristic compressive cube strength of concrete in N/mm<sup>2</sup>.

## Modulus of Elasticity (E<sub>c</sub>) (IS 456 Ch 2 Pg 16)

$$E_c = 5000 \sqrt{f_{ck}} \text{ N/mm}^2$$

where  $f_{ck}$  is the characteristic compressive strength of concrete in N/mm<sup>2</sup>. Modulus of elasticity of steel is 200 GPa. (IS 456 Ch 2 Pg 16)

## Poisson's Ratio

Ratio of lateral strain to longitudinal strain strain is known as poisson's ratio.

It varies from 0.1 to 0.3 for concrete.

## Shrinkage (IS 456 Ch 2 Pg 16)

The volumetric changes of concrete structures due to the loss of moisture by evaporation is known as shrinkage of concrete.

The total shrinkage strain may be taken as 0.003 for design.

## Care (IS 456 Ch 2 Pg 16)

Care of concrete is defined as deformation of a structure under sustained or continuous load.

Unit weight of PCC and RCC can be taken as 24 kN/m<sup>3</sup> and 25 kN/m<sup>3</sup> respectively.

## Objectives of Design

- \* Stability
- \* Strength
- \* Sustainability
- + Aesthetics
- \* Economy.

## Design Codes and Handbooks.

- 1) They ensure adequate structural safety, by specifying certain essential minimum requirements for design.
- 2) They render the task of the designer relatively simple.
- 3) They ensure a measure of consistency across different designers.
- 4) They have some legal validity.

## Basic Code for Design

- \* Published by Bureau of Indian Standards, New Delhi.
- IS 456 : 2000 Plain and reinforced concrete - code of practice
- IS 875 : 1981 Code of practice for design loads (other than earthquakes) for buildings and structures

## Parts 1 - 5

IS : 875 (Part 1) - 1981	Dead loads - unit weight of building materials and stored materials
IS : 875 (Part 2) - 1981	Applied loads (live loads), materials
IS : 875 (Part 3) - 1981	Wind loads
IS : 875 (Part 4) - 1981	Snow loads
IS : 875 (Part 5) - 1981	Special loads and combinations.

→ IS 1893 : 2000 Criteria for Concrete Reinforced Design of Structures  
Design Handbooks

- SP 16 : 1980 Design Aids  
→ SP 24 : 1983 Implementing Handbook  
→ SP 34 : 1987 Handbook on concrete reinforcement and detailing.  
→ SP 33 : 1982 Design for concrete mix

### Loads

For the purpose of calculating max stresses in any structure or member of a structure, the following loads should be taken into account.

- \* Dead load - the wt of material of the structure and all the materials supported by the structure permanently. → IS 875 (Part I)
- \* Live loads - these are the loads which temporarily rest at one place such as furniture, material loads, moving loads etc → IS 875 (part II)
- \* Wind loads → IS 875 part III
- \* Snow load → IS 875 part IV
- \* Earthquake loads (seismic loads) - IS 1893 : 2000

### Characteristic load (IS 456 part 2 page 6.1)

- \* The characteristic load means that value of load which has a 95% probability of not being exceeded during the life of a structure.

- \* The man working load that the structure has to withstand and for which it is to be designed is called characteristic load.

## Design Philosophies:

- i) Working stress method.
- ii) Load factor method.
- iii) Limit State Method.

## Working Stress Method:

- \* First theoretical Method
- \* Olded Method
- \* In this method structure is analyzed and designed by elastic theory
- \* Assumes both steel and concrete are elastic
- \* obeys Hooker law.
- \* The stress strain curve for both the concrete and steel is linear
- \* It means that the stress is directly proportional to strain upto point of collapse.
- \* Modulus ratio is used to determine stresses in steel and concrete.

$$m = \frac{E_s}{E_c} ; \quad \begin{aligned} m &\Rightarrow \text{modulus ratio} \\ E_s &\Rightarrow \text{modulus of elasticity of Steel} \\ E_c &\Rightarrow \text{modulus of elasticity of Concrete} \end{aligned}$$

$$m = \frac{280}{300}$$

$\sigma_{c'}$  is the permissible compressive stress due to bending in concrete at N/mm<sup>2</sup> (16.456 N/mm<sup>2</sup>)

- \* In this method, the ultimate strength of concrete and yield strength or yield proof strain of steel is divided by factor of safety / safety factor to obtain permissible stress.

### Advantages

- \* Reduces calculation effort.
- \* Reasonably reliable.
- \* Resulting in better acceptability.

### Disadvantages

- \* It assumes concrete is elastic which is not true as the concrete behaves elastoplastic at low loads & becomes brittle at high loads.
- \* Factor is provided only for stresses and not for loads.
- \* It doesn't consider acceptability.
- \* No account for shrinkage & creep.
- \* Gives uneconomical sections.
- \* No provision for uncertainty of loads.
- \* Pays no attention to the conditions that arise at the time of collapse.

### Assumption of working stress method (IS 456 Clause B-1)

### Limit State Method (IS 456 Clause B-1) pg 47)

- \* the object of limit state design is based on the concept utilizing an acceptable probability that a structure will not become undesirable in its function for the use for which it is intended.

- \* Limit state is defined as the acceptable limits of safety and serviceability requirements before failure
- \* The aim of this method is that the structure should be able to withstand safely all the loads that are liable to act on it all along its life and satisfy serviceability requirements of limiting deflection and cracking
- \* Stress-strain curve is non-linear

### Limit State Method of Design

- \* A structure is said to have reached its limit state when the structure as a whole or part becomes unfit for use for one reason or another during its expected life
- \* The limit state of a structure is the condition of its being unfit for its intended use

Two major limit states exist are :-

- 1) Ultimate strength limit state or limit state of collapse
- 2) The serviceability limit state which deals with conditions such as deflection, cracking of the structure under service loads.

### Limit State of Collapse (IS 450: 2000, Pt 5)

→ Limit state of collapse :

Failure (IS 450 Pt 5 page)

collapse:

→ Limit state of compression  
(IS 450 Pt 5 page) collapse

→ Limit state of shear  
(IS 450 Pt 5 page)

→ Limit state of collapse - torsion  
(IS 450 Pt 5 page)

### Limit State of Serviceability

→ Limit state of serviceability  
in durability

→ Limit state of deflection  
(IS 450 Pt 5 page)

→ Limit state of cracking  
(IS 450 Pt 5 page)

→ Limit state of overstrength

Comparison b/w working stress method & Limit state method

### Working Stress Method

Limit State Method.

- \* It is based on elastic theory.  
i.e., elastic region is considered for design. It is also known as elastic method.
- \* Design is based on safe working stress which lies within elastic region.
- \* Design stresses are used in limit state method.

Safe working stress / permissible stress =  $\frac{\text{Ultimate stress}}{\text{Factor of Safety}}$

fos for concrete = 3

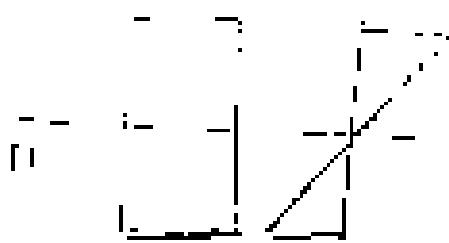
fos for steel = 1.8

Design strength = Characteristic strength / partial safety factor

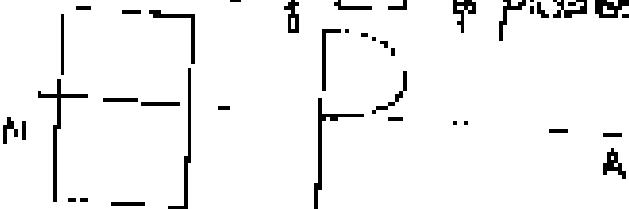
psf for concrete = 1.5

psf for steel = 1.15

- \* Stress variation in compression zone is assumed to be linear



\* Stress variation in compression zone is non-linear. It is a combination of linear & parabolic



- \* Structure is over stiff & uneconomical

\* Structure is safe and economical.

- \* No serviceability requirements are checked

\* Serviceability requirements are checked in this method.

Partial Safety Factor for Loads and Material Strengths.

Partial Safety factor ( $\gamma_L$ ) = 1.456 (for loads).

It is generally used for design considerations.

Partial safety factors are applied both to loads on the structure

Partial safety factor for materials ( $\gamma_m$ ) = Characteristic strength/ $f_{ck}$  and to Strength of materials. Design strength = Characteristic strength/ $\gamma_m$

Design load =  $\gamma_m \times \text{char. load}$ .

Partial safety factor ( $\gamma_L$ ) for loads,  $\gamma_L = 1.456$  (for loads).

Partial safety factor ( $\gamma_m$ ) for materials ( $\gamma_m = 1.3$  for materials).

Partial Safety Factor is used in Limit State Method of design.

Design Values ( $\gamma_L \times \text{characteristic value} / \gamma_m$ )

For Materials ( $\gamma_L \times \text{characteristic value} / \gamma_m$ )

Design strength for material,  $f_d = \frac{f}{\gamma_m}$

$f \rightarrow$  Characteristic strength.

$\gamma_m \rightarrow$  psf

For loads,  $(\gamma_L \times \text{characteristic load}) / \gamma_m$

Design load,  $F_d = F \times \gamma_L$

$F \rightarrow$  Characteristic load

$\gamma_L \rightarrow$  psf

Factor of Safety or Safety factor.

It is the factor used in the design of structures so that

the value of the stresses coming on the structure due to applied loads does not exceed the design strength of the materials used in the structure. It is the margin of value taken for security consideration against the failure of the structure.

The Safety factor is the ratio of the max. value of load on stress that the structure can withstand without failure to the applied load as shown on the structure.

$$SF = \frac{\text{max stress/force}}{\text{applied stress/force}}$$

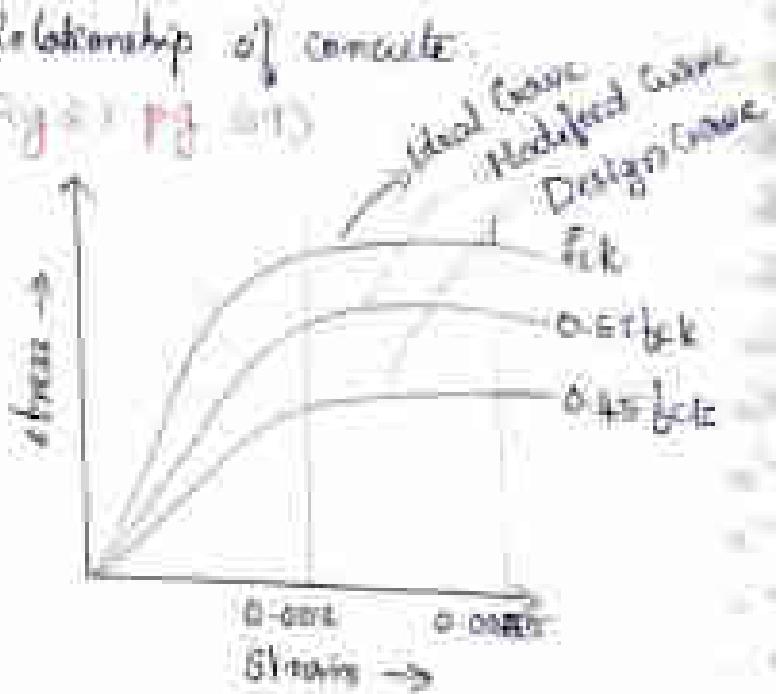
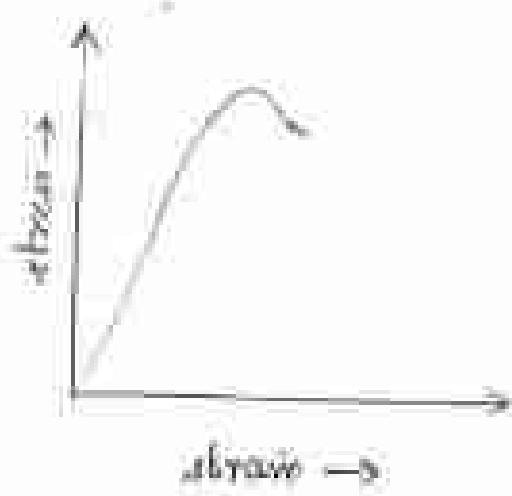
The main purpose of SF is to make the strength of the structure so that it remains safe and free from all failures.

It also remains ready for all unaccountable failure effects.  
SF represents the load carrying capacity of the structure actual loads.

It is used in working stress method.

Stress - Strain Relationship of concrete

(IS 456 Pg 211)



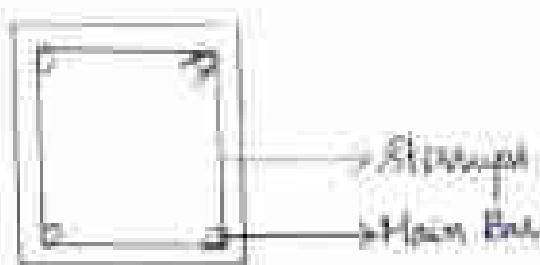
$$\text{Design Strength} = \frac{0.67 f_{ck}}{1.5}$$

$$= 0.45 f_{ck}$$

Representation shows diagram given for Reinforcement

(It can be pg 11 pg 10)

### Analysis of Singly Reinforced Beam



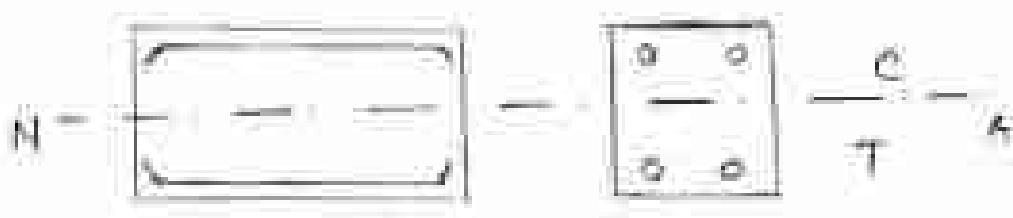
### Singly Reinforced Beams

If the beam is provided with longitudinal reinforcement in tension zone only then it is called singly reinforced beam.



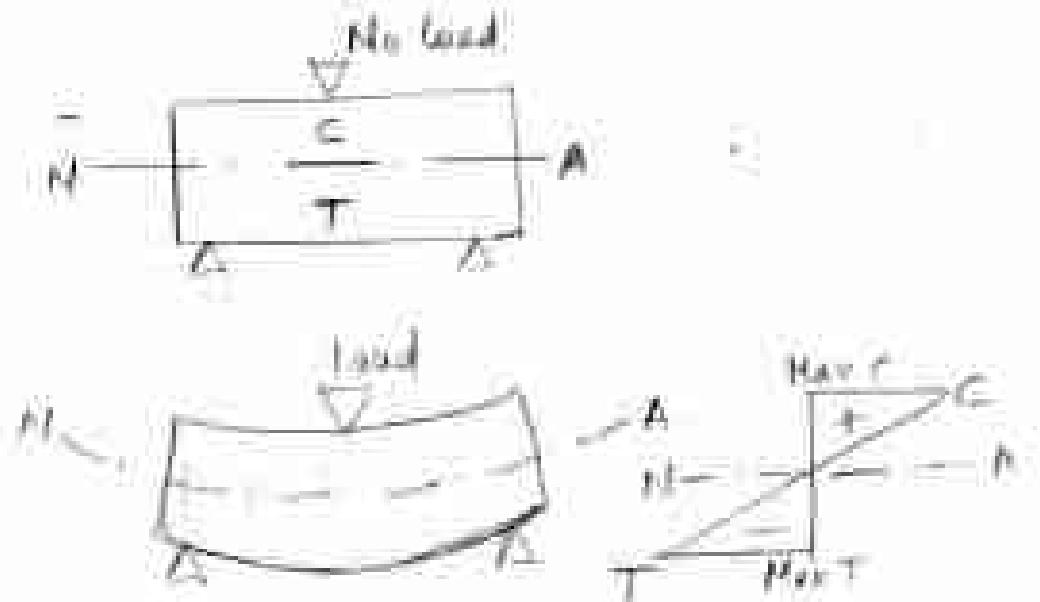
### Doubly Reinforced Beams

If the beam is provided with steel reinforcement both in tension zone and compression zone, it is known as doubly reinforced beam.



### Neutral Axis

NA of a beam is the line of intersection of the neutral layer with the beam section. This is the line dividing the cross section into compression and tension zone.



location of NA

For finding the depth of NA, consider the effects of compressive and tensile forces.

$$C = T$$

$$0.36 f_{ck} \bar{x}_n b = f_{ct} \bar{y}_t b$$

$$0.36 f_{ck} \bar{x}_n b = f_{ct} \bar{y}_t \cdot 0.25$$

$$\text{depth of NA, } \frac{\bar{x}_n}{d} = \frac{0.82 f_{ct} \bar{y}_t}{0.36 f_{ck} b}$$

(See 4th edition Fig 9.2)

Effective Cover ( $d_c$ )

Distance from centre of reinforcement to extreme fibre

Clear Cover ( $c$ )

Distance from end of reinforcement to extreme fibre

Effective Depth ( $d$ )

It is the distance from extreme compression fibre to centre reinforcement.

Overall depth ( $D$ )

$$D = \text{eff depth} + \text{eff cover}$$

$$= d + d_c$$

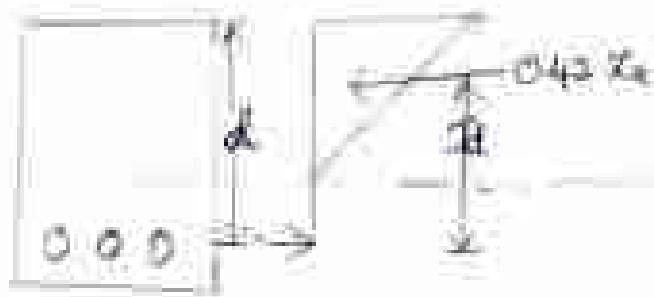


### Deviation ( $I_d$ )

This is the distance b/w line of action of resultant compression and resultant tension. It is represented as  $I_d$ .

Resultant compressive force acting at a distance of  $0.42\lambda_a$  below the top fibre.

$$\therefore I_d = d - 0.42\lambda_a$$



### Moment of Resistance ( $M_R/M_u$ )

It is the twisting moment offered by a beam against BM at the section. The resulting moment is due to the moment of couple formed by compression & tension.

Moment of Resistance =  $C \cdot I_d$  or  $T \cdot I_d$

Resultant compressive force =  $0.36 \text{ kN/m}^2$

Resistant tensile force =  $40 \times 10^{-3} \text{ kN/m}^2$

Moment of resistance in terms of tensile force

$$M_{Rn} = T_n \cdot I_{eff}$$

$$= A_{st} \cdot f_{st} \times \left( d - 0.42 Z_u \right) \\ f_{st} = 0.67 f_y$$

$$= A_{st} \cdot 0.67 f_y \times \left( d - 0.42 Z_u \right)$$

$$= A_{st} \cdot 0.67 f_y \times d \left( 1 - \frac{0.42 Z_u}{d} \right)$$

$$f_y \times 0.67 f_y \times d \left( 1 - \frac{0.42 \times 0.67 f_y}{0.36 f_{ck} b d} \right)$$

$$= A_{st} \cdot 0.67 f_y d \left( 1 - \frac{0.36 f_y}{0.36 f_{ck} b d} \right)$$

$$= A_{st} \cdot 0.67 f_y d \left( 1 - \frac{f_y}{f_{ck} b d} \right)$$

$$\boxed{M_{Rn} = 0.67 f_y d^2 \left( 1 - \frac{f_y}{f_{ck} b d} \right)}$$

Moment of resistance in terms of compressive force

$$M_{Rn} = C \cdot I_d$$

$$= 0.36 f_{ck} Z_u b \times \left( d - 0.42 Z_u \right)$$

$$= 0.36 f_{ck} Z_u b \left( 1 - \frac{0.42 Z_u}{d} \right)$$

$$\therefore C = \frac{f_y d}{f_{ck}} \quad \boxed{f_{ck} = 0.36 f_{ck} Z_u \left( 1 - \frac{0.42 Z_u}{d} \right) b d^2}$$

IS 456 pg 76.

limiting value of Moment of Resistance

Substitute  $Z_{max}$  for  $Z_u$ .

(pg 76 IS 456)  
limiting value of  $Z_{max}$ .  $\boxed{M_{Rn,m} = 0.36 f_{ck} \frac{Z_{max}}{d} \left( 1 - \frac{0.42 Z_{max}}{d} \right) b d^2}$

$$\left. \begin{array}{l} \text{for } Z_c = 0.50 \\ Z_c = 1.5 \\ Z_c = 5.00 \end{array} \quad \begin{array}{l} M_{Rn,m} = 0.168 + 0.67 b d^2 \\ M_{Rn,m} = 0.137 f_{ck} b d^2 \\ M_{Rn,m} = 0.133 f_{ck} b d^2 \end{array} \right\} \quad \text{IS 456 pg 76}$$

Balanced, Under-reinforced and Over reinforced sections.

### Balanced Section

Rcc section in which the steel reaches the yield strain ( $\epsilon_y$ ) simultaneously as the concrete reaches failure strain is called balanced section. In a balanced section,

$$\frac{x_u}{d} = \frac{x_{umax}}{d} \quad x_u = x_{umax}$$

### Under Reinforced Section

Under reinforced section is the one in which the quantity of steel provided is less than what is required for a balanced section.

In an under reinforced section, steel reaches yield strain earlier than concrete reaches failure strain. It gives sufficient warning before failure of the structure.

$$\frac{x_u}{d} < \frac{x_{umax}}{d}$$

### Over reinforced section

Over reinforced section is the one in which the quantity of steel provided is more than what is required for a balanced section.

In over reinforced section, failure strain in concrete reaches earlier than steel reaches yielding strain. The structure fails without any warning.

$$\frac{x_u}{d} > \frac{x_{umax}}{d}$$

Steps for finding Moment of Resistance ( $M_u$ /M<sub>u</sub>)  
*(IS 456 Annexure G-11 Pg 46)*

1. Determine depth of R.A from the following eqn.

$$\frac{x_u}{d} = \frac{0.84 \text{ by ASI}}{0.36 \text{ fact}}$$

2. Find the value of  $\frac{x_{umax}}{d}$  given by

3. Compare  $\frac{x_u}{d}$  and  $\frac{x_{umax}}{d}$  *(IS 456 G-11 Pg 46)*

If  $\frac{x_u}{d} > \frac{x_{umax}}{d}$ ; Over reinforced section

$$\frac{x_u}{d} = \frac{x_{umax}}{d}; \text{ Balanced section}$$

$\frac{x_u}{d} < \frac{x_{umax}}{d}$ ; Under reinforced section  
*(IS 456 Annexure G-11 Pg 46)*

4. Find the moment of resistance of the section.

For under reinforced section

$$M_u = 0.84 \text{ by ASI} \left[ 1 - \frac{0.42 \text{ by ASI}}{\frac{b d^2}{64 f_y k_s}} \right] \text{ (IS 456 Annexure G-11 Pg 46)}$$

For balanced section

$$M_{u_{un}} = 0.36 \frac{x_{umax}}{d} \left[ 1 - 0.42 \frac{x_{umax}}{d} \right] b d^2 f_y k_s$$

*(IS 456 Annexure G-11 Pg 46)*

For Over reinforced section

$$M_{u_{or}} = 0.36 \frac{x_{umax}}{d} \left[ 1 - 0.42 \frac{x_{umax}}{d} \right] b d^2 f_y k_s$$

The section should be redesigned.

*(IS 456 Annexure G-11 Pg 46)*

Q. Calculate the ultimate moment carrying capacity of a reinforced section with breadth 250 mm, eff. depth 400 mm,  $A_{st} = 3600 \text{ mm}^2$ . Use  $f_y = 415 \text{ N/mm}^2$  and  $f_c = 45 \text{ N/mm}^2$

Given Data :-

$$b = 250 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$A_{st} = 3600 \text{ mm}^2$$

$$f_y = 415 \text{ N/mm}^2 (\text{Fe 415})$$

$$f_c = 45 \text{ N/mm}^2 (\text{FC 45})$$

① Depth of M.A.

(IS 456 Annexure G (1) P10)

$$\frac{x_u}{d} = \frac{0.47 f_y A_{st}}{0.36 \times f_c b d}$$

$$= \frac{0.47 \times 415 \times 3600}{0.36 \times 45 \times 250 \times 400}$$

$$= 1.605$$

②  $\frac{Z_u(\text{max})}{d}$

$$\frac{Z_u(\text{max})}{d} = 0.48 ; f_y = 415 \text{ N/mm}^2$$

(IS 456 P10)

③ Comparing  $\frac{x_u}{d}$  &  $\frac{Z_u(\text{max})}{d}$

$$\frac{x_u}{d} > \frac{Z_u(\text{max})}{d}$$

It is an over reinforced section.

$$\begin{aligned}
 Q) \text{ Moment} &= 0.36 \frac{\text{X}_{\text{max}}}{d} \left[ 1 - 0.42 \frac{\text{X}_{\text{max}}}{d} \right] b d^2 f_{ck} \\
 &\quad (\text{Ans. from Q-11 Pg 91}) \\
 &= 0.36 \times 0.48 \left( 1 - 0.42 \times 0.48 \right) 240 \times 400^2 \times 20 \\
 &= 10310.16 \text{ N-mm} \\
 &= 10.31 \text{ kNm}
 \end{aligned}$$

Since the beam is over reinforced, it has to be redesign.

- Q) Calculate the ultimate moment carrying capacity for a rectangular beam with breadth 300 mm. Overall depth 450 mm. Assume  $f_{ck} = 20 \text{ N/mm}^2$  (M20) and  $f_{ck} = 250 \text{ N/mm}^2$  (F250).

Given Data:-

$$b = 300 \text{ mm}$$

$$D = 450 \text{ mm}$$

$$\text{Net } A_s = 1600 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2 \text{ (M20)}$$

$$f_{ck} = 250 \text{ N/mm}^2 \text{ (F250)}$$

$$\text{Assume eff cover} = 40 \text{ mm}$$

$$\text{eff. depth, } d = D - \text{eff. cover}$$

$$= 450 - 40$$

$$= 410 \text{ mm}$$

Q) Depth of NA (Ans. 1) Answer (Q-11 Pg 91)

$$\begin{aligned}
 \frac{\text{X}_n}{d} &= \frac{0.47 \times 410}{0.36 \times 20 \times 300} \\
 &= \frac{0.47 \times 260 \times 1600}{0.36 \times 20 \times 300 \times 410} = 0.34
 \end{aligned}$$

$$\textcircled{3} \quad \frac{T_{max}}{d} = 0.535e \quad (\text{Is } 450 \text{ Pg 46})$$

$$\textcircled{4} \quad \text{Compare } \frac{T_a}{d} \text{ and } \frac{T_{max}}{d}$$

$$\frac{T_a}{d} < \frac{T_{max}}{d}$$

$\therefore$  It is safe under Monitored section  
 (Is 450 Annexure 3.1 Pg 46)

$$\textcircled{5} \quad \text{Moment of resistance} \quad (\text{Is 450 Annexure 3.1 Pg 46})$$

$$M_u = 0.87 f_y A_s f_d \left[ 1 - \frac{b_1 + b_2}{b_1 b_2} \right]$$

$$= 0.87 \times 250 \times 1600 \times 415 \left[ 1 - \frac{600 \times 250}{200 \times 400 \times 20} \right]$$

$$= 11,92,000 \text{ Nmm}$$

$$= 11.92 \text{ kNm}$$

For an RCC beam 200x400 mm is reinforced with 3 nos of 16 mm dia bars of Fe 450. Find the safe number of layers of bars of Fe 450 if the beam can carry safely over a span of 5 m.

Take  $M_{Ed}$  correctly.

Given Data:

$$b = 200 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$f_y = 415 \text{ N/mm}^2 (\text{Fe 450})$$

$$M_y = 20 \text{ N-mm}^2 (M_{Ed})$$

$$l = 5 \text{ m}$$

Assume  $d = 40 \text{ mm}$

$$d - D - d_s = 400 - 40 - 40 = 320 \text{ mm}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.125 \text{ mm}^2$$

① Depth of M.A. (IS 456 Annex G-1.1 pg 96)

$$\frac{x_u}{d} = \frac{0.83 b_f A_s}{0.36 b_f d}$$

$$= \frac{0.83 \times 415 \times 603.125}{0.36 \times 207.00 \times 360} = 0.420$$

②

$\frac{x_{umax}}{d}$

$$\frac{x_{umax}}{d} = 0.42 ; F_a 415 \quad (\text{IS 456 pg 70})$$

③ Compare  $\frac{x_{umax}}{d}$  and  $\frac{x_u}{d}$

$$\frac{x_u}{d} < \frac{x_{umax}}{d}$$

∴ The section is under-reinforced.

④ Moment of Resistance (IS 456 Annex G-1.1 pg 96)

$$M_u = 0.83 b_f d \left( 1 - \frac{b_f A_s}{b d} \right)$$

$$= 0.83 \times 415 \times 603.125 \times 360 \left( 1 - \frac{415 \times 603.125}{207.00 \times 360} \right)$$

$$= 64.71204 \times 10^6 \text{ N-mm}$$

$$= 64.712 \text{ kNm}$$

⑤ Load

$$M_u = \frac{w l^2}{8} \quad (\text{ndl})$$

$$w = \frac{M_u \times 8}{l^2} = \frac{64.712 \times 8}{5^2} = 20.726$$

Q. A  $\boxed{1}$  beam 200 mm wide, and 400 mm deep upto the centre of reinforcement. Find the area of reinforcement required if it has to develop a moment of 25 kNm. Use M<sub>20</sub> concrete and Fe 450 steel.

Given Data:-

$$b = 200 \text{ mm} \quad f_{ck} = 25 \text{ N/mm}^2$$

$$d = 400 \text{ mm} \quad f_y = 450 \text{ N/mm}^2$$

$$M_u = 25 \text{ kNm}$$

$$\textcircled{1} \quad \text{Factored BM} = 1.5 \times 25 \quad (\text{for } \times 3\%)$$

$$= 37.5 \text{ kNm}$$

② Area of Reinforcement

$$M_u = 0.87 f_y b t d \left[ 1 - \frac{h + \delta}{h + 6\delta} \right] \quad (\text{IS 456 Annexure D pg 26})$$

$$37.5 \times 10^6 = 0.87 \times 450 \times 400 \left( 1 - \frac{h + 45}{200 + 400 + 20} \right)$$

$$26.545 = Act \left[ 1 - 2.59375 \times 10^{-4} \text{ At} \right]$$

$$2.59375 \times 10^{-4} \text{ At}^2 - At + 26.545 = 0$$

$$\begin{aligned} At &= 352.155 \text{ mm}^2 \\ At &= 279.95 \approx 280 \text{ mm}^2 \end{aligned} \quad \left. \right\} \text{ least}$$

$$\therefore \text{Take } At = 280 \text{ mm}^2$$

③ Lamda

$$\lambda = 0.45 + \frac{f_y}{f_{ck}} \quad (\text{IS 456 Eqn. 6.1-1 pg 26})$$

$$\lambda = 0.45 + \frac{450}{25} \quad (\text{IS 456 Annexure D-1 pg 26})$$

④ Depth of NA

$$\frac{L_s}{d} = 0.87 \times \frac{f_y}{f_{ck}}$$

$$\frac{1}{0.36 \text{ sec. std}}$$

$$= \frac{0.87 \times 415 \times 280}{0.36 \times 29 \times 280 \times 10^6}$$

$$= 0.1156$$

$$\underline{\underline{= 0.1156}}$$

Q) Compare  $\frac{x_a}{d}$  and  $\frac{x_{\text{max}}}{d}$

$$\frac{x_a}{d} < \frac{x_{\text{max}}}{d}$$

∴ The section is under-reinforced

The condition assumed is correct.

- Q) Analyse a rectangular beam 800mm width and 500mm eff. depth. Determine ultimate M.R of the tension reinforcement of 4 nos. 16 mm  $\phi$ . Use  $M_{R0}$  constants and Fe 415 steel.

Given Data :-

$$b = 800 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$A_{st} = 4 \times \pi / 4 \times 16^2 = 804.24 \text{ mm}^2$$

$$f_y = 425 \text{ N/mm}^2 (\text{Fe 415})$$

$$f_{ck} = 20 \text{ N/mm}^2 (M_{R0})$$

① Depth of NA

(IS 456 Annex Cr-1.1 pg 76)

$$\frac{x_a}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

$$② \frac{X_{max}}{d} \left( IS 456 Pg 76 \right)$$

$$\frac{X_{max}}{d} = 0.45 \times 4.45$$

$$③ \text{ Compare } \frac{X_u}{d} \text{ and } \frac{X_{max}}{d}$$

$$\frac{X_u}{d} < \frac{X_{max}}{d}$$

∴ The section is under reinforced.

$$(IS 456 Annex G-1-1 Pg 94)$$

(i) MOR

$$M_u = 0.87 \text{ by Ref. } \left( 1 - \frac{63 \text{ N/mm}^2}{f_y \times 1000} \right)$$

$$= 0.87 \times 4.45 \times 8.04 \cdot 24 \times 500 \left( 1 - \frac{425 \times 204 \cdot 94}{300 \times 500 \cdot 24} \right)$$

$$= 129.035 \text{ Nmm}$$

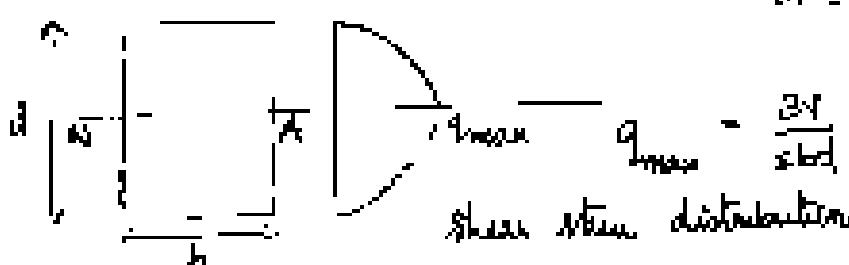
$$M_u = 129 \text{ kNm}$$

=====

Limit state of collapse in shear and bond-shear stressed  
in beams - types of reinforcement - shear strength of RC  
beam - IS code accommodation for shear design -  
design of shear reinforcement - examples.

design of shear reinforcement -  
Bond and development length - anchorage for reinforcement  
Bars - code recommendations regarding installation of  
reinforcement.

A beam loaded with transverse loads & subjected to SF and BM.  
 The shear force at any section is equal to the rate of change  
 of BM. SF results into shear stresses across the cross section. Shear  
 stress is parabolic with zero at top and bottom and max. shear  
 stress occurs at  $M = \frac{3V}{2bd}$  where  $V = S.F$  at the section  
 $b = \text{width of section}$   
 $d = \text{depth of section}$ .



*Shear stress distribution in a rectangular cut*

John L. Jones & P.C. Beard

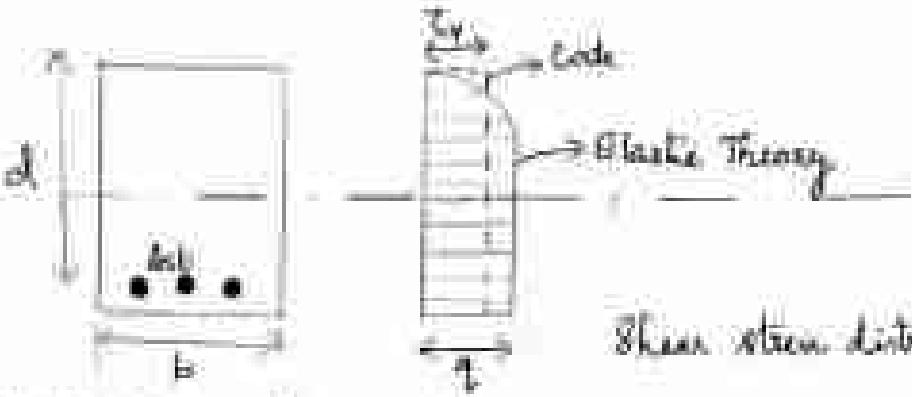
#### 1) Edge Based Approach. (Flexible Tracing)

1) Stress Based Approach (Elasticity Theory):  
 Shear stress is parabolic in the compression zone with zero at the top and max. at the N.A. The value of shear stress is constant in the tensile zone and is equal to max. shear stress ( $\tau$ ).  
 The max. value of shear stress ' $\tau$ ' as per elastic theory is  $\tau = \frac{V}{b d t}$ .

*against. The shear force at the section*

Learn some depth factors

b and d = Dimensions of the section.



Shear stress distribution in RCC beam

### (i) IS Code Approach

As per IS code 456:2000, the shear based approach depicts the true behaviour of the RCC beam in shear. Here the eqn  $\frac{q \cdot v}{b d}$  has been simplified & recommends the use of  $\frac{b d}{2}$  shear strength ( $V_s$ ) for RCC beams. The nominal shear stress ( $\tau_n$ ) or avg. shear stress distribution is given by

$$\tau_n = \frac{V}{b d} \quad (\text{IS 456 Cl 40.1 Pt 2})$$

Limit state of collapse gives the following

$80-90\%$  of SF resisted by web and remaining by flange

### Types of Shear Failure

#### i. Diagonal Tension Failure

If occurs under large SF and small BM. Consider zone or near supports.

#### ii. Flawed Shear Failure

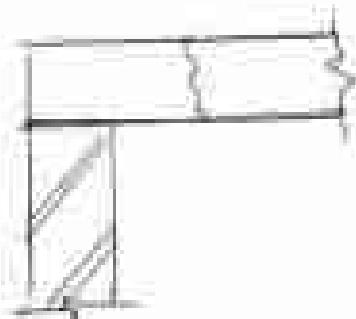
If occurs under large BM and small SF. Ans between the supports and modulus the cracks inclination vary from  $45^\circ$  to  $90^\circ$  gradually.

### 3. Diagonal Compression Failure.

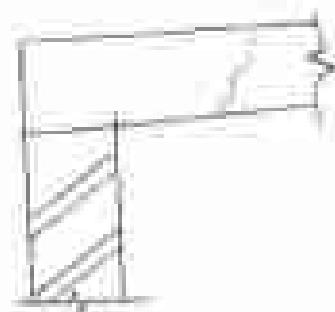
It also occurs under large SF and due to it there may be compression cracked diagonal cracks may be found.



Diagonal tension  
failure



Lateral shear  
failure



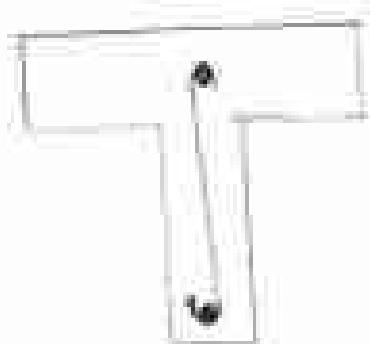
Diagonal compression  
failure

### Type of Shear Reinforcement

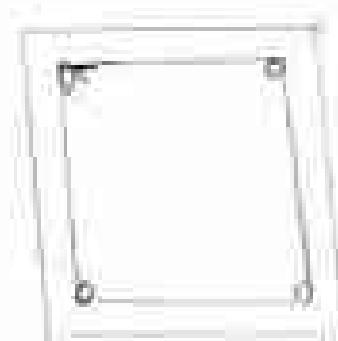
The following are the types of shear reinforcement used:

1. Vertical Stirrups
2. Bent up bars along with stirrups
3. Inclined Stirrups

### Vertical Stirrups

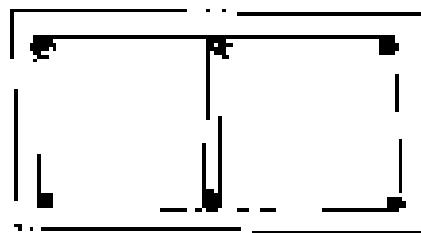


Single legged stirrup

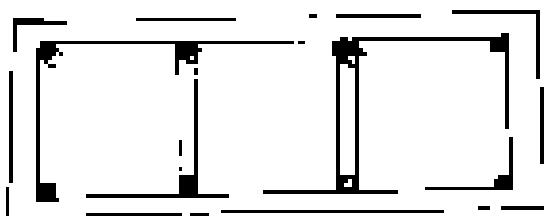


Two legged stirrup

(4) There are two steel bars vertically placed around the tensile reinforcement at suitable spacing along the length of the beam. Their diameter varies from 8mm to 16mm. The free end of the stirrups are anchored in the compression zone of the beam to the anchor bars. In the compressive reinforcement Depending upon the magnitude of SF to be resisted different stirrup shapes may be one legged, two legged, four legged and so on.



One legged stirrup.

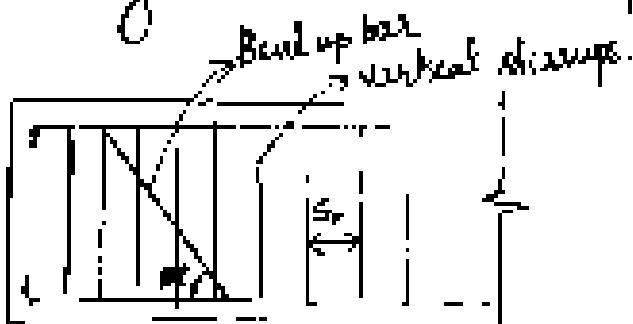


Two legged Stirrup.

It is desirable to use closely spaced stirrups for better prevention of diagonal cracks.

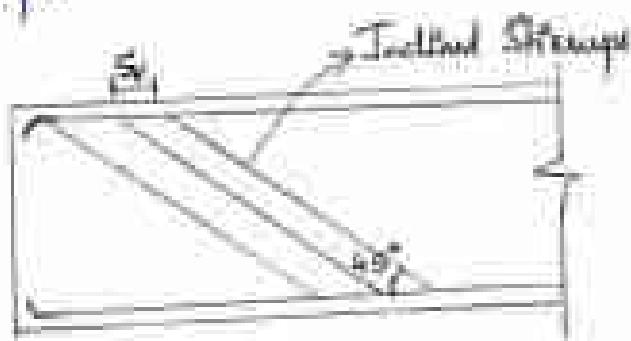
The spacing of stirrups near the support is less as compared to spacing near the mid-span since SF is max. at the supports.

But up bars along with vertical stirrups



- \* Some of the longitudinal bars in a beam can be bent up near the supports where they are not required to resist BM (at the supports).
- \* These bent up bars resist diagonal tension (Diagonal shear force near supports).
- \* Equal number of bars are to be bent on both sides to maintain symmetry.
- \* These bars are usually bent up at  $45^\circ$ .
- \* This system is used for heavier SF.
- \* The total shear resistance of the beam is calculated by adding the contributions of bent up bars and vertical stirrups.
- \* The contribution of bent up bars is not greater than half of total shear reinforcement.

### Tied and Strapped



- \* Provided at  $45^\circ$  for resisting diagonal tension.
- \* Provided throughout the length of the beam.

Helical / spiral ties / Hoop reinforcement of column  
Circular ties

## ⑥ Shear strength of R.C. Beam.

The shear resistance of a R.C. beam depends upon the following:-

1. Grade of Concrete

Shear resistance varies with the grade of concrete.

2. Percentage of Tensile Reinforcement.

Shear strength varies with % of tensile reinforcement ( $A_s$ ).

Necessity for minimum Shear reinforcement

- \* Safety against any sudden failure of a beam.  
If the concrete over collapsed and bond to the tension zone is lost.
- \* Prevent brittle shear failure which can occur without shear stress.
- \* Prevent failure caused by tension due to thermal stresses and restraint cracking of beam.
- \* Hold the reinforcement in the place while placing concrete.
- \* Act as necessary ties for compression steel and make them effective.

②  
Steps for design of shear reinforcement (IS-456 Ed 3 Pg 73)  
Final State of Design : Shear

Given :- loads

A pair of beam

Material - concrete grade and type of steel

Area of tensile sheet (A<sub>s</sub>)

Procedure :-

i) Calculate  $V_{U,D}$  at the critical section of the beam due to the given loads.

ii) Determine nominal shear stress ( $\tau_v$ )

$$\tau_v = \frac{V_u}{bd} \quad (\text{IS-456 Ed 3 Pg 73})$$

where  $V_u \rightarrow$  SF due to design loads

b  $\rightarrow$  breadth of member

d  $\rightarrow$  off depth

iii) Depending upon the grade of concrete and % of steel ( $\frac{\text{Area of steel}}{\text{Area of concrete}}$ ), find out shear strength of concrete ( $\tau_c$ ) (IS-456 Table 11 Pg 73)

iv) Compare  $\tau_v$  and  $\tau_c$  max (IS-456 Table 10 Pg 73)

If  $\tau_v > \tau_c$  max undergo the action.

v) Compare  $\tau_v$  and  $\tau_c$

$\rightarrow$  If  $\tau_v < \tau_c$

Nominal shear reinforcement is to be provided in form of vertical stirrups.

The spacing of vertical stirrups gives as (Non Shear Reinforcement)

$$\frac{A_{sv}}{b \cdot S_r} \geq 0.4 \quad (\text{IS-456 Ed 3 Pg 14 Pg 40})$$

(8)

$$\rightarrow d_f > t_c$$

Some reinforcement is to be designed as follows:-

\* Calculate  $V_{us}$

$$V_{us} = V_u - \zeta b d \quad (\text{IS 456 Cl 4.4 pg 15})$$

\* If vertical stirrups are provided then their spacing is given by the following equation

$$s_v = \frac{0.45 \text{ by } A_{sv} \cdot d}{f_y}$$

$A_{sv}$   $\rightarrow$  total c/s area of stirrups legs  
on bent up bars within a distance  $s_v$

$s_v$   $\rightarrow$  Spacing of stirrups or bent up bars  
along the length of the member

\* If bent up bars are used, then calculate SF taken by  
bent up bars  $V'_{us}$ ;  $V'_{us} = \frac{V_{us} + V_{us}}{2}$

$$V_{us} = \frac{0.45 \text{ by } A_{sv} \cdot d}{f_y} \quad (\text{IS 456 Cl 4.4 pg 15})$$

$$\text{or } (\text{IS 456 Cl 4.4 pg 15})$$

for enclosed stirrups or a series of bars bent up at  
diff. c/s.

$$V_{us} = 0.45 \text{ by } A_{sv} \cdot d \cdot f_y$$

$$(\text{IS 456 Cl 4.4 pg 15})$$

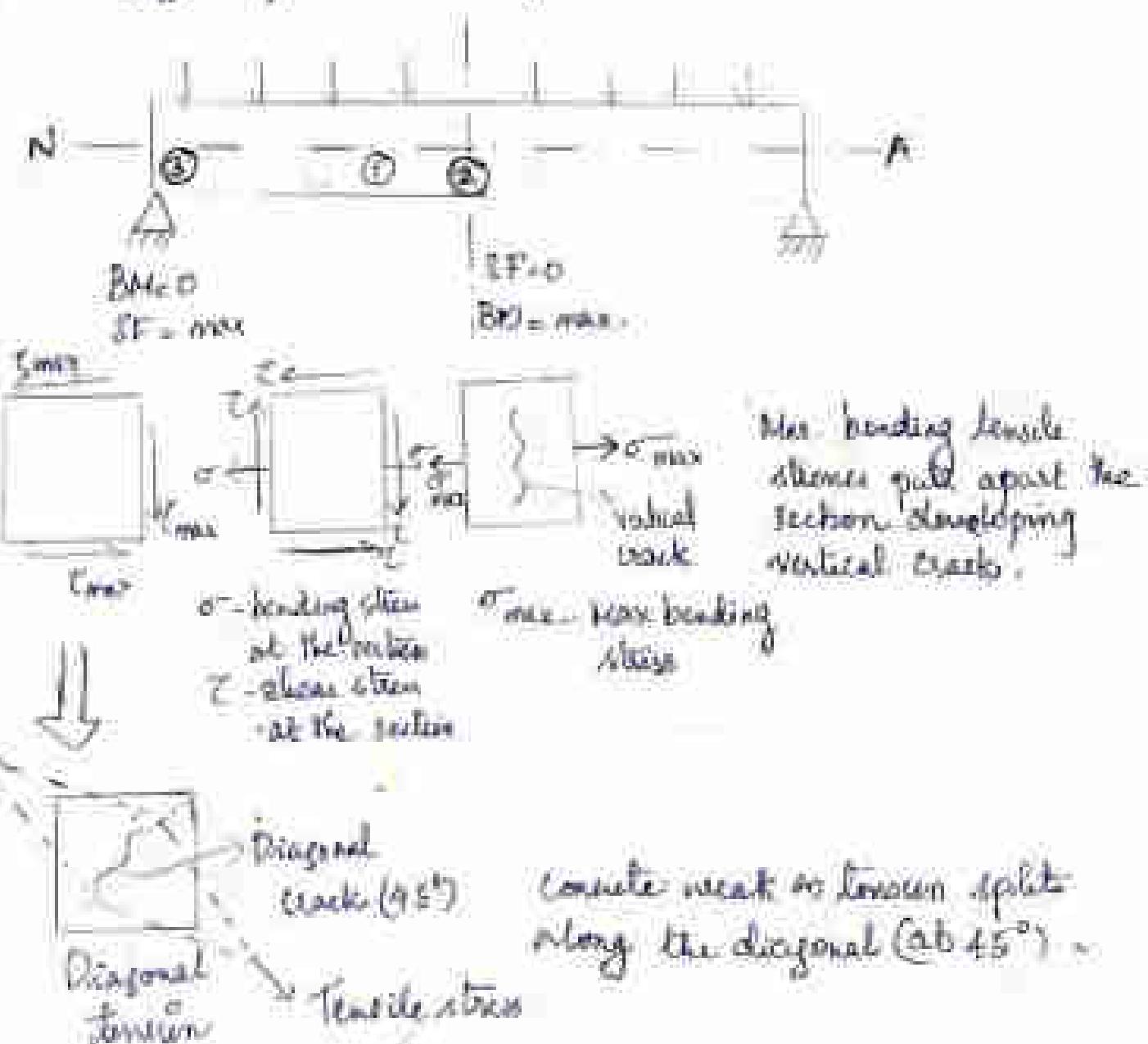
for single bar or single group of bent bars all  
bent up at same cross section

or  $\rightarrow$  If the enclosed stirrup or bent  
bar and the axis of member not  
less than  $45^\circ$ .

For the balanced ST ( $V_{us} = V_{uc}$ ), design the vertical stirrups. (In Reinforced Systems)

- Spanning of stirrups should not be  $\geq 0.15d$  or  $300\text{ mm}$  which ever is less. (IS 456 (26-5-1-5 pg 47))
- Spanning can be varied across length by calculating the distance from supports up to which the shear reinforcement is to be designed, and no rest of the length plain shear reinforcement may be provided.

Effect of shear :- Diagonal Tension.



Q10 A simple supported beam having bottom wide end and depth 450 mm (eff. depth) is reinforced with 4 nos of 18 mm  $\phi$  bars. Design shear reinforcement if M20 grade of concrete and Fe450 steel is used. Beam is subjected to SF = 60 kN.

Given Data :-

$$b = 300 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 18^2 = 1017.876 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$V = 150 \text{ kN}$$

$$\text{Factored SF, } V_u = 1.5 \times 150 = 225 \text{ kN}$$

Planned Shear force,  $C_s$

( $F_a + F_c - C_s - P_d$ )  $\rightarrow$

$$C_s = \frac{V_u}{bd}$$

$$= \frac{225 \times 10^3}{300 \times 450} = 12.8 \text{ N/mm}^2$$

Design shear strength of concrete,  $C_c$  ( $\leq 40\% \text{ of all } V_u$ )

$$pt = \frac{100 A_{st}}{bd} = 100 \times \frac{1017.876}{300 \times 450} = 0.9047$$

By interpolation

$$0.15 \rightarrow 0.56$$

$$1 \rightarrow 0.62$$

$$\frac{0.56 - 2}{0.75 - 0.15} = \frac{0.46 - 0.42}{0.15 - 1}$$

$$\frac{0.56 - 2}{-0.45} = \frac{-0.06}{-0.24}$$



$$\begin{array}{r}
 0.567 \\
 -0.15 \\
 \hline
 0.567 \\
 -0.074 = 0.567 \\
 \hline
 0.567
 \end{array}$$

$$T_{\text{max}} = 0.5 \text{ N/mm}^2 \quad (\text{Given fail 20 Pg 18})$$

$$T_y < T_{\text{max}}$$

Hence OK.

$$T_y > T_c$$

Thus reinforcement to be provided

$$\text{Shear strength of reinforcement, } V_{us} = V_u - T_{c,b,d}$$

$$\begin{aligned}
 &= 225 \times 10^3 - 0.576 \times 250 \times 450 \\
 &= 15745 \text{ N}
 \end{aligned}$$

Use form of 2 legged stirrup (vertical) ~~(Assumption)~~

$$\text{Shear strength of reinforcement, } V_{us} = \frac{0.37 \text{ by hand}}{S_v}$$

$$\begin{aligned}
 A_{sv} &= 24 \frac{\pi r^2}{h} \quad (\text{35456, Pg 23}) \\
 &= 100.53 \text{ mm}^2
 \end{aligned}$$

$$S_v = \frac{0.37 \text{ by hand}}{V_{us}}$$

$$= \frac{0.37 \times 415 \times 100.53 \times 450}{15745}$$

$$= 103 \text{ mm}$$

Nominal shear reinforcement (IS 456 Cl. 20.5.1.6 Pg 47)

$$\frac{A_{sv}}{b s_y} = \frac{0.4}{0.3765}$$
$$\frac{100 \times 550}{250 \times 35} = \frac{0.4}{0.17 \times 4.15}$$

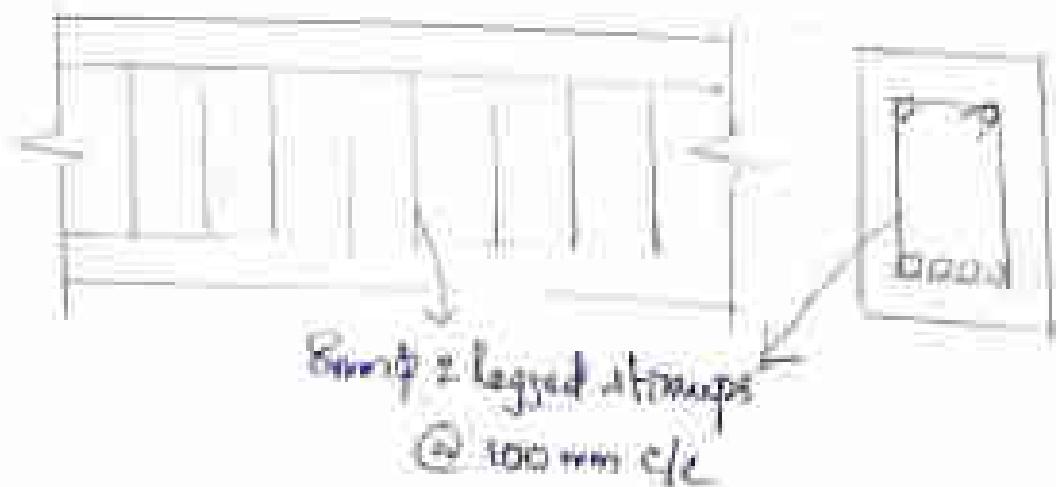
$$S_y = \frac{100 \times 550 \times 0.37 \times 4.15}{0.4 \times 1.25} = 362.96 \text{ mm}$$

Check for spacing (IS 456 Cl. 20.5.1.5 Pg 47)

Spacing of stirrups should be min. of the following

- i)  $203 \text{ mm} \cong 100 \text{ mm}$
- ii)  $0.16d = 0.16 \times 450 = 351.5 \text{ mm}$
- iii) 200 mm
- iv) 362 mm

∴ Provide 2 legged 8mm dia bars as stirrups  
@ 100 mm c/c



Ques. The beam 1750 x 400 mm cantilever is carrying a load of 15 kNm. The beam is reinforced with 4 bars of 32 mm dia. Clear span of beam is 4 m. Design the shear reinforcement like M25 concrete and mild steel bars.

$$b = 1750 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$A_s = 4 \times \frac{\pi}{4} \times 32^2 = 150 \times 32^2 \text{ mm}^2$$

$$f_y = 240 \text{ N/mm}^2$$

$$f_{ck} = 20.14 \text{ N/mm}^2$$

$$w = 15 \text{ kNm} = 15 \text{ kN/m}$$

$$V = \frac{wL}{2} + \frac{w \times 4}{2} = 35 \times 10^3 \text{ N}$$

$$\sqrt{V} = 1.5 \times 35 \times 10^3 \\ = 45 \times 10^3 \text{ N}$$

Nominal shear stresses ( $\tau_{n1}, \tau_{n2}, \tau_{n3}, \tau_{n4}$ )

$$\tau_{n1} = \frac{\sqrt{V}}{bd} = \frac{45 \times 10^3}{250 \times 400} = 0.45 \text{ N/mm}$$

Design shear strength  $\tau_d$  (IS 456 pg 73 Table 11)

$$\tau_d = \frac{10046}{bd} = \frac{10046 \times 250 \times 400}{1000000} = 1.520$$

by interpolating  
 $\tau_d = 0.72$

$$\tau_{nmax} = 2.0 \text{ N/mm} \quad (\text{IS 456 Table 20})$$

$\tau_d < \tau_{nmax}$  Hence OK

Compare  $\tau_d$  and  $\tau_c$

$$\tau_c > \tau_d$$

No shear reinforcement is required

But as per IS 456, nominal shear reinforcement must be provided.

Nominal shear reinforcement (IS 456 Cl 26.5+6 Pg 4)

$$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.47 \times 15}$$

Using 2 legged shear stirrups

$$A_{sv} = 2 \times \frac{\pi d^2}{4} = 100.53 \text{ mm}^2$$

$$\frac{100.53}{0.47 \times 50} = \frac{0.416}{0.47 \times 150}$$

$$\frac{0.4021}{S_v} = 1.837 \times 10^{-3}$$

$$S_v = 212.66 \text{ mm} \approx 200 \text{ mm}$$

Check for spacing (IS 456 Cl 26.5+6 Pg 4)

Spacing of the stirrups should be min of the following

i) 200 mm

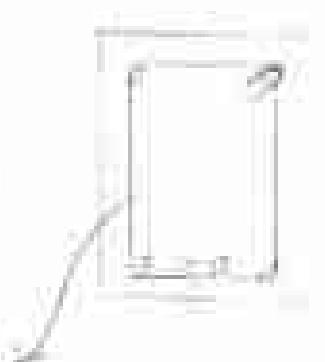
ii) 0.75 \* d = 0.75 \* 400 = 300 mm

iii) 300 mm

Provide 2 legged shear of stirrups @ 200 mm c/c



1 legged shear  
@ 200mm c/c



2 legged shear  
@ 200mm c/c

A simply supported beam (see a free end) of 111 depth is reinforced with 5 bars of 25 mm dia. of concrete grade of M 15. If a load of 30 kN is provided on an off span of 6 m. Out of the given bars, 12 bars can be fixed up safely near the supports. Design the shear reinforcement for the beam. Use R.C.C. concrete and Fe 450.

Given Data:-

$$l = 300 \text{ m}$$

$$d = 600 \text{ mm}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 25^2 = 1177.6 \text{ mm}^2$$

$$q = 30 \text{ kN/m}$$

(2 bars fixed up  
only 10 bars remaining  
at supports)

Assume width of the support as 400 mm

Shear force  $V = \frac{wL}{2}$

$$= \frac{30 \times 400 \times 6}{2} = 360 \text{ kN}$$



$$\text{Design SF, } V_u = 204 \times 1.5$$

$$= 306 \text{ kN}$$

$$= 306 \times 10^3 \text{ N}$$

allowable shear stress

$$\tau_v = \frac{V_u}{bd} \quad (15 \text{ kg/cm}^2 \text{ or } 1.5 \text{ MPa})$$

$$= \frac{306 \times 10^3}{400 \times 600}$$

$$= 1.25 \text{ MPa}$$

$$= 1.25 \text{ MPa}$$

⑩

$\tau_{cmax} = 2.8 \text{ N/mm}^2$  (IS 456 Table 20)

$\tau_c < \tau_{cmax}$

Hence OK

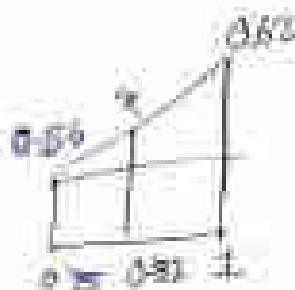
Design shear strength,  $\tau_c$  (IS 456 Table 14)

$$\frac{f_t + \text{rec } f_t}{\text{ad}} = \frac{100 \times 11.5 \times 6}{250 \times 600}$$

$$= 0.22 \text{ N/mm}^2$$

By interpolation:

$$\frac{0.62 - 0.56}{1 - 0.75} = \frac{x - 0.56}{0.82 - 0.75}$$



$$x = 0.58 \rightarrow \text{FoS} = 0.78 \text{ N/mm}^2$$

Compare  $\tau_c$  and  $\tau_s$ .

$$\tau_c > \tau_s$$

Since  $\tau_c > \tau_s$ , shear reinforcement is required.

Strength of shear reinforcement (IS 456 Pg 75)

$$V_{sd} = V_u - \tau_{sd} b d$$

$$= 3.56 \times 10^3 - 0.35 \times 600 \times 300 = 39.0$$

$$= 211600 \text{ N}$$

$$\frac{V_{sd}}{2} = 105800 \text{ N}$$

Bent up bars, (IS 456 Pg 75)

$$V_{sd} = 0.84 b y t u \text{ N/mm}^2$$

Shear resistance  $V_{sd}$  for two bent up bars

$$V_{sd} = 2 [0.84 \times 30 \times 300 \times 300]$$

$$\Delta_{SV} = \frac{R}{N} + 20 \quad (\text{max shear load up})$$

$$= 470.87 \text{ mm}$$

Nominal  $V_{us} = 270.87 + 4.65 + 470.87 + 3 \times 45$

$$= 850634.108 \text{ N}$$

$$V_{us} > V_{us}^{\prime} \quad \text{but } V_{us} < \frac{V_{us}}{2}$$

$$\therefore V_{us} = \frac{V_{us}}{2} = 115800 \text{ N}$$

$V_{us} \rightarrow$  shear taken by backup bars.

Balance shear area have to designed for vertical stirrups.

Balance of shear to be carried by vertical stirrups,

$$V_{us} - V_{us}'$$

$$V_{us} = 231650 - 115800$$

$$= 115800 \text{ N.}$$

### Design of Vertical Stirrups

Assume 8 nos of 8 legged stirrups.

$$A_{sv} = \frac{R}{N} \frac{\pi \times 8^2}{4} = 100.630 \text{ mm}^2$$

$$V_{us} = 0.37 \frac{b_s A_{sv} d}{s_y} \quad (\text{Ex-406 pg-73})$$

$$SV = \frac{0.37 \times 417 \times 100.63 \times 600}{16800}$$

$$= 122.064$$

$$\approx 130 \text{ mm}$$

(b) Nominal shear reinforcement ( $\rightarrow$  A.S.C. 21.26 rev. 1.5 Pg 48)

$$\frac{A_{sv}}{b s_y} = \frac{0.4}{0.47}$$

$$\frac{100.53}{300 \times 34} = \frac{0.4}{0.47 \times 415}$$

$$S_{sv} = 308.46 \text{ mm}$$

Check for Spacing ( $\rightarrow$  A.S.C. 21.26 rev. 1.5 Pg 47)

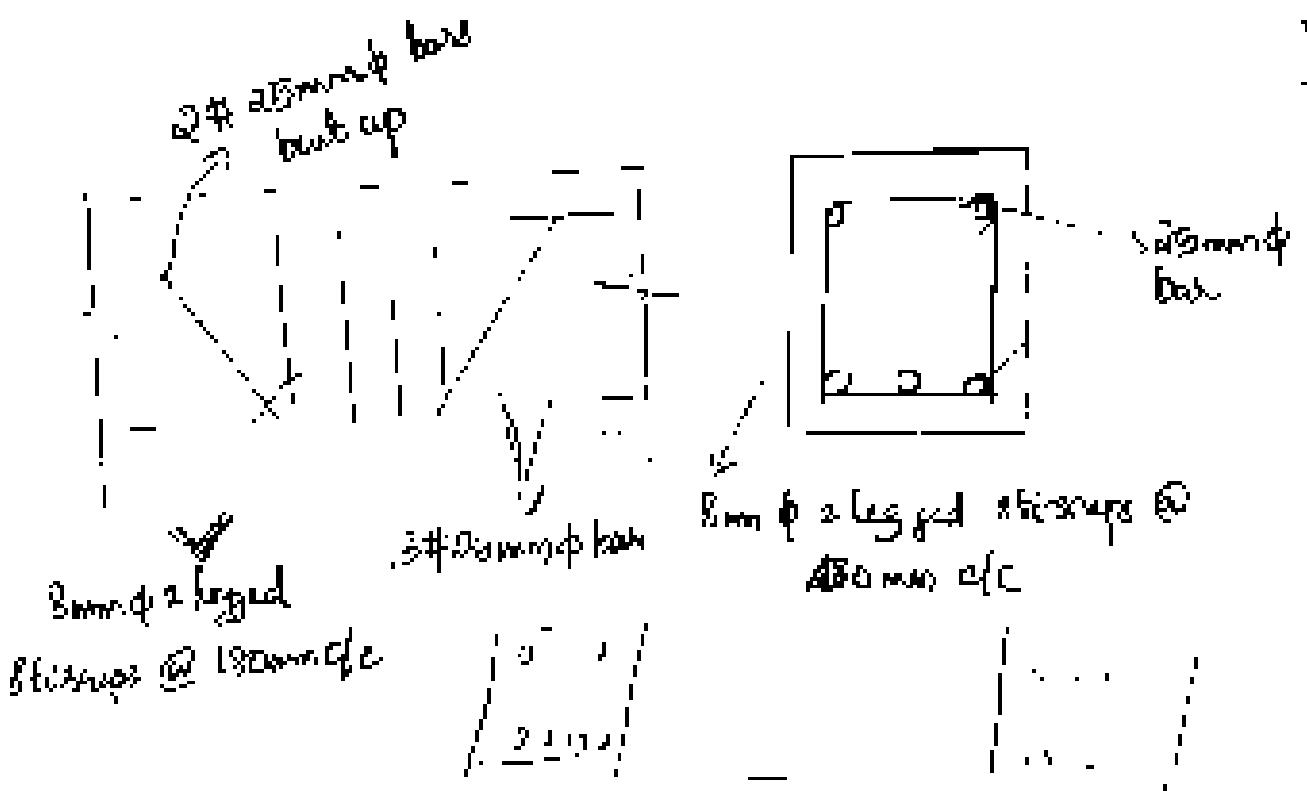
i) 180 mm

ii)  $0.75d = 450 \text{ mm}$

iii) 300 mm

iv) 362 mm

Provide 8 mm  $\phi$  @ legged stirrups @ 180 mm c/c.



In the above example if only vertical strips are to be used as shear reinforcement and no bars are bent up.  
Given Data :-

$$b = 300 \text{ mm}$$

$$d = 600 \text{ mm}$$

$$A_t = 6 \times \frac{\pi \times 25^2}{4} = 3453.3 \text{ mm}^2$$

$$W = 30 \text{ kN/m}$$

$$\text{SF}_v V = \frac{Wl}{2} = \frac{30 \times 5 \times 6}{2}$$

$$= 450 \text{ kN}$$

$$\text{Design SF}_v V_n = 824.815$$

$$= 330 \text{ kN}$$

Reinforced shear stress,

$$\tau_v = \frac{V_n}{bd} \left( 1.5 + 5.6 \times 1.40 + P_d^{0.5} \right)$$

$$= \frac{330 \times 10^3}{300 \times 400} = 1.35 \text{ N/mm}$$

$$\tau_{max} = 0.8 \text{ N/mm}^2 \quad (\text{IS 456 Table 26})$$

$$\tau_v < \tau_{max}$$

Hence OK.

Design shear strength, or  $\phi \tau$  (IS 456 Table 19 Pg 73)

$$\phi \tau = \frac{100 A_s b}{bd^2}$$

$$= \frac{100 \times 2.454 \cdot 26}{300 \times 400}$$

$$= 1.361$$

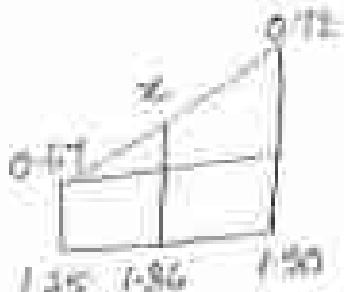
For M.R.D. grade concrete

$$0.72 - 0.67$$

$$1.50 - 1.25$$

$$x = 0.67$$

$$1.30 - 1.25$$



$$\sigma_c = \tau_c = 0.67 \text{ N/mm}^2$$

$$\sigma_c < \sigma_{\text{concrete}}$$

Hence OK

Design of Vertical Stirrups.

$$\text{Shear resistance by concrete} = \tau_c b d$$

$$= 0.67 \times 300 \times 600$$

$$= 124200 \text{ N}$$

$$\therefore \text{Shear to be taken by stirrups, } V_{us} = V_u - \tau_c b d$$

$$= 236000 - 124200$$

$$= 211800 \text{ N}$$

Using Span of a legend stirrups

$$\text{Area} = 2 \times \frac{\pi \times 8}{4} = 100.53 \text{ mm}^2$$

$$\text{Span of stirrups, } S_v = \frac{0.87 \text{ m}}{\text{Area}} \text{ (IS 456 Pg 265)}$$

$$= \frac{0.87 \times 100.53 \times 600}{211800}$$

$$= 100.09 \approx 100 \text{ mm}$$

Spanning for nominal shear reinforcement (IS 456 Cl 265)

$$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.176}$$

Pg 48

$$S_y = 0.84 \frac{\text{kg}}{\text{mm}^2} A_{sv}$$

$\therefore 0.84$

$$= \frac{0.84 \times 4(5.6)(0.05)}{0.4 \times 200}$$

$$= 302.4 \text{ mm}$$

Check for spacing (IS 456 Cl 26.5.1.5 pg 41)

The spacing should be least of the following:

i)  $0.75d = 0.75 \times 800 = 600 \text{ mm}$

ii) 300mm

iii) 100 mm

iv) 302mm

∴ Provide 100mm of 2 legged stirrups @ 100 mm c/c.

## BOND AND DEVELOPMENT LENGTH

### Bond

The term bond refers to the adhesion between Concrete and Steel which resist the slipping of steel bar from concrete. It is the bond which is responsible for transfer of stresses from steel to concrete thereby providing composite action of steel and concrete as per.

The bond develops due to setting of concrete on drying which results in gripping of the steel bars.

The bond resistance in RC is obtained by following mechanism :-

1. Chemical adhesion

It is due to glue like property of the substance formed after setting of concrete.

2. Frictional Resistance

It is due to friction b/w steel and concrete.

3. Gripping action

It is due to gripping of steel by the concrete on clamping.

4. Mechanical Interlock

It is provided by irregularities on ribs based on the surface of deformed bars.

The bond is assumed to be perfect in the design of Reinforced concrete.

The bond b/w steel and concrete can be increased by the following methods:-

\* Using deformed or twisted bars

\* Using thick concrete mix

\* Adequate compaction & curing of concrete for proper setting.

\* Providing hooks at the end of the reinforcing bars.

where,  $L_d$  is the embedded length of steel bar  
of characteristic strength of the bar  
 $L_d$  design bond stress by Lumb's ultimate method  
 $\phi$  diameter of bar

+ 'L\_d' is the development length.

+ It is the minimum length of bar which must be  
embedded in concrete beyond any section to develop its  
full strength.

This is also called as anchorage length or safe. of axial  
tension or axial compression and development length  
in case of flexural tension or flexural compression.

The permissible bond stress  $\phi$  depends upon the grade  
of concrete and type of steel.

The values of permissible bond stress are given in  
IS 456 Cr. 26-2.1.1. pg 43.

#### NOTE

For deformed bars,  $L_d \geq 6c$  more than plain bars  
It is easier to pull a bar than to break it inside.  
Therefore permissible bond stress for plain and deformed  
bars in compression is taken as 25% more than that for  
the bars in tension.

There are two types of bond failure

- ① Anchorage bond failure.
- ② Flawed bond failure.

### Anchorage Bond (Development length)

For understanding the concept of bond and development length, let us consider a steel bar embedded in concrete.



The bar is subjected to a tensile force  $T$ . Due to this tensile force, the steel bar will tend to come out and slip out of the concrete. This tendency of slipping is resisted by the bond stress developed over the surface of the bar.

Bond stress ( $C_{bd}$ ) is the shear stress developed along the contact surface between the reinforcing steel and the surrounding concrete which prevents the bar from slipping out of concrete.

To avoid slipping

$$T \leq C_{bd} \times 2\pi \frac{\Phi}{2} \times L_d \quad \left\{ \begin{array}{l} \text{Surface Area} \\ = 2\pi \frac{\Phi}{2} \times L_d \end{array} \right.$$

$$0.87 b_y \times \frac{\pi}{4} \Phi^2 \leq C_{bd} \times 2\pi \times \frac{\Phi}{2} \times L_d \quad \left\{ \begin{array}{l} T = 0.87 b_y \\ \times \frac{\pi}{4} \Phi^2 \end{array} \right.$$

$$0.87 b_y \times \frac{\pi}{4} \Phi^2 \leq C_{bd} \times 2\pi \times \frac{\Phi}{2} \times L_d \quad \left\{ \begin{array}{l} T = 0.87 b_y \\ \times \frac{\pi}{4} \Phi^2 \end{array} \right.$$

$$L_d \geq \frac{0.87 b_y \Phi}{4 C_{bd}}$$

$T = 456 \text{ kN}$

## Flexural Bond

In a simple beam, at the critical sections i.e. at free end of the support, at point of inflection and at point of high SF, high bond stress may develop due to large variations in BM. These bond stresses are called flexural bond stress. Should be checked carefully at critical sections.

Consider a beam subjected to flexural bending.

Consider two sections at a distance  $\Delta x$  and  $\Delta x + \Delta z$  along the length of the beam subjected to moment  $M$  and  $M + \Delta M$  at  $x$  &  $x + \Delta x$  resp. Let  $T$  and  $T + \Delta T$  be the tension.



$\Delta x$  Flexural bond stress  
Flexural bond stress is tension reinforcement  $\sigma$

$$\sigma_{\text{bond}} = \frac{V}{A \cdot b \cdot a}$$

where  $V$  = SF factored at the section  
 $A$  = area of bar  
 $a$  = lever arm

Development Anchorage length,  $L_d \leq \frac{M_1}{V} + l_0$   
 where  $M_1$  = moment of reaction (IS 456 Cl 20.2.2 Pg 44)  
 at the section

$V$  = factored SF at the section

$l_0$  = additional length provided for safety

## ANCHORAGE OF REINFORCEMENT BARS

(IS 456 Cl 20.2.2 Pg 43)

The development length of bars can be provided in the form of straight bar if available otherwise it may be partially straight and buttably hooked.

The anchorage is normally provided in the form of bends and hooks.

Anchorage bars in tension (IS 456 Cl 20.2.2 Pg 43)



hook =  $16\phi$



bend  
 $n=2$ ; mild steel bars  
 $n=4$ ; deformed bars

$\rightarrow L_g \rightarrow$

1 Anchorage value

bend

Anchorage value

Anchoring Bars in compression (IS 456 Cl 20.2.2 Pg 43)

Anchoring shear reinforcement (IS 456 Cl 26.2.4 Pg 43)

## Calculation of Reinforcement - Code Recommendations

(IS 456:2000, P 45) To economy the design of bars are cut short at the section beyond which it is no longer required to extend to get direct development length? Please  $\rightarrow$  calculate  
Is App C (A.2.3.6) demand  
of reinforced

$$L_d \geq \frac{M_i}{f_y} + l_0$$

If it is not satisfied, the following measure may be adopted for satisfying the check.

- \* By reducing dia of main steel to lower value of  $L_d$ , but keeping the area of steel unchanged
- \* By increasing the value of  $L_d$  by protracting the length of the bar over the beam.
- \* By increasing the number of bent up bars

Checking for development length is essential at the following sections:

- \* At simple supports
- \* At continuous supports
- \* At point of contraflexure
- \* At point of bar cut off
- \* On beams of very short spans
- \* On abutments and terminus of bridge.

## Splicing of Reinforcement (I=456 Chapt 2 Pg 44)

- \* When the available length of bar is inadequate or has to be extended then splicing of reinforcement is done.
- \* The splicing of bars means joining two reinforcement bars to transfer the forces effectively from the terminating bar to the other bar.
- \* Therefore the concrete at a point of splicing is subjected to high stresses which may cause cracking of concrete.
- \* Splicing of reinforcement is done by following methods:
  - i) Lapping of bars. → two pieces of steel bars are overlapped to form a continuous bar.
  - ii) Mechanical joint
  - iii) Welding the bars.

## Module - 3.

Singly reinforced beams - Doubly reinforced beams -  
Cantilever beams - T beams - Design for tension.

### Singly Reinforced beams:

Singly reinforced beams have steel reinforcement in tension zone.

### Doubly reinforced beams:

Doubly reinforced beams have steel reinforcement provided in tension as well as in compression zone.

### Basic rules for design of beams:

1. Effective span [IS 456; Cl. 22] 19.34

The eff. span of beam is taken as follows:-

→ Simply Supported Beam

The effective span of a member is

- clear span + eff. depth of beam
  - + center to center distance b/w supports
- whichever is less.

→ Continuous Beam:-

If the width of fl. support is less than  $\frac{1}{12}$  of the clear span

the effective span shall be as in Cl. 22.2

if the supports are wider than  $\frac{1}{12}$  of the clear span or  
600mm whichever is less, the effective span shall be taken as  
600mm.

For end span with one end fixed and the other  
swinging or for intermediate spans, the effective span

shall be clear span between supports

② For end span wider one end free and the other continuous  
the effective span shall be  $\frac{\text{length}}{2}$  plus half the  
effective depth of the beam or later on the clear span  
plus half the width of the discontinuous support, whichever is greater.

③ In case of spans with roller or rockered bearings, the eff. span shall always be the distance b/w the centres of bearing.

#### → Cantilever Beams

The eff. length of a cantilever shall be taken as its length to the free end of the support plus half the eff. depth except where it forms the end of a continuous beam where the length to the centre of supports shall be taken.

#### 2. Effective Depth [IS 456; CI 35.0] Pg 24

Effective depth of a beam is the distance b/w the centres of the ends of tension reinforcement and the max compression reinforcement.

#### 3. Control of Deflection [IS 456; CI 43.21] 37

→ Simply supported beam,  $\frac{\text{Span}}{\text{eff. depth}} = 20$  } Span and girder depth from these values.

→ Cantilever,  $\frac{\text{Span}}{\text{eff. depth}} = 7$  } than these values.

→ Continuous Beam,  $\frac{\text{Span}}{\text{eff. depth}} = 26$  } than these values.

#### 4. Depth of neutral axis, [IS 456; ANNEX 6 & 7]

$$x_n = 0.87 \text{ ft.}$$

0.36 ft. to

If  $x_n > x_{\text{allow}}$  → over reinforced

$x_n < x_{\text{allow}}$  → under reinforced.

$x_{\text{allow}} = x_{\text{allow}} + \text{balanced}$

Under Reinforced

M.R = 0.17 by listed  $(1 - \frac{\text{let. by}}{\text{ft. but.}})$  OR 0.57 by let  $(1 - 0.42 x_n)$

Over Reinforced  $M.R = 1.00 \text{ ft.} (1 - 0.42 \frac{x_{\text{allow}}}{\text{ft. but.}})$  OR 0.36 by let  $(1 - 0.42 x_n)$

M.R = 0.34  $\frac{\text{ft. but.}}{\text{ft. but.}} (1 - 0.42 \frac{x_{\text{allow}}}{\text{ft. but.}})$  OR 0.36 by let  $(1 - 0.42 x_n)$

## Singly reinforced beam - Problems

1. A singly reinforced RC beam of section 250 x 500mm is reinforced with four bars of 16mm dia with an effective cover of 50mm. The beam is simply supported over a span of 8m. Find the max permissible load on beam. Use M<sub>20</sub> concrete and Fe<sub>450</sub> steel.

Given Data:-

$$D = 500 \text{ mm}$$

$$\text{Eff. Cover} = 50 \text{ mm}$$

$$\text{Eff. depth} = 500 - 50 = 450 \text{ mm}$$

$$\text{Span}, l = 8 \text{ m}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 450 \text{ N/mm}^2$$

$$A_{st} = 3 \times \frac{\pi \times 16^2}{4} = 603.085 \text{ mm}^2$$

$$b = 250 \text{ mm}$$

Assume it to be an under reinforced section.

(i) Depth of N.A.

[IS 456: ANNEXURE I]

pg 14

$$\frac{x_e}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

$$\frac{x_e}{d} = \frac{0.87 \times 450 \times 0.3 \times 10.5}{0.36 \times 20 \times 250 \times 450}$$

$$\frac{x_e}{d} = 0.16$$

[from page 40]

[IS 456]

$$\frac{x_{min}}{d} = 0.67$$

$$\frac{x_e}{d} < \frac{x_{max}}{d} \rightarrow \text{Under reinforced section.}$$

④ Moment of resistance

(B-45C April 2011)

$$M.R = 0.67 \times f_y \times \text{Area} \left( d + \frac{4f_y}{f_{ck} b_d} \right)$$

$$= 0.67 \times 250 \times 60 \times 105 \times 450 \left( 450 - \frac{250 \times 60}{30 \times 350} \right)$$

$$= 55.02 \text{ kNm}$$

⑤ Max. BM

$$M_u = \frac{W_a t^2}{8}$$

$$55.02 = \frac{W_a \times 6^2}{8}$$

$$W_a = 6.024 \text{ kNm}$$

⑥ permissible udl

$$w = \frac{W_a}{l^2} = \frac{6.024}{7.5^2} = 4.032 \text{ kN/m}$$

- 2 A simply supported beam of effective span of 7.5m and loaded by a super imposed load of 20 kN/m, grade of concrete is M<sub>20</sub> and Fe 415 steel, adopt a ~~size~~<sup>area of stiffener</sup> of 350 mm. Design the UDL using limit state method.

Given data:-

$$L = 7.5 \text{ m}$$

$$w = 20 \text{ kN/m}$$

$$f_{ck} = 30 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$b_d = 350 \text{ mm}$$

① Overall depth  $\frac{\text{span}}{\text{depth}} = 2.0$

$$\text{Depth} = \frac{75}{0.05} = 0.375 \text{ m}$$

$$= 375 \text{ mm}$$

② Find the load.

$$\text{Dead load} = b \times D \times \text{Density}$$

$$= 0.35 \times 0.375 \times 25$$

$$= 3.28 \text{ kN/m}$$

$$\therefore \text{Total load, } \omega = D_L + L.L$$

$$= 3.28 + 3.0$$

$$= 33.28 \text{ kN per/m}$$

$$\therefore \text{Factored load, } \omega_u = 1.5 \times \omega$$

$$= 1.5 \times 33.28$$

$$= 49.92 \text{ kN/m}$$

③ Determine effective depth.

$$\text{Eff. depth, } d = \sqrt{\frac{M_u}{R_u \times b}}$$

$$\therefore \begin{cases} \text{For } f_{ck} = 415 \text{ N/mm}^2 \\ R_u = 0.149 \text{ kN} \end{cases}$$

$$\therefore \begin{cases} \text{For } f_{ck} = 500 \text{ N/mm}^2 \\ R_u = 0.138 \text{ kN} \end{cases}$$

$$= 0.138 \times 30$$

$$\text{Max. moment, } M_u = \frac{\omega_u l^2}{8} = 2.76$$

$$\therefore \frac{49.92 \times 7.5^2}{8} = 351 \text{ kNm}$$

$$\therefore d = \sqrt{\frac{351 \times 10^6}{2.76 \times 350}}$$

$$= 3.60 \text{ m}$$

$$= 3600 \text{ mm}$$

Ans -

④ Calculate Act  
Concrete as balanced section  
 $M_u = M_k$

$$M_k = 0.84 \text{ by def of } \left( 1 - \frac{A_{st} f_y}{f_{ck} b d} \right)$$

$$301 \times 10^6 = 0.84 \times 350 \times A_{st} \times 400 \left( 1 - \frac{A_{st} \times 250}{350 \times 250 \times 700} \right)$$

$$= 152250 A_{st} \left( 1 - \frac{500 \times 10^{-6} A_{st}}{350 \times 700} \right)$$

$$301 \times 10^6 = 152250 A_{st} - 7.16 A_{st}^2$$

$$7.16 A_{st}^2 - 152250 A_{st} + 301 \times 10^6 = 0$$

$$A_{st} = 16151.53 \text{ mm}^2$$

$$A_{st} = 2682.31 \text{ mm}^2$$

of % steel,

$$\rho_s = \frac{100 A_{st}}{b d}$$

$$= \frac{(100 \times 2682.31)}{350 \times 700} = 1.08\%$$

3. A rectangular section of eff size 600 x 300 mm used a simply supported beam for effective span of 8m. What is the max. load can be allowed on the beam if more of steel is provided only in tension side. Use concrete & Fe 45 Steel. Details of steel to be furnished

① Depth of N.A. Assume the section to be a balanced section

$$Z_u \text{ limit} = \frac{Z_u}{d} = 0.42$$

$$Z_u = 0.42 d$$

$$= 0.42 \times 550$$

$$= \underline{\underline{231 \text{ mm}}}$$

[Is 456 Annex 3(1)]

② Moment of resistance (0.92%  $\Sigma_{\text{Area}} (1 - 0.42 Z_{\text{Umax}}) b f y$ )

$$M.R = 0.92 b f y I_{\text{unit}} (d - 0.42 Z_{\text{Umax}})$$

$$= 0.92 \times 26 \times 350 \times 264 (550 - 0.42 \times 231)$$

$$= \underline{\underline{292.12 \text{ kNm}}}$$

③ Determine the load.

$$M_u = \frac{W_u l^2}{8}$$

$$292.12 = \frac{W_u l^2}{8}$$

$$W_u = \underline{\underline{36.5 \text{ kN}}}$$

$$\begin{aligned} W &= \frac{W_u}{1.5} \\ &= \underline{\underline{24.34 \text{ kN/m}}} \end{aligned}$$

④ Calculate A<sub>st</sub>

for balanced section,

$$T = C$$

$$0.36 \text{ bcf } b \times Z_{\text{Umax}} = 0.37 f_y A_{\text{st}}$$

$$A_{\text{st}} = \frac{0.36 \times 26 \times 350 \times 231}{0.37 \times 415} = \underline{\underline{142.62 \text{ mm}^2}}$$

④ 2nd shell

$$P = \text{new } P$$

Int.

$$\text{Int} = \frac{1}{2} \rho_0 \rho_1$$

constant

$$\frac{1}{2} \rho_0 \rho_1$$

2. ultraparabolic model based on highly supported by  
the existing shallow water theory and its application  
is clearly indicated that it can better describe the  
ultralow wave setup than the simple and the old model  
described above.

$$\begin{aligned} & \text{Setup and } \text{Setup} \\ & \text{Setup and } \text{Setup} \\ & \text{Setup and } \text{Setup} \end{aligned}$$

$$\begin{aligned} & \text{Setup and } \text{Setup} \\ & \text{Setup and } \text{Setup} \end{aligned}$$

$$\text{Setup and } \text{Setup}$$

i) Take  $\lambda = 6.0m$

③ Factored Load ( $w_n$ ) and Factored Moment ( $M_n$ )

$$\text{Self weight of beam} = D \times b \times 2.5 = 0.5 \times 0.15 \times 2.5 = 3.75 \text{ kN/m}$$

(unit weight of  
RCI = 25 kN/m<sup>3</sup>)

$$\text{Imposed load} = 15 \text{ kN/m}$$

$$\begin{aligned}\text{Total load, } w &= 18 + 3.75 \\ &= 18.75 \text{ kN/m}\end{aligned">$$

$$\begin{aligned}\text{Factored Load, } w_n &= 1.5 \times w \\ &= 1.5 \times 18.75\end{aligned">$$

$$w_n = 28.125 \text{ kN/m}$$

$$\begin{aligned}\text{Factored Moment, } M_n &= \frac{w_n l^2}{8} \\ &= \frac{28.125 \times 6^2}{8}\end{aligned">$$

$$M_n = 122.3 \text{ kNm}$$

④ Minimum effective depth required.

IS 456 Annex pg 96

$$d_{eq} = \sqrt{\frac{M_n}{R_a \cdot b}}$$

$$= \frac{122.3 \times 10^6}{9.45 \times 250}$$

$$= 376.55 \text{ mm}$$

$$\begin{cases} R_a = 0.132 \text{ fck} \\ = 0.132 \times 25 \\ = 3.3 \text{ fck} \end{cases}$$

$$d_{eq} < d_{assumed} (450 \text{ mm})$$

Hence OK

Since the depth of section is more than that reqd for a balanced section the section is designed as under reinforced section

Adopt :  $D = 500 \text{ mm}$  and  $b = 150 \text{ mm}$

Assuming clear cover as 20mm, 2 nos of 18 mm dia bars

$$d = 500 - 20 - 20 \times 2 = 460 \text{ mm}$$

(a) Area of sheet metal.

{T.S. area, answer to (i)}

The weight of copper sheet

$$M_1 = 0.446 \text{ kg/m}^2 (1 - 0.40)$$

$$12.5 \times 10^3 \times 0.446 \times 0.40 \times 0.25 \left( \frac{0.02 \times 0.01}{0.02 \times 0.01 \times 0.01} \right)$$

$$12.5 \times 10^3 \times 0.6502 \times 0.40 \times 0.25 \times 10^6$$

$$M_1 = 275020 \text{ kg} = 275.02 \text{ t}$$

$$A_1 = 6.05 \times 10^{-3} \text{ m}^2$$

$$A_2 = 4.52 \times 10^{-3} \text{ m}^2$$

$$\text{Take } A_2 = 4.52 \times 10^{-3} \text{ m}^2$$

(b) Minimum area of sheet ( $A_1$ ) {T.S. area, answer to (i)}

$$A_1 = 0.446 \text{ kg/m}^2$$

$$= \frac{0.446}{0.01}$$

$$A_1 = \frac{0.446 \times 10^3}{0.01} = \frac{0.446 \times 10^3 \times 45.2}{0.01}$$

$$= \frac{0.446 \times 10^3 \times 45.2 \times 10^6}{0.01}$$

$$\text{Hence, } 0.6 -$$

Using sheet of base,

$$A_1 = \frac{0.446 \times 10^3}{0.01} = 44.6 \text{ m}^2$$

$$\text{Area of sheet} = \frac{M_1}{A_1} = \frac{275}{44.6} = 6.16 \text{ m}^2$$

Perimeter of sheet base

$$A_1 \text{ periphery} = 17.48 + 16.25$$

(c) Check for reflection:

$$\frac{1}{2} \times \frac{17.48 \times 16.25}{17.48 + 16.25} = 13.12$$

$$0.56 f_y \left[ \frac{A_f}{A_f + p_{max}} \right]$$

Pg 34 26.956  
Fig 4

$$f_y = 0.56 \times 415 \left( \frac{P_{12}}{P_{12}^c} \right)$$

$$= 212 \text{ N/mm}^2$$

Interpolating for  $f_y = 212 \text{ N/mm}^2$  & pt = 0.75

Modified factor

$$k_1 = 1.25$$

$$f_y = 410, h_1 = 1.25$$

$$f_y = 240, k_1 = 1.25$$



$$\left( \frac{l}{d} \right)_{max} = \frac{212}{135} = 1.57$$

$$= 25.6$$

$$\left( \frac{l}{d} \right)_{per} = \frac{6000}{482} = 12.4$$

$$\left( \frac{l}{d} \right)_{max} > \left( \frac{l}{d} \right)_{permitted}$$

$$\frac{240-135}{1.25-1.25} = \frac{212-135}{2.5-1.25}$$

$$\frac{50}{-0.15} = \frac{-25}{1.25-1.25}$$

$$50 \approx -67.5 = -2.5$$

$$50 \approx -3.3 \text{ or } 3.3$$

Hence OK

$$k_1 = \frac{1.25-1}{240}$$

② Design for shear

$$[25.456, 24.40]$$

$$k_1 = 1.25$$

Shear force,  $V_u = \frac{W_u L}{2} = \frac{27.2 \times 5.72}{2} = 78.5 \text{ kN}$

L = clear span  
= 5.72 m

Horizontal shear stress  $\tau_h$

$$[25.456, 24.40]$$

$$C = 40.0$$

$$\tau_h = \frac{V_u}{W_u}$$

$$= \frac{78.5 \times 10^3}{40.0 \times 482}$$

$$\tau_h = 0.48 \text{ N/mm}^2$$

- Design area change ( $\Delta A$ )  $\rightarrow$   $\Delta A = A_{\text{new}} - A_{\text{old}}$
- for  $\mu = 0.1$  and  $H = 0.01$   $\rightarrow \Delta A = 0.0001 \text{ m}^2$
- $T_1 = 45^\circ \text{ C}$   $\rightarrow T_2 = 25^\circ \text{ C}$
- $T_1 < T_2$   $\rightarrow \Delta A > 0$
- $T_1 > T_2$   $\rightarrow \Delta A < 0$

Hence shear reinforcement of longitudinal

$$\text{Area to be added by reinforcement} = \frac{\Delta A \cdot f_y}{f_y - f_u} = \frac{0.0001 \times 400}{400 - 350} = 8 \text{ mm}^2$$

$$= 8 \text{ mm}^2 = 0.000008 \text{ m}^2$$

$$V_{\text{new}} = V_{\text{old}} + \Delta V$$

• Area of shear reinforcement ( $A_s$ )

Area of shear reinforcement

$$A_s = \frac{V_{\text{new}} - V_{\text{old}}}{f_y}$$

$$= \frac{100 - 40}{400}$$

$$= \frac{60}{400} = 0.15 \text{ m}^2$$

$$= 15000 \text{ mm}^2$$

$$= \frac{15000}{1000000} = 0.015 \text{ m}^2$$

$$S_{\text{new}} = S_{\text{old}} + \Delta S$$

• Main longitudinal reinforcement  $\rightarrow$   $S_{\text{new}} = S_{\text{old}} + \Delta S$

$$= \frac{15000}{1000000} = 0.015 \text{ m}^2$$

$$= 15000 \text{ mm}^2$$

$$= \frac{15000}{1000000} = 0.015 \text{ m}^2$$

The spacing should be least of the following

[IS 456]

or 25 mm

$$0.05d + 0.75 \times 46.2 + 346 = 300$$

i.e) 300 mm.

∴ Spacing  $S_s = 300$  mm.

Provide two 2-legged stirrups @ 100 mm c/c throughout  
the length of the beam.

Provide 2 ft min. of anchor bars in the compression zone.

④ Check for developed length [IS 456 page 44]

Using

$$\frac{M_c}{M_u} > l_d$$

not provided

$$M_c = 0.876 \text{ kNm} \left( 1 - \frac{d/d_g}{\text{bulge}} \right)$$

$$= 0.876 \times 415 \times 92.2 \times 46.2 \left( 1 - \frac{492 \times 46.2}{210 \times 415 \times 46.2} \right)$$
$$= 13486 \times 92.2 \text{ Nmm}$$

$$M_u = 126.35 \text{ kNm}$$

Using

$$M_u = 126.35 \text{ kNm}$$

No bond or hook,  $l_b = 0$

$$\frac{M_c + l_b}{M_u} = \frac{13486 \times 92.2}{126.35} = 1770 \text{ mm}$$

$$\text{Development length, } l_d = \frac{\phi b \cdot 37.4}{+ Y_{bd}} \quad [\text{PD 42 IS 456}]$$

or 25 mm

$$= \frac{20 \times 0.876 \times 46.2}{4 \times 2.35} \quad \text{CL}_d = 80/14 + 14$$
$$= 16.86 + 14$$
$$= 30.86 \text{ mm}$$
$$= 2.24 \text{ kNm}$$

$\frac{M_c + l_b}{M_u} > l_d$ , Hence OK

⑤ Design Summary:

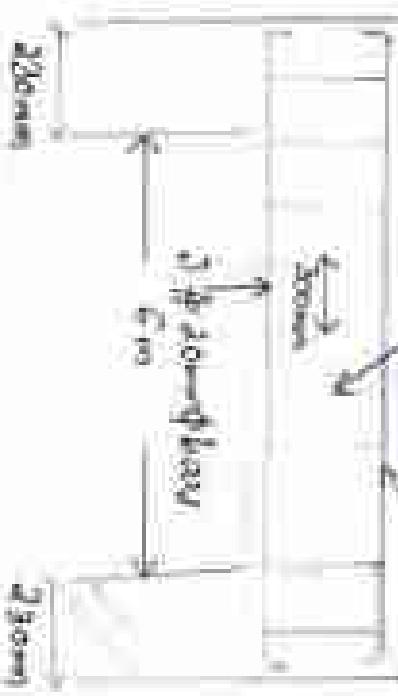
Beam size =  $260 \times 420$  mm

Main bars = 3 # 20 mm  $\Phi$

Stirrups = 8 nos 6 mm  $\Phi$  2-legged @ 300 mm c/c

# Subject Name : Design of Civil Engineering Project

(14)



## Design of Cantilever Beam

- \* Max. value of  $\frac{M}{L}$  permitted is 7
- \* The apparent value of depth =  $\frac{1}{3}$  of span
- \*  $M_a = \frac{wL^2}{2}$
- \*  $V_a = wLx\delta$

- \* The usual dimension leading formula is as follows
- \*  $\text{Assumption is to be taken at left}$
- \* Design stress = 0.6 of actual stress

### Spanwise beam - Problems

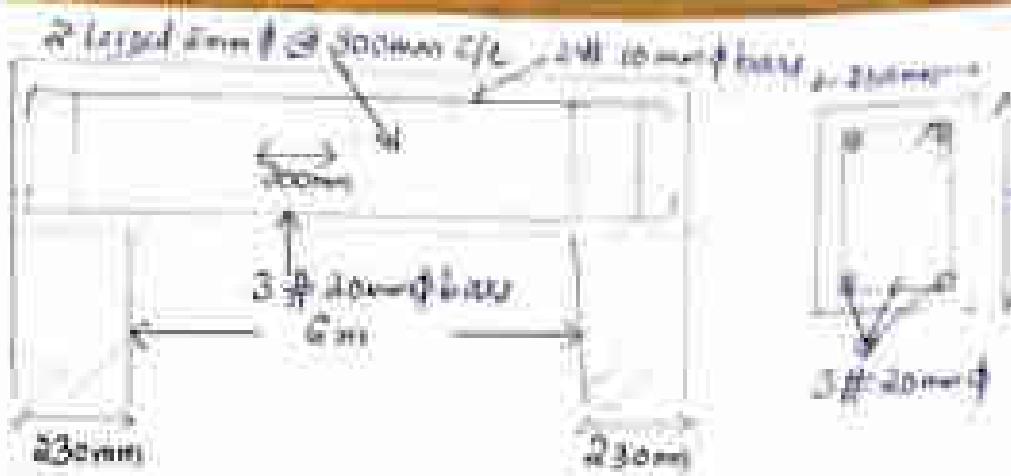
- \* A portion of a guest house consists of a cantilever beam of uniform span  $L = 6\text{m}$  supported at one end. The beam supports a uniform load of  $3\text{kN/m}$  acting downwards. The max. eccentricity of load is  $2\text{m}$ . Design an appropriate beam.

Given data:

$$L = 6\text{m}$$

$$w = 3\text{kN/m}$$

$$E = 2.5 \times 10^5 \text{ N/mm}^2$$



## Design of Cantilever Beam:

- \* Max. value of  $\frac{M}{I}$  permitted is  $\frac{1}{4}$   
i.e. the approx. value of depth =  $\frac{1}{4}$  th of span
- \*  $M_u = \frac{w_u l^2}{8}$
- \*  $V_u = w_u \times l$
- \* For usual downward loading tension is at top, hence reinforcement is to be given at top.
- \* Design shear = 0.87 by all supports

## Cantilever beam - Problems

1. A portion of a guest house consists of a cantilever beam of effective span 3m spaced at 2.5m c/c. The beam supports 12.5-mm thick slab. If an slab is 0.5 m high. Use M<sub>u</sub> = 1000 kg/cm<sup>2</sup> and F<sub>ck</sub> = 35 N/mm<sup>2</sup>. Design an un-reinforced beam by four methods
  - a) Direct Method
  - b) Moment Distribution Method
  - c) R.F.D. Method
  - d) SFD Method

① Dimensions

(IS 456 Q 23.4.1 pg 31)

$$\text{for cantilever } \frac{l}{d} = T$$

$$\frac{3}{d} = T$$

$$d = 3 \cdot 4285 \text{ m. } \Sigma 0.450 \text{ m. } = 450 \text{ mm}$$

Assume clear cover of 50 mm.

$$D = (d - 450 \text{ mm}) \times 0.450 + 0.05 \\ = 0.500 \text{ m. } = 500 \text{ mm}$$



$$b = \frac{1}{2} D \text{ or } \frac{1}{3} d \text{ of depth}$$

$$b = \frac{0.500}{2} = 0.250 \text{ m. } = 250 \text{ mm}$$

unit weight  
per..

② Load Calculations.

$$D.L = \text{thickness} \times \text{c/c distance} \times 25$$

$$= 0.120 \times 2.5 \times 25$$

$$= 7.5 \text{ kN/m}$$

$\frac{1}{2}$  c/c distance.

$$L.L = 1.5 \times 2.5$$

$$= 3.75 \text{ kN/m}$$

$$-f_L = 7.5 + 3.75$$

$$= 11.25 \text{ kN/m}$$

$$\text{Factored } P_L, w_u = 1.5 \times 11.25$$

$$= 16.875 \text{ kN}$$

$$\text{Shear force, } V_u = (w_u) \times l$$

$$= 16.875 \times 3$$

$$= 50.625 \text{ kN}$$

$$\therefore =$$

2.

$$M_0 = \frac{0.25 \times I^2}{d}$$

$$M_0 = 0.25 \times 10^2$$

$$M_0 = 2.5 \text{ kilonewton}$$

(3) Moment of resistance ( $M_R$ )

$$M_{R,\text{lim}} = 0.25 \cdot T_{\text{max}} \left( 1 - 0.47 \frac{T_{\text{max}}}{d} \right)$$

$\left\{ \begin{array}{l} \text{for } d < 450 \\ \text{for } d \geq 450 \end{array} \right.$

$$= 0.25 \times 0.47 \left( 1 - 0.47 \times 0.47 \right) \times \frac{350 \times 450^2 \times 20}{350 \times 450^2 \times 20} = 0.47$$

$$M_R \text{ limit} = 131.68 \text{ kilonewton}$$

$$M_R < M_{R,\text{lim}}$$

Section is singly reinforced

(4) for singly reinforced section

$$M_R = 0.25 \text{ by first } \left[ \frac{\text{Art. ft}}{\text{Inch foot}} \right]$$

$$0.25 \times 10^6 = 0.47 \times 415 \times 40 \times 1000 \left( 1 - \frac{40 \times 0.47}{20 \times 450 \times 400} \right)$$

$$75.45 \times 10^6 = 16.2672 \times 400 \times 400 \times 400$$

$$18.86 \times 10^6 = 1624.12 \times 400 \times 400$$

$$\text{Ans. } 18.86 \text{ kip/inch}^2$$

$$\delta_R = 216 \text{ in. or } 18 \text{ mm}$$

Stress of fiber at 18 mm

$$\delta_R = 18 \text{ mm}$$

$$18.86 \text{ kip/inch}^2$$

$$pl = \frac{100 \text{ kN}}{6d}$$

$$= \frac{100 \times 804.24}{6 \times 70 \times 450} = 0.7141$$

#### ④ Design of shear reinforcement

+ Nominal shear stress ( $\tau_c$ )

[IS 456, Cl 40-17]  
Pg 72

$$\tau_c = \frac{V_u}{6d} = \frac{50.62 \times 10^3}{6 \times 70 \times 450} = 0.45 \text{ N/mm}^2$$

[IS 456 Pg 73]

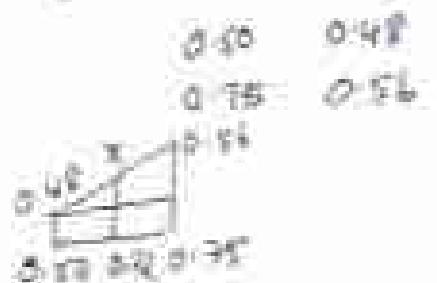
$$\tau_{cmax} = 0.5 \text{ N/mm}^2$$

$$\tau_c = 0.55 \text{ N/mm}^2$$

$$\tau_c < \tau_{cmax}$$

$$\tau_c > \tau_v$$

Provide nominal stirrups of 2 legged  
6mm Ø at 300mm c/c



$$\frac{0.55 - 0.45}{0.55 - 0.45} = \frac{0.72 - 0.55}{0.72 - 0.55}$$

$$S/R = \frac{0.55}{0.55 - 0.45}$$

$$3.45 - 1.675 = 1.775$$

$$1.775 \times 1.775 = 3.14$$

$$\approx 0.55$$

#### ⑤ Development length

$$l_d = \frac{\phi \times 0.87 f_y}{4 \tau_{vd}}$$

[ $\tau_{vd}$  IS 456 Pg 49  
 $= 20 / (1.1 + 1.5)$   
 $= 12450 + 1.2$

$$= \frac{12 \times 0.87 \times 415}{4 \times 1.2}$$

$$= 12.1 \text{ m}$$

$$= 76.2 \text{ m} \rightarrow$$

J

Wings in condition, some fading due to age of the  
birds, a number of birds were taken by shooting  
with the shotgun and the gun shot.

Spotted Tern

♂ 1000 ad.

1000 - 1000 Adm.

1000 - 1000 Adm.

1000 Dotted Tern (T. macroura) 1000

Spotted Tern

depth

depth = 1000 - 2000 - 3000

Young, 1000 - 1000 ad. found and most numerous at the  
bottom, clear down to 1000, then gradually less and  
less up through.

1000 - 1000 ad. 1000

1000 - 1000 ad. found and

1000 - 1000 ad.

1000 - 1000 ad. found

1000 - 1000 ad. found and

1000 - 1000 ad.

1000 - 1000 ad.

1000 - 1000 ad. found and

1000 - 1000 ad.

1000 - 1000 ad. found

1000 - 1000 ad. found = 1000 - 1000 ad.

② Minimum depth required (d<sub>min</sub>) from Annex to Pg 96

$$d_{min} = \sqrt{\frac{M_u}{R_u b}} \quad R_u = 0.13 \text{ kN/mm}$$

$$= 0.13 \times 20$$

$$= 2.6$$

$$= \sqrt{\frac{94.5 \times 10^6}{0.13 \times 2.6}}$$

$$= 370 \text{ mm} \approx 414 \text{ mm}$$

Hence OK.

③ Area of steel (A<sub>s</sub>)

(IS 456 ANNEX C(i))

$$M_u = 0.87 b_y A_s + d \left[ 1 - \frac{4(1.67)}{bd f_y} \right]$$

$$94.5 \times 10^6 = 0.87 \times 115 \times A_s + 414 \left[ 1 - \frac{4(1.67) \times 415}{350 \times 41.6 \times 20} \right]$$

$$94.5 \times 10^6 = 14.94747 A_s (1 - 2.02 A_s)^2$$

$$94.5 \times 10^6 \Rightarrow 14.94747 A_s - 2.02 A_s^3$$

$$A_s = 4246.33 \text{ mm}^2$$

$$A_s = 442.80 \text{ mm}^2$$

Take A<sub>s</sub> = 742.80 mm<sup>2</sup>

④ Minimum area of steel (A<sub>s</sub>)

(IS 456 C1.26.5 H)

$$A_s = \frac{0.45 bd}{b y}$$

$$= \frac{0.45 \times 250 \times 414}{415} = 211 \text{ mm}^2 \approx 742.80 \text{ mm}^2$$

Hence OK!

Allowing for over lap = A<sub>s</sub> =  $\frac{742.80}{1.5} = 495.20 \text{ mm}^2$

$$\text{Ratio of base} = \frac{A_s}{A_p} = \frac{742.80}{400} \approx 2$$

∴ Provide 400 mm  $\times$  base

## Reinforced concrete

### Shear force

(i) Design of shear reinforcement

\* Nominal shear stress ( $\tau_n$ )

[ASCE pg 11  
CL 2.6.1]

$$\frac{\tau_n}{\tau_s} = \frac{V_u}{V_{n,s}} = \frac{0.001f_y}{0.001f_y} = 1$$

$\tau_n = 0.001f_y$

i. Design shear strength ( $V_n$ )

[ASCE pg 70]

$$A_s \text{ at supports} = \frac{V_n}{0.001f_y}$$

$$p = \frac{(0.001f_y A_s)}{L} = \frac{0.001f_y A_s}{25.9844} = 0.1 f_y$$

Max. shear capacity and  $p = 0.1 f_y$

$$\begin{aligned} \tau_s &= 0.001f_y \\ \tau_s &\leq \tau_c \\ \tau_c &> \tau_s \end{aligned}$$

[ASCE  
CL 2.6.1]

$$\tau_c > \tau_s$$

Max. shear reinforcement is required

\* Shear is resisted by shears ( $V_u$ )

$$V_u = V_n - F_{bd}$$

$$0.001f_y A_s - 0.001f_y A_s + 1.4f_y = 11.25 \text{ kN}$$

i. Design shear strength

Design shear strength

\* Spacing of bars

10 mm

[ASCE pg 17  
CL 2.6.1]

1.000 mm thickness, 16 bars

• Max Spacing w.r.t min reinforcement

$$S_r = 0.17 \text{ dia fy}$$

$$= 0.17 \times 6$$

$$= \frac{0.17 \times 100 \times 53}{0.48350}$$

$$= 36.2 \text{ mm}$$

The spacing should be least of the following

i)  $0.75 d = 0.75 \times 414 = 310.5 \text{ mm}$

ii) 300 mm

Provide 3 nos of 2 legged stirrups @ 300mm c/c therefore the length of the beam

Provide 2@15 mm & another bar.

④ Development length / establishment of bar

The bar provided at min reinforcement can be extended at a distance of  $1.75m$  ( $\frac{1}{2} l$ ) from the free of support.

$$\text{Development length, } l_d = \frac{\phi \times 0.87 \text{ fy}}{42 + f_{\text{ult}}}$$

$$= \frac{16 \times 0.87 \times 415}{42 + 172}$$

$$= 75.2 \text{ mm} < 175 \text{ mm}$$

Hence OK

⑤ Check for deflection

(Ref 5.2 15.4.5C b/g)

$$f_1 = 0.337$$

$$F_s = 0.337 \times \left[ \frac{\text{Dist. end}}{\text{Dist. mid}} \right] \times \text{Cross section} \left[ \frac{M_u}{504} \right]$$

$$= 802 \text{ N/mm}^2$$

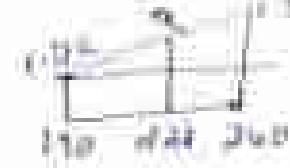
Designing for  $f_c = 22.4 \text{ N/mm}^2$

$$f_c = 22.4 \text{ N/mm}^2$$

$$f_t = 240 \text{ N/mm}^2$$

$$\gamma_{\text{concrete}} = 1.56$$

$$\left(\frac{l}{d}\right)_{\text{max}} = 18.5 - 10.5$$



$$\left(\frac{l}{d}\right)_{\text{permitted}} = \frac{3000}{500} = 7.2$$

$$\left(\frac{l}{d}\right)_{\text{min}} > \left(\frac{l}{d}\right)_{\text{permitted}}$$

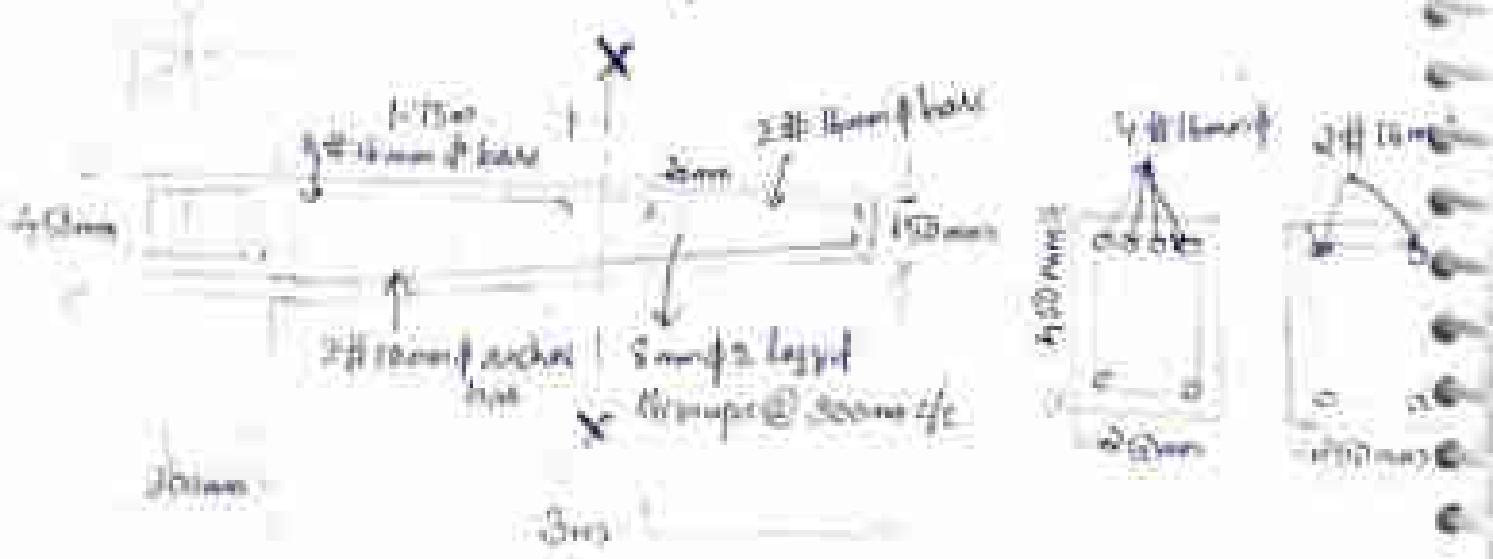
Hence OK

## ② Design Summary

Beam size :  $b \times 250\text{mm}$   $D = 450 \text{ mm}$  at fixed end  
 $D + 150 \text{ mm}$  at free end

Main bars : 16 mm  $\phi$

Shimape : 8 mm  $\phi$  @ 300mm c/c 2 legged  
Tie rods bar : 2 mm  $\phi$



$$M_u = 496.61 \text{ kNm}$$

## Design for tension

Reinforced concrete sections are also subjected to torsional moments which cause twisting or warping of the section. Some of the examples of structures subjected to torsional moments are as follows:-

Ring beam provided at the bottom of the elevated circular water tank.

Flange beams

Beams supporting a circular slab.

Beams curved in plan

Code Approach :- [IS 456 Page 75]

As per IS code, the design for torsion is based on the calculation of an equivalent shear force and an equivalent bending moment. The effect of torsional moment (in the form of additional shear force and additional bending moment) is added to the actual bending moment and shear force at equivalent bending moment and equivalent shear force.

Equivalent bending moment and equivalent shear force are calculated as follows:-

$$\text{Equivalent BM. } M_e = M_u + M_T$$

where

$M_u$  = ultimate BM

$M_T$  = additional BM which considers the effect of torsional moment

$$M_T = T_u \cdot \left( \frac{1 + D/b}{2} \right)$$

where

$T_u$  = Torsional moment on the applied

$D$  = total depth of the beam

$b$  = breadth of the section.

$$\text{Equivalent SF. } V_e = V_u + V_T$$

where

$V_u$  = ultimate shear force

$V_T$  = additional shear force caused due to torsional moment

&  $\tau$

$$V_T = 1.6 \frac{T_u}{b}$$

IS 456 Pg 25

### Design Procedure

#### 1. Equivalent shear force

$$V_e = V_u + V_T$$

$$T_u = 1.6 \frac{T_u}{b}$$

#### 2. Equivalent torsional shear stress ( $\tau_{eq}$ ) (Ans.)

$$\tau_{eq} = \frac{V_e}{cI}$$

for value of  $cI$  to be less than  $\Sigma cI$   
Ans. Pg 25

$M_c < M_u$ , the longitudinal steel add to resist  $M_u$   
is calculated and distributed

If  $M_c > M_u$ , the longitudinal reinforcement shall be  
provided on the flexural compression zone of the beam  
moment  $M_c$ .

$M_c = M_f - M_u$ , taken as acting opposite  
to moment  $M_u$

① Transverse Reinforcement [IS 456 pg 75 Cl 4.4.2]

Two legged closed loops enclosing the corner  
longitudinal bars shall have an area of  $A_{tr}$  given by

$$A_{tr} = \frac{T_u S_v}{b_i d_i (0.87 b_y)} + \frac{V_u S_u}{0.5 d_i (0.87 b_y)}$$

but the total transverse reinforcement shall not be  
less than

$$\frac{(T_u - T_c) b_s}{0.87 b_y}$$

Where,  $T_u$  = torsional moment

$$V_u = SF$$

$S_v$  = spacing of shims

$b_i$  = the thickness of corner bar in the direction

of width

$d_i$  = the distance b/w corner bar in the direction

of depth

$b_y$  = breadth of the member

$T_u$  = Equivalent Steel Area as specified in cl 4.7.1

$T_c$  = shear strength of concrete as per table 19.

(iii) \* If  $\tau_{ve} > \tau_c$ , section should be rectangular

3. Shear strength of concrete  $\tau_c$  without reinforcement is calculated from the table

Compare it with  $\tau_{ve}$

\* If  $\tau_{ve} > \tau_c$ , then nominal shear reinforcement is provided in the form of stirrups such that

$$\frac{A_{sv}}{b \cdot s_y} \geq \frac{0.4}{0.876g} \quad [IS 456 pg 48 A 2]$$

$$s_y \geq 0.876g \cdot A_{sv}$$

Where

$$A_{sv} = \frac{0.4b}{c/s}$$

$c/s$  = c/s area of stirrups leg  
 $s_y$  = characteristic strength of shear reinforcement

$b$  = breadth of beam

$s_y$  = spacing of stirrups

\* If  $\tau_{ve} > \tau_c$  is the equivalent shear stress is greater than shear strength of concrete, then longitudinal reinforcement in the form of longitudinal and transverse reinforcement is provided.

① Longitudinal Reinforcement [IS 456 Pg 75 C 4-4-2]

The longitudinal reinforcement shall resist equivalent BM, i.e.

$$M_L = M_u + M_f$$

$$M_u = BM \text{ at the } c/s$$

$$M_f = T_a \left[ \frac{1 + p\%}{1.4} \right]$$

## Distribution of Torsional Reinforcement

When a member is subjected to torsion, the torsion reinforcement shall be provided as below.

### (a) The transverse reinforcement (IS 456, Cl. 26.5.1.7)

Transverse reinforcement shall be rectangular closed stirrups placed perpendicular to the axis of the member. The spacing of the stirrups shall not exceed the least of  $2\sqrt{d}$ ,  $2\sqrt{11}$  and 300 mm.

### (b) The longitudinal reinforcement (IS 456, Cl. 26.5.1.8)

The longitudinal reinforcement shall be placed as close as is practicable to the corner of the C/S and in all cases, there shall be at least one longitudinal bar in each corner of the C/S.

If the C/S of the member exceeds 450 mm, additional longitudinal bars shall be provided to satisfy the longitudinal reinforcement and spacing requirement of minimum reinforcement as per requirement of IS 456 Cl. 26.5.1.3.

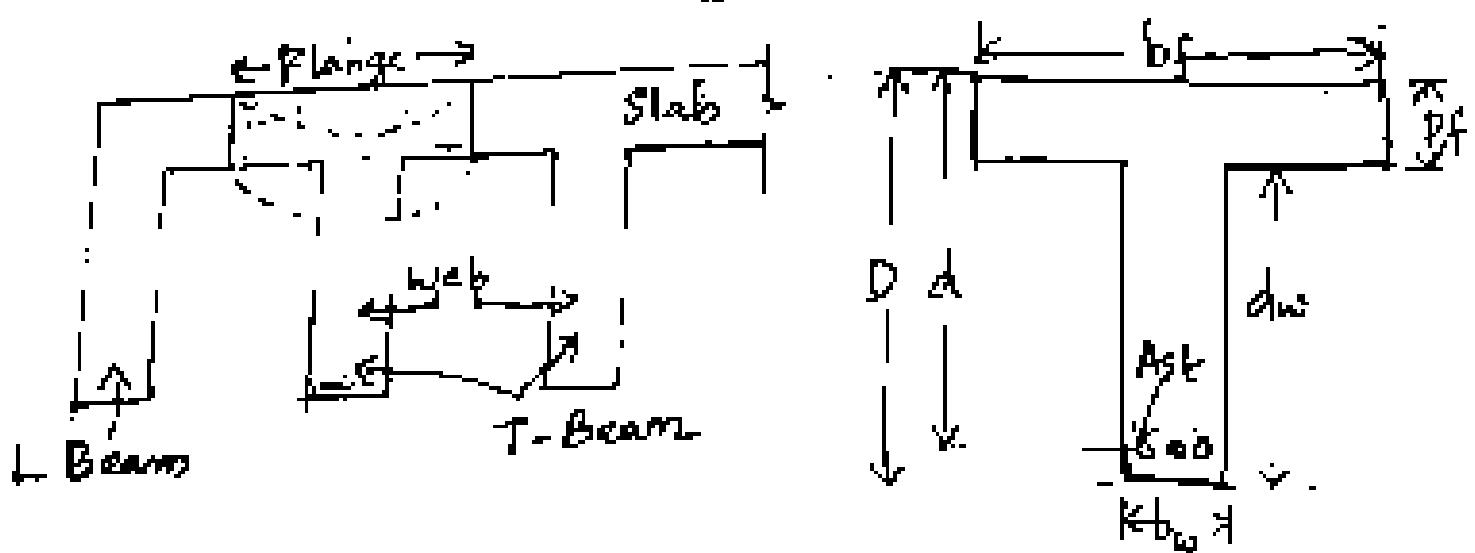
### Tables on design for torsion

Tables on design for torsion for a beam of dimensions  $150 \times 450$  mm subjected to a factored BM of 150 kNm, using  $\sigma_{yf} = 300$  MPa and factored torsional moment factored shear force of 100 kN and factored torsional moment of 50 kNm for M20 concrete and Fe 415 steel.



## Design of T-Beams.

In RCC construction, slabs and beams are cast monolithically. In such construction, a portion of the slab acts integrally with the beam and bends along with the beam under the loads. The portion of the slab which acts integrally with the beam to resist bending is called as Flange of the T-beam or L-beam. The portion of the beam below the flange is called Web or Rib of the beam. The intermediate beams supporting the slab are called as T-beams and the end beams are called as L-beams. The flange of the beam contributes in resisting compression.



Width of web (bw)

(2)

The ratio of width of web to depth of web is kept at  $\frac{1}{3}$  to  $\frac{1}{2}$ .

Thickness of flange (Df)

The thickness of flange of T beam is equal to the width or depth of slab forming flange of beam.

Overall depth of beam (D)

Overall depth of a flanged beam is equal to the sum of depth of flange and the depth of web. It is generally limited as  $\frac{1}{12}$  to  $\frac{1}{15}$  th of the span.

Effective width of flange (bf)

The off width of compression flange of flanged beam can be calculated as follows: [IS:456 E. 2.2.1.2] (Pg: 37)

$$\text{for T beams, } bf = \frac{d}{2} + bw + 6D_f$$

$$\text{L beams, } bf = \frac{d}{12} + bw + 3D_f.$$

Depth of N.A. and M.R.

There are two cases that may arise

① N.A. falls in the flange i.e.  $x_a < D_f$

② N.A. falls in the web i.e.  $x_a > D_f$

Case-I :  $x_a \leq D_f$

2

Depth of N.A.,

If  $x_a \leq D_f$ , the section will behave as a rectangular section having width equal to bf. Depth of N.A. can be found from the equation used for rectangular section after replacing  $b$  from the equation.

$$\text{for } bf \text{ is } \frac{x_a}{d} = \frac{0.833 D_f}{0.25 b k_b f_d}$$

## MR ( $M_u$ )

Comparing  $Z_u$  and  $Z_{max}$ , and determining the type as follows:-

- (i)  $Z_u < Z_{max}$ , the section is under-reinforced and MR of the section is given by,

$$M_u = 0.77 b_f d \left( 1 - \frac{b_f d}{3k_b d} \right)$$

- (ii)  $Z_u = Z_{max}$ , the section is balanced and is calculated as

$$M_{u(bal)} = 0.36 b_f d \times \frac{Z_{max} \times d^2}{d} \left( 1 - \frac{b_f d}{3k_b d} \right)$$

$$M_{u(bal)} = 0.87 b_f d \left( 1 - \frac{b_f d}{3k_b d} \right)$$

- (iii)  $Z_u > Z_{max}$ , the section is over-reinforced or is 45% if should be redesigned.

### Problems on T-beams

- Find the moment of resistance of a T-beam having a web width of 240mm, effective depth of 400mm, flange width of 740mm and flange thickness = 100mm. The beam is reinforced with 5 nos of 16mm dia bars. The  $M_u$  is given by the data

given below

$$b_w = 240 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$b_f = 740 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$b_f = 415 \text{ N/mm}^2$$

$$b_w = 20 \text{ kN/mm}^2$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3 \text{ mm}^2$$

(3)

Assuming the NA to fall in the flange

[IS 456 ANNEX  
Gr. 1]

$$\frac{Z_u}{d} = \frac{0.176y A_{st}}{0.36 f_y d}$$

$$Z_u = \frac{0.176 \times 415 \times 1005.3}{0.36 \times 20 \times 740}$$

$$= 68.1 \text{ mm} < D_f$$

Hence NA lies in the flange

$$Z_{max} = 0.48d$$

$$= 0.48 \times 400$$

$$= 192 \text{ mm}$$

$$Z_u < Z_{max}$$

Hence the section is under-reinforced.

[IS 456 ANNEX Gr. 1]

Moment of resistance ( $M_u$ )

$$M_u = 0.176y A_{st} d \left( 1 - \frac{6y A_{st}}{bf d f_y} \right)$$

$$= 0.176 \times 415 \times 1005.3 \times 400 \left( 1 - \frac{415 \times 1005.3}{740 \times 400 \times 20} \right)$$

$$= 184,75,3789.7 \text{ N-mm}$$

$$M_u = 184,75 \text{ kNm}$$

A T-beam floor system has 120 mm thick slab supported by 7 beams. The width of beam is 300 mm and effective depth in beams. The concrete of beam is 300 mm and diameter of bars is 20 mm. The beam is reinforced with 8 bars of 20 mm diameter. The grade of concrete and reinforcement is 45 grade. The beam is 3.6 m long at 3 m c/c. The eff. span of beam is 3.6 m.

⑤ Given data :-

$$b_w = 300\text{mm}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$d = 90\text{mm}$$

$$f_y = 460\text{N/mm}^2$$

$$D_f = 120\text{mm}$$

$$L = 3.6\text{m}$$

$$R_F = \frac{\pi K}{4} \times 20^2 \times 253 =$$

i) Effective width of flange ( $b_f$ ) (IS 456 PG 37)  
CL 2.3, 1.9.

$$b_f = \frac{L}{6} + b_w + 1.5 D_f$$

$$b_f = L - 3.6\text{m}$$

$$b_f = \frac{3600}{6} + 300 + 1.5 \times 120 \\ = 1620\text{m}$$

Clear span to the left & right of the beam

$$= 3000 - 300 - 300 \\ = 2400\text{mm}$$



$$b_f \geq 0.5(L_1 + L_2) + b_w$$

$$b_f \geq 0.5(2400 - 2700) + 300 = 600\text{mm}$$

$$b_f \geq 300\text{mm}$$

$$\therefore b_f = 1620\text{mm}$$

$$= 2400\text{mm}$$

ii) Depth of NE

[IS 456 ANNEX 2]

Determining the NE to fall in the flange

$$x_u = 0.53 f_y A_t$$

$$\frac{x_u}{d} = \frac{0.53 f_y A_t}{0.86 f_{ck} b_f d}$$

$$x_u = \frac{0.53 \times 415 \times 2513}{0.86 \times 20 \times 1620}$$

$$x_u = 756\text{mm} < D_f \therefore \text{NE lies in flange}$$

$$x_{u,\max} = 0.48 d = 0.48 \times 900 \\ = 230.4\text{mm} > x_u \rightarrow \text{Under depth}$$

$\rightarrow$  Under depth

i) Moment of resistance ( $M_u$ )

[IS 456 Annex E]  
(x 1.7)

$$M_u = 0.37 \text{ by Art. 1 - Art 14}$$

$$= 0.37 \times 15 \times 2613 \times 560 \left( 1 - \frac{2513 \times 415}{20 \times 1620 \times 580} \right)$$
$$= 496.61 \times 10^6 \text{ N-mm}$$

$$\underline{M_u = 496.61 \text{ kNm}}$$

### Design for tension

Reinforced concrete sections are also subjected to torsional moments which cause twisting or warping of the section. Some of the examples of structures subjected to torsional moments are as follows:-

- \* Ring beam provided at the bottom of the elevated Galleria structure.
- \* L-beams
- \* Beams supporting a cantilever slab
- \* Beams curved in plan

ii) Code Approach :- [IS 456 page 75]

As per IS code, the design for tension is based on the calculation of an equivalent shear force and an equivalent bonding moment. The effect of torsional moment (in the form of additional shear force and additional bonding moment) is added to the actual bonding moment and shear force to get equivalent bonding moment and equivalent shear force.

The equivalent bonding moment and equivalent shear force is calculated as follows:-

## Situations Demanding doubly reinforced beams.

Doubly reinforced sections are generally required in situations where the cross sectional dimensions of the beam are restricted by architectural or other considerations.

When singly reinforced section is not adequate in view of moment resisting capacity.

Doubly reinforced sections are also used in a situations where reversal of the moments is likely occur in some beam sections with varying loads undergo change of sign which makes compression zone as tension zone and vice versa.

Safety against reversal of stresses in structures due to wind forces, seismic forces and temperature stresses. When the loads are eccentric or beam subjected to accidental lateral loads.

- \* In case of continuous beams or slabs the sections at supports are generally designed as doubly reinforced sections.

$$\text{Ans. } f_{sc} = 0.87 \times 250 = 217.5 \text{ N/mm}^2$$

For higher grade steel  
consider  $A_s/A$  and find out  $f_{sc}$   
from table F of SP 16 or  
 $\text{Ans. } f_{sc} = 0.87 f_y$

$$\text{No of bars} = \frac{\text{Act}}{\text{Ag}} = \frac{535}{201} = 2.6623$$

$\frac{\pi \times 16}{4}$

Using 16 mm dia bars,  $A_g = 201 \text{ mm}^2$

No. of bars reqd =  $\frac{\text{Act}}{\text{Ag}} = \frac{535}{201} = 2.6623 \approx 3$

4. Design a rectangular beam  $230\text{mm} \times 600\text{mm}$  over span of 5 m. The super imposed load on the beam is 20 kN/m. The effective cover to reinforcement is taken as 50 mm. Use M20 concrete and Fe 415 steel.

Given data :-

$$b = 230 \text{ mm} \quad D = 600 \text{ mm} \\ d = 600 - 50 = 550 \text{ mm} \quad d' = 50 \text{ mm}$$

$$l = 5 \text{ m}$$

Superposed load =  $20 \text{ kN/m}$ .

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\text{M}_u = 1.7$$

Modular ratio  
factor

(3) Design or factored load:

$$\text{Self weight} = 0.23 \times 0.6 \times 1 \times 25 = 3.45 \text{ kN/m}$$

$$\text{Impact load} = 50 \text{ kN/m}$$

$$\text{Total Load, } \underline{\omega} = 3.45 + 50 = 53.45 \text{ kN/m}$$

$$\begin{aligned} \text{Design / factored load, } \underline{\omega_u} &= 1.5 \times \underline{\omega} \\ &= 1.5 \times 53.45 = 80.175 \text{ kN/m} \end{aligned}$$

(4) Design moment ( $M_u$ ) and Design shear ( $V_u$ ):

$$\begin{aligned} M_u &= \frac{\underline{\omega}_u l^2}{8} = \frac{80.175 \times 6^2}{8} \\ &= 250.55 \text{ kNm} \end{aligned}$$

$$V_u = \frac{\underline{\omega}_u l}{2} = \frac{80.175 \times 6}{2} = 240.5 \text{ kN}$$

(5) Limiting moment of resistance ( $M_{ult,m}$ ) (ANNEX G1 A.1 15.4.5)

$$M_{ult,m} = 0.36 \frac{X_{max}}{d} \left( 1 - 0.42 \frac{X_{u,max}}{d} \right) b d^2 / \text{ek}$$

$$\frac{X_{max}}{d} = 0.48$$

$$\begin{aligned} M_{ult,m} &= 0.36 \times 0.48 \left( 1 - 0.42 \times 0.42 \right) 230 \times 650^2 / 20 \\ &= 1019674.7 \text{ Nmm} \\ &= 101.96 \text{ kNm} \end{aligned}$$

$$M_u > M_{ult,m}$$

The section is doubly Reinforced.

(6) Calculate  $M_{ur}$

(Fig 12 SP 16)

$$M_{ur} = M_u - M_{ult,m}$$

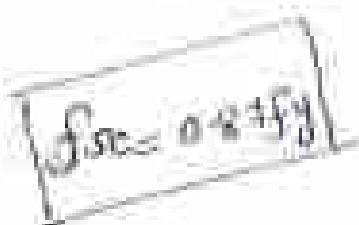
$$= 250.55 - 101.96$$

$$M_{ur} = 148.58 \text{ kNm}$$

$$\begin{aligned} M_{ur} &= M_{u,r} - M_{ult,r} \\ &= M_{u,r} - M_{ult,r} \end{aligned}$$

Q) Calculate  $\frac{d'}{d}$  &  $f_{sc}$

(SP 16 Table F)



$$\frac{d'}{d} = \frac{60}{550} = 0.09 \approx 0.1$$

$$f_{sc} = 363 \text{ N/mm}^2$$

⑥ Area of compression steel ( $A_{sc}$ ) (Pg 12 SP 16)

$$M_{u2} = f_{sc} A_{sc} (d - d')$$

$$68.53 \times 10^6 = 363 \times A_{sc} (550 - 52)$$

$$A_{sc} = \frac{68.53 \times 10^6}{363 (550 - 52)} = 228.6 \text{ mm}^2$$

Using 16 mm  $\phi$  bars.

$$\text{Area of one bar, } \frac{\pi}{4} \times 16^2 = 201 \text{ mm}^2$$

$$\text{No of bars} = \frac{A_{sc}}{A\phi} = \frac{228.6}{201} = 1.13 \approx 2$$

∴ Provide 2 no 16 mm  $\phi$  bars

$$A_{st}(\text{per bar}) = \pi \times \frac{\pi}{4} \times 16^2 \\ = 402 \text{ mm}^2$$

⑦ Area of steel ( $A_{st}$ )

(IS 456 ANNEX 6)

$$M_{u,lim} = 0.87f_y A_{st} d \left( 1 - \frac{4y_{eff}}{b c h} \right)$$

$$171.96 \times 10^6 = 0.87 \times 415 \times A_{st} \left( 1 - \frac{4(3) A_{st}}{20 \times 230 \times 550} \right)$$

$$A_{st} = 1128.3 \text{ mm}^2$$

$$M_{u2} = 0.87f_y A_{st2} (d - d') \quad (\text{SP 16 Pg 12}) \\ = 0.87 \times 415 \times A_{st2} (550 - 52)$$

$$A_{st,2} = \underline{380 \text{ mm}^2}$$

(3)

$$A_{st} = A_{st,1} + A_{st,2}$$

$$A_{st} = 1128.3 + 380$$

$$= \underline{\underline{1508.3 \text{ mm}^2}}$$

using 20mm φ bars,

$$A_{sf} = \frac{\pi \times 20^2}{4} = \underline{\underline{314.16 \text{ mm}^2}}$$

$$\text{No. of bars} = \frac{A_{st}}{A_{sf}} = \frac{1508.3}{314} = 4.825$$

Provide 5 # 20mm φ bars.

$$A_{sf(\text{prov})} = 5 \times \frac{\pi}{4} \times 20^2 = 1570 \text{ mm}^2$$

1st of them 2 bars can be centered at a distance of  
0.081 =  $0.08 \times 500 = 40 \text{ mm}$  from supports

$$3.081 = 0.08 \times 500 = 40 \text{ mm} \quad (\text{fig 4})$$

Check for deflection

$$\rho_1 = \frac{w_1 k_1}{6d} = \frac{0.01 \times 1570}{230 \times 500} = 1.261$$

$$\rho_c = \frac{100 A_{sc}}{6d} = \frac{100 \times 402}{230 \times 500} = 0.311$$

$$\rho_s = 0.58 b_3 \left[ \frac{A_{st} M_d}{A_{st} p_{sd}} \right] = 0.58 \times 415 \times \left[ \frac{1508}{1570} \right]$$

$$\rho_s = 23.1 \text{ N/mm}^2$$

$$\text{For } \rho_1 = 1.261 \text{ and } \rho_s = 23.1 \text{ N/mm}^2$$

$$\begin{cases} k_1 = 0.97 \\ \text{Deflection factor} \end{cases}$$

(ii) for  $\rho_c = 0.8\%$  and  $f_s = 25 \text{ N/mm}^2$

Modification factor  $k_c = 1.09$

$$\left(\frac{l}{a}\right)_{\max} = 20 \times k_f \times k_c \\ = 20 \times 0.48 \times 1.09 = 21.36$$

$$\left(\frac{l}{a}\right)_{\text{prev}} = \frac{5000}{500} = 9.09$$

$$\left(\frac{l}{a}\right)_{\max} > \left(\frac{l}{a}\right)_{\text{prev}}$$

Hence OK

④ Design for shear

(IS 456 Cl 40)

$$V_u = 200.4 \text{ kN}$$

$$\# \text{ Nominal section } \left(\frac{l}{a}\right)_{\text{v}} = \frac{V_u}{k_f \cdot b d} = \frac{200.4 \times 10^3}{1.58 \times 100 \times 230 \times 500} = 1.58 \text{ mm}^{-1}$$

For M20 concrete,  $\tau_{cmax} = 2.8 \text{ N/mm}^2$  (IS 456 Pg 73)

$\tau_v < \tau_{cmax}$ , Hence OK.

In Design shear strength ( $\tau_v$ )

Two bars are available at distance 400mm from support

So only 3 bars remaining at supports.

$$A_{st} \text{ at supports} = 3 \times \frac{\pi}{4} \times 20^2$$

$$= 942 \text{ mm}^2$$

$$\rho_e = \frac{100 \text{ kN}}{bd} = \frac{100 \times 942}{230 \times 500}$$

for  $P_t = 0.75\%$  and M20 concrete

$$\tau_c = 0.56 \text{ N/mm}^2$$

$$\tau_v > \tau_c$$

Hence shear reinforcement had to be provided

1. Area of reinforcement ( $V_{us}$ ) (IS 456 pg 73 (3))

$$\begin{aligned}V_{us} &= T_u - T_{sd} \\&= 200.4 \times 10^3 - (0.54 \times 230 \times 10^3) \\&= 129560\text{N}\end{aligned}$$

2. Area of stirrups ( $A_{sv}$ )

Using 2 legged 8 mm & vertical stirrups,

$$A_{sv} = \frac{2 \times K_{sv} f_y^2}{4} = \underline{\underline{100.53 \text{mm}^2}}$$

3. Spacing of Stirrups ( $S_v$ ) (IS 456 pg 73 (40.1))

$$S_v = \frac{0.87 f_y A_{sv}}{V_{us}} = \frac{0.87 \times 415 \times 100.53 \times 572}{129560}$$

$$= 154 \text{mm} \approx 150 \text{mm}$$

4. Spacing as per min. requirement (IS 456 pg 49 Cl. 26.5.1D)

$$S_v = \frac{0.37 f_y A_{sv}}{0.46} = \frac{0.37 \times 415 \times 100.5}{0.46 \times 230}$$

$$= 394 \text{mm}$$

The max. spacing should be less than the following,

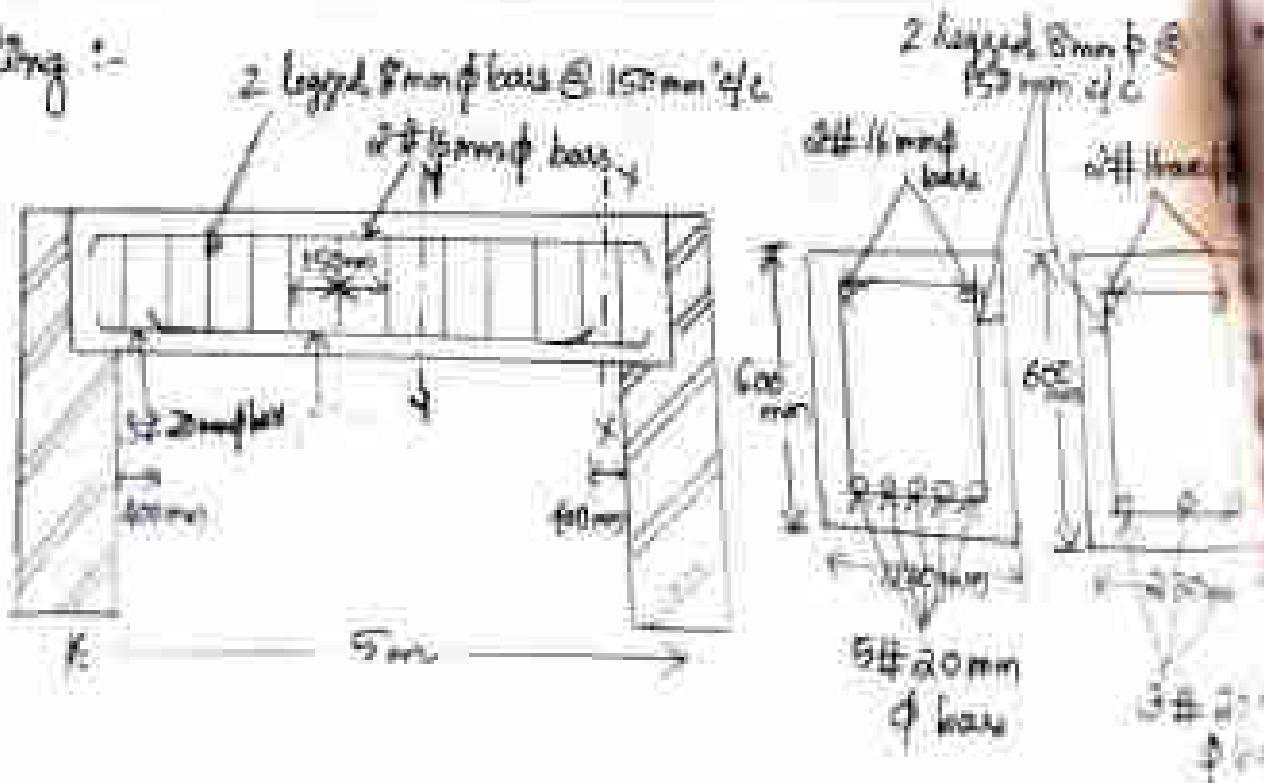
$$(i) 0.76 d = 0.76 \times 250 = 190 \text{mm}$$

(IS 456 Cl. 26.5.15)

(ii) 300 mm.

∴ Provide 2 legged 8 mm & bars @ 150 mm c/c

⑥ Detailing :-



### Design of T-Beams.

In RCC construction, slab and beams are cast monolithically. In slab construction, a portion of the slab acts integrally with the beam and carries along with the beam under the loads. The portion of the slab which acts integrally with the beam to resist the loads of the beam is called Web or Rib of the beam. The beam below the flange is called Flange of the beam. The slab and the beam together are called as T-beam.

$$\begin{aligned}
 \textcircled{6} \quad \% \text{ of } 5\text{ml} \\
 pt &= \frac{100 \text{ kg} +}{6d} \\
 &= \frac{100 \times 1842.02}{50 \times 560} \\
 &= 0.957\%
 \end{aligned}$$

4. A rectangular reinforced concrete beam is simply supported on two masonry walls 230 mm thick and 6 m apart (c/c). The beam is carrying an imposed load of 15 kN/m. Design the beam without any cracking checks. Use M20 concrete and Fe 415 steel.

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$$\text{Apparent load} = 15 \text{ MN/m}^2$$

$$I_{\text{c}} = 25 \text{ A/mm}^2$$

= 475 km

④ Depth of Seaw (assume)

~~Span/Dep~~ = 5%

四

$$\frac{P}{Q} = \frac{1}{12} = \frac{6000}{12} = 500 \text{ min}$$

Assumed eff. concn = 50 ppm

$$\therefore \text{eff. depth} = 500 - 50 = 450 \text{ mm}$$

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effective span ( $l$ )

② Effective approach  
is known as least of the following

The off shore is least of the  
leeward corner of supplies

$$\text{Center to center of supports} = 6 \text{ m}$$

$$1) \text{ Center to center of supports} = 5.74 + 0.45 = 6.19 \text{ m}$$

$$2) \text{ Clear span } + d = 6.19 + 0.45 = 6.64 \text{ m}$$

11) Clear span + d = 2(11) + 2(4) = 30 m = 50%

$\therefore$  Take  $L = 6\text{ m}$

(ii) Factored Load ( $W_a$ ) and Factored Moment ( $M_a$ )

$$\begin{aligned} \text{Self weight of beam} &= 0.6 \times 3.25 \times 25 \\ &= 3.125 \text{ kN/m} \end{aligned} \quad \left. \begin{array}{l} \text{Self weight = } \\ \text{R.C.C. + steel = } \end{array} \right.$$

$$\text{Imposed load} = 15 \text{ kN/m}$$

$$\begin{aligned} \text{Total load, } w &= 18 + 3.125 \\ &= 18.125 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} \text{Factored load, } w_a &= 1.5 \times 18 \\ &= 1.5 \times 18.125 \end{aligned}$$

$$w_a = 27.2 \text{ kN/m}$$

$$\begin{aligned} \text{Factored Moment, } M_a &= \frac{w_a L^2}{8} \\ &= \frac{27.2 \times 6^2}{8} \end{aligned}$$

$$M_a = 122.3 \text{ kNm}$$

IS 456 Annex Pg 76

(iii) Minimum effective depth required.

$$d_{eq} = \sqrt{\frac{M_a}{R_a \times b}}$$

$$\begin{cases} R_a = 0.138 \text{ kN/mm} \\ = 0.138 \times 25 \\ = 3.45 \end{cases}$$

$$\begin{aligned} &= \frac{122.3 \times 10^6}{3.45 \times 250} \\ &\approx 376.65 \text{ mm} \end{aligned}$$

$$d_{eq} < d_{assumed} (450 \text{ mm})$$

Hence ok

Since the depth of section is more than that reqd for a balanced section the section is designed as under. Reinforced section. Adopt  $w = 500\text{mm}$  and  $b = 250\text{mm}$ .

Assuming clear cover as 40mm,  $b_{eff} = 210\text{mm}$  & bars  $d = 500 - 8 - 20 = 472 \text{ mm}$

⑤ Area of Steel reqd.

[IS 456, Clause 6.1.1]

For under reinforced section,

$$M_u = 0.87 f_y b d \left(1 - \frac{f_y A_s}{f_{ck} b}\right)$$

$$122.3 \times 10^6 = 0.87 \times 415 \times 416 \times 462 \left(1 - \frac{415 \times A_s}{390 \times 462 \times 25}\right)$$

$$122.3 \times 10^6 = 166505 / 416 - 23.77 A_s$$

$$A_s = 6958.7 \text{ mm}^2 + 51032.11 / 1 = 0$$

$$A_s = 6126.02 \text{ mm}^2$$

$$A_s = 632.87 \text{ mm}^2$$

$$\text{Take } A_s = 632.87 \text{ mm}^2$$

⑥ Minimum area of steel ( $A_s$ ) [IS 456 Clause 6.1.1]

$$\frac{A_s}{b d} = \frac{0.85}{64} = 0.85 \times 250 \times 462$$

$$A_s = \frac{0.85 b d}{64} = \frac{0.85 \times 250 \times 462}{64}$$
$$= 236 \text{ mm}^2 < 632 \text{ mm}^2$$

Hence OK.

Using 20 mm dia,

$$A_s = \frac{\pi}{4} \times 20^2 = 314 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_s}{A_s} = \frac{632}{314} = 2.0 \approx 3$$

Provide 3 # 20 mm dia

$$\text{Net provided} = 3 \times 314 = 942 \text{ mm}^2$$

⑦ Check for deflection

$$\frac{P_l \cdot \frac{100 \cdot b l}{b d}}{240 \times 462} = \frac{100 \times 942}{240 \times 462} = 0.81\%$$

$$f_c = 0.58 f_g \left[ \frac{\text{Act load}}{\text{Live load}} \right] \quad \left\{ \begin{array}{l} \text{Fig 38, IS 456} \\ \text{Fig 4} \end{array} \right.$$

$$f_c = 0.58 \times 415 \left( \frac{83.3}{94.2} \right) \\ = 212 \text{ N/mm}^2$$

Interpolating for  $f_c = 212 \text{ N/mm}^2$  ( $\rho_{pt} = 0.8\%$ )

$$k_1 = 1.23$$

Modifying factor

$$f_g = 190, k_2 = 1.35$$

$$f_g = 240, k_2 = 1.2$$

$$\left( \frac{l}{d} \right)_{\text{max}} = 20 \times 1.23 \\ = 25.6$$



$$\left( \frac{l}{d} \right)_{\text{per}} = \frac{6000}{462} = 12.9$$

$$\frac{240-190}{1.2-1.35} = \frac{212-190}{x-1.35}$$

$$\left( \frac{l}{d} \right)_{\text{max}} > \left( \frac{l}{d} \right)_{\text{permitted}}$$

$$\frac{50}{-0.15} = \frac{34}{x-1.35}$$

$$50x - 67.5 = -3.3 \\ 50x = -3.3 + 67.5 \\ 50x = 64.2 \\ x = \frac{64.2}{50}$$

Hence OK

① Design for shear

[IS 456, Cl. 40]

$$k_4 = 1.28$$

$$\text{Shear force, } V_u = \frac{W_u L}{2} = \frac{37.2 \times 5.73}{2} \\ = 105.5 \text{ kN}$$

$\left\{ \begin{array}{l} \text{clear span} \\ = 5.73 \text{ m} \end{array} \right.$

Nominal shear stress ( $\tau_v$ )

[IS 456 Pg 72  
Cl 40.1]

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{105.5 \times 10^3}{250 \times 4.62}$$

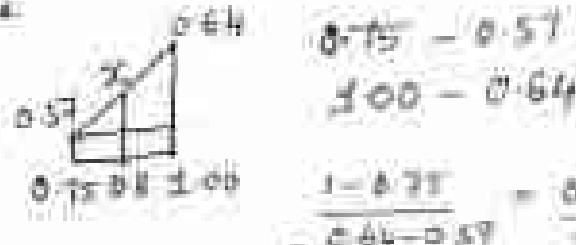
$$\tau_v = 0.48 \text{ N/mm}^2$$

2

\* Design shear strength of ( $\tau_c$ ) Concrete { pg 73 IS 456  
Table 19 }

for  $\rho_s = 0.8\%$  and  $M_{25}$  concrete

$$\tau_c = 0.58 \text{ N/mm}^2$$



$$\therefore \tau_{cmax} = 0.58 \text{ N/mm}^2 \quad \{ \text{pg 73 IS 456} \}$$

$\tau_c < \tau_{cmax}$  { Table 20 }

$$\tau_c > \tau_c$$

Hence shear reinforcement is required.

Shear to be carried by reinforcement =  $V_{us}$ . [IS 456 pg 73]

$$V_{us} = V_u - \tau_c b d \quad \{ \text{cl 40 4} \}$$

$$= 76.5 \times 10^3 - (0.58 \times 350 \times 350) \text{ N}$$

$$V_{us} = 11510 \text{ N}$$

\* Area of shear reinforcement ( $A_{sv}$ )

Design 8 mm of 2 legged stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.57 \text{ mm}^2$$

\* Spacing of stirrups,  $S_v = \frac{0.37 b y}{V_{us}}$  [IS 456 pg 73]  
[cl 40 4]

$$= \frac{0.37 \times 350 \times 120.3}{11510}$$

$$S_v = 1456 \text{ mm}$$

\* Max. Spacing as per norm. Reinforced [IS 456 pg 48]

$$S_v = \frac{0.12 A_{sv} b}{0.46} \quad \{ \text{cl 26.5.1.6} \}$$

$$= \frac{0.12 \times 100.57 \times 350}{0.46 \times 350}$$

$$S_v = 36.2 \text{ mm}$$

The spacing should be least of the following

[IS 456  
Cl. 22-34.5]

- i)  $0.15d = 0.15 \times 467 = 69.55\text{ mm}$   
ii) 300 mm

∴ Spacing  $S_p = 300\text{ mm}$ .

Provide 8 nos of 2 legged stirrups @ 300 mm c/c throughout the length of the beam.

Provide 2 # 10-mm anchor bars in the compression zone.

④ Check for development length

[IS 456 page 44]

Using  $\frac{M_c + l_d}{V_u}$

Not provided

$$M_c = 0.876y \cdot 4t \cdot d \left( 1 - \frac{0.876y}{4t \cdot d} \right)$$

$$= 0.876 \times 415 \times 942 \times 467 \left( 1 - \frac{942 \times 4.5}{260 \times 415 \times 25} \right)$$

$$= 13556.936 \text{ N-mm}$$

$$M_c = 135.56 \text{ kNm}$$

Using  $V_u = 14500 \text{ N}$

No bend on back,  $l_o = 0$

$$\frac{M_c + l_d}{V_u} = \frac{13556.936}{14500} = 1.736 \text{ mm}$$

Development length,  $l_d = \frac{\phi 0.876y}{4t \cdot d}$  [Pg : 42 IS 456]  
Cl. 22-21

$$= \frac{20 \times 0.876 \times 4.5}{4 \times 2.24} = \frac{1.6 \times 20 + 1.4}{2.24} = 2.04 \text{ m}$$

$$\frac{M_c + l_d}{V_u} > l_d, \text{ Hence OK}$$

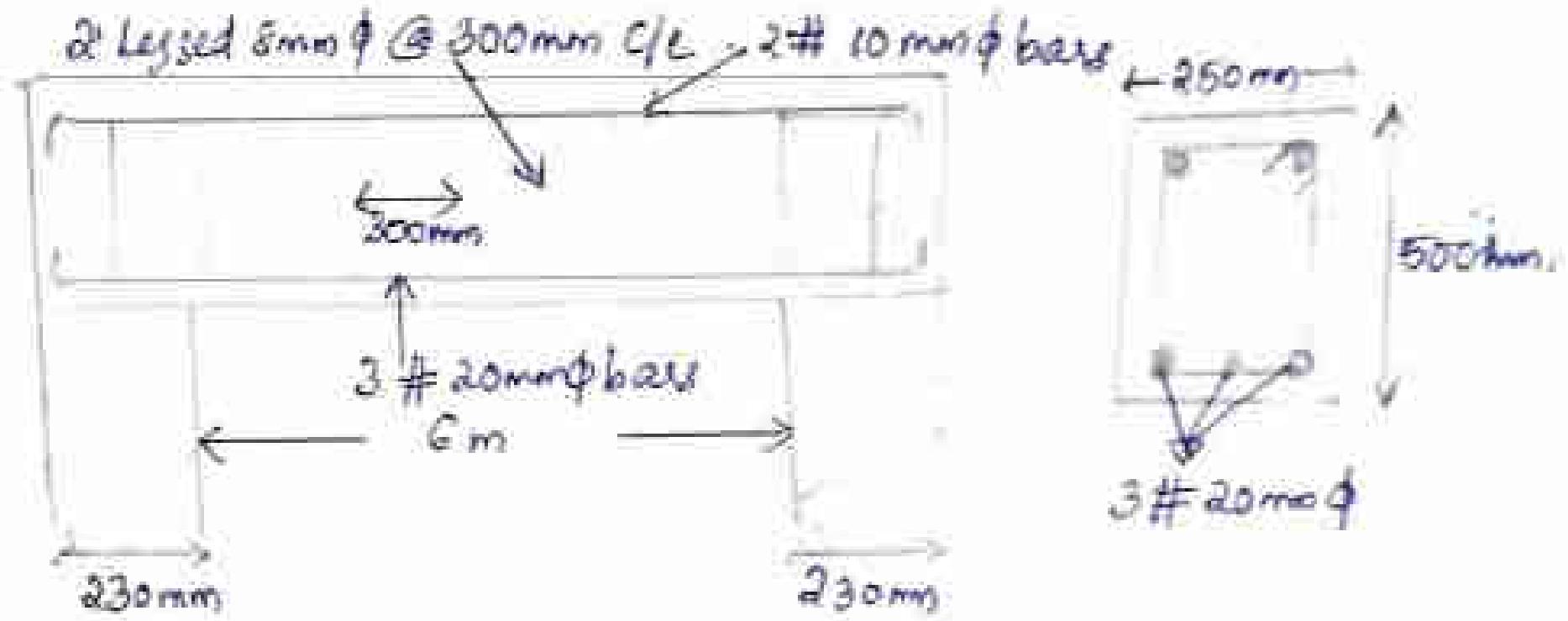
⑤ Design Summary

Beam size = 250 x 550 mm

Main bars = 2 # 20-mm

Stirrups = 8 nos 2 legged @ 300-mm-c/c

Anchor bars = 2 # 10-mm



## Design of Cantilever Beams.

- \* Max. value of  $\frac{l}{d}$  permitted is 7
- in the approx. value of depth =  $\frac{1}{7}$  th of span
- \*  $M_u = \frac{W_u l^2}{a}$

The stirrups shall not exceed the least of  
1,  $\frac{2+4}{4}$  and 300 mm.

The longitudinal reinforcement (IS 456 Cl. 26.5.1*a*)

The longitudinal reinforcement shall be placed  
as close as is practicable to the corner of the C/S and  
in all cases, there shall be at least one longitudinal  
bar in each corner of the flange.

If the C/S of the member exceeds 450 mm, additional  
longitudinal bars shall be provided to satisfy the  
requirement of minimum reinforcement and spacing given  
in IS 456 Cl. 26.5.1.3.

### Problem on design for tension.

Determine the reinforcement requirement for a beam of  
size 300 mm x 600 mm subjected to a factored B.M. of 150 kNm,  
factored shear force of 100 kN and factored horizontal moment  
of 50 kNm. Use M<sub>20</sub> concrete and Fe 415 steel.

Given Data:-

$$b = 300 \text{ mm}$$

$$M_u = 150 \text{ kNm}$$

(a)

$$T_a = 50 \text{ kNm} \quad T_c = 50 \text{ kNm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$D = 600 \text{ mm}$$

$$V_a = 100 \text{ kN}$$

$$f_{ct} = 0.14 \text{ N/mm}^2$$

(b) Effective depth,  $b_e = 200 \text{ mm}$  and  $\phi = 0.7$  bar at 200 mm

Assume clear cover of 20 mm

$$\text{Effective depth} = 600 - 20 - \frac{20}{2}$$

$$= 560 \text{ mm}$$

(c) Equivalent shear,  $V_e$

$$V_e = V_a + \frac{1.5 T_a}{b}$$

$$= 100 + \frac{1.5 \times 50}{0.3}$$

$$V_e = 361.667 \text{ kN}$$

[IS 456 Pg 36  
Cl 41.3.1]

(d) Equivalent shear stress ( $\tau_{eq}$ ) [IS 456 Cl 4.8.1]

$$\tau_{eq} = \frac{V_e}{bd} = \frac{361.67 \times 10^3}{300 \times 570}$$

$$\tau_{eq} = 2.14 \text{ N/mm}^2$$

For M<sub>u</sub> concrete,  $\tau_{concrete} = 2.8 \text{ N/mm}^2$  [IS 456 Pg 75  
Table 20]

$\tau_{concrete} > \tau_{eq}$ , Hence OK

(e) Shear strength of concrete ( $\tau_c$ ) [IS 456 page 73 Table 18]

Assume area of steel as 0.5%

$$\tau_c = 0.48 \text{ N/mm}^2$$

$$Z_e < Z_u$$

Hence longitudinal and transverse reinforcements are provided as torsional reinforcement

Longitudinal Reinforcement

[IS 456 pg 82-75]

(a)

Cl 8-4-2

41-4-2

+ Equivalent B.M (M<sub>e</sub>)

$$M_e = M_u + M_p$$

$$= M_u + T_{u,n} \left( 1 + \frac{0.1b}{h} \right)$$

$$= 150 + 50 \left( 1 + \frac{600}{300} \right)$$

$$M_e = \underline{\underline{258.2 \text{ kNm}}} \quad 1-7$$

Since  $M_e > M_p$ , there is no need of compression reinforcement due to longitudinal moment.

+ Area of steel required for equivalent B.M (M<sub>e</sub>) [IS 456 - Annexure 4]

$$M_e = 0.87 f_y A_{st} t \left( 1 - \frac{f_y A_{st}}{f_{ck} b t} \right)$$

$$0.362 \times 10^6 = 0.87 \times 415 \times A_{st} \times 570 \left( 1 - \frac{415 \times A_{st}}{20 \times 300 \times 570} \right)$$

$$0.06916 A_{st}^2 + 5.72 A_{st} - 6590.25 = 0 \quad 20 \times 300 \times 570$$

$$\underline{\underline{A_{st} = 1390 \text{ mm}^2}}$$

\* Check for A<sub>st</sub>

$$M_{st} \rightarrow \frac{A_{st} - 0.95}{b d} f_y$$

From SP 16

$$\frac{M_e}{b d^2} \leq \frac{0.95 \times 10^6}{200 \times 570^2} = 0.40 \text{ N/mm}^2 \quad M_{st} \rightarrow 0.6 \times 0$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

[From table 2]  $\rho_t = 0.814\%$

$$\rho_t = \frac{100 A_{st}}{b d}$$

$$\text{Page 48 } \therefore A_{st} = \frac{0.814 \times 300 \times 570}{100} = \underline{\underline{1392 \text{ mm}^2}}$$

Using 80 mm of bars

$$A_{sf} = \frac{\pi}{4} \times 20^2 = \underline{\underline{314.15 \text{ mm}^2}}$$

$$\text{No. of bars} = \frac{A_{st}}{A_{sf}} = \frac{1392}{314.15} = 4.4 \frac{2}{5}$$

- (i) Provide 5# 20 mm bars on tension side  $\left(\frac{5 \times 20^2}{100} = 200 \text{ mm}^2\right)$   
 Also provide 2# 12 mm  $\phi$  bars on the compression side as per  
 (ii) Side face reinforcement

If the depth of the reinforcement is more than 450 mm,  
 side face reinforcement is required as per IS 456 Cl. 25.2  
 and 26.5.1.7 on page 47 and 48.

$$\text{Area of side face reinforcement} = 0.05\% \text{ on each face}$$

$$= 0.05 \times \frac{300 \times 600}{100}$$

Hence provide one bar of 12 mm  $\phi$  on each face in the middle ( $\frac{\pi \times 12^2}{4} = 113 \text{ mm}^2$ )

- (iii) Transverse Reinforcement. (IS 456 Pg 75 Cl. 41.4.3)

Spacing of stirrups =  $S_v$

$$A_{sv} = \frac{T_a S_v + V_a S_v}{b_i d_i (Q_{sd})^{0.5} \cdot 1.5 (P_s - b)}$$

$b_i$  = c/c distance b/w corner bars in the direction of breadth.

$$= 300 - 20 - 20 - \frac{12}{2} - \frac{12}{2} = 248 =$$

$d_i$  = c/c distance b/w corner bars in the direction of depth.

$$= 600 - 24 - 20 - \frac{20}{2} - \frac{12}{2} = 544 =$$

Closing 10 mm  $\phi$  2 legged stirrups.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 10^2 = 157 \text{ mm}^2$$

$$0.27 \times A_{sv} = \frac{T_a S_v}{b_i d_i} + \frac{V_a S_v}{2.5 d_i}$$

$$0.27 \times 157 = \frac{50 \times 10^6 \times S_v}{248 \times 544} + \frac{100 \times 10^3 \times S_v}{2.5 \times 544}$$

For designing, the slab is considered as beam of 1m width.

### Design steps of slabs:-

1. Assume overall depth
2. Calculate effective span [IS 456 Cl. 22.5 page 34]
3. Calculate dead load and live load. Convert it into factored load, for finding load, assume slab as beam of 1m width.
4. Find the BM and SF
5. Check the effective depth using the equation [IS 456 Annex 6.1.1]

$$M_{\text{allow}} = 0.36 \frac{I_{\text{max}}}{d} \left( 1 - 0.42 \frac{I_{\text{max}}}{d} \right) f_{ck} b d^2$$

$$M_{\text{allow}} = R_u \times b \times d^2$$

$$d = \sqrt{\frac{M_{\text{allow}}}{R_u \times b}}$$

$$R_u = 0.138 \text{ fck} \rightarrow f_c 415$$

$$R_u = 0.133 \text{ fck} \rightarrow f_c 500$$

$$R_u = 0.149 \text{ fck} \rightarrow f_c 250$$

6. Calculate the area of main steel and find spacing between main reinforcement bars [SP 16 Page 230]
7. Check for minimum reinforcement [IS 456 Cl. 36.5.2.1 Pg 33]
8. Check for max. bar diameter [IS 456 Cl. 36.9.2.2 Pg 46]
9. Check for min. spacing of main bars [IS 456 Cl. 36.4.3 Pg 46]
  - a. Spacing  $\geq 3d$
  - b. Spacing  $\geq 300 \text{ mm}$  whichever is less
10. Calculate the area and spacing of distribution bars [IS 456 Cl. 36.5.2.1 Pg 33]

11. Check for max. spacing b/c distribution bars  
[IS 456, Cl. 26.3.3 page 46]

Spacing  $\neq 5d$

Spacing  $> 450 \text{ mm} \rightarrow$  withdraw 3 smaller

12. Check for deflection. [IS 456, Cl. 23.2.1 page 34]

13. Check for shear. [IS 456, Cl. 40 page 72]

### Problems - One Way Slab.

4. Determine the ultimate moment of resistance of a 150mm thick slab reinforced with 10mm dia bars at 200mm c/c. The cover is 25mm. Use M20 concrete and Fe 415 steel. Given Data

$$D = 150 \text{ mm}$$

$$d = 150 - 25 = 125 \text{ mm}$$

$$A_{st} = 10 \text{ mm}^2 \text{ @ } 200 \text{ mm c/c}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

#### ① Calculate $A_{st}$ .

$$A_{st} = \frac{b \cdot A_f}{\text{spacing}}$$

$$\left. \begin{array}{l} b = 120 \text{ for} \\ = 1000 \text{ mm} \end{array} \right\}$$

Given:  $10 \text{ mm dia}$

$$= \frac{1000 \times \pi \times 10^2}{4} = 314 \text{ mm}^2$$

#### ② Depth of NA ( $x$ )

[IS 456 ANNEX]

$$x_u = \frac{0.81 f_y d t}{0.36 f_{ck} b} = \frac{0.81 \times 415 \times 125}{0.36 \times 20 \times 1000} = 14.6 \text{ mm}$$

$$x_{min} = 0.45 d = 0.45 \times 125 = 60 \text{ mm}$$

$x_u < x_{min} \therefore$  Section is under reinforced

## Want of Resistance

$$M_u = 0.84 \text{ by Astd } \left( i - \frac{I_y A_{st}}{b d^2} \right)$$

$$= 0.84 \times 415 \times 313 \times 105 \left( i - \frac{415 \times 293}{20 \times 1000 \times 125} \right)$$

$$= 46565.20 \text{ Nmm}$$

$$M_u = 16.56 \text{ kNm}$$

Design a simply supported roof slab for a room  $4\text{m} \times 10\text{m}$   
in size the slab is carrying an imposed load of  $25 \text{ kN/m}^2$   
the  $M_u$  min and the 415 steel. Thickness of wall = 230 mm.

$$l_y = 10 \text{ m}$$

$$l_x = 4 \text{ m}$$

$$\frac{l_y}{l_x} = \frac{10}{4} = 2.5 > 2 \text{, One way slab}$$

(IS 456 Cl. 23.2.1 page 37)  
 Overall Depth  $\left\{ \frac{550}{d} \leq 20 \right\}$

$$\frac{l_x}{d} < 20$$

$$d = \frac{4000}{20} = 200 \text{ mm} \rightarrow \text{Take } d = 160 \text{ mm}$$

Assume 40 mm clear cover and  $\phi$  of main bar as 8 mm.

$$D = d + \phi \text{ of main bar} + \text{clear cover}$$

$$= 160 + 20 + \frac{8}{2} = 184 \text{ mm} \approx 180 \text{ mm}$$



[IS 456 Cl. 23.2 Pg. 34]

① Effective Span

least of

\* c/c distance b/w supports

$$= 4000 + \frac{160}{2} + \frac{160}{2} = 4320 \text{ mm}$$

\* Clear span + eff. depth =  $4000 + 160 = 4160 \text{ mm}$

\* Clear span + eff. span =  $4160 \text{ mm}$

## ④ Load Calculation

$$\begin{aligned} DL &= 25 \text{ kN/m} \\ &= 25 \times 1 > 0.18 \\ &= 4.5 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} LL &= 25 \text{ kN/m}^2 \\ &= 25 \times 1 \text{ m} \\ &= 25 \text{ kN/m} \end{aligned}$$

$$TL = 4.5 + 2.5 = 7 \text{ kN/m}$$

Factored load,  $w_u = 1.5 \times 7$   
 $= 10.5 \text{ kN/m}$

## ⑤ Calculation of BM and SF

$$BM = \frac{w_u l^2}{8} = \frac{10.5 \times 4.16^2}{8} = 22.41 \text{ kNm}$$

$$SF = \frac{w_u l}{2} = \frac{10.5 \times 4.16}{2} = 21.24 \text{ tN}$$

## ⑥ Check for effective depth

$$d_{\text{reqd}} = \sqrt{\frac{M_u}{R_u \times b}}$$

For Fe 415,  $R_u = 0.138 f_{ck} = 0.138 \times 20 = 2.76 \text{ N/mm}^2$

$$d_{\text{reqd}} = \sqrt{\frac{22.41 \times 10^3}{2.76 \times 1000}} = 90.70 \text{ mm}$$

90.70 mm < 160 mm  $\therefore d_{\text{reqd}} < d_{\text{prov}}$

Hence safe

Area of main reinforcement:

$$M_u = 2.93 \times 10^6 \text{ N-mm} \quad (1-1/4\pi)$$

$$\frac{M_u}{bd^2} = \frac{2.93 \times 10^6}{1000 \times 160^2} = 0.83$$

From Table 2 IS 456 pg 41, for  $f_y = 415 \text{ N/mm}^2$ ,  $f_t = 20 \text{ N/mm}^2$

$$P_t = 0.252 / \frac{0.15 - 0.241}{0.90 - 0.264} = 0.252 / \frac{0.15 - 0.241}{0.90 - 0.264} = 0.252 / 0.99$$

$$p_t = \frac{100 \text{ kN}}{bd} = \frac{0.15 - 0.241}{0.264 - 0.241} = \frac{0.15 - 0.241}{0.90 - 0.264}$$

$$0.252 = \frac{100 \times 415}{1000 \times 160}$$

$$A_{st} = 4.12 \text{ mm}^2 = \underline{\underline{4.12 \text{ cm}^2}}$$

$$\frac{0.1}{0.16} = \frac{0.15 - 0.241}{0.90 - 0.264} = \frac{0.1}{0.16} = \frac{0.15 - 0.241}{0.90 - 0.264}$$

$$0.12 = \frac{0.15 - 0.241}{0.90 - 0.264} = 0.12 = \frac{0.15 - 0.241}{0.90 - 0.264} = 0.12 = \frac{0.15 - 0.241}{0.90 - 0.264} = 0.12 = \underline{\underline{0.12}}$$

④ Spacing of main bars:

item 57, Table 96 } for  $A_{st} = 4.12 \text{ cm}^2$   
Table 96 } Using 8 mm Ø bars  
pg 236 }

$$\text{Spacing} = 12 \text{ cm} = 120 \text{ mm}$$

$$\text{spacing} = \frac{1000 \text{ kN}}{8 \text{ kN}}$$

$$= \frac{1000 \times 8}{8} = \frac{1000}{8} = 125$$

$$= 12.5 \text{ mm} / \text{bar}$$

⑤ Check for minimum reinforcement [IS 456 Cl 26.5-2.1 page 42]

$$\text{Min } A_{st} = 12 \times 1.60$$

$$= \frac{0.12 \times 1000 \times 180}{100} = \underline{\underline{216 \text{ mm}^2}}$$

$$216 < A_{st} \text{ prov. } (412 \text{ mm}^2)$$

Hence OK.

⑥ Check for max. bar diameter [IS 456 Cl 26.5-2.2 pg 42]

$$\text{bar } \phi \neq \frac{L}{D}$$

$$s \neq \frac{L}{6} \times 180$$

Hence OK

- (ii) Check for max. spacing between main bars  
[IS 456 Cl 26.3.3 pg 46]

Spacing should be less than least of the following

$$\rightarrow 3d = 3 \times 160 = 480 \text{ mm}$$

$$\rightarrow 300 \text{ mm}$$

$$\text{Spacing } 180 \text{ mm} < 300 \text{ mm}$$

Hence safe

- (iii) Area of Distribution bars [IS 456 Cl 4.6.5.2.1 Pg 47]

$$\text{Area of distribution bars} = 0.12 / bD \quad (\text{Minimum reinforcement})$$

$$= \frac{0.12 \times 100 \times 180}{100}$$

$$= 21.6 \text{ mm}^2$$

- (iv) Spacing of distribution bars

(IS 456 Cl 4.6.5.2.1 Pg 47) Using Span length  $\delta$   
for A<sub>st</sub> = 2.16 cm<sup>2</sup>

$$\text{Spacing} = \delta / A_{st} = 23.8 \text{ mm}$$

- Less than spacing of distribution bars

Spacings equal to the width of plinths

$$1.1 + 1.1 + 1.1 = 3.3 \text{ m}$$

$$1.1 + 1.1 + 1.1 = 3.3 \text{ m}$$

Provide 8 nos. of bars at 120 mm c/c as main reinforcement along shorter span (4m) and provide 1 nos. of bars at 220 mm c/c as distribution bars along longer span (10 m)

(1)

② Check for deflection [IS:456 Cl 23.2.1 pg 37]

$$f_b = 0.254 \text{ N/mm}^2$$

$$f_s = 0.56 f_y \left[ \frac{\text{Act. u.d.}}{\text{Act. p.u.}} \right]$$

$$f_s = 0.56 \times 415 \times \left( \frac{216}{403.2} \right) = 128.94 \text{ N/mm}^2$$

$$f_s = 180 \text{ N/mm}^2$$

$$f_s = 145.4 \text{ N/mm}^2$$

$$\left( \frac{d}{a} \right)_{\max} = 20 \times 2 \\ = 40$$

$$\left( \frac{d}{a} \right)_{\text{prov}} = \frac{4160}{460} = 26$$

$$\left( \frac{d}{a} \right)_{\max} > \left( \frac{d}{a} \right)_{\text{prov}}$$

Hence ok.

③ Check for development length [IS:456 pg 44]

$$M_r = 0.8463 A_e d \left( 1 - \frac{6 + 4t}{600 b d} \right)$$

$$A_e d = \text{Act. min.} = 216 \text{ mm}^2$$

$$M_r = 0.8463 \times 2415 \times 216 \times 160 \left( 1 - \frac{6 + 4t}{600 \times 160 \times 40} \right) \\ = 92.128 \text{ kNm}$$

$$V = 21.14 \text{ kN/m}$$

Providing no hooks,  $l_d = 0$

[IS 456 pg 4]

$$\frac{M_1 + l_0}{v} = \frac{12.128 + 0}{21.84} = 0.553 \text{ m}$$
$$= 553 \text{ mm}$$

Developed length,  $L_d = \frac{\sigma_s A}{4 \tau_{bd}}$

[IS 456 pg  
and 42]

$$= \frac{0.87 f_y A_s}{4 \times 1.92}$$
$$= 376.09 \text{ mm}$$

$$\frac{M_1 + l_0}{v} > L_d$$

Hence OK.

(ii) Check for shear.

[IS 456 Cl 40, page 72]

\* Nominal shear stress ( $\tau_v$ )

$$\tau_v = \frac{V_u}{bd} = \frac{21.54 \text{ kN}}{1000 \times 160} = 0.136 \text{ N/mm}^2$$

\* Design shear strength of concrete ( $\tau_c$ ) At supports =  $0.136 \text{ N/mm}^2$

$$\rho_t = \frac{300 \text{ Ast}}{bd} = \frac{300 \times 216}{1000 \times 160} = 0.135\% = \frac{216}{160}$$

For M<sub>20</sub> concrete {IS 456 Table 19}

$$\tau_c = 0.28 \text{ N/mm}^2 \quad [K=1.2 \text{ From } 25.4 \text{ pg 72}]$$

$$\tau_c \cdot K = 0.28 \times 1.2 = 0.336 \text{ N/mm}^2$$

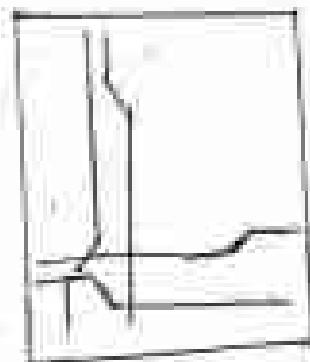
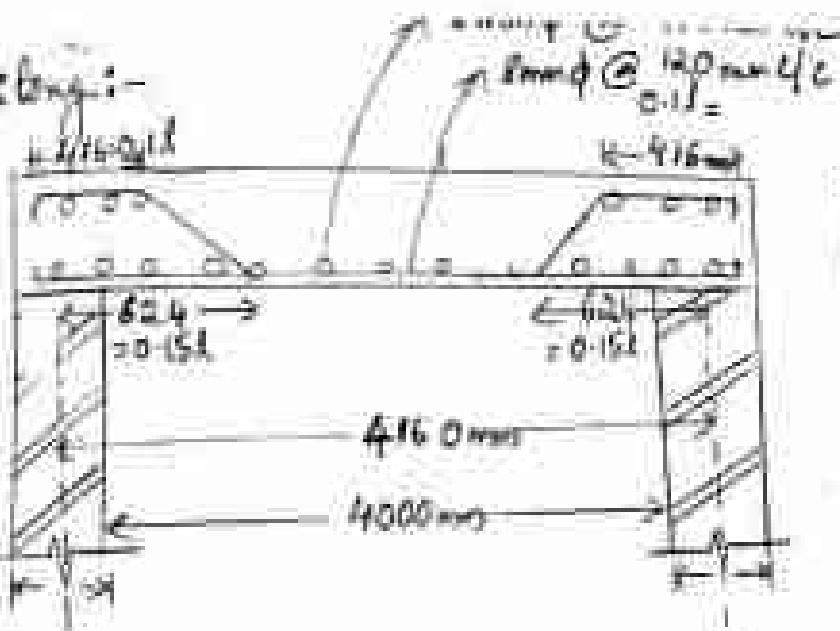
$$\tau_{max} = 2.8 \text{ N/mm}^2 \quad [\text{IS 456 Pg 73 Table 10}]$$

$$\tau_v < \tau_{max}$$

$$\tau_v < \tau_c \cdot K$$

Hence Safe

Design :-



Design a corridor slab which is supported on two opposite sides only. The clear distance b/w the support is 3m. Support width = 230mm.  $\sigma_u = 4 \text{ kN/mm}^2$ . Use M20 concrete & Fe415 steel.

### ① Overall depth

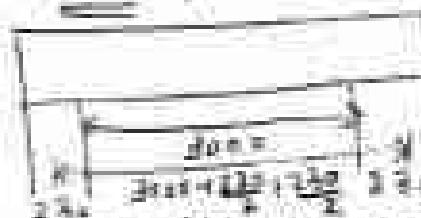
$$\frac{d}{2} = 20$$

$$\frac{3000}{d} = 20$$

$$\text{effective depth, } d = 150 \text{ mm}$$

$$\text{Overall depth, } D = d + c_c + \frac{\Phi}{2}$$

$$= 150 + 20 + \frac{20}{2} = 170 \text{ mm}$$



### ② Effective Span

Effective span is the least of  
\* the distance b/w the supports

$$= 3000 + \frac{230}{2} + \frac{230}{2} = 3230 \text{ mm}$$

\* clear span + eff. depth

$$= 3000 + 150 = 3150 \text{ mm}$$

Take eff. span = 3150 mm

(1)

Distribution of Slabs - One way slab - Two way slab - Load distribution - Cantilever slab - Continuous slab (Detailing)

Slabs used in floors and roofs of buildings are mainly supported with the supporting beams. They carry the distributed load primarily by bending. The thickness of slab is very small as compared to its length and width. The thickness of slab is very small as compared to its length and width.

Slabs are classified on the basis of  $\frac{b_y}{l_x}$  ratio.

i)  $b_y \rightarrow$  length of the longer span.

ii)  $l_x \rightarrow$  length of the shorter span

i) One way slab

ii) Two way slab.

One way slab

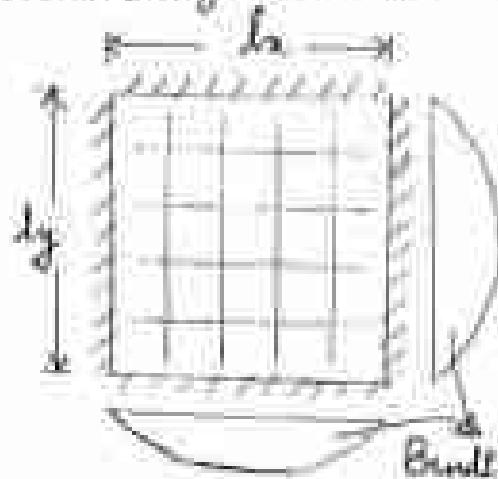
One way slabs are those slabs in which the  $\frac{b_y}{l_x}$  ratio is greater than 2 ( $\frac{b_y}{l_x} > 2$ ). The type of slab is also called as slab bending in one direction, as the bending takes place only bending in one direction, and reinforcement is provided in the shorter span. Therefore, main reinforcement is provided in the shorter span the one-way slab is analyzed by taking it to be a beam of  $l_x$  width.

$b_y > l_x$

Bending in shorter direction.

## ② Two way slab.

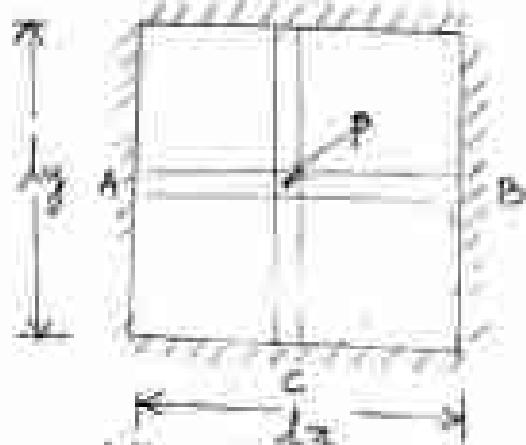
The slab which is supported on all four edges and  $\frac{L_y}{L_x}$  ratio less than or equal to 2 ( $L_y/L_x \leq 2$ ) is called a two way slab. Two way slab is also called as slab spanning in two directions because bending takes place in both directions. The transverse reinforcement is provided along both the directions in a two way slab (longitudinal & transverse direction).



Bending takes place along both directions

Load distribution in slab

Consider a slab as shown in fig subjected to a unit width load where  $L_y$  is the longer span and  $L_x$  is the shorter span.



Consider two middle steps AB and CD of unit width along  $L_y$  and  $L_x$  let  $w_y$  is the load carried by the slab in the longer direction and  $w_x$  is the load carried by the slab in the shorter direction.

$$\therefore \omega = w_x + w_y \quad \text{--- (1)} \quad (3)$$

The deflection of the centre point 'P' is given by,

$$\Delta_p = \frac{5}{384} \frac{w_x l_x^4}{E I} \quad \text{--- (2) (for strip AB)}$$

$$\Delta_p = \frac{5}{384} \frac{w_y l_y^4}{E I} \quad \text{--- (3) (for strip CD)}$$

Equating (2) and (3),

$$\frac{5}{384} \frac{w_x l_x^4}{E I} = \frac{5}{384} \frac{w_y l_y^4}{E I}$$

$$\frac{w_x}{w_y} = \left( \frac{l_y}{l_x} \right)^4$$

$$w_x = w_y \left( \frac{l_y}{l_x} \right)^4$$

Substituting value of  $w_x$  in (1)

$$\omega = w_y + w_y \left( \frac{l_y}{l_x} \right)^4$$

$$w_y = \frac{w}{\left[ 1 + \left( \frac{l_y}{l_x} \right)^4 \right]}$$

$$\text{and } w_x = \frac{w \left[ \frac{l_x}{l_y} \right]^4}{\left[ 1 + \left( \frac{l_y}{l_x} \right)^4 \right]}$$

$$\frac{l_y}{l_x} = 2 \quad w_y = \frac{w}{(1+2^4)} = 0.05w$$

$$w_x = \frac{w \cdot 2^4}{(1+2^4)} = 0.45w$$

$$\frac{l_y}{l_x} = 1.5 \quad w_y = \frac{w}{(1+1.5^4)} = 0.16w$$

$$w_x = \frac{w \cdot 1.5^4}{1+1.5^4} = 0.54w$$

$$\text{ii) For } \frac{b_y}{l_x} = 1 \quad w_y = \frac{w}{1+1^2} = 0.5w$$

$$w_x = \frac{w \cdot 1^2}{1+1^2} = 0.5w$$

From the above values it is clear that

- i) If  $\frac{b_y}{l_x} \geq 2.0$ , the load carried by shorter span is 95% and that carried by longer span is only 5%. So such a slab can be designed as slab supported on two edges with bending in one direction only i.e., shorter direction.
- ii) If  $\frac{b_y}{l_x} < 2$ , the load is shared by both the spans. Such a slab is designed as two way slab having bending in both directions.
- iii) One way slab is subjected to larger BM's as load is carried by one span i.e., shorter span only. While the 2 way slab is subjected to smaller BM's since the load is distributed over both the directions.
- iv) The depth of one-way is more than the depth of 2 way slab as the max. BM is more in one way slab.

#### One way slab

- \*  $b_y/l_x \geq 2$
- \* Bending takes place in one direction only i.e., shorter span
- \* Depth required is more
- \* Main reinforcement is provided along shorter span
- \* Less economical as thickness is more and the amount of steel is more.

#### Two way slab

- \*  $b_y/l_x \leq 2$
- \* Bending takes place in both the directions
- \* Depth required is less
- \* Main reinforcement is provided along both the spans
- \* More economical since thickness is less and area of reinforcement is less.

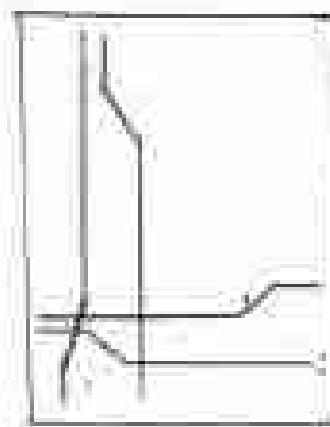
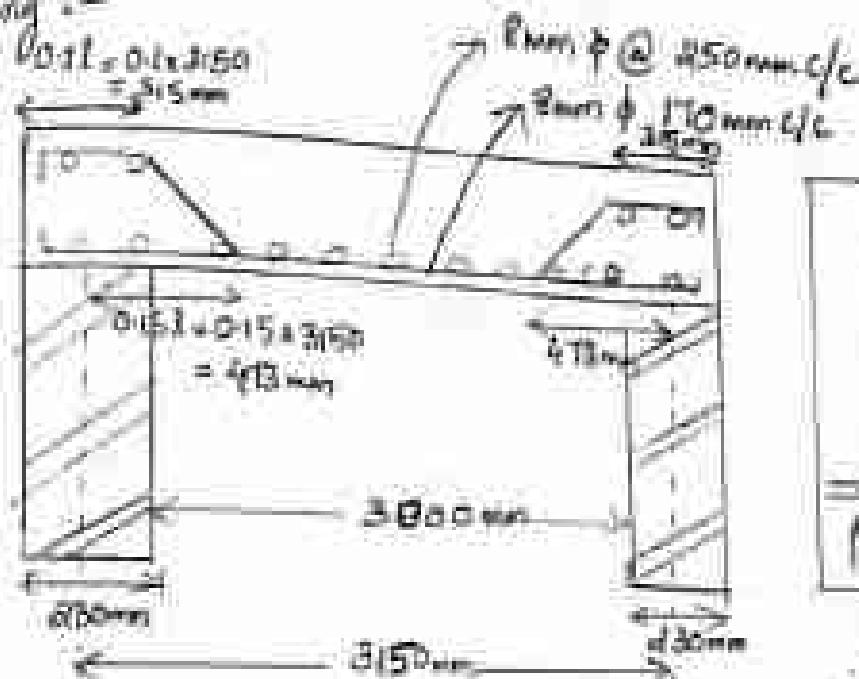
$$\tau_w = \tau_{\text{concrete}}$$

$$\tau_w < \tau_{\text{ck}}$$

Hence Safe

(19)

Detailing :-



### Cantilever Slab or Chajja (Balcony)

One way cantilever slab or chajja is designed as a continuous beam of one metre width. The points to be considered in the design of one way cantilever slab are as following:-

1. The eff. span of the cantilever slab shall be taken as its length upto the face of the support plus half the eff. depth except where it forms the part of continuous slab where the length from c/c of supports is taken.
2. The effective depth at fixed end is more and is assumed to be about  $\frac{\text{Span}}{12}$  to  $\frac{\text{Span}}{10}$ .
3. The sag at free end is minimum and is kept  $1/2$  to  $1/3$  of the depth at fixed end.
4. The main reinforcement is provided at the top and distributed at appropriate points.

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## Problem - Cantilever Slab

2. Design a cantilever slab for an overhanging of 1.2m. The imposed load on slab consist of 1 kN/m<sup>2</sup> of live load and weight of finishing is 200 N/m<sup>2</sup>. Use M20 concrete and Fe415 Steel Aggregates.

$$l = 1.2 \text{ m}$$

$$\text{Imposed load} = 1 \text{ kN/m}^2$$

$$\text{Weight of finishing} = 200 \text{ N/m}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\textcircled{1} \quad \text{Overall depth } d_s = 415 \text{ mm}$$

$$\frac{l}{d} = 10.7$$

$$\frac{1200}{d} = 10.7$$

$$d = \frac{1200}{10.7} = 110 \text{ mm}$$

Assume 20 mm Cc and 8 mm φ bars

$$\text{Overall depth, } D = d + Cc + f_t = 110 + 20 + \frac{9}{2} = 140.5 \text{ mm}$$

$$\textcircled{2} \quad \text{Effective Span}$$

$$\begin{aligned} l_e &= \text{clear span} + \frac{d}{2} = 1200 + \frac{110}{2} = 1305 \text{ mm} \\ &= 1200 + \frac{110}{2} = 1305 \text{ mm} \end{aligned}$$

$$\textcircled{3} \quad \text{Load Calculations}$$

$$L_L = 1 \text{ kN/m}^2 \times 1 \text{ m} = \underline{\underline{1 \text{ kN/m}}}$$

$$\text{Weight of finishing} = 200 \text{ N/m}^2 = 0.2 \text{ kN/m} \times 1 \text{ m} = \underline{\underline{0.2 \text{ kN/m}}}$$

$$D_L = 20 \text{ N/m} \times 1 \times 0.17 = 3.4 \text{ kN/m}$$

$$\text{Total load, } w = \frac{6.3}{4.46} = \underline{\underline{1.4 \text{ kN/m}}}$$

$$\text{Total factored load, } W_u = 1.5 \times 5.555 \\ = 8.325 \text{ kN/m} \\ = 8.45$$

(2)

BM & SF

$$BM = \frac{W_u L^2}{8} = \frac{8.325 \times 1.25^2}{2} = \underline{\underline{5.608 \text{ kNm}}}$$

$$SF = \frac{W_u L}{4} = 8.325 \times \frac{1.25}{4} = \underline{\underline{2.581 \text{ kN}}}$$

Check for effective depth

$$d_{\text{req}} = \sqrt{\frac{M_u}{R_u b}}$$

$$= \sqrt{\frac{5.608 \times 10^6}{276 \times 1000 \times 52 \times 10^{-3}}} = \underline{\underline{48.9 \text{ mm}}}$$

$$R_u = 0.138 \text{ fcu}$$

$$= 0.138 \times \frac{12}{276} = \underline{\underline{0.246 \text{ N/mm}^2}}$$

$d_{\text{req}} < d_{\text{pro}} (150 \text{ mm})$

Hence Safe.

[T5.456 ANSWER]

Area of main reinforcement

$$A_u = 0.87 \times \frac{4 \text{ Ast}}{52 \times 100} \left[ 1 - \frac{45.8t}{450 \text{ Ast}} \right]$$

$$6.608 \times 10^6 = 0.87 \times 4 \times 15 \times 100 \times 450 \left[ 1 - \frac{45.8t}{20 \times 1000 \times 450} \right]$$

$$433.25 \text{ Ast} \left[ 1 - \frac{45.8t}{10800} \right]$$

$$433.25 \text{ Ast} = \frac{45.8t}{10800}$$

$$6.608 \times 10^6$$

$$45.8t^2 - 433.25 \text{ Ast} = 0$$

$$45.8t^2 - 433.25 \text{ Ast} = 0$$

$$45.8t^2 = 433.25 \text{ Ast}$$

$$t = \frac{433.25 \text{ Ast}}{45.8}$$

$$t = 9.47 \text{ mm}$$

(2)

## ③ Check for min. dist.

[IS 456 Cl 26.3.2]

$$\text{min. dist} = 0.12 \times 60$$

$$= 0.12 \times 1000 \times 150$$

$$= \frac{100}{100} \times 216 > \frac{136}{180 \text{ mm}} > 15.676 \text{ mm}$$

$$\therefore \text{Provide min. dist} = \underline{\underline{216 \text{ mm}}}$$

## ④ Spacing of main reinforcement

$$\text{Using } 8 \text{ mm } \phi \text{ bars, } A_f = \frac{\pi d^2}{4} = 50.3 \text{ mm}^2$$

$$\text{Spacing of 8mm bar} = \frac{1000 A_f}{A_{st}}$$

$$> \frac{1000 \times 50.3}{200 \times 216} = 0.74 \text{ mm} \approx 74 \text{ mm}$$

## ⑤ Check for spacing of main reinforcement

[IS 456 Cl 26.3.3 pg 46]

Spacing should be less than least of the following

$$* 3d = 3 \times 120 = 360 \text{ mm}$$

$$* 300 \text{ mm}$$

$$\text{Spacing} = 270 \text{ mm} < 300 \text{ mm}$$

Hence Ok.

Provide 8mm  $\phi$  bar @  $\underline{\underline{270 \text{ mm}}}$  c/c as main reinforcement  
 Also provide 8mm  $\phi$  bar @  $\underline{\underline{270 \text{ mm}}}$  c/c as distribution bars. Since min bar  $\rightarrow$  min dist permitted

## ⑥ Check for development length (IS 456 pg 47)

$$\text{development length, } l_d = \frac{0.876 \times 4}{4 \times 1.92}$$

$$= \frac{0.876 \times 413 \times 9}{4 \times 1.92}$$

$$= 1.342 \approx 1.34$$

$$l_d = \underline{\underline{376 \text{ mm}}}$$

② Check for deflection

[IS 456 page 35]

(c)

$$f_c = 0.58 \text{ by } \left[ \frac{\text{41 kgd}}{\text{61 psiv}} \right]$$

$$\text{61 psiv} = \frac{1000 \text{ kN}}{\text{Span}} = \frac{1000 \times 10^3}{240 - 230} = \frac{100 \times 10^3}{20} = 5000 \text{ N/mm}^2$$

$$f_t = 0.58 \times 415 \left[ \frac{216}{\frac{+20}{+26.2}} \right] = \frac{237.73}{232.56} \text{ N/mm}^2$$

$$p_t = \frac{100 \text{ kN}}{64} = \frac{100 \times 10^3}{160} = \frac{0.125}{0.125} \text{ N/mm}^2$$

for  $p_t = 0.125 \text{ N/mm}^2$  and  $f_t = 232.56 \text{ N/mm}^2$

$K = 2$

[IS 456 Part 3  
page 58]

$$\left( \frac{l}{d} \right)_{max} = \sqrt{2} = 14$$

$$\left( \frac{l}{d} \right)_{psiv} = \frac{1250}{120/40} = 105.8$$

$$\left( \frac{l}{d} \right)_{max} > \left( \frac{l}{d} \right)_{psiv}$$

Hence OK

③ Check for shear

[IS 456 Cl 40 pg 72]

\* Nominal shear stress ( $\tau_v$ )

$$\tau_v = \frac{V_u}{bd} = \frac{(0.48 \times 10)^3}{1000 \times 120} = 0.0653 \text{ N/mm}^2$$

\* Design shear strength of concrete ( $\tau_c$ )

[IS 456 Table 19]

$$\frac{1000 \text{ kN}}{64} = \frac{100 \times 10^3}{64} \text{ psiv} \quad p_t = 0.125 \text{ N/mm}^2$$

$$\frac{1000 \text{ kN}}{64} = \frac{1000 \times 10^3}{64} \text{ N/mm}^2 \quad \tau_c = 0.28 \text{ N/mm}^2$$

$$\frac{1000 \text{ kN}}{64} = 0.125 \quad \tau_c K = 0.28 \times 1.3 \quad [K = 1.3 \text{ IS 456}, \text{ pg 72}]$$

$$= 0.364 \text{ N/mm}^2$$

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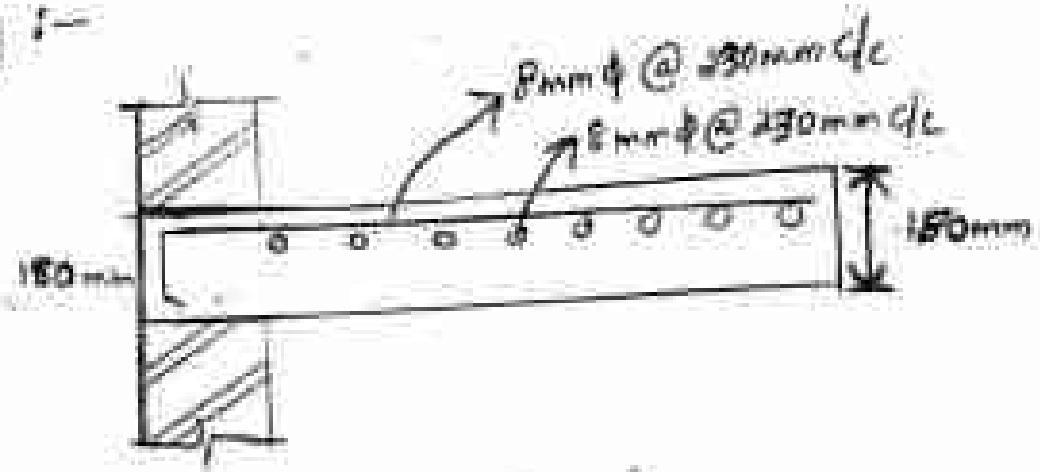
$$T_{c,max} = Q \cdot S \cdot N / \text{mod}$$

$$T_c < T_{c,max}$$

$$T_c < T_{c,k}$$

Hence Safe

Doubtless :-



### Continuous Slabs

The beams and slabs are cast monolithic in RC construction. The T-beams are spaced at regular intervals and the slab continuous over these supports.

Design Procedure:-

① Assume a depth of  $\frac{1}{30}$  of the span.

② Effective span =  $\frac{1}{12}$  of clear span

③ Find the design moment & shear force

$$M_{des} = \frac{w_d l^2}{10} - \frac{w_e l^2}{9}$$

$$V_{des} = (0.6 w_d + 0.6 w_e) l$$

Factored DL =  $\frac{1}{4} \times 30 = 7.5$  kN/m

④ Check for moment.

⑤ Check for shear.

(2)

$$\tau_{c,max} = 2.4 \text{ N/mm}^2$$

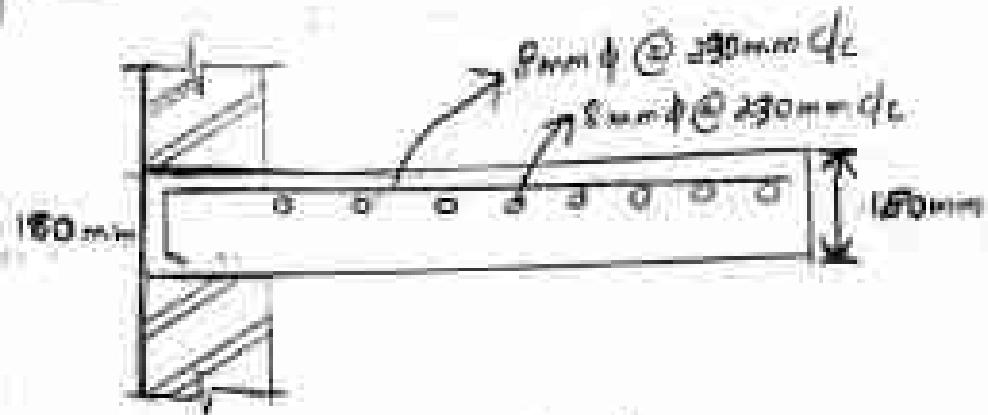
(IS 456 Table)

$$\tau_v < \tau_{c,max}$$

$$\tau_v < \tau_{c,k}$$

Hence Safe

Detailing :-



### Continuous Slabs

The beams and slabs are cast monolithic in RC construction. The T-beams are spaced at regular intervals and the slab is continuous over these supports.

Design Procedure:-

① Assume a depth of  $\frac{1}{30}$  of the span.

② Effective span =  $\frac{1}{12}$  of clear span [IS 456 Cl. 22.2 p. 11]

③ Find the design moment & shear force.

$$M_{max} = \frac{w_e l^2}{10} - \frac{w_e l^2}{9} \quad \left[ \text{Factored DL} \times \text{eff. Span} + \text{UDL} \right]$$

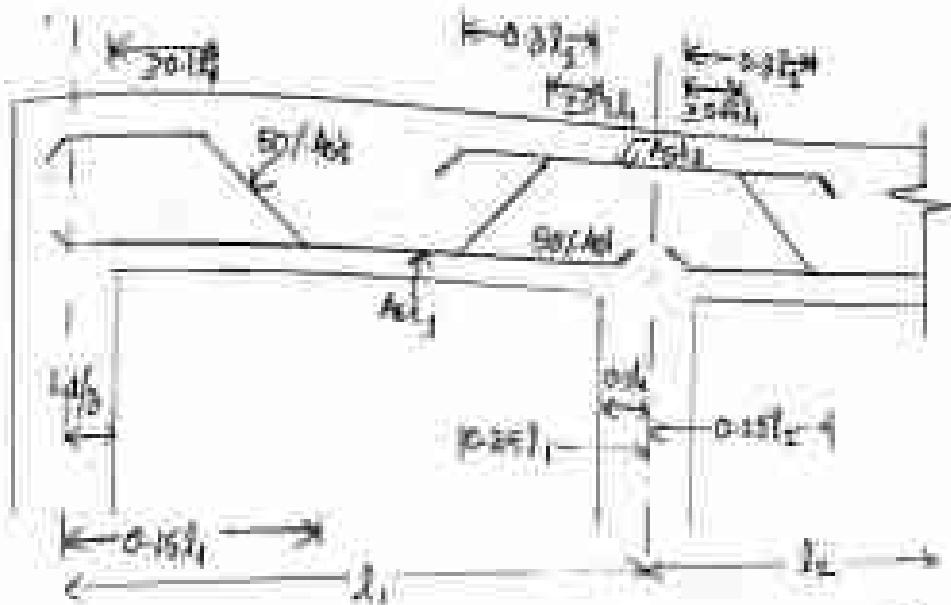
$$V_{max} = (0.6 w_d + 0.6 w_s) \left[ \text{Factored DL} \times \text{eff. Span} + \text{UDL} \right]$$

④ Check for moment

⑤ Check for shear

- i) Check for deflection.  
 ii) Design of distribution steel  
 iii) Reinforcement details.

✓



ONE CO-EFFICIENTS [IS-456 pg 36  
Table 12 part B]

Q12

The BM coefficients and SF coefficients for continuous slab is as shown below in the table.

BM Coefficients (Table 12)

Type of load	Span Moments		Support Moments	
	Near middle of end span	At middle of int. span	At support next to the end support	At other interior supports
Dead and imposed load (fixed)	$+Y_{12}$	$+Y_{16}$	$-Y_{10}$	$-Y_8$
Dead load fixed	$+Y_{10}$	$+Y_{12}$	$-Y_9$	$-Y_9$

## SF Coefficients (Table B)

Type of Load	At End Support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
Dry and Imposed load (fixed)	0.4	0.6	0.95	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

## MODULE - V

### Two Way Slabs - Limit State of Serviceability

Two way slabs are those slabs which are supported on all the four edges and having  $b_1/b_2 \leq 2.0$  in these types of slab bending moment is the same direction & longer direction as well as shear force in the both directions & longer direction [IS 456 ANNEX D].

There are two kinds of two way slabs:- [IS 456 ANNEX D]

1) Restrained slabs

2) Unrestrained slabs

#### Restrained Slabs

When a two way slab is loaded, the corners get lifted up. These corners can be prevented from lifting by providing fixity at the supports by beams or walls. Such type of slabs in which the corners are prevented from lifting are called as restrained slabs. In these slab special corner reinforcement is provided at the edges to prevent cracking of the corner. These are also called as slabs with corners held down.

The Indian standard specifies two ways of providing fixity in the case of restrained slabs.

- (i) The slab is simply supported on four edges by beams and the beams and slabs are cast monolithic (at the same time). Beams and slabs are cast monolithic (at the same time).
- (ii) Slab is supported on the edges by the four walls and there are proper structure walls also above the slabs.

#### Unrestrained Slabs

The slabs in which corners are not prevented from lifting are called as unrestrained slabs or slabs with corners not held down.

① BM coefficients : [IS 456 Table 26 and Table 27 pg 9]

$$\boxed{M_x = \alpha_x w d_x^2} \quad [\text{IS 456 pg 9}]$$
$$\boxed{M_y = \alpha_y w d_y^2} \quad \text{[Eq]} \quad \text{[Eq]}$$

Retained slabs  $\alpha_x, \alpha_y \rightarrow$  Table 26

Unretained slabs  $\alpha_x, \alpha_y \rightarrow$  Table 27

② Span to depth ratio [IS 456 Cl. 2.4.1 Pg 31]

For two way slab the depth is assumed  
the basis of span ratios based on deflection, concrete  
depth.

(Taking shorter spans into consideration).

For SS Slabs (upto 3.5m)

$$\frac{\text{Span}}{\text{depth}} = 35 \quad \begin{cases} \rightarrow \text{mild steel} \\ 35 \times 0.8 \end{cases}$$
$$\frac{\text{Span}}{\text{depth}} = 28 \quad \rightarrow \text{Fe 415, Fe 500}$$

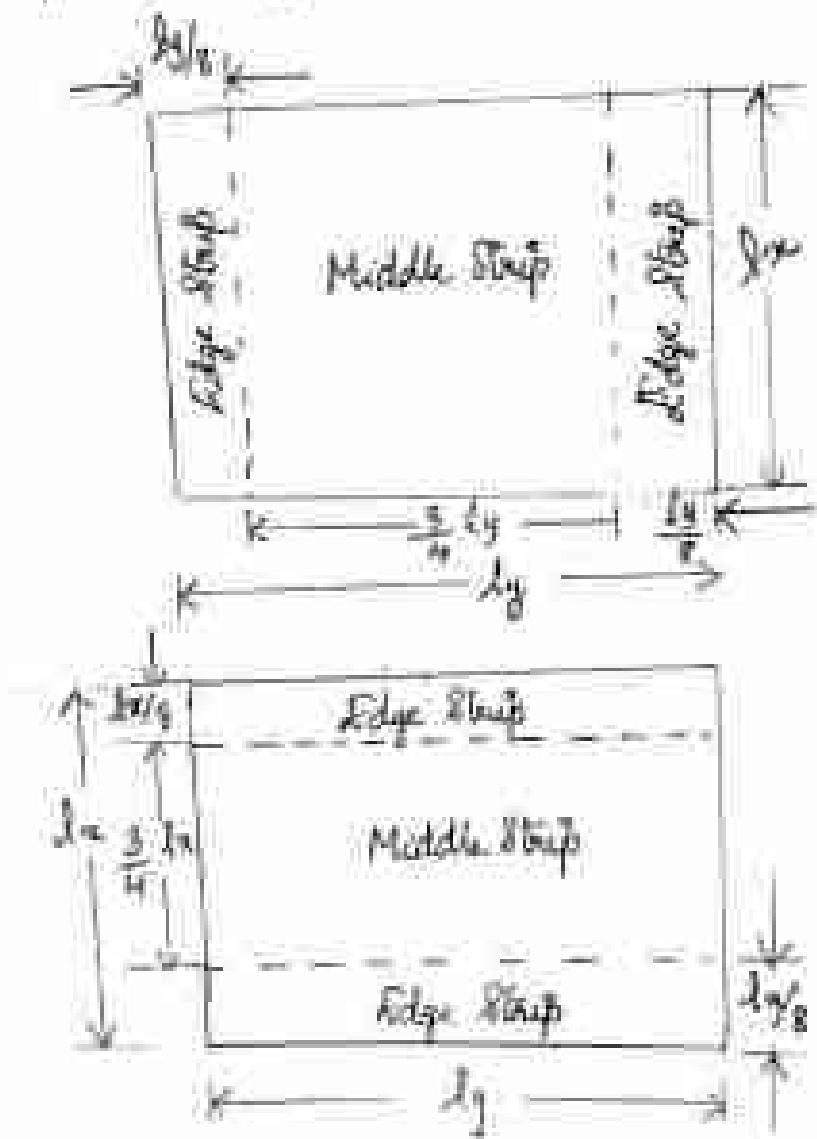
For continuous slabs (upto 3.5m)

$$\frac{\text{Span}}{\text{depth}} = 40 \quad \begin{cases} \rightarrow \text{mild steel} \\ 40 \times 0.8 \end{cases}$$
$$\frac{\text{Span}}{\text{depth}} = 32 \quad \rightarrow \text{Fe 415, Fe 500}$$

③

## Modelling of two way slabs

Slabs are considered as divided in each direction into middle strips and edge strips.



## Problems - Two Way Slab

Draw a reinforced concrete slab for a room of clear dimensions  $4m \times 5m$ . The slab is simply supported on walls of width 300mm. The slab is carrying a live load of  $4 kN/m^2$  and floor finish of  $2 kN/m^2$ . Use M<sub>20</sub> concrete and Fe415 steel. The corners of slab are held down. (restrained slab)

(b) Given Data :-

$$l_x = 4m$$

$$l_y = 5m$$

$$f_c = 4 \text{ kN/m}^2$$

$$F.F. \approx 3 \text{ kN/m}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\textcircled{1} \quad \frac{l_y}{l_x} = \frac{5}{4} = 1.25 < 2$$

Hence It is a two way slab.

$$\textcircled{2} \quad \frac{l_x}{d} = 25 \quad [\text{IS 456 U 24, Pg 57}]$$

$$\frac{4000}{d} = 25$$

$$d = \frac{4000}{25} = \underline{\underline{160 \text{ mm}}}$$

Assume overall depth = 180 mm

Assuming clear cover = 15 mm and  $\phi$  of m<sup>2</sup> bar = 8 mm.

$$d = 180 - 15 - 8/2$$

$$= \underline{\underline{161 \text{ mm}}}$$

Effective Span [ IS 456 Cl. 22.2 Pg 34 ]

Effective span in  $\text{L}_x$  direction is least of



i) c/c distance =  $4 + \frac{0.2}{8} + \frac{0.2}{8} = 4.3 \text{ m.}$

ii) Clear span + eff. depth. =  $4 + 0.161 = 4.161 \text{ m.}$

∴ Effective span in  $\text{L}_x$  direction,  $\text{L}_x = 4.161 \text{ m.}$

Effective span in  $\text{L}_y$  direction is least of



i) c/c distance =  $5 + \frac{0.2}{8} + \frac{0.2}{8} = 5.3 \text{ m.}$

ii) Clear span + eff. depth. =  $5 + 0.161 = 5.161 \text{ m.}$

∴ Effective span in  $\text{L}_y$  direction,  $\text{L}_y = 5.161 \text{ m.}$

④ Design load ( $w_u$ )

for slab,  $b = 1600 \text{ mm}$

Self wt. of slab = 25 kN

$$\text{Self wt. of slab} = 25 \times 1 \times 0.18 = 4.5 \text{ kN/m}$$

$$\text{Finish load} = 1 \text{ kN/m}^2 \times 2 \text{ m} = 2 \text{ kN/m}$$

$$\text{L.L.} = 4 \text{ kN/m}^2 \times 1 \text{ m} = 4 \text{ kN/m}$$

$$\text{T.L., } w = 4.5 + 4 = 8.5 \text{ kN/m}$$

$$\text{Factored load, } w_u = 1.5 \times 8.5 = 14.25 \text{ kN/m}$$

⑤ Design BM and shear

$$\frac{d_y}{L_x} = \frac{5}{4} = 1.25$$

Since the slab is supported on all 4 edges and corners ~~are held down~~  
[IS 456 Table 26 Pg 91]

$$d_x = 0.075$$

$d_y = 0.074$  [IS 456 Table 26 Pg 91]  
Four edges discontinuous.



$$1.2 \rightarrow 0.072$$

$$1.3 \rightarrow 0.079$$

$$d_y = 0.076 \quad 1.2 \quad 1.05 \quad 1.13$$

$$1.2 - 1.2 = \frac{1.25 - 1.2}{2 \times 0.072} = \frac{0.05}{0.144} = \frac{3.57}{14.4} = 0.25$$

$$1.3 - 1.3 = \frac{1.35 - 1.3}{2 \times 0.072} = \frac{0.05}{0.144} = \frac{3.57}{14.4} = 0.25$$

$$\frac{0.076 - 0.072}{2 \times 0.072} = \frac{0.04}{0.144} = \frac{2.78}{14.4} = 0.19$$

(5)

$$M_x = w_a l_a^2$$

$$M_y = w_y l_a l_z^2$$

$$M_x = 0.075 \times 14.25 \times 4.161^2 \\ = 18.5 \text{ kNm}$$

$$M_y = 0.056 \times 14.25 \times 4.161^2 \\ = 18.8 \text{ kNm}$$

$$\text{SF. } V_u = \frac{w_a l_a}{2} \\ = \frac{14.25 \times 4.161}{2} = 29.65 \text{ kN}$$

(6) Minimum depth reqd ( $d_{req}$ )

$$d_{req} = \sqrt{\frac{M_{u2}}{R_u b}} \quad R_u = 0.133 \text{ fck} \\ = 0.133 \times 20 \\ = 2.76 \text{ m} =$$

$$= \sqrt{\frac{18.9 \times 10^6}{2.76 \times 1000}}$$

$$d_{req} = 82 \text{ mm} < d_{prov} (161 \text{ mm})$$

Hence Safe

(7) Design of main reinforcement

(i) along shorter span in X direction (Concrete Tension)

width of  $a_t$  in this  
 $M_{u2} = 2.76 \text{ m}$ 

$$M_{u2} = 0.87 \text{ by fact } d \left[ 1 - \frac{b_y d t}{b_0 f_{ck}} \right]$$

$$= 2.76 \times 14.25 \times 10^6 = 0.17 \times 415 \times 2.76 \times 161 \left[ 1 - \frac{415 \times 161}{5000 \times 10} \right]$$

$$= 3.67 \text{ m} =$$

$$= 2.5 \times 10^6 = 56129 \text{ or } 401 \left[ 1 - \frac{5000 \times 10}{1250 \times 10} \right] \\ = 56129.05 \text{ kN} - 7.49 \text{ kN}$$

$$A_{st} = 7428.36 \text{ mm}^2$$

$$A_{st} = 332.58 \text{ mm}^2$$

∴ Take  $A_{st} = 332.58 \text{ mm}^2$

Area of bars,  $A_{sf} = \frac{\pi d^2}{4} = 56.3 \text{ mm}^2$

$$\text{Spacing} = \frac{1000 A_{sf}}{A_{st}} = \frac{1000 \times 56.3}{332}$$
$$= 159.1$$
$$\approx 150 \text{ mm}$$

[IS 456 Cl. 26.3.3. Pg 46]

\* Check for spacing  
spacing should be less than least of the following

i)  $3d = 3 \times 16 = 48 \text{ mm}$

ii)  $300 \text{ mm}$

∴ Spacing =  $150 \text{ mm} < 300 \text{ mm}$

Hence OK

[IS 456 Cl. 26.5.1. Pg 47]

\* Check for min.  $A_{st}$

min.  $A_{st} = 0.421 \text{ b.D}$

$$\therefore \frac{0.42 \times 1000 \times 50}{100} = 216 \text{ mm}^2$$

$$A_{st} \text{ provided} = \frac{1000 \times A_{sf}}{\text{spacing}} = \frac{1000 \times 56.3}{150}$$
$$= 335 \text{ mm}^2$$

$A_{st} \text{ provided} > A_{st, \text{min}}$

Hence Suf.

∴ Provide 8 mm of @ 150 mm c/c

in the middle strip of width 3.87 m.

- Q) (i) Along longer span in y direction  
 width of middle strip =  $\frac{3}{4} \times 2.15 = 3.125$   
 Effective depth along y direction.



$d_s = 153 - \frac{3}{2} - \frac{3}{2} = 153 \text{ mm}$  Reinforcement along longer span

$A_{sy} = 0.87 \times 153 \times 2.15 \left( 1 - \frac{3.125}{153} \right)$

$(0.87 \times 10^6) = 0.87 \times 153 \times 2.15 \times 153 \left[ 1 - \frac{4.5}{153} \right]$

$= 55240.65 \text{ As}_y$

$13.87 \times 10^6 = 55240.65 \text{ As}_y = 7.49 \text{ As}_y$

$$As_y = 7116.15 \text{ mm}^2$$

$$As_t = 259.07 \text{ mm}^2$$

$\therefore$  Take  $As_t = 259.07 \text{ mm}^2 > As_{t\max} (215 \text{ mm}^2)$

Using 8 mm  $\phi$  bars

$$As_f = \frac{\pi}{4} \times 8^2 = 50.3 \text{ mm}^2$$

$$\therefore \text{Spacing} = \frac{1000 As_f}{As_t} = \frac{1000 \times 50.3}{259.07} = 194 \text{ mm}$$

\* Check for spacing.

Spacing should be the less than least of following

i)  $3d = 3 \times 153 = 459 \text{ mm}$

ii) 300mm.

$$\therefore \text{Spacing} = 194 \text{ mm} < 300 \text{ mm}$$

Hence OK

\* Check for max. bar diameter (Is 456 Cl 26.6)

$$\text{bar } \phi \neq \frac{1}{5} D$$

$$8 \neq \frac{1}{5} \times 150$$

$$8 \neq 30 \text{ mm}$$

Hence OK

\* Provide 8 mm Ø bars at 190 mm c/c along 'y' direction in the middle strip of width 3.12 m. ⑨

(i) Reinforcement in edge strip. [IS 456 Cl 26.5.2.1 pg 48]

$$A_{st,min} = 0.12 \cdot 1.60$$

$$= \frac{0.12 \times 1000 \times 190}{100} = \underline{\underline{216 \text{ mm}^2}}$$

Using 8 mm Ø bars.

$$\text{Spacing} = \frac{1000 A_{st}}{A_{st}} = \frac{1000 \times 2013}{216} \\ = 232 \approx \underline{\underline{230 \text{ mm}}}$$

\* Check for spacing. [IS 456 Cl 26.5.3 pg 46]

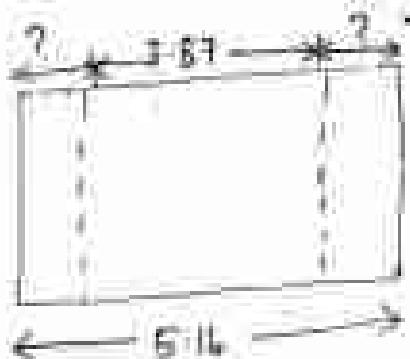
spacing should be less than least of following.

$$0.5 d = 5 \times 16 = 80 \text{ mm}$$

(ii) 450 mm.

$\therefore$  Spacing = 230 mm < 450 mm

Hence OK



$$\frac{5.11 - 3.67}{2} = 0.645 \text{ m} \quad \text{along x direction}$$



$$\frac{4.16 - 3.67}{2} = 0.52 \text{ m} \quad \text{along y direction}$$

\* Provide 8 mm Ø bars @ 230 mm c/c in the edge strip of width 0.645 m along x direction and edge strip of width 0.52 m along y direction.

⑧ Check for shear

$$V_u = \frac{W_s I_s}{2} = \frac{14.25 \times 4.45}{2} = 31.45 \text{ kN}$$

Normalized Shear stress,  $\tau_v = \frac{V_u}{bdy} = \frac{29.65 \times 10^3}{1000 \times 153} = 0.193 \text{ N/mm}^2$

[IS 456 Cl. 4.8]

$d \rightarrow$  eff depth along ly direction  
+ Design shear strength concrete [IS 456 Cl. 40.2 pg 71]  
 $\phi_s \phi_f$  steel, pt =  $\frac{\text{load at prov}}{\text{load}}$

$$\text{At prov. } \frac{1000 \times \frac{\pi}{4} \times 8^2}{150} = 335 \text{ mm}$$

$$\text{pt} = \frac{1000 \times 335}{1000 \times 161} = 0.208 \%$$

Design shear strength,  $\tau_c = 0.33 \text{ N/mm}^2$

For slab, design shear strength =  $\tau_c K$   
=  $0.33 \times 1.2$   
=  $0.396 \text{ N/mm}^2$

Max. shear stress,  $\tau_{\text{max}} = 2.1 \text{ N/mm}^2$

[IS 456 Table 2]

$$\tau_v < \tau_{\text{max}}$$

$$\tau_v < \tau_c K$$

Hence shear reinforcement is

⑨ Check for deflection [IS 456 Cl. 23.6]

$$\text{pt} = 0.208 \approx 0.21\%$$

$$f_b = 0.55 \cdot 64 \left[ \frac{A_t + 1.6 A_s}{A_t + p A_s} \right]$$

for  $f_s = 240 \text{ N/mm}^2$ ,  $\rho_t = 0.21\%$

(1)

Modification factor,  $k_t = 1.6$

$$(V_d)_{\max} = 20 \times 1.6 = 32.$$

$$(V_d)_{\text{prov}} = \frac{4160}{161} = 25.8$$

$$(V_d)_{\max} > (V_d)_{\text{prov}}$$

Hence Safe

Tensional Reinforcement at corners

Since the slab is restrained, there is a need to provide additional reinforcement at the corners.

$$\text{Mesh Size} = \frac{l_y}{6} = \frac{41.61}{5} \approx 8.32 \text{ m}$$

$$\begin{aligned}\text{Area of tensional reinforcement} &= \frac{3}{4} \times 46.1 \text{ max.} \\ &= \frac{3}{4} \times 33.5 \\ &= 251.25 \text{ mm}^2\end{aligned}$$

Using 8mm Ø bars.

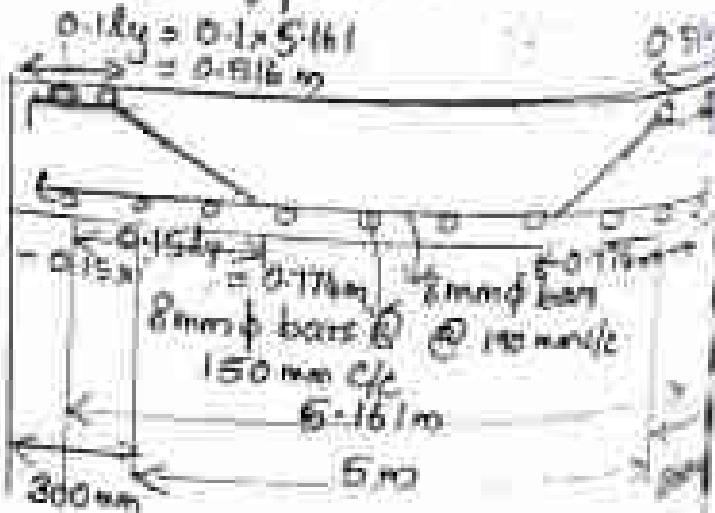
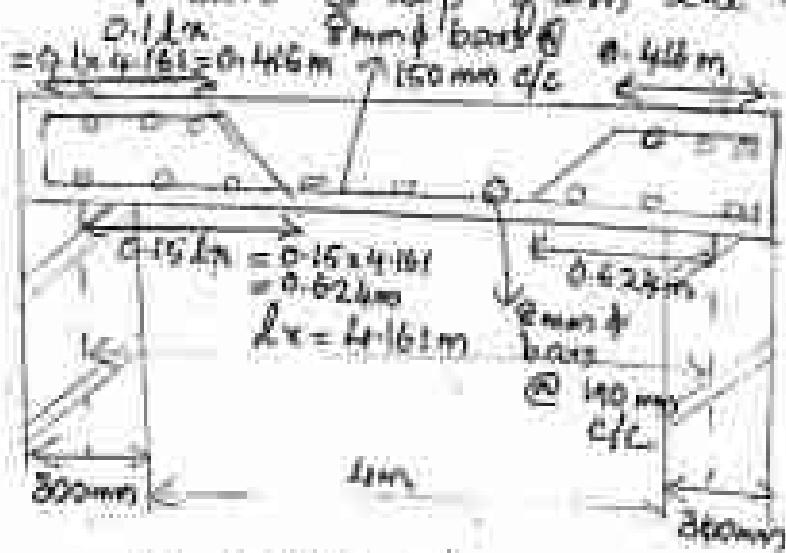
$$A_\phi = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4} = 50.3 \text{ mm}^2$$

$$\text{Spacing} \rightarrow \frac{1600 \times A_\phi}{A_t} = \frac{1600 \times 50.3}{251.25} = 200.17 \approx 200 \text{ mm}$$

Provide : 8mm mesh of bars at 200mm c/c  
in a mesh of size 840 mm at the corners

## (ii) Arrangement of reinforcement.

- a) Bending half of the main bars at a distance of  $0.15L_y$  from Centre of support along  $x$  direction and available length of bars @ top from centre of support
- b) Bending half of the main bars at a distance of  $0.15L_y$  from the Centre of Support along  $y$  direction and available length of bars @ top from the centre of support



a) Section along  
short span



b) Section along long span



## Limit State of Sustainability

(13)

One of the very important aspect of the design of any structure is its sustainability. If a structure is strong enough to support the loads but shows excessive deflection and cracking leading to undesirable conditions, then it becomes unsustainable. Therefore it is very important to satisfy the limit state of sustainability. Limit state of sustainability ensures that there should not be any excessive deflection and cracking at service loads, thereby the definition of the structure. The limit state of sustainability (deflection and cracking) is applicable at service loads, while limit state of collapse which is applicable at ultimate loads. Therefore, limit state of sustainability is determined on the basis of safety analysis or working stress method.

Limit state of sustainability consist of the following limit states:-

- (i) limit state of deflection [IS 456 Annexure page 22]
- (ii) limit state of cracking [IS 456 Annexure page 45]
- (iii) limit state of vibration.

### Limit State of deflection

This limit state ensures that the deflection of a structure will not adversely affect the appearance or efficiency of the structure or finishes or fixtures. In any structure, the deflection must be kept constant.

- (i) short term deflection
- (ii) long term deflection

It is essential to consider both the short term and long term deflection to calculate total deflection.

Short term deflections are elastic deformations that any structure undergoes immediately after it is loaded.

- (b) → long term deflections occur over a long period and are mainly due to shrinkage and creep.  
 → long term deflections are due to basic causes like short term deflections.

[25.456] Article C

### Short Term Deflection ( $\Delta_s$ )

Short term deflection is calculated using short modulus of elasticity of concrete ( $E_s$ ) and an effective moment of inertia ( $I_{eff}$ ) given by the following eqn:

$$I_{eff} = \frac{I_g}{\left(1 - \frac{M_c}{M_g} \frac{x}{d}\right) \frac{b}{b}}$$

Where,

$I_g$  = moment of inertia of cracked section.

$M_c$  = cracking moment, equal to  $\frac{f_c}{f_y} I_{gr}$  where

$I_{gr}$  = the modulus of rupture of concrete,  $I_{gr} = \frac{M_g}{x}$  the moment of inertia of the gross section about centroidal axis, neglecting the reinforcement, and the distance from centroidal axis of gross, neglecting the reinforcement. As extreme fiber is tension.

$M_g$  = Max moment under service load

$x$  = lever arm.

$x$  = depth of M.H

$d$  = effective depth

$b_c$  = breadth of web

$b$  = breadth of compression face

For continuous beams, deflection shall be calculated using the values of  $I_g$ ,  $I_{gr}$  and  $M_g$  modified by the following

$$\chi_a = k_1 \left[ \frac{x_1 + x_2}{x} \right] \times (1 - k_1) \chi_0$$

$X_c = \text{modified value of } X$

$X_1, X_2 = \text{values of } X \text{ at the supports}$

$X = \text{value of } X \text{ at mid span}$

$k_i = \text{coefficient given in Table 25}$

$X = \text{value of } L, T_g, \text{ or } M_g \text{ as appropriate}$

Table 25 Value of Code coefficients,  $k_i$

$k_i$	0.0000	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
$X_1$	0	0.03	0.08	0.12	0.18	0.22	0.26	0.31	0.37	0.44

$$k_i \text{ is given by } k_i = \frac{M_1 + M_2}{M_{12} + M_{21}}$$

where,  $M_1, M_2 = \text{support moments}$

$M_{12}, M_{21} = \text{fixed end moments}$

### $T_g$ Long Term Deflection

Long term deflection is the sum of deflection due to shrinkage and deflection due to creep.

#### Deflection due to shrinkage ( $\Delta_s$ )

Deflection due to shrinkage occurs over a long period of time and depends upon the environmental conditions (humidity and temperature) at the time of curing. It also depends on many other factors such as type of concrete, shape and size of element, ratio of concrete to aggregate, shape and size of aggregates etc. The deflection caused by shrinkage ( $\Delta_s$ ) is correlated in accordance with equation C of IS 456 pg 22

$$\Delta_s = k_s t_{ex}^2$$

where  $t_{ex} = \text{age depending upon the support conditions}$

- 0.5 - continuous  
 0.125 - 23 members at one end.  
 0.085 - members continuous at one end.  
 0.065 - fully continuous members.  
 $\psi_{cs} \rightarrow$  charge capacity equal to  $\frac{E_{cs}}{D}$   
 $\psi_{cs}$   $\rightarrow$  ultimate strength strain of concrete.

When  $\frac{E_{cs}}{D} = \psi_{cs}$  ultimate strength strain of concrete.

$$K_u = 0.72 > \frac{P_u - P_c}{\sqrt{P_c}} \leq 1.0 \text{ for } 0.15 \leq \frac{E_{cs}}{E_{c,s}} \leq 0.25$$

$$= 0.65 \times \frac{P_u - P_c}{\sqrt{P_c}} \leq 1.0 \text{ for } P_u - P_c \geq 1.0$$

where

$$P_c = \frac{100 A_s l}{b d} \text{ and } P_u = \frac{100 A_s l c}{b d}$$

and  $D$  is the total depth of the section and  $l$  is the length of span.

### ③ Deflection due to creep ( $\Delta_c$ ) (IS 456 ANNEX C page 86)

Crep is the long term deformation under sustained stress which depends upon the magnitude of the sustained or permanent load.

The deflection due to permanent loads are (perm) may be obtained from the following equation:

where

$$\Delta_{c,spans} = \alpha_{c,spans} (\text{perm}) - \alpha_c (\text{perm})$$

$\alpha_{c,spans}$  = Critical plus creep deflection of permanent loads obtained using elastic analysis with an effective modulus of elasticity.

$\alpha_c (\text{perm})$  = short term deflection due to permanent load using  $E_c$

$$E_c = \frac{E_c}{1 + \theta}; \theta \text{ being creep coefficient}$$

$$E_c = 5000 \sqrt{f_{ck}} \text{ N/mm}^2$$

	Age of concrete	Coefficient
1	7 days	2.2
2	28 days	1.0
3	1 year	1.1

$$\text{Total deflection} = \Delta_e + \Delta_s + \Delta_c$$

Limit State of Cracking short term deflection long term deflection

Cracking is a very common phenomenon observed in reinforced concrete structures the tensile strength of concrete is very low, so it is liable to crack even under very small loads. Cracking not only affects the appearance, but also the effect of durability of the structure. It may result in loss of strength, thus affecting the safety of the structure. The cracking of the concrete should not adversely affect the appearance and durability of the structure. IS 456 2000 has prescribed the acceptable limits of the crack under working loads. These vary with the type of structure and the environmental conditions of the place. If the crack width falls within these limits, then it may be safely assumed that the cracks will not affect the durability of the structure. Various acceptable limit of crack width as per IS 456 (19-25 pg 67)

The surface crack widths should not be more than 0.2mm in members where cracking is not harmful and has no adverse effect either on the durability of the structure or on the environment.

up coefficient in members, which are exposed continuously to moisture or harmful weather or are in contact of soil or ground water, crack width should not exceed 0.2mm.

⑧ (ii) For structures in very aggressive environment (such as those listed in Table 3 of IS 456:2000) the surface crack width should not exceed 0.1mm.

In addition to the above, IS 456:2000 also specifies limit state of cracking for flexural members and compression members in Cl. 43; pg 76 as explained below:

③ For flexural members [IS 456 Cl 43.1 pg 76]

The spacing requirements of reinforcement to be as per Cl. 24.3.2 pg 45.

The following shall apply in spacing of bars:

a) The horizontal distance b/w two bars of main reinforcement shall be less than the greater of the following:

i) dia of bar, if dia are equal.

ii) dia of larger bar, if dia are unequal.

iii) 5mm more than nominal max. size of CA.

b) Greater horizontal distance than specified above in a) shall be provided whenever possible.

When needle vibrators are used, the horizontal distance b/w bars of a group may be reduced to  $\frac{2}{3}$ rd of nominal max. size of bars provided sufficient space is left b/w group of bars to enable the vibrators to be immersed.

c) Where 2 or more rows of bars are there, the bars shall vertically in line and min vertical distance b/w the bars shall be greater of the following:

→ 15 mm

→  $\frac{2}{3}$  rd nominal max. size of CA.

→  $\frac{2}{3}$  rd max. size of bars

for compression members [IS 456 Cl. 43.2 pg 76] (9)

Cracking in a compression member subjected to design axial load more than 0.2 fact  $A_c$  need not be checked where  $A_c$  is the gross cross area of the member.

A compression member subjected to tensile loads than 0.2 fact may be considered as flermal member for the purpose of crack control as per Cl. 26.3.2, pg 45. (min. distance b/w individual bars)

Calculation of crack widths [IS 456 ANNEX Pg 95]

The design crack width can be calculated by following equation, provided the strain in the tensile reinforcement is limited to  $0.8 \delta_{y}/E_s$ , where  $E_s$  is the modulus of elasticity of steel and it taken as  $2 \times 10^5 \text{ N/mm}^2$ .

Design crack width,

$$w_{cr} = \frac{3a_{cr} E_m}{1 + 2(C_{cr} - C_{min})}$$

where  $w_{cr}$  is the design surface crack width.

$a_{cr}$  is the distance of the point under consideration to the surface of the nearest longitudinal bar.

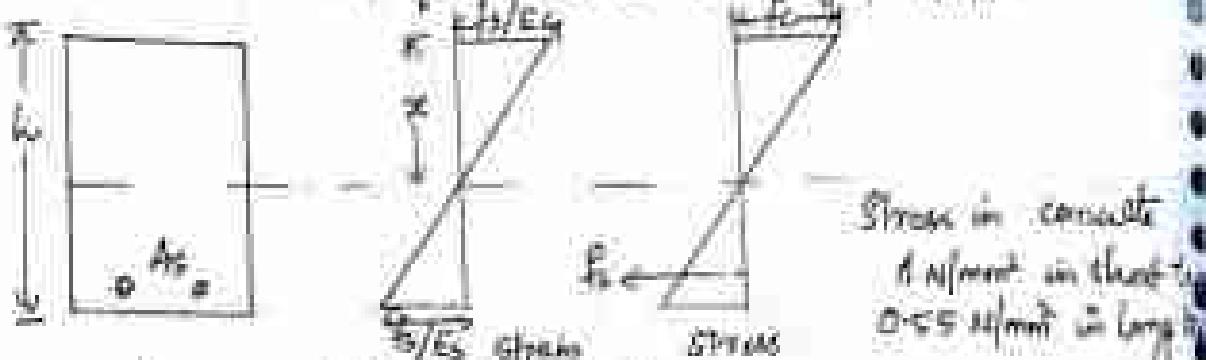
$C_{min}$  is the min. cover to the longitudinal bar.

$h$  = Overall depth of the member

$z$  = depth of N.T.

$E_m$  = avg. steel strain at the point under consideration

- (2) The avg. steel strain  $\epsilon_m$  may be calculated on the basis of following assumptions -
- \* The concrete and steel are both considered to be fully elastic tension and compression.
  - \* The elastic modulus of steel may be taken as  $200 \times 10^3 \text{ N/mm}^2$ .
  - The elastic modulus of concrete is  $5800 \sqrt{f_c} \text{ N/mm}^2$ .



Stress in concrete  
A/mm<sup>2</sup> in that layer  
 $0.55 \text{ N/mm}^2$  in long

$b$  = overall depth of section

$x$  = depth from the compression face to the NA

$f_c$  = max. compressive stress in concrete

$f_t$  = tensile stress in reinforcement

$E_s$  = modulus of elasticity of reinforcement

$$\epsilon_m = \epsilon_s - \frac{b(h-x)(a-x)}{3E_s h(d-x)} \quad (\text{for rectangular section})$$

where

$A_s$  = area of tension reinforcement

$b$  = width of the section at the centroid of the tension steel

$\epsilon_s$  = strain at the level considered, calculated ignoring the stiffening of the concrete in the tension zone

$a$  = distance from compression face to the point where the crack is being calculated

$d$  = effective depth

Problems - Crack width.

(2)

- 1) simply supported beam 500mm x 100mm having a span of 6m reinforced with 5 # 25 mm diameter bars on the tension side, calculate the design surface crack width at the following locations  
 Max concrete and Fe 450 steel is used. BM of section = 300x10 Nm
- At the bottom corner on the tension face
  - Under the middle bar on the tension face
  - At any point midway b/w 2 bars on the tension face.
- Given data :-

$$b = 500 \text{ mm}$$

$$D = h = 100 \text{ mm}$$

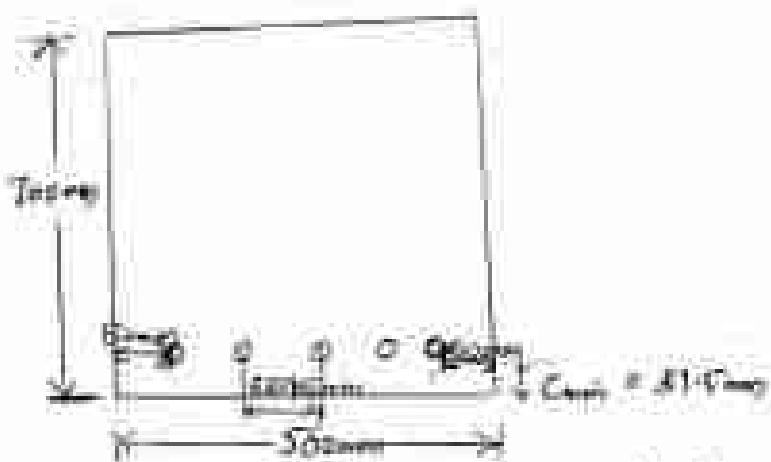
$$l = 6 \text{ m}$$

$$f_{ct} = \frac{2M}{bh^2} > 25 = 24.57 \text{ N/mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

- ① Arrangement of reinforcing bars in the beam.



Assume 50mm clear cover on each side.

$$\text{Clear spacing b/w bars} = \frac{500 - (50 \times 2) - (25 \times 2)}{4} \\ = 38.15 \text{ mm}$$

$$C_{min} = \text{clear cover} - \phi f_2 = \frac{50 - 25}{2} = \underline{\underline{37.5 \text{ mm}}}$$

(ii)

### ② Location of NA

Max. allowable stress in concrete

$$\frac{b z^2}{2} = m A c t [d - z]$$

$$d = 700 - \left( \frac{200}{50} \right) 50 \\ = 680 \text{ mm} \\ = 650 \text{ mm}$$

Grade of concrete

$m = \text{modular ratio}$

	$\sigma_{c,c}$
M <sub>45</sub>	5
M <sub>25</sub>	7
M <sub>35</sub>	8.5
M <sub>50</sub>	10

$$\frac{200}{3 \times 7} = \frac{200}{21} = 10$$

$$500 \times \frac{z^2}{2} = 10 \times 2455 \pi [6335 - z]$$

228.22 mm

$$z = 228.60 \text{ mm}$$

—

### ③ Moment of inertia of cracked section

$$I_{cr} = \frac{b z^2}{2} + m A c t (d - z)^2$$

$$= \frac{500 \times 228.60^2}{2} + 10 \times 2455 (500 - 228.60)^2$$

$$= 3.5 \times 10^9 \text{ mm}^4 = 3.51 \times 10^9 \text{ mm}^4$$

$$W_{cr} = \frac{\sigma_{c,c} E_m}{1 + 2(\alpha_{cr} - \alpha_m)}$$

IS 456

Annexure

pg 45

### ④ Design of crack width

i) Design surface crack width at bottom course on the tension face (pt A).

$$\alpha_{cr} = \sqrt{50^2 + 31.5^2}$$

$$\alpha_{cr} = 62.5 \text{ mm}$$

Strain calculation at point A

$$\frac{\text{Stress}}{\text{Strain}} = E_c$$

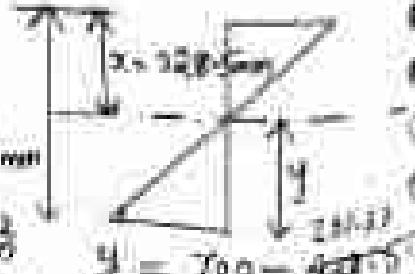


$$\frac{M}{I} = \frac{F}{x} = \frac{E}{\epsilon}$$

$$\text{Strain at point A (}\epsilon_A\text{)} = \frac{f_c}{E_c} = \frac{M \cdot y}{I_{cr}}$$

$$E_c = 5000 \sqrt{f_{ck}} = 5000 \times \sqrt{20} = 20360.6 \text{ N/mm}^2$$

$$(E_c \text{ is } 456 \text{ GPa pg 6}) = 20360.6 \text{ N/mm}^2$$



$$\epsilon_1 = \frac{M_y/I_{cr}}{E_c} = \frac{\frac{300 \times 10^3 \times 471.5}{2.31 \times 10^3 \times 3.64 \times 10^9}}{22260.6}$$

$$= 0.016$$

$$= 0.65 \times 10^{-4}$$

Max strain at the point of consideration ( $\epsilon_m$ ) [IS 456 ANNEX P Pg 95]

$$\epsilon_m = \epsilon_1 - \frac{b(1-2)(a-z)}{3E_s A_s(1-\alpha)} \quad [a = \frac{D_s h}{700} = \frac{0.016}{700}]$$

$$= \frac{0.016}{3 \times 210 \times 2455} \frac{22260.6 (100 - \frac{22.5}{22.5})}{500 \times 10^{-4}} \times 37.5$$

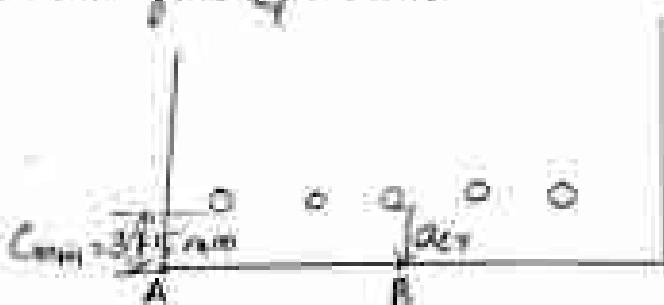
$$\epsilon_m = \frac{0.016}{0.016} = 1$$

Design crack width at A,

$$w_{cr} = \frac{3c_{cr} \epsilon_m}{1 + 2(c_{cr} - c_{min})} \quad [IS 456 ANNEX P Pg 95]$$

$$= \frac{3 \times 62.5 \times \frac{0.016}{100}}{1 + 2(62.5 - 37.5)} = \frac{2.67 \text{ mm}}{100 - 22.5 = 77.5} = 0.415 \text{ mm}$$

Design surface crack width under the middle bar on the tension face (Point B)



$$c_{cr} + c_{min} = 37.5 \text{ mm}$$

Strain at B is same as that at A,

$$\epsilon_m = 0.8 \times 10^{-4} \times 0.016$$

24)

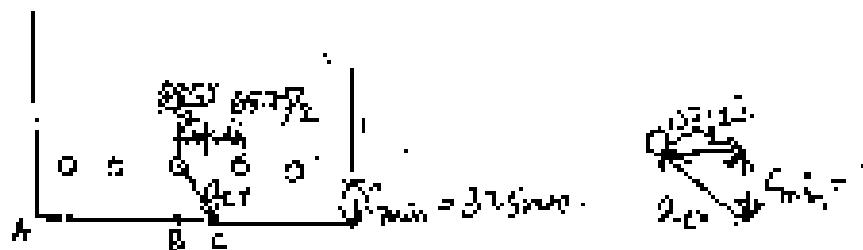
Design surface crack width at B.

[IS 456

ANNEX F Pg 95]

$$W_{cr} = \frac{3 \alpha_{cr} \varepsilon_m}{1 + 2(\alpha_{cr} - \alpha_{min})} = \frac{3 \times 31.6 \times 6.87 \times 10^{-4}}{1 + 2 \left( \frac{37.5 - 37.5}{700 - 236.7} \right)} \\ = 0.0765 \text{ mm}$$

(ii) Design surface crack width at point C mid way between two bars on the tension face (Pt C).



$$\alpha_{cr} = \sqrt{\left(\frac{67.5}{2}\right)^2 + \left(\frac{37.5}{2}\right)^2} = 50.87 \text{ mm}$$

Avg strain at C is same as that at A and B

$$\varepsilon_m = 6.87 \times 10^{-4}$$

Design surface crack width at C,

$$W_{cr} = \frac{3 \alpha_{cr} \varepsilon_m}{1 + 2(\alpha_{cr} - \alpha_{min})} \quad [IS 456 ANNEX \\ Pg 95]$$

$$= \frac{3 \times 50.87 \times 6.87 \times 10^{-4}}{1 + 2 \left( \frac{50.87 - 37.5}{700 - 236.7} \right)} \\ = 0.0845 \text{ mm}$$

- columns - introduction - classification - effective length
- short column - long column - reinforcement - IS specification
- limit state of collapse & compression - design of axially loaded short columns - design examples with rectangular ties and lateral reinforcement
- short columns - types - proportioning - loads - distribution of loads
- axial tensions - design and detailing of dog legged flans
- concepts of Head - Ritter type stain (detailing only).

### COLUMNS

Columns are an important element of every reinforced concrete structures there are used to transfer the load of superstructure to the foundation safely. Mainly columns, piers and pedestals are used as compression members in buildings, bridges, supporting systems of tanks, factories and many more structures.

A column is defined as a vertical compression member which is mainly subjected to axial load and the effective length of which exceeds three times at least lateral dimension.

The compression member whose effective length is less than three times its least lateral dimension is called as Pedestal. The compression member which is subjected to horizontal and is subjected to axial loads is called as short. Piers are used in houses.

## ② Effective length of the column.

[IS 456 ANNEX E : Pg 42]

The effective length of a column is defined as the length of the column which the part is buckling under the action of loads. This is also defined as the length till the point of contraflexure (Change of moment sign or point of zero moment) of the buckled column.

The unsupported length of a column is the clear height between the floor and the lower level of the ceiling. Under the action of loads the column buckles or deflects as shown in Fig.



The deflected shape depends upon the type of end supports or degree of end restraint the design of column is done on the basis of effective length. IS 456-2000 gives various values of effective length for various end conditions in terms of 25 unsupported lengths. [Table 28 - Page 44]

The table 28 (Pg 44 IS 456) shows theoretical recommended value of effective length. The end conditions are ideal. Theoretical value corresponds to ideal conditions recommended values give the practical aspect.

## Slenderness Ratio.

[IS 456 & 2513 Pg 41]

The slenderness ratio of a compression member is defined as the ratio of eff length to the least lateral dimension.

$$\text{Slenderness ratio} = \frac{\text{Effective length}}{\text{least lateral dimension}}$$

Columns are classified as following two types  
depending upon the slenderness ratio:

- (i) Short Columns
- (ii) Long Columns.

### Short Column

The column is considered as short when the slenderness ratio of column i.e. ratio of effective length to its least lateral dimension ( $\frac{l_{eff}}{b}$ ) is less than or equal to 12.

$$\frac{l_{eff}}{b} \leq 12$$

### Long Column

If slenderness ratio of the column is greater than 12, it is called as long or slender column.

$$\frac{l_{eff}}{b} > 12$$

### Classification of columns

Columns are classified based on different criteria:

Shape of cf

Material of construction

Type of loading

Slenderness ratio

Type of lateral reinforcement

### Shape of cf

With the basis of the cf of the column, the column can be classified as following:

- (1) Square      
- (2) Rectangular      
- (3) Circular      
- (4) Pentagonal      
- (5) Hexagonal      
- (6) Octagonal      
- (7) T. Shape or L shape       

### Material of construction

Columns may be classified as following as per the materials used for construction.

#### 1) Timber columns

- \* Used for light loads
- \* Used in small houses and wooden houses
- \* These are called as posts

#### 2) Masonry columns

- \* Used for light loads

#### 3) RCC columns

- \* Used for all types of buildings and other RCC structures like tanks, bridges etc.

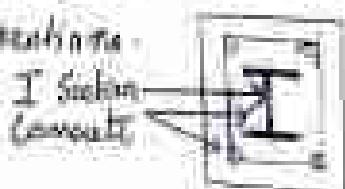
#### 4) Steel columns

- \* Used for heavy loads

#### 5) Composite Columns

- \* Used for heavy loads

- \* Consist of steel sections like joists (I or H sections) embedded in RCC sections.



## Type of loading

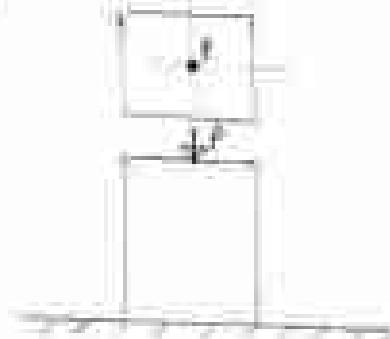
(5)

- Column may be classified as follows, based on type of loading
- axially loaded columns
- Eccentrically loaded columns

### Axially loaded Columns

The columns which are subjected to loads acting along the longitudinal axis or centroid of the column section are called axially loaded columns.

Axially loaded column is subjected to direct compressive stress only and no bending stress develops irrespective of the column section.

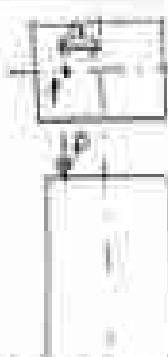


### Eccentrically loaded columns

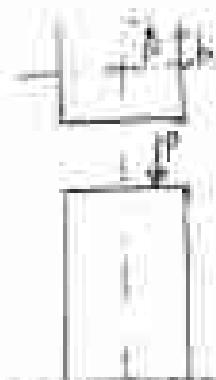
Eccentrically loaded columns are those columns in which the loads do not act on the longitudinal axis of the column they are subjected to direct compressive stress & bending stresses.

Eccentrically loaded columns may be subjected to lateral bending or axial bending depending upon the line of action of load with respect to the two axes of the column section.

(5)



Eccentrically loaded  
column (uniaxial  
bending)



Eccentrically load  
column (biaxial  
bending)

### Slenderness Ratio.

① Short Column

$$\frac{L_{eff}}{b} \leq 12$$

② Long Column

$$\frac{L_{eff}}{b} > 12$$

Difference b/w short column and long column

Short Column

① SR =  $\frac{L_{eff}}{b} \leq 12$

②  $\frac{L_{eff}}{R_{crit}} = \frac{L_{eff}}{R_{crit}} \leq 40$   
lost radius of gyration

③ Buckling tendency is  
very low

⑤ The load carrying capacity is  
high as compared to long  
columns of the same size

⑥ Failure is by crushing

Long Column

① SR =  $\frac{L_{eff}}{b} > 12$

②  $\frac{L_{eff}}{R_{crit}} > 40$

③ Long and slender columns  
buckle easily

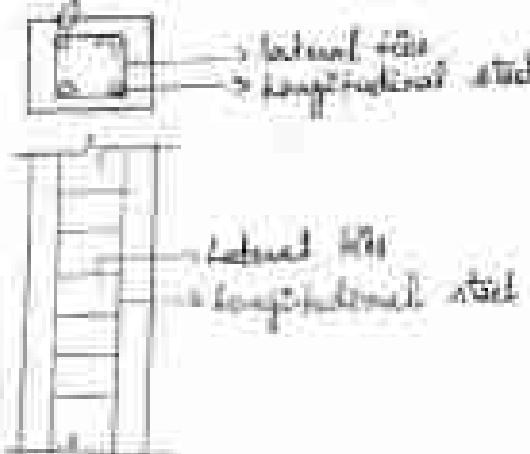
④ The load carrying capacity  
of the long column is less  
as compared to short column  
of the same size

⑤ Failure is generally by  
buckling

## Type of Lateral Reinforcement

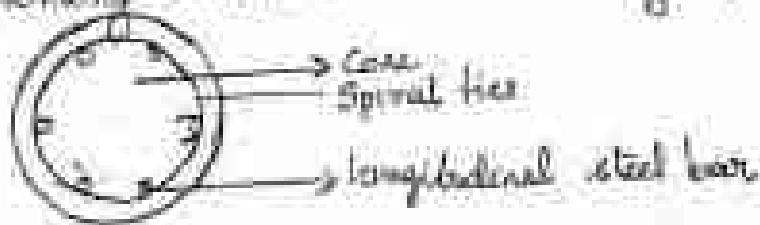
- As the column has longitudinal and lateral reinforcement, they can also be classified according to the manner in which the longitudinal steel is laterally supported or tied.
- Column with longitudinal steel and lateral ties.
  - Column with longitudinal steel and spiral ties.
  - Column with longitudinal steel and lateral ties.

In this type of arrangement, the longitudinal bars are tied laterally at suitable intervals with the help of lateral ties.



## Column with longitudinal steel and spiral/helical ties.

In this type of arrangement, the longitudinal bars tied continuously with the help of spiral reinforcement. Columns with helical or spiral reinforcement are better in giving lateral support to bars as compared to links ties because the buckling resistance and ductility of the column.



## Reinforcement in a Column

Concrete is strong in compression. Thus a column can be made up of plain concrete but it is always desirable to use R.C.C. columns instead of plain concrete columns because of following reasons:

- 1) A plain concrete column requires very large area as compared to R.C.C. columns. For a particular load, the cross area of an R.C.C. column will be much thinner than that of plain concrete. Thus by using R.C.C. columns a lot of space can be saved as the size of column will be less.
- 2) A minimum area of steel is always provided in the column whether it is required for carrying load or not. It is done to resist tensile stresses which may be caused due to eccentricity of loads (it is practically impossible to have a perfectly axially loaded structure).

There are two types of reinforcement provided in R.C.C. columns :-

- 1) Longitudinal Reinforcement
- 2) Transverse Reinforcement

### Longitudinal Reinforcement

The longitudinal reinforcement consists of steel bars placed longitudinally in a column. It is also called as main steel. The functions of longitudinal reinforcement are as follows:-

- 1) To share the compressive loads along with concrete, thus reducing the overall size of the column and saving more space.

and tensile stresses developed due to any moment (J) accidental eccentricity.

To impact directly to the column.  
To reduce the effect of creep and shrinkage to continuous  
without loading applied for a long time.

### Transverse Reinforcement

Transverse reinforcement provided along the lateral direction  
to the column in the form of ties or spirals enclosing  
main steel. The function of transverse steel are as  
following :

- 1) To hold the longitudinal bars in position.
- 2) To prevent buckling of the main longitudinal bars.
- 3) To resist diagonal tension caused due to transverse shear  
developed because of any moment or load.
- 4) To impact directly to the column.
- 5) To prevent longitudinal splitting or bulging out of concrete  
confining it in the core.

STATE OF COLLAPSE: COMPRESSION - Assumptions [IS 456 pg 69 and 70]

Design of Axially Loaded Short Column [Cl 3.2.1 and 37.1]  
Determine the cross section and reinforcement required for  
axially loaded column of effective length 3m and  
axial load 2300 kN. Use M20 concrete and Fe45 steel  
given Data :-

$$L_{eff} = 3m$$

$$\text{Axial load, } P_u = 2300 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

(1) Assume it as axially loaded shear column.

[IS 456 Pg 31 (43)]

$$P_a = 0.4 f_{ck} A_c + 0.67 f_y A_s \leq$$

As per IS 456 Pg 48 Cl 26.3 (a)

Cb area of reinforcement  $\neq 0.8 \text{ of } A_g$   
 $\neq 6 \text{ of } A_g$

Assume % of stirrups as 8% of total cb area.

$$\text{or } A_{sc} = 8\% A_g \quad A_{sc} \rightarrow \text{Area of stirrups}$$

$$A_{sc} = 2.8\% A_g \quad A_{sc} \rightarrow \text{Area of concrete}$$

$$2300 \times 10^3 = 0.4 \times 20 \times \frac{48}{100} A_g + 0.67 \times 415 \times \frac{\Sigma A_s}{100} \leq$$

$$A_g = 171628.9 \text{ mm}^2$$

Assume it as square column.

$$\text{over } a^2 = 171628.9$$

$$a = 414.58 \text{ mm} \approx 415 \text{ mm}$$

i. Provide 4.5 x 4.5 mm square column.

$$\begin{aligned} \text{Area of side} & \quad A_{sc} = \frac{4}{100} \times 171628.9 = 3432.578 \text{ mm}^2 \\ \text{reinforcement} & \end{aligned}$$

$$\text{Area of concrete, } A_c = \frac{98}{100} \times 171628.9 = 168196.392 \text{ mm}^2$$

(2) Check for slenderness ratio [IS 456 Cl 26.1.2 Pg 41]

$$\frac{L_{eff}}{b} = \frac{3000}{415} = 7.27 < 12 \quad D = 415 \text{ mm}$$

Hence it is a short column.

(3) Check for  $\ell_{min}$ .

$$\ell_{min} > \frac{1}{500} + \frac{2}{50} \quad \ell_{min} = \frac{20.07 D}{500} \quad [\text{IS 456 Cl 37.3 Pg 31}]$$

$$\ell_{min} = \frac{1}{500} + \frac{D}{30} \quad [\text{IS 456 Cl 37.3 Pg 31}]$$

$$[\text{IS 456 Cl 25.4 Pg 40}]$$

Minimum eccentricity = 20 mm [IS 456 Cl. 25.4 Pg. 42] (1)

$$0.05 D \geq 20 \text{ mm}$$

$$\frac{1}{600} + \frac{D}{30} \geq \frac{3000}{600} + \frac{415}{30} = 19.13$$

$$0.05 D = 0.05 \times 415$$

$$= \frac{20.75}{30} \geq 20 \text{ mm} \quad e_{min} \leq 0.05 D$$

$$\frac{1}{600} + \frac{2}{30} = 19.83 < 20.13$$

Hence condition is satisfied - axially loaded column

### Design of longitudinal reinforcement

$$\text{Area of longitudinal reinforcement} = \alpha^2 \\ = D^2 - 415 \times 415 = 172425 \text{ mm}^2$$

$$A_e = \frac{\alpha_f}{100} A_g = \frac{168.780.5}{100} \text{ mm}^2$$

$$A_{ox} = \frac{2}{100} A_g = 344.5 \text{ mm}^2$$

Assume 20 mm  $\phi$  bars. (1) [IS 456 Cl. 25.5.3.1(g) Pg. 48]

$$\text{No. of bars} = \frac{A_e}{A_f} = \frac{2444.5}{6 \times \frac{3.14 \times 25^2}{4}} = 7.01 \leq 8 \text{ numbers}$$

(i) Close bar spacing [IS 456 Cl. 24.5.3.1(g) Pg. 48]

Spacing  $\neq 30 \text{ mm}$ .

$\phi$	$\phi$	$\phi$	
$\phi$			
	$\phi$		

Assume clear cover = 40 mm.

$$\text{Spacing} = \frac{D - (2 \times 20) - (3 \times \phi)}{2} = \frac{415 - (2 \times 40) - (3 \times 25)}{2} \\ = 130 \text{ mm}$$

130 mm  $< 250 \text{ mm}$

Hence OK.

## ④ Design of transverse reinforcement [IS 456 Ch 2, pg 25]

Diameter of ties should not be less than

$$① \text{The largest dia of bars} = \frac{1}{4} \times 45 = 11.25 \text{ mm}$$

### ② Ties

$\therefore$  Use 8 mm  $\phi$  bars.

## ⑤ Spacing of lateral bars. [IS 456 Ch 2, pg 49]

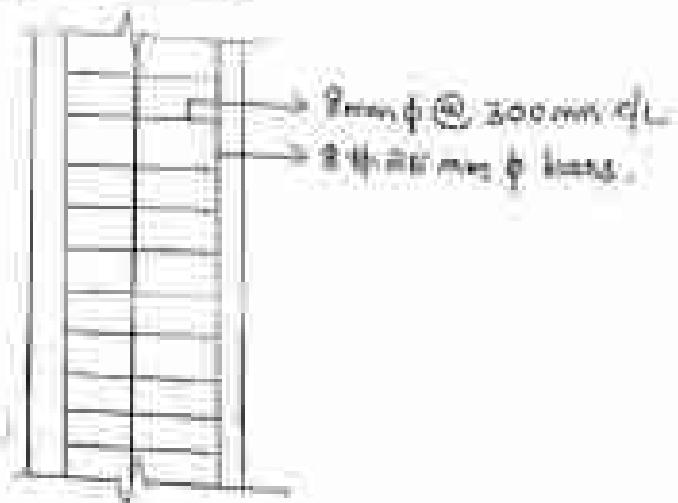
The spacing should not be more than the following

① The least lateral dimension = 415 mm

② No. of smallest  $\phi$  of longitudinal bar =  $16 \times 25 = 400$  mm

③ 300 mm.

$\therefore$  Provide 8 mm  $\phi$  bars @ 300 mm c/c



⑥ Check for

Design a circular RC column to carry an axial load of 1800 kN  
40 grade concrete & Fe 415 steel. Effective length = 3m.

Given data :-

$$P_u = 1800 \text{ kN} \quad P_a = 1.5 \times 1800 = 1800 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$L_{eff} = 3 \text{ m}$$

Assume it is axially loaded short column. [IS 456 Cl. 39.4 Pg 71]

$$P_u = 1.05 (0.4 \text{ fact A}_{sc} + 0.67 \text{ f}_y A_g)$$

Assume  $A_{sc} = 10\% \text{ of } A_g$  and  $A_{sc} = 99\% \text{ of } A_g$

$$1800 \times 10^3 = 1.05 \left[ 0.4 \times 20 \times \frac{99}{100} \times A_g + 0.67 \times 415 \times \frac{1}{100} \times A_g \right]$$

$$A_g = 160.212 \times 10^3 \text{ mm}^2$$

$$\therefore A_{sc} = 10\% \text{ of } A_g = \frac{1}{100} \times 160.212 \times 10^3 \\ = 16.0212 \text{ mm}^2$$

$$\therefore A_c = 99\% \text{ of } A_g = \frac{99}{100} \times 160.212 \times 10^3 \\ = 153.609.88 \text{ mm}^2$$

Since it is a circular column.

$$\frac{\pi D^2}{4} = 160.212 \times 10^3$$

$$D = 461.55 \text{ mm} \quad \underline{\underline{460 \text{ mm}}}$$

Take  $D = 450 \text{ mm}$ .

Check for slenderness ratio.

$$\frac{L_{eff}}{b} = \frac{3000}{450} = 6.67 < 12$$

Hence it is a short column.

④ Check for Column

$$e_{min} \leq 0.05D \quad e_{max} \geq 0.05D$$

[IS 456 Cl 39.3 Pg 41]

$$0.05D = 0.05 \times 460 \quad e_{min} = \frac{1}{500} + \frac{1}{30} = \frac{3000 + 460}{500 \times 30} = 21.33 \\ \approx 22$$

[IS 456 Cl 35.4 Pg 41]

$$e_{max} = \frac{1}{500} + \frac{D}{30} = 21.33 + 0.05 \times 460 = 23$$

$$e_{max} = 26.13 \geq 23 \quad e_{min} = 21.33 < 23 \text{ mm}$$

$$l_{min} = 20 \text{ mm} \quad \text{this eccentricity} = 20 \text{ mm}$$

[IS 456 Cl 25.4 Pg 41]

$$0.05D \geq 20 \text{ mm}$$

$$23 \text{ mm} \geq 20 \text{ mm}$$

Hence the condition is satisfied - axially loaded column

⑤ Design of longitudinal reinforcement

$$A_g = \frac{\pi D^2}{4}$$

$$= \frac{\pi}{4} \times 460^2 = 166190.25 \text{ mm}^2$$

$$A_{sc} = \pm 1 \text{ of } A_g = \frac{1}{100} \times 166190.25 = 1661.902 \text{ mm}^2$$

$$A_c = 99.1 \text{ of } A_g = \frac{99}{100} \times 166190.25 = 164538.94 \text{ mm}^2$$

Assume 30 mm dia bars,

$$\text{no. of bars} = \frac{A_{sc}}{\pi d^2} = \frac{1661.902}{\pi \times 20^2} = 4.26 \approx 6 \text{ nos}$$

$$\text{As provided} = 6 \times \frac{\pi \times 20^2}{4} = 1865.44 \text{ mm}^2 \approx A_g = \frac{\pi \times 20^2}{4}$$

⑥ Check for splicing [IS 456 Cl 26-5.3.1(g) Pg 41]

Splicing  $\nless 300 \text{ mm}$

$$\text{Splicing} = \frac{\pi D}{6} = \frac{3.1460}{6}$$

$$= 240.16 \text{ mm} \lessdot 300 \text{ mm}$$



Hence Safe

Angle of transverse reinforcement / helical ties [IS 456 Cl. 26.5.2 (c) pg 42]

Diameter of ties should not be less than.

$$\text{D}_t = \frac{1}{\pi} \times \text{largest dia of longitudinal bar} = \frac{1}{\pi} \times 25 = 5 \text{ mm.}$$

Pitch 6mm.

Provide 8mm  $\phi$  bars.

2) Pitch [IS 456 Cl. 29.4.1 pg: 41]

$$\text{Core diameter } D_c = D - (cc \times 2)$$

Assume clear cover = 50mm

$$D_c = 450 - (50 \times 2)$$

$$= 350 \text{ mm.}$$



$$\text{Area of core} = \left( \frac{\pi \times 350^2}{4} \right) - 1875$$

$$= 99902.60 \text{ mm}^2$$

$$\text{Volume of core per pitch} = 99902.60 \text{ mm}^3$$

$$\text{Volume of helical reinforcement} = \frac{\pi \times 8^2 \times \pi}{4} (360 \times 1)$$

$$= 55555.61 \text{ mm}^3$$

$$\frac{\text{Volume of helical reinforcement}}{\text{Vol. of core per pitch}} \leq 0.36 \left[ \frac{A_g}{A_c} - 1 \right] \frac{f_{ck}}{f_g}$$

$$\frac{55555.61}{99902.60} \leq 0.36 \left[ \frac{166190.90 - 1}{99902.60} \right] \frac{20}{415}$$

$$\frac{0.556}{P} \leq 0.0288$$

$$\frac{0.556}{0.0288} \leq P$$

$$P \geq 19.30 \text{ mm}$$

④ Check for pitch [IS 456 Cl. 24.5.22 (b) (ii) pg 39]

Pitch should not be more than:

i) 15 mm

ii)  $\frac{1}{6} \times \text{less diameter} = \frac{1}{6} \times 360 = 60 \text{ mm}$

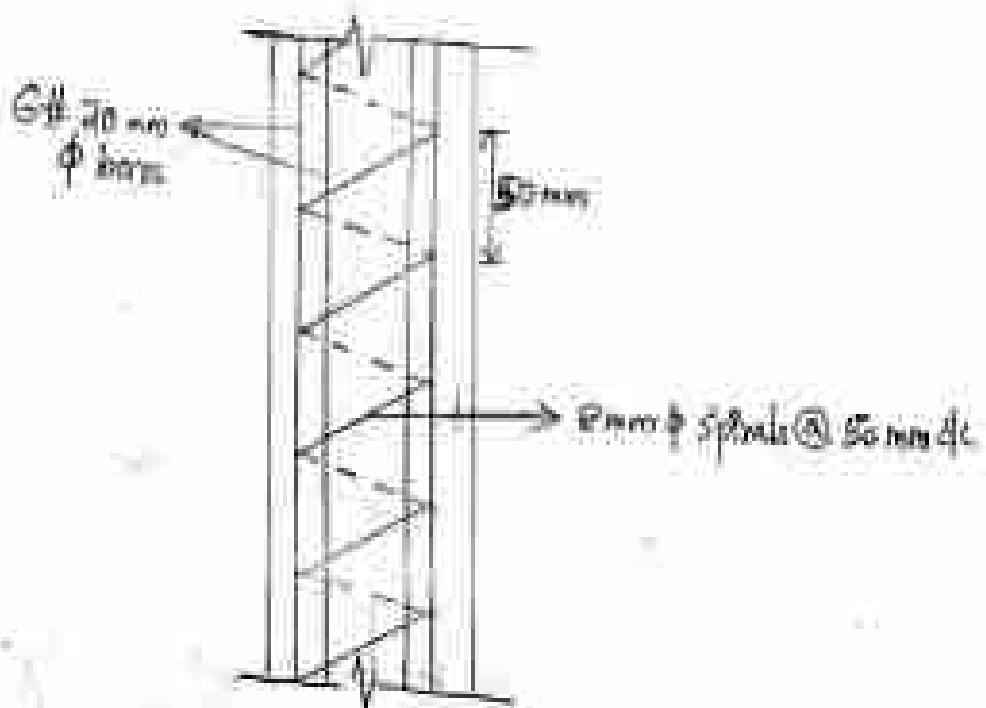
Pitch should not be less than:

i)  $3 \times \phi$  of Reinforcement =  $3 \times 8 = 24 \text{ mm}$

ii) 25 mm.

Provide helical reinforcement of 8 mm  $\phi$  at 50 mm c/c

460 mm



Given a column  $300 \times 400 \text{ mm}$  which has an effective length  $\approx 3.6 \text{ m}$  and subjected to a factored load  $1100 \text{ kN}$ . Use M25 Concrete and Fe415 Steel. Assume an effective cover of  $60 \text{ mm}$ .

Given Data

$$b = 300 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$L_{\text{eff}} = 3.6 \text{ m}$$

$$P_u = 1100 \text{ kN}$$

$$\rho_{\text{st}} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\text{Eff. cover} = 60 \text{ mm}$$

$$A_g = 300 \times 400 = 120000 \text{ mm}^2$$

$$\begin{aligned} \text{Assume } A_{\text{sc}} &= 2\% \text{ of } A_g \\ &= \frac{2}{100} \times 120000 = 2400 \text{ mm}^2 \end{aligned}$$

D) Slenderness ratio

$$\frac{L_{\text{eff}}}{b} = \frac{3600}{300} = 12 \quad \text{It is a short column.}$$

It is a short column.

E) Check for Column. [IS 456 Cl 9.3 & Cl 25.4 Pg 71 & 42]

$$C_{\text{min}} = \frac{V_{\text{sd}}}{P/30} = \frac{3600}{500} + \frac{400}{30} = 20.5 \text{ mm}$$

$$0.05 D = 0.05 \times 400 = 20 \text{ mm}$$

$$L_{\text{eff}} > 0.05 D \therefore \text{Check } C_{\text{min}}/b = 20.5/400 = 0.051 \leq 0.05 \text{ Hence OK}$$

$$C_{\text{min}} = \frac{V_{\text{sd}}}{P/30} = \frac{3600}{500} + \frac{30}{30} = 17.2 \text{ mm}$$

$$0.05 b = 0.05 \times 300 = 15 \text{ mm}$$

$$L_{\text{eff}} > 0.05 D \therefore \text{Check } C_{\text{min}}/b = 17.2/300 = 0.05 \leq 0.05 \text{ Hence OK}$$

$\therefore$  It is an axially loaded column.

④ Design of longitudinal reinforcement

$A_f = 1600 \text{ mm}^2$

$$A_{sf} = 2400 \text{ mm}^2$$

Provide 20mm  $\phi$  bars.

$$\text{no. of bars} \times \frac{A_s}{A_f} = \frac{2400}{\frac{1600}{2}} \Rightarrow 7.5 \approx 8 \text{ bars}$$

∴ Provide 8 no 20mm  $\phi$  bars.

⑤ Check for spacing [IS 456 Cl 26.5.3.1 (g) pg 48]

Spacing  $\geq 300 \text{ mm}$ .

$$\text{Spacing} = \frac{400 - (60 \times 2) - (20 \times 3)}{2} = 110 \text{ mm}$$

$$\text{Spacing} = 110 \text{ mm} \leq 300 \text{ mm}$$

Hence Ok.

$$\text{Spacing} = \frac{300 - (60 \times 1) - (20 \times 3)}{2} = 60 \text{ mm} \leq 300 \text{ mm}$$

Hence Ok.

⑥ Design for transverse reinforcement/lateral ties

[IS 456 Cl 26.5.3.2 (c) pg 48]

Diameter of ties should not be less than

i)  $\frac{1}{4} \times \text{largest dia. of longitudinal bars} = \frac{1}{4} \times 20 = 5 \text{ mm}$

ii) 6mm

∴ Provide 8mm  $\phi$  bars.

⑦ Spacing of lateral ties [IS 456 Cl 26.5.3.2 pg 48]

The spacing should not be more than the following.

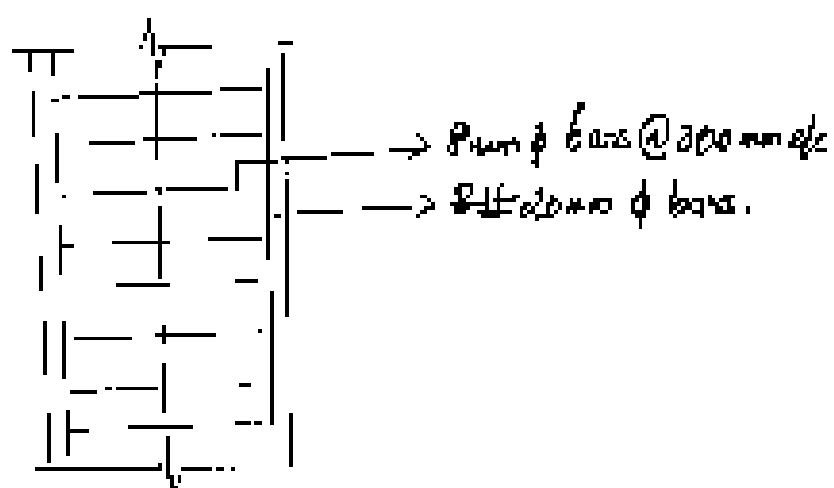
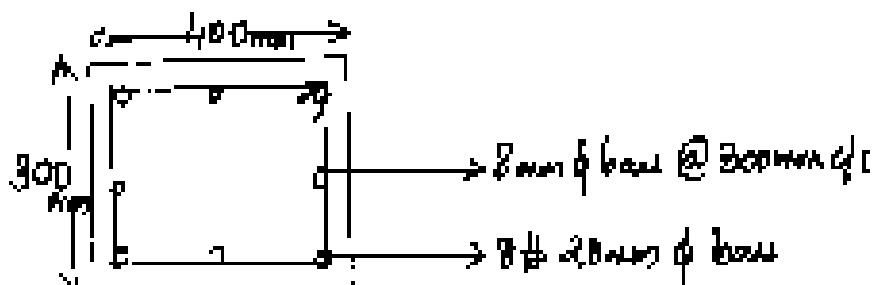
i) The least lateral dimension = 300 mm.

$$\text{Smallest dia of longitudinal bars} = 6 \times 30 \\ = 320 \text{ mm}$$

14

contd.

Provide 8 nos  $\phi 16$  bars @ 300 mm c/c.



Design a short RCC column to carry an axial load of 200 kN if it is 4 m long, effectively held in position and restrained against rotation at both ends. Use M<sub>20</sub> concrete and Fe 450 - steel.

Solution:

$$f_s = 40 \text{ N/mm}^2$$

$$P = 200 \text{ kN} \quad P_u = 1.5 \times 1600 = 2400 \text{ kN}$$

$$f_{ck} = 30 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Effective length: [IS 456 Table 23 Pg 24]

$$l_{eff} = 0.66 l$$

$$\geq 2.66 \times 4 = 2.66 \text{ m}$$

$$\textcircled{2} \quad \textcircled{2} \quad P_a = 0.4 f_{ck} A_c + 0.67 b_f y_b c \quad [\text{IS 456 Cl 39.3 pg 70}]$$

Assume  $A_{sc} = 1/4 \text{ of } A_g$ .

$$A_c = 99.1 \text{ mm}^2$$

$$P_a = 0.4 f_{ck} \frac{99.1}{100} A_g + 0.67 b_f \frac{1}{100} w_d$$

$$0.4 \times 400 \times 10^3 = 0.4 \times 20 \times 0.99 \times A_g + 0.67 \times 415 \times 0.01 A_g$$

$$A_g = \underline{\underline{224.299 \text{ mm}^2}}$$

Assume it as a square column.

$$A_g = a^2 = 224.299$$

$$a = 473.63 \text{ mm}$$

$$\approx \underline{\underline{500 \text{ mm}}}$$

$\therefore$  Provide  $500 \text{ mm} \times 500 \text{ mm}$  square column.

\textcircled{3} Slenderness ratio

$$\frac{l_{eff}}{d} = \frac{4000}{500} = 5.2 < 12$$

Hence it is a short column.

\textcircled{4} Check for Lateral  $[\text{IS 456 Cl 89.2 pg 71}]$

$$e_{min} < 0.05D$$

$$[\text{IS 456 Cl 264 pg 247}] \quad e_{min} = \frac{l}{500} + \frac{D}{30} = \frac{4000}{500} + \frac{500}{30} = 24.67 \text{ mm}$$

$$0.05D = 0.05 \times 400 = 20 \text{ mm} < e_{min}$$

$$\text{So check, } \frac{e_{min}}{D} = \frac{24.67}{500} = 0.049 < 0.05 \quad \left[ \frac{e_{min}}{D} < 0.05 \right]$$

Hence the column is designed as axially loaded column.

\textcircled{5} Design for longitudinal steel

$$A_g \text{ provided} = a^2 = 500^2 = 250000 \text{ mm}^2$$

$$\therefore A_{sc} = 1/4 A_g = 62500 \text{ mm}^2$$

(2)

$$\text{number of bars} = \frac{A_{st}}{A_f} = \frac{2500}{\frac{E \times 20}{4}} = 7.95 \approx 8$$

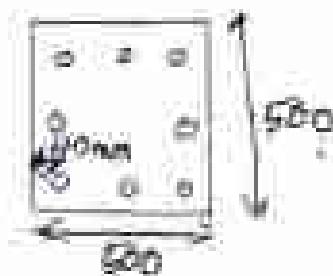
Provide 8 # 20 mm of bars

i) Check for spacing [IS 456 Cl 26.5.3.1 (g) pg 42]

Spacing  $\neq 300$  mm

Assume 40 mm clear cover

$$\text{Spacing} = \frac{500 - (40 \times 2) - (3 \times 40)}{2} = 180 \text{ mm}$$



Spacing = 180  $\leq$  300 mm

Hence Ok.

Design for transverse reinforcement / lateral ties

[IS 456 Cl 26.5.3.2 (c) pg 49]

Diameter of ties should not be less than

i)  $\frac{1}{n} \times \text{largest dia of longitudinal bar} = \frac{1}{8} \times 20 = 5 \text{ mm}$

ii) 6 mm

$\therefore$  Provide 6 mm of bars

Spacing of lateral ties [IS 456 Cl 26.5.3.2 pg 49]

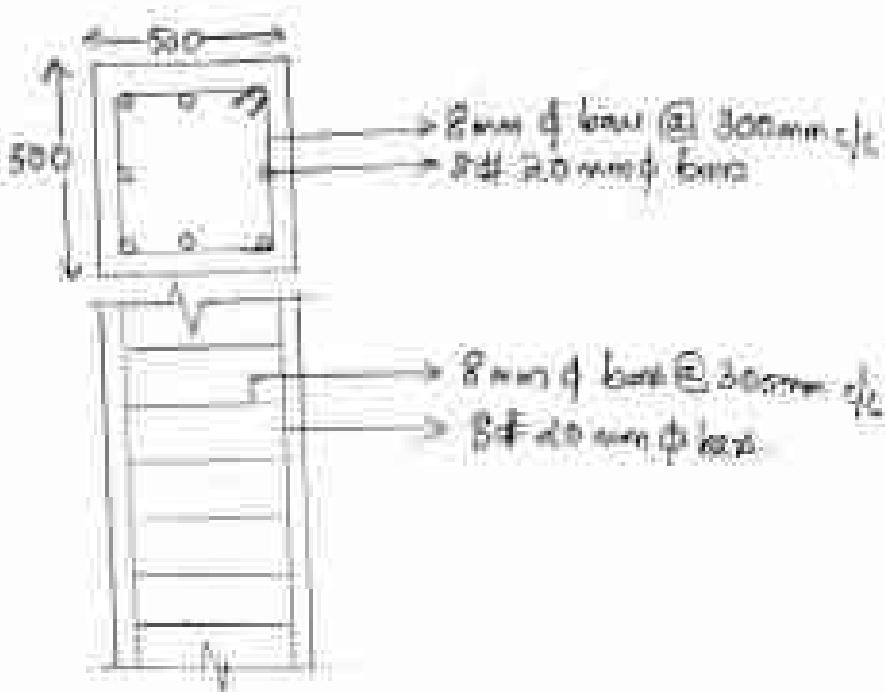
the spacing should not be more than the following

i) the last lateral dimension = 500 mm

ii)  $16 \times \text{smallest dia of longitudinal bar} = 16 \times 20 = 320 \text{ mm}$

iii) 300 mm.

$\therefore$  Provide 6 mm of bars @ 300 mm c/c.



5. Design a column of size 450mm x 600mm and having 3m unsupported length. The column is subjected to a load of 2000 kN and is effectively held in position but not restrained against rotation. Use M420 concrete and Fe450 reinforcement. Data:-

$$b = 450 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$l = 3 \text{ m}$$

$$P = 2000 \text{ kN}, \quad P_u = 2000 \times 1.5 = 3000 \text{ kN}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

① Effective length

$$l_{eff} = l/1$$

$$\approx l/3 = \underline{\underline{2 \text{ m}}}$$

② Slenderness Ratio

$$SR = \frac{l_{eff}}{b} = \frac{3000}{450} = 6.67 < 12$$

$\therefore$  It is a short column

## Q. Check for Column

$$e_{min} = \frac{l}{600} + \frac{D}{30} \quad [IS 456 Cl 25.4 pg 24]$$

$$= \frac{3000}{600} + \frac{600}{30} = 25 \text{ mm}$$

$$l_{min} = 20 \text{ mm} \quad [IS 456 Cl 25.4 Pg 42]$$

$25 \text{ mm} > 20 \text{ mm}$

$$0.05 D = 0.05 \times 600$$

$$= 30 \text{ mm}$$

$$e_{min} < 0.05 D \quad [IS 456 Cl 37.3 pg 71]$$

$$e_{min} = \frac{l}{600} + \frac{b}{30} \quad [IS 456 Cl 25.4 pg 24]$$

$$= \frac{3000}{600} + \frac{450}{30} = 25 \text{ mm}$$

$$l_{min} = 20 \text{ mm} \quad [IS 456 Cl 25.4 Pg 42]$$

Eccentricity }  $25 \text{ mm} > 20 \text{ mm}$ .

$$0.05 b = 0.05 \times 450 = 22.5 \text{ mm}$$

$$e_{min} < 0.05 b \quad [IS 456 Cl 37.3 pg 71]$$

Hence it is an axially loaded short column

## Design of Longitudinal Reinforcement

$$A_g = 450 \times 600 = 270000 \text{ mm}^2$$

$$\text{Ans. } A_{sc} = 4.1 \text{ of } A_g$$

$$A_{sc} = \frac{1}{100} \times A_g = \frac{1}{100} \times 270000 \\ = 2700 \text{ mm}^2$$

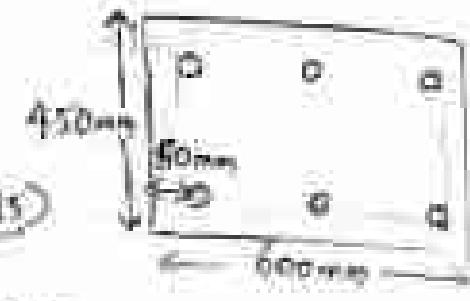
Using 25 mm  $\phi$  bars

$$\text{Number of bars} = \frac{A_{sc}}{\phi} = \frac{2700}{\frac{\pi}{4} \times 25^2} = 5.5 \approx 6$$

Provide 2 no. of 25 mm  $\phi$  bars

④ Check for spacing [IS 456 Cl. 26.5.3.1(b) Pg 44]

Spacing  $\neq$  300 mm  
Assume 50 mm clear cover



$$\text{Spacing} = \frac{600 - (2 \times 50) - 2 \times 25}{2}$$

$$= 312.5 \text{ mm} < 300 \text{ mm}$$

Hence OK

$$\text{Spacing} = \frac{450 - (4 \times 50) - 2 \times 25}{1}$$

$$= 300$$

Hence OK

⑤ Design for transverse reinforcement / lateral ties [IS 456 Cl. 26.5.3.2 Pg 44]

Diameter of ties should not be less than

i)  $\frac{1}{4}$  x largest dia of longitudinal bar

$$= \frac{1}{4} \times 25 = 6.25 \text{ mm}$$

ii) 6 mm

∴ Provide 8 mm  $\phi$  bars

⑥ Spacing of lateral ties [IS 456 Cl. 26.5.3.2 Pg 44]

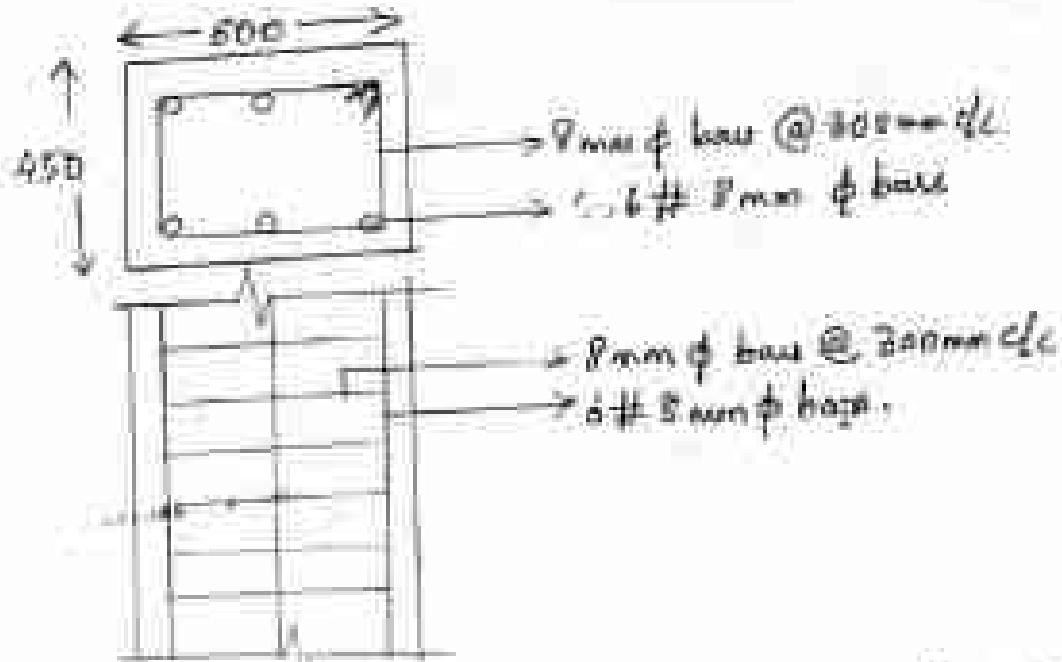
The spacing should not be more than the following

i) The least lateral dimension = 450 mm

ii) 6 x smallest dia of longitudinal bar =  $6 \times 25 = 150 \text{ mm}$

iii) 300 mm

∴ Provide 8 mm  $\phi$  bars @ 300 mm c/c



Design a circular column of diameter 400 mm subjected to load of 1200 kN. The column is having spiral tie. The column is 3m long and is effectively held in position at both ends but not restrained against rotation. Use M25 concrete and Fe 415 steel.

Given data :-

$$d = 3m$$

$$P = 1200 \text{ kN} \quad P_u = 1.5 \times 1200$$

$$D = 400 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

effective length

[IS 456 Table 21 Pg 94]

$$l_{eff} = 3.0 \text{ m}$$

$$l_{eff} = 1 \times 3 = 3 \text{ m}$$

slenderness ratio

$$SR = \frac{l_{eff}}{b} = \frac{3000}{400} \approx 7.5 < 12$$

Hence it is a short column.

### ③ Check for Lateral

$$C_{min} = \frac{1}{500} + \frac{D}{30} = \frac{2000}{500} + \frac{400}{30} = 14.33$$

$$C_{min} < 0.05D \quad [IS 456 Cl 25.4 Pg 71]$$

$$0.05D = 0.05 \times 400 \\ = 20 \text{ mm}$$

$$C_{min} < 20 \text{ mm}$$

$$C_{min} = 20 \text{ mm} \quad [IS 456 Cl 25.4 Pg 71]$$

$$\frac{C_{min}}{D} = \frac{20}{400} = 0.05 < 0.05$$

∴ It is designed as axially loaded column.

### ④ Design for longitudinal reinforcement

$$A_g = \frac{\pi D^2}{4} = \frac{\pi \times 400^2}{4} = 12566.37 \text{ mm}^2$$

Assume  $A_{sc} = 1.1 \text{ of } A_g$

$$A_{sc} = \frac{1}{100} \times 12566.37 = 125.6637 \text{ mm}^2$$

Assume 20mm φ bars,

$$\text{Number of bars} \Rightarrow \frac{A_{sc}}{A_g} = \frac{125.6637}{\frac{\pi \times 20^2}{4}} = 3.99 \approx 4$$

But min number of longitudinal bars in a circular column = 6.

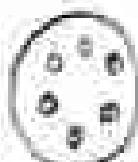
∴ Provide 6 # 20mm φ bars.

### ⑤ Spacing [IS 456 Cl 26.5.3.1 (g) Pg 47]

Spacing ≠ 300mm

$$\text{Spacing} = \frac{\pi D}{6} = \frac{\pi \times 400}{6} = 209 \text{ mm} < 300 \text{ mm}$$

Hence OK



## (2)

### Design of transverse reinforcement/helical ties

{ IS 456 (1) 26.5.3.2 (c) pg 49}

• Diameter of ties should not be less than:

•  $D_t \geq \frac{1}{4} \times D_c = \frac{1}{4} \times 200 = 50 \text{ mm}$

•  $6 \text{ mm}$

Provide 3 nos of bars.

### (3) Pitch

(IS 456 Cl 37.4.1 Pg 71)

Core Diameter,  $D_c = D - (c + r_2)$

Assume clear cover = 50mm

$$D_c = 400 - (50 + 25) \\ = 300 \text{ mm}$$

$$\text{Area of core} = \left( \frac{\pi}{4} \times 300^2 \right) - 1257 \\ = 69428.83 \text{ mm}^2$$

Volume of core per pitch =  $69428.83 \times p$ .

$$\text{Volume of helical reinforcement} = \frac{\pi}{4} \times r^2 \times n (300 - r) \\ = 46110.79 \text{ mm}^3$$

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core per pitch}} \leq 0.36 \left( \frac{A_t}{A_c} - 1 \right) \frac{d_t}{69} \\ \frac{46110.79}{69428.83p} \leq 0.36 \left( \frac{125663.7}{69428.83} \right) \frac{25}{415}$$

$$\frac{0.664}{p} \leq 0.075$$

$$p \geq 37.95 \text{ mm}$$

====

50mm

(2) Check for pitch [IS 456 (A. 26.5.8)(d)(D) pg 45]

Pitch should not be more than

i) 75 mm

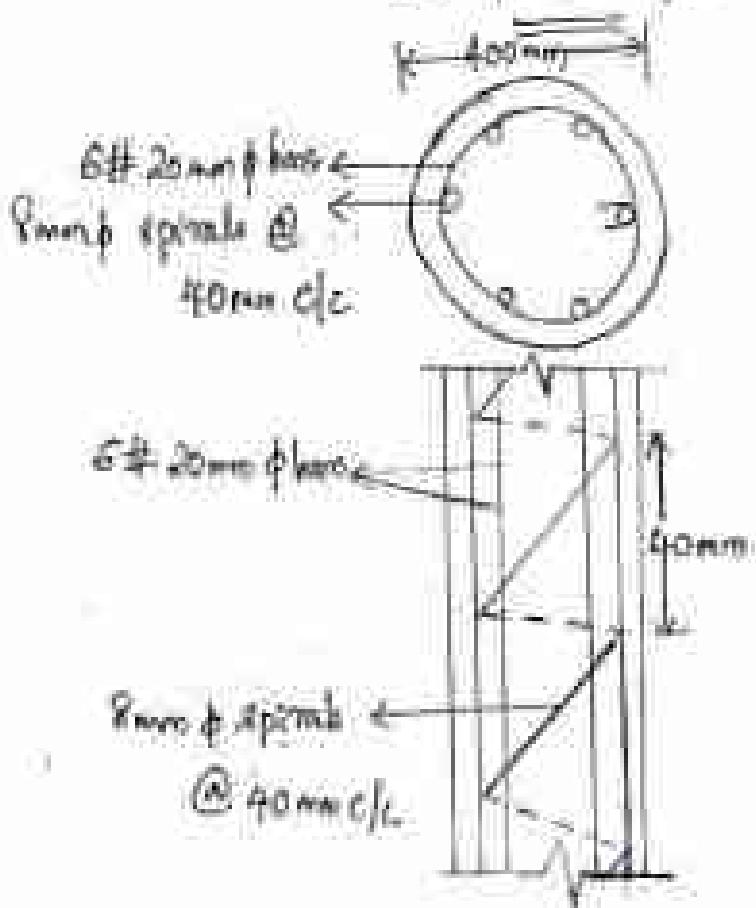
ii)  $\frac{1}{4} \times \text{core diameter} = \frac{1}{4} \times 300 = 75 \text{ mm}$

Pitch should not be less than

i)  $3 \times \text{dia of transverse reinforcement} = 3 \times 8 = 24 \text{ mm}$

ii) 25 mm

∴ Provide helical reinforcement of 8 mm  $\phi$   
at 40 mm C/c.



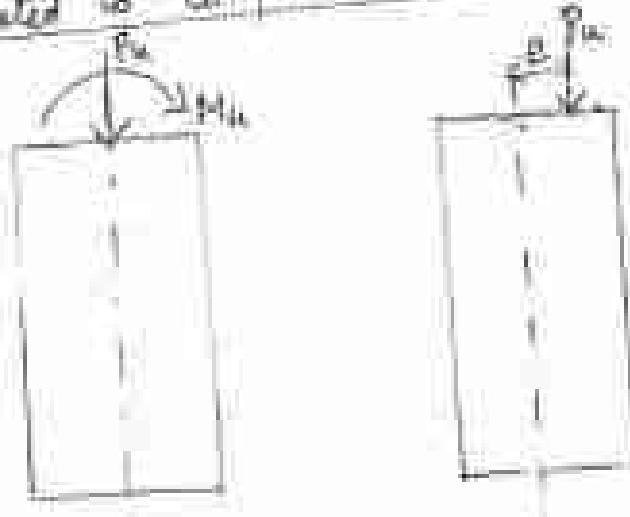
## Column Subjected to Axial Compression and bending Moment

As per IS 456:2000, all columns should be designed for ultimate eccentricity as it is not possible to have a perfectly axially loaded column. Due to imperfections in construction and non-homogeneity of materials, some eccentricity will always be there which will result in bending of column. If  $e_{min} \leq 0.05D$ , the column is axially loaded column, but if  $e_{min} > 0.05D$  or some moment is coming on the section, then the column should be designed for bending in addition to compression.

### Modes of Failure

- (i) Cracking or compression failure without tension.
- (ii) Cracking or compressive failure with tension.
- (iii) Tension failure.

### Column Subjected to Compression and bending



$$M_u = P_u \cdot e$$

$$e = \frac{M_u}{P_u}$$

Design of columns subjected to compression and horizontal bending

1. Design a R.C. column 500mm square c/s carrying a factored load of 1300 kN at an eccentricity of 120 mm. Use M<sub>20</sub> concrete and Fe 415 steel.

Given Data:

$$b = D = 500 \text{ mm}$$

$$e = 120 \text{ mm}$$

$$P_u = 1300 \text{ kN}$$

$$f_{ck} = 25 \text{ MPa}$$

$$f_y = 415 \text{ N/mm}^2$$

$$\textcircled{1} \text{ Calculation of } \frac{M_u}{P_u}$$

$$130 \times 10^3 = \frac{M_u}{500}$$

$$M_u = 156 \text{ kNm}$$

\textcircled{2} Design of longitudinal reinforcement

$$A_s = \frac{\pi}{4} \times 500^2 = 250000 \text{ mm}^2$$

Assume  $A_{sc} = 1\% \text{ of } A_s$

$$= \frac{1}{100} \times 250000 = 2500 \text{ mm}^2$$

Using - 2 nos.  $\phi 20$  bars

$$\text{number of bars} = \frac{A_{sc}}{A_s} = \frac{2500}{250000} = 0.01 = 1 \text{ bar}$$

Provide 8# 20 mm  $\phi$  bars

Check for spacing [IS 456 Cl 26.5.3(9) pg 48]

(2)

Spacing  $\leq$  300mm

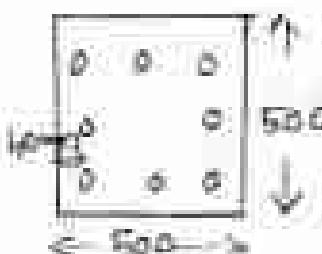
Assume 40mm clear cover

$$\text{Spacing} = \frac{500 - (40 \times 2)}{2 \times 20}$$

$$= \underline{\underline{180\text{mm}}}$$

$$\text{Spacing} = 180\text{mm} < 300\text{mm}$$

Hence OK



Design for transverse reinforcement / lateral ties

[IS 456 Cl 26.5.3(10) pg 49]

Diameter should not be less than

$\geq$  longest side of longitudinal bar  $\times \frac{1}{1000} = 5 \times \frac{1}{1000} = 5\text{mm}$

$\geq 6\text{mm}$

$\therefore$  Min. dia of bars:

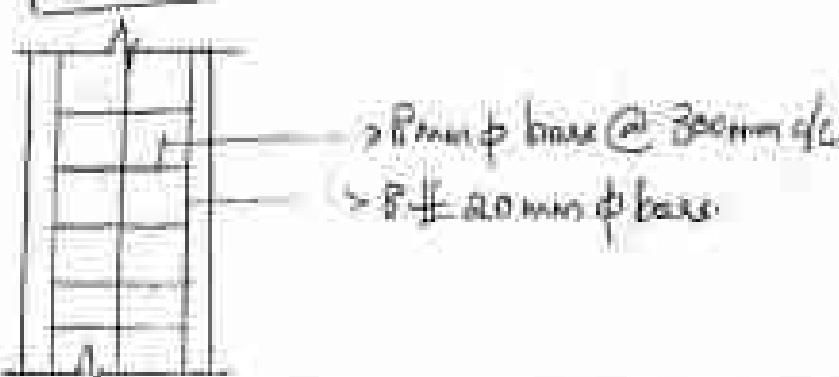
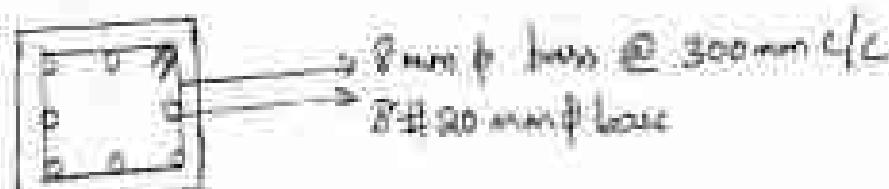
Spacing of lateral ties [IS 456 Cl 26.5.3.2 Pg 49]

spacing should not be more than the following.

At least lateral dimension = 50mm

Max. Smallest dia of longitudinal bar  $= 10 \times 20 = 200\text{mm}$

300mm  $\therefore$  Min. Spacing b/w bars  $\geq 300\text{mm c/c}$



(Q) Design a column 300 x 400 mm with ends as pinned at the top and bottom. Effective length 3.6 m. It has been subjected to a factored load of 119 kN and a factored moment of 230 kNm about the major axis. Use M24 concrete & Fe415 steel. Assume an effective cover of 30 mm.

$$b = 300 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$I_{\text{eff}} = 2\pi \frac{D^3}{3} \text{ mm}^4$$

$$r_g = 115 \text{ mm}$$

$$P_u = 1100 \text{ kN}$$

$$M_{u_0} = 230 \text{ kNm}$$

Eff. cover = 60 mm.

$$L_{\text{eff}} = 3.6 \text{ m}$$

$$L_{\text{eff}} = 3600 \text{ mm}$$

$$\textcircled{1} \quad A_g = 200 \times 0.06 \\ = 12000 \text{ mm}^2$$

Assume  $A_{\text{eff}} = \pi r^2$  of  $\frac{D}{2}$

$$= \frac{\pi}{4} \times 120000 = 37680 \text{ mm}^2$$

\textcircled{2} Slenderness ratio

$$\frac{L_{\text{eff}}}{b} = \frac{3600}{300} = 12 = 12$$

It is a short column

=====

Check for eccentricity [IS 456 Cl 5.9.3 & Clause 4]

(a)

[Pg 31 and Pg 42]

$$e_{max} = \frac{l}{500} + \frac{b}{30}$$

$$\therefore \frac{3600}{500} + \frac{400}{30} = 20.53 \text{ mm}$$

$$0.05D = 0.05 \times 400 \\ = 20 \text{ mm}$$

$$e_{max} > 0.05D$$

$$e_{min} = \frac{l}{500} + \frac{b}{30}$$

$$= \frac{3600}{500} + \frac{300}{30} = 17.2 \text{ mm}$$

$$0.05D = 0.05 \times 300 \\ = 15 \text{ mm}$$

$$e_{min} > 0.05D$$

$\therefore$  The column is laterally loaded column

(ii) Design of longitudinal reinforcement

$$A_f = 12000 \text{ mm}^2$$

$$A_{sc} = 2400 \text{ mm}^2$$

Assume 30mm Ø bars

$$\text{Number of bars} = \frac{A_{sc}}{\phi d} = \frac{2400}{157 \times 20} = 7.63 \approx 8 \text{ nos.}$$

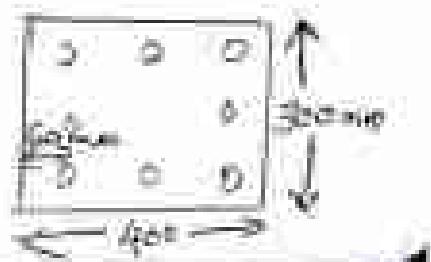
$\Rightarrow$  Provide 8 no 30mm Ø bars

(iii) Check for spacing [IS 456 Cl 6.7.3.1(g) Pg 48]

Spacing  $> 300 \text{ mm}$

$$\text{Spacing} = \frac{400 - (60 \times 2) - (20 \times 3)}{2} \\ = 110 \text{ mm} < 300 \text{ mm}$$

∴ Spacing OK



(34)

$$\text{Spalling} = \frac{300 - (6 \times 2)}{2(0.6 \times 3)}$$

$$= 60\text{mm} < 300\text{mm}$$

Hence OK

(6) Design for transverse/lateral tie [IS 456 Cl 24.5.3.2(c) Pg 49]  
 Diameter of tie should not be less than

i)  $\frac{1}{4}$  times dia of longitudinal bar =  $\frac{1}{4} \times 25 = 6.25\text{mm}$

(ii) 6 mm

or Parallel 8 mm Ø bars

(7) Spacing of lateral tie [IS 456 Cl 26.5.2.2 Pg 49]

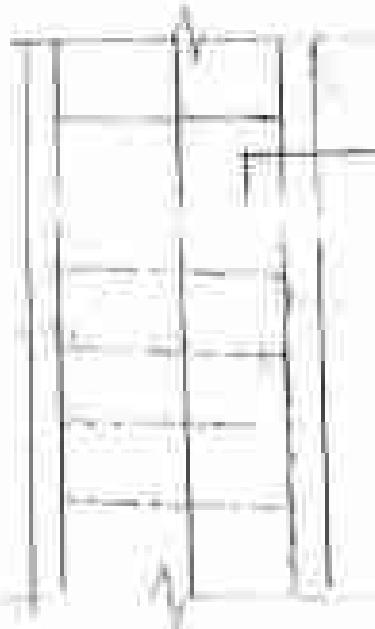
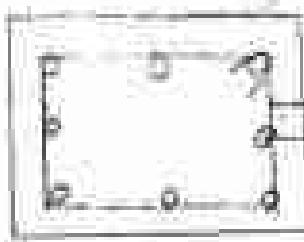
The spacing should not be more than the following.

i) The least lateral dimension = 300mm

ii) 1.6 times dia of longitudinal bar =  $1.6 \times 25 = 40\text{mm}$

iii) 300 mm

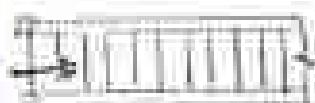
or Parallel 8 mm Ø @ 300mm CfC



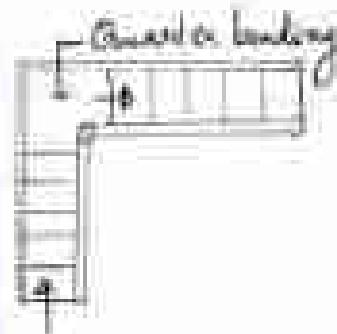
## Staircase

(3)

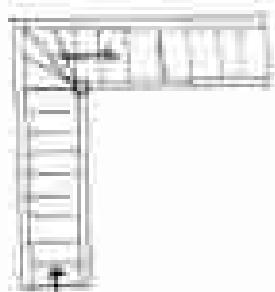
Staircase is a means of giving access to different floors or parts of a building. Stair cases are used in almost all buildings. It consists of a number of steps arranged in a way that a person can move from one level to another. The arrangement of steps is as per the convenience, standards and space available. Commonly used stair cases are as follows:



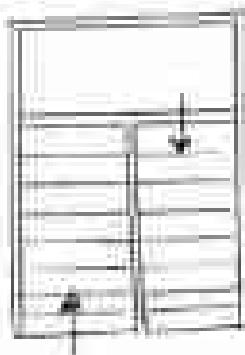
1. Straight Stairs



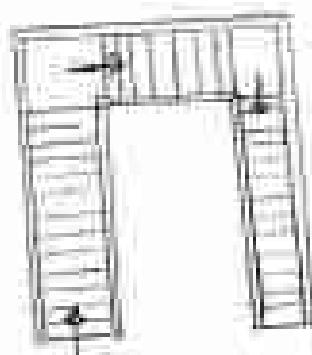
2. Quarter-turn Stair.



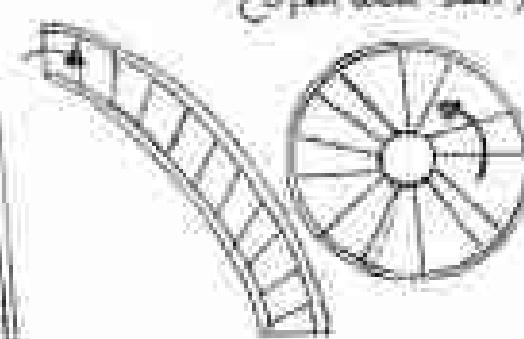
3. Half turn Stair  
(Open well stair)



4. Day lit Stair



5. Open newel Stair  
with quarter space  
landing



6. Spiral  
Stair

55

## Terminology

### Flight

Flight is the length of the staircase between two landings. It is the sloping and projection (slab) of the stairs. The number of steps in a flight varies from 3 to 12.

### Landing

Landing is the intermediate horizontal portion provided in a staircase. It is provided for resting while climbing and entering or exiting a staircase.

### Rise



The vertical height of a step is called rise or rise height. It varies from 150mm to 180mm for residential building and 120 to 150 mm for public building.

### Run



The horizontal distance for two rises on a step is called as run. The width of a run is kept as 200mm to 300mm for residential building and 300 to 350mm for public building.

### Bossing and Nosing



The horizontal distance for two runs is known as nosing and the portion projecting out from the floor surface is called as nosing. Nosing is provided when the available horizontal distance for a stair is less.

### Head Room

It is the clear height available for the flight and other above it.

### Gaffit

It is the bottom edge of the wall slab.

