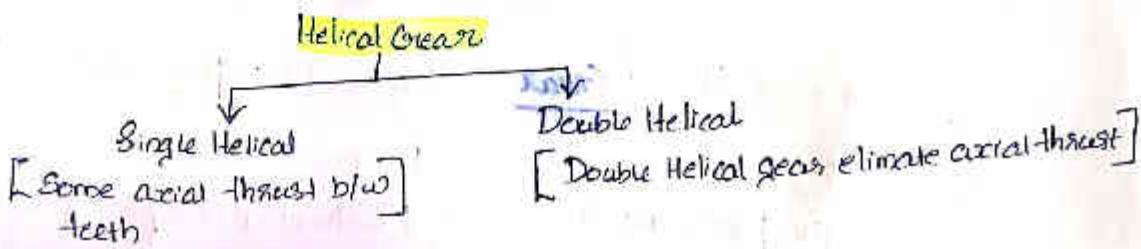


# Module 4 Helical Gears DESIGN

①

- Helical gears are used to transmit motion b/w parallel or crossed shafts or b/w a shaft and a rack by meshing teeth that lie along Helix angle to the axis of the shaft.
- Helix angle - Meshing of teeth occur such that two or more teeth of Advantage each gear are always in contact.
- Helical Gear teeth is form of Helix.



## Advantages & disadvantages

- Gradual engagement & disengagement of teeth reduces the noise and dynamic load.
- Drive is smooth, shock less and without noise, vibration & impact.
- Suitable transmitting low & medium power at high speed.
- Gears can be surface hardened to higher BHN
- Due to helical shape, Gears are subjected to axial, radial & tangential load
- Axial load Eliminated by using Herringbone Gears.

*Comparison of  
B/w Spur &  
Helical Gears*

### **Spur Gears**

- Spur gears straight teeth
- used with parallel shaft
- operate low speed
- Load suddenly applied
- Operation is noisy
- Dynamic stresses are high
- pitch line velocity up to 8 m/s
- It begins to engage the contact extends across the entire tooth on a line parallel to gear axis

### **Helical Gears**

- Helical gears their teeth cut in the form of a helix
- parallel & non-parallel shaft
- High speed
- Gradually applied
- Smooth
- Low
- 20-30 m/s
- Here contact begins at one end of entering tooth & gradually extends along a diagonal line across tooth faces as gear rotates

## Application of Helical Gear

- Turbine, Automobiles.

## Terms used in Helical Gears

- Helix angle ( $\beta$ ): It is the angle made by the helices with axis of rotation.
- Axial pitch ( $P = P_c$ ): It is same as the circular pitch and therefore defined as the circular pitch in the plane of rotation.
- Normal pitch ( $P_n = P_{cn}$ ): It is distance b/w similar faces on adjacent teeth along a helix on the pitch cylinder normal to both

## Proportion of Helical Gear (Figure 12.8, page 213)

$\beta$  - Helix angle

$F_t$  - Torque producing force

$F_a$  - End thrust

$F_n$  - Normal force

$P_c$  - Circular pitch

$P_n$  - Normal circular pitch

$b$  - Face width

$m$  - Module,  $m_n$  - Normal module in plane normal to tooth

$$P_n = P_c \cos \beta \quad [\text{page 211, eq 12.19(a)}]$$

$$P_n = \frac{\pi d}{Z} \times \cos \beta = \pi m \cos \beta \quad (P = Pd/Z, m = d/Z)$$

Helix angle ( $\beta$ )  $\rightarrow$  For Single helix  $20^\circ < \beta < 35^\circ$ , Herringbone gears  $\beta \leq 45^\circ$

- Normal diametrical pitch  $P_n = \frac{P}{\cos \beta}$  [211],  $P_n = \frac{Z}{d \cos \beta}$

- Normal Module  $m_n = m \cos \beta = \frac{d}{Z} \cos \beta$  [211, eq 12.19(c)]

- No: calculated teeth  $Z = \frac{d}{m_n} \cos \beta = d \cos \beta / P_n$  [211, eq 12.19(d)]

- pitch circle diameter  $d = m Z = \frac{Z}{P} = \frac{Z}{P_n \cos \beta} = \frac{Z m_n}{\cos \beta}$  [211, eq 12.19(e)]

- $\tan \alpha_0 = (\tan \alpha_0 \cos \beta)$  [211, 12.22(b)]

- Centre distance  $a = \frac{(Z_1 + Z_2)}{2 \cos \beta} m_n$  [211, 12.20]

- Axial thrust  $F_a = F_t \tan \beta$  [211, 12.21]

## Face width (b)

In order that contact be maintained across the entire face width of gears min value of face width  $b = \frac{P}{\tan \beta}$

$$b_{\min} = \frac{1.15 \pi m^2}{\tan \beta} = \frac{1.15 \pi m_n}{\sin \beta} \quad [213, 12.23(b)]$$

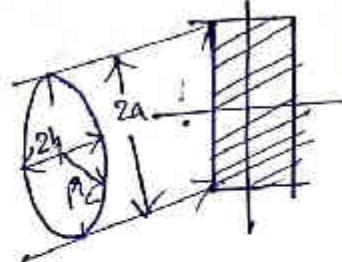
$$b_{\min} \geq \frac{2.3 \pi m^2}{\tan \beta} = \frac{2.3 \pi m_n}{\sin \beta} \quad [213, 12.23(d)]$$

$$b = 20m_n \text{ to } 30m_n \quad [213, 12.23(g)]$$

## Virtual or Formative Number of teeth ( $Z_e$ )

(2)

A plane normal to the gear teeth intersects the pitch cylinder to form an ellipse. The gear-tooth profile generated in this plane using the radius of curvature of ellipse would be spur gear having the same properties as an actual helical gears.



$$\text{Radius of curvature of an ellipse } R_e = \frac{d}{\cos^2 \beta}$$

{ Virtual No: of teeth on equivalent spur gear in Normal plane called Virtual or Formative on equivalent No: of teeth }

$$Z_e = P_o \times 2R_e = \frac{Z}{\cos \beta} \times \frac{2d}{2\cos^2 \beta} \Rightarrow Z_e = \frac{Z}{\cos^3 \beta}$$

(page 367, eq 12.22(a))

$Z_e$  = Actual No: of teeth on helical gear  
 $\beta$  = Helix angle

## Force Analysis of Helical Gears

(page 312, Refer figure 12.7)

For Derivation  
Look Text Book

- Tangential Force  $F_t = \frac{2M_F}{d}$

Radial component  $F_r = F_t \tan \beta$  [212]

- Axial thrust  $F_a = F_t \tan \beta$  [212]

$$F_{au} = \frac{F_t \tan \beta}{\cos \beta}$$

$$\left[ \frac{F_a}{F_t} = \frac{F_t \cos \beta \sin \beta}{F_t \cos \beta \cos \beta} \Rightarrow F_a = F_t \tan \beta \right]$$

$$\frac{F_a}{F_t} = \frac{F_t \sin \beta}{F_t \cos \beta \cos \beta} = \frac{\tan \beta}{\cos \beta}$$

- component  $F_t \tan \beta$  along the gears axis called end thrust.

$$F_u = \frac{\tan \beta}{\cos \beta} F_t$$

$$F_u = F_t \cos \beta \cdot \frac{1}{\cos \beta} \frac{\tan \beta}{\cos \beta}$$

$$F_u = F_t \sin \beta$$

## Strength of Helical Gear

Lewis eq for  
Helical & Herringbone  
Gears.

$$F_t = \frac{\sigma_d C v b Y}{P_{dew}} = \frac{\sigma_d C v b Y_m n}{C_w}$$

(page 314, eq 12.24(a))

where  $\sigma_d$  = Allowable static stress,  $C_w$  = wear & lubrication factor

### Derivation

$$F_t = \sigma_d C v b Y_m$$

$$F_{bu} = \sigma_d C v b_n Y_m n$$

$$\frac{F_{bu}}{\cos \beta} = \sigma_d C v b_n Y_m n \frac{b}{\cos \beta} \quad F_{bu} = F_t / \cos \beta$$

$$\left[ F_t = \frac{\sigma_d C v b Y_m n}{C_w} \right]$$

where

$$m_n = m \cos \beta$$

$$b_n = b / \cos \beta$$

# DESIGN METHODOLOGY FOR HELICAL GEARS

## Method (i)

If pitch diameter is known

$$F_t = \frac{\sigma d C_v b Y_m n}{C_w} \quad [214, 12.24(a)]$$

where  $k = \frac{b}{mn}$

$$\therefore F_t = \frac{\sigma d C_v k Y_m n}{C_w}$$

$$F_t = \frac{\sigma d C_v k Y_m n^2}{C_w}$$

$$m_n = \frac{F_t C_w}{\sigma d C_v k Y} \Rightarrow m_n = \sqrt{\frac{F_t C_w}{\sigma d C_v k Y}}$$

## Dynamic tooth load

$$F_d = F_t + k_3 \alpha (C_b \cos^2 \beta + F_t) \cos \beta \quad [214, eq. 12.26(a)]$$

## Endurance Strength

$$F_s = \sigma_{end} b Y_m n \quad [214, 12.26(b)] \Rightarrow F_s \geq F_d \text{ for safety}$$

## wear tooth load

$$F_w = \frac{d_1 b Q k}{\cos^2 \beta} \geq F_d \quad [214, 12.26(c)]$$

$$K = \frac{\sigma_{essinal}^2}{1.40} \left[ \frac{1}{E_1} + \frac{1}{E_2} \right]$$

$$Q = \frac{2d_2}{d_2 + 1} = \frac{2z_2}{z_2 + z_1}$$

## DESIGN PROCEDURE FOR HELICAL GEAR

- If same as Spur Gear (Refer Module 3 Note)
- Module ( $m$ ) replaced with normal Module ( $m_n$ ) & all calculation based on
- Lewis factor included in Virtual No. of teeth ( $z_e$ )

A Helical Gear of 250mm dia transmits a torque of 200 Nm. The pressure angle in a plane normal to the teeth is  $20^\circ$ . Helix angle is  $30^\circ$ . Determine Gear tooth loads.

Solution

$$G.D \Rightarrow d = 250 \text{ mm}, M_t = 200 \text{ Nm}, \alpha_n = 20^\circ, \beta = 30^\circ$$

(a) Tangential force  $F_t = \frac{2M_t}{d} = \frac{2 \times 200 \times 10^3}{250} = 1600 \text{ N}$

(b) Axial / Thrust force  $F_a = F_t \tan \beta = 1600 \times \tan 30^\circ = 1600 \text{ N} \quad [212]$

(c) Radial component  $F_r = \frac{F_t \tan \alpha_n}{\cos \beta} = \frac{1600 \times \tan 20^\circ}{\cos 30^\circ} = 823.57 \text{ N} \quad [212]$

## Method (ii)

If the pitch diameter is unknown

Torque =  $F_t d$

$$M_t = F_t \times d/2 \Rightarrow F_t = \frac{2M_t}{d}$$

$$\frac{\sigma d C_v b Y_m n}{C_w} = \frac{2M_t}{d}$$

$$M_t = \frac{\sigma d C_v (k m_n) Y_m n}{2C_w} \left( \frac{m_n z}{\cos \beta} \right) \quad | \begin{array}{l} k = b/m_n \\ d = m_n z \\ \hline \end{array}$$

$$= \frac{\sigma d C_v k Y_m n^2}{2C_w \cos \beta}$$

$$m_n = 3 \sqrt{\frac{2M_t C_w \cos \beta}{\sigma d C_v k Y_m n}}$$

$$Y = \pi Y \\ k = b/m_n \\ C_v = \text{Veloc. Factor}$$

$$C = \frac{e}{k_1} \left[ \frac{1}{E_1} + \frac{1}{E_2} \right]$$



### Problem of known Diameter/

- Design a pair of equal diameter  $20^\circ$  stub teeth and helical gears to transmit 37.5kW with Moderate Shock 1200 rpm. The two shafts are parallel and 0.45m apart. Each gear is made of Steel with  $\beta = 30^\circ$

Solution

G.I.D

$\alpha = 20^\circ$  stub teeth,  $P = 37.5 \times 10^3$  W

piston speed  $N_1 = 1200$  rpm, Service - Moderate Shock

Centrel distance = 0.45m = 450mm

Diameter Equal, Helix angle  $\beta = 30^\circ$  API

$$\boxed{i = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}} \rightarrow \text{Here } d_1 = d_2 \\ i = 2$$

$\therefore N_1 = N_2, z_2 = z_1$

Centr. distance  $a = \frac{d_1 + d_2}{2}$  [page 203, eq 12.3(a)]

$$450 = \frac{d_1 + d_1}{2} \Rightarrow d_1 = 450\text{mm} = d_2$$

Step 1 Check weaker part (piston/Gear)

Given (Steel) is material for both pinion & Gear

Assume pinion & Gear made up of Steel, C45, untreated (234, T12.7)

$$\sigma_{dg} = \sigma_{dp} = 233.4 \text{ MPa}, BHN_1 = BHN_2 = 200$$

Here pinion is weaker part ( $\sigma_{dp} < \sigma_{dg}$ )

Design Based on pinion.

Formative / Virtual No. of teeth

Assume  $z_1 = 20$  teeth,  $z_2 = i \times z_1 = 1 \times 20 = 20$  teeth

Virtual No. of teeth  $\Rightarrow \boxed{Z_e = \frac{Z}{\cos \beta}}$  (page 211, eq 12.22(a))

$$Z_{e1} = \frac{z_1}{\cos^3 \beta} = \frac{20}{\cos^3 30} = 30.79 \approx \underline{\underline{31}}$$

$$Z_{e2} = \frac{z_2}{\cos^3 \beta} = \underline{\underline{31}}$$

Step 2 To find Module (m)

Here diameter is known

$$M_n = \left[ \frac{F_t c_w}{\sigma_d c_v b Y} \right]^{\frac{1}{2}} \quad [\text{page 214, eq 12.24(b)}]$$

- Where  $V = \frac{\pi d N}{60} = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.450 \times 1200}{60} = 28.27 \text{ m/s}$
- page 214  $\Rightarrow$  when  $V > 20 \text{ m/s}$   $C_V = \frac{5.55}{5.55 + \sqrt{V}} = \frac{5.55}{5.55 + \sqrt{28.27}} = 0.5107$
- $F_t = 1000 \text{ PCs}$  [page 205, eq 12.07(a)]  
 $= \frac{1000 \times 975 \times 1.5}{28.27} = 1989.74 \text{ N}$  | Moderate Shock, Assume 8-10 kHz  
 Service ( $s=1.5$ ) [Table 12.8, page 235]
- $Y = \pi y_1 = \pi \left[ 0.175 - \frac{0.95}{Z_{e1}} \right] \rightarrow Z_{e1} = 20^{\circ} \text{ stub teeth}$   
Below eq 12.24(b)  $= \pi \left[ 0.175 - \frac{0.95}{31} \right] = 0.4535$
- $k = \frac{b}{M_n} = 15 \text{ (Assume)}$
- $C_w = 1.15$  [Table 12.21, page 241],  $\sigma_{d \text{ pinion}} = 233.4 \text{ MPa}$
- $M_n = \left[ \frac{1989.74 \times 1.15}{233.4 \times 0.5107 \times 15 \times 0.4535} \right]^{\frac{1}{2}} = 1.68 \text{ mm}$

From Table 12.2 (page 229)  $\Rightarrow$  Standard module  $M_n = 2 \text{ mm}$

Trial I [check Module]

Lewis eq  $F_t = \frac{\sigma_d c_v b Y M_n}{C_w}$  [page 214, eq 12.24(a)]

Where  $F_t = 1989.74 \text{ N}$ ,  $C_w = 1.15$ ,  $C_V = 0.5107$ ,  $b = 15 M_n = 15 \times 2 = 30 \text{ mm}$   
 $\therefore M_n = 2 \text{ mm}$

Here No: of teeth on pinion  $\Rightarrow z_1 = \frac{d_1 \cos \beta}{M_n}$  [page 211, eq 12.19(a)]

$$= \frac{450 \times \cos 30}{2} = 194.8 = 195 \text{ teeth}$$

$$\underline{z_2 = z_1 = 195 \text{ teeth}} \Rightarrow z_e = \frac{z}{\cos^3 \beta} \Rightarrow (211, 12.22(a))$$

$$Y = \pi y_1 = \pi \left[ 0.175 - \frac{0.95}{Z_{e1}} \right] \quad \underline{Z_{e1} = \frac{z_1}{\cos^3 \beta} = \frac{195}{\cos^3 30} = 300 = Z_{e2}}$$

$$Y = \frac{0.175 - 0.95}{300} = 0.5398$$

Sub: Lewis eq

$$1989.74 = \frac{\sigma_d \times 0.5107 \times 30 \times 0.5398 \times 2}{1.15}$$

$$\sigma_d = 138.34 \text{ MPa} \leq 233.4 \text{ MPa}$$

$m_n = 2 \text{ mm}$  (Satisfactory)

Face width  $b = 15m_n = 15 \times 2 = 30 \text{ mm}$   
 pitch circle dia of pinion = 450 mm ( $d_1$ )  
 pitch circle dia of gear = 450 mm ( $d_2$ )

### Step 3 Dynamic tooth load ( $F_d$ )

$$F_d = F_t + K_3 V \frac{(C_b \cos^2 \beta + F_t) \cos \beta}{K_3 V^2 + \sqrt{C_b \cos^2 \beta + F_t}} \quad (\text{page 214, eq 12-26(a)})$$

Where  $F_t = 1989.74 \text{ N}$

$K_3 = 20.67$  (below  $\alpha$ )

$V = 28.27 \text{ m/s}$

$b = 30 \text{ mm}$

$\beta = 30^\circ$

To find  $C$

Page 237  $\Rightarrow$  Table 12-14

$V > 26 \text{ m/s} \Rightarrow e = 0.0127$

$\alpha = 20^\circ$  Stub (Steel-Steel combination)

$$e = 0.01 \text{ mm}, C = 118.7 \text{ N/mm} \quad (\text{page 236})$$

$$e = 0.0127 \text{ mm}, C = ? \quad (\text{12-12})$$

$$C = \frac{0.0127 \times 118.7}{0.01} = 150.75 \text{ N/mm}$$

$$F_d = 1989.74 + (20.67 \times 28.27) \left[ \frac{150.75 \times 30}{\cos^2 30} + 1989.74 \right] \cos 30$$

$$(20.67 \times 28.27) + \sqrt{150.75 \times 30 \times \cos^2 30 + 1989.74} \quad C = 150.75 \text{ N/mm}$$

$$F_d = 6130.51 \text{ N}$$

### Step 4 To find Endurance Strength

$$F_s = \sigma_{en} b Y M_n \geq F_d \quad \Rightarrow \text{page 214, eq (12-26(a))}$$

where  $b = 30 \text{ mm}, Y = 0.5398, m_n = 2 \text{ mm}$

Table 12-15  $\Rightarrow$  page 238  $\Rightarrow$  Steel - BHN = 300  $\Rightarrow \sigma_{en} = 345 \text{ MPa}$

$$F_s = 345 \times 30 \times 0.5398 \times 2 = 11.17 \times 10^3 \text{ N}$$

$F_w > F_d$  Material is safe against static tooth load

### Steps (Wear load) $F_w$

$$F_w = \frac{d_1 b @ k}{\cos^2 \beta} \geq F_d \quad [\text{page 214, eq 12-26(c)}]$$

where  $d_1 = 450 \text{ mm}$

$b = 30 \text{ mm}$

$$\alpha = \frac{2d_2}{d_1 + d_2} \quad (\text{Below ea}) = \frac{2 \times 450}{450 + 450} = \underline{\underline{1}}$$

- For  $\alpha = 20^\circ$  and Steel-Steel combination having  $BHN_1 = BHN_2 = 200$

$$k = 0.539 \quad [\text{page 239, Table 12-16}]$$

$$F_w = \frac{450 \times 30 \times 1 \times 0.539}{\cos^2 30} = \underline{\underline{9702 \text{ N}}}$$

$F_w > F_d$ , Material safe against wear

Assuming H = 10 [from service and steady load, service factor  $C_{s1}$ ]

$$F_1 = \frac{10(0) \times 15 \times 1}{41.89} \quad \dots \text{Th. 12.8/Pg. 2.85}$$

$$F_1 = 358 \text{ N} \quad \dots \text{Th. 12.8/Pg. 2.85}$$

$$\text{From factor, } F_{\text{face}} Y = \pi M_n \quad \dots \text{Below eq. 12.24(c), Pg. 2.14}$$

12. A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10000 rpm and has 80 mm pitch diameter. The gear has a pitch diameter of 320 mm. If gears are made of cast steel having allowable static strength of 100 MPa; determine module and face width from static strength considerations and check the gears for wear, given  $\sigma_{u1} = 618 \text{ MPa}$ .
- Solution:  $P = 15 \text{ kW}, \alpha = 20^\circ$ , stub teeth, helix angle,  $\beta = 45^\circ$ , pinion speed,  $N_1 = 10000 \text{ rpm}$ , pitch diameter of pinion,  $d_1 = 80 \text{ mm}$ , pitch diameter of gear,  $d_2 = 320 \text{ mm}$ ,  $\sigma_{u1} = \sigma_{u2} = 100 \text{ MPa}$ ,  $\sigma_{c1} = 616 \text{ MPa}$ .

- a.  $m_n = ?$    b.  $b = ?$    c.  $F_{\text{w}} = ?$

We know that,  $i = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$

$$i = \frac{d_2}{d_1} = \frac{320}{80} = 4$$

Identifying weaker part (pinion/gear):

We know that strength factor =  $\sigma_{r,y}$

Since both pinion and gear are made of same material, the pinion is weaker, i.e.  $\sigma_{r,y1} < \sigma_{r,y2}$ .

Hence design is based on pinion.

\* Assume

$$z_1 = 20 \text{ teeth}$$

- \* Virtual number of teeth,  $z_v = \frac{z}{\cos^3 \beta} \quad \text{[Page 2.11, Eq. 12.22(a)]}$

$$z_v = \frac{z_1}{\cos^3 \beta} = \frac{20}{\cos^3 45}$$

$$z_v = 56.37 \approx 57$$

a. To find module ( $m_n$ )

$$\text{Since the diameter is known, we have } m_n = \left[ \frac{F_{\text{c}} C_m}{\sigma_u C_o K^2} \right]^{1/2} \quad \text{[Page 2.14, Eq. 12.24(b)]}$$

- \*  $\varphi = \frac{\pi d_1 N_1}{60} = \varphi = \frac{\pi \times 0.08 \times 10000}{60} = 41.89 \text{ rad/s}$

- \* For  $v > 20 \text{ m/s}$ ,  $C_v = \frac{5.55}{5.55 + \sqrt{v}} = \frac{5.55}{5.55 + \sqrt{41.89}} = 0.4616$

$$F_r = \frac{1000 P C_s}{v} = \frac{1000 \times 15 \times 10^3}{20} = 750000 \text{ N} \quad \text{[Page 2.05, Eq. 12.7(a)]}$$

Since  $b > b_{\min}$ , the calculated value of  $b$  is safe.

\* Check for  $\sigma_{u1}$ :

We know that,

$$F_1 = \frac{\sigma_u C_b Y m_n}{C_m} \quad \text{[Page 2.14, Eq. 12.24(a)]}$$

Here

$$Y = \pi v_1 \quad \text{(Below eq. 12.24(a))}$$

$$= \pi \left[ 0.175 - \frac{0.55}{z_1} \right] \quad \text{[Page 2.04, Eq. 12.5(c)]}$$

[Page 2.11]  
Eq. 12.21(q)

Number of teeth on pinion,  $z_1 = \frac{d_1 \cos \beta}{m_n} = \frac{80 \times \cos 45}{1.25} = 45.25 \approx 46$  teeth

Therefore, standard module,

$$m_n = \sqrt{\frac{358 \times 1.15}{100 \times 0.4616 \times 15 \times 0.4974}} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

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$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

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$$m_n = 1.09 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$m_n = 1.25 \text{ mm} \quad \dots \text{Th. 12.21/Pg. 2.41 DHB}$$

$$= \pi \left[ 0.175 - \frac{0.95}{130.10} \right]$$

$$Y = 0.5268$$

$$\therefore \text{Eq. (ii) yields, } 358 = \frac{\sigma_{et} \times 0.4616 \times 19 \times 0.5268 \times 1.25}{1.15}$$

$\sigma_{et} = 71.28 \text{ MPa} < 100 \text{ MPa}$   
Since calculated values are < permissible values, the assumed values are satisfactory.

b. To find face width (b)  
 $b = 19 \text{ mm}$

c. To find dynamic tooth load ( $F_d$ )

$$\text{We know that, } F_d = F_t + \frac{K_3 v (C b \cos^2 \beta + F_t) \cos \beta}{K_3 v + \sqrt{(C b \cos^2 \beta + F_t)}}$$

$[ \text{Eq. 14, Eq. 12.26(c)} ]$

$$\text{here } K_3 = 20.67 \quad (\text{Refer Eq. 12.26(a)})$$

① $v > 26 \text{ m/s}$	Error, $e = 0.0127 \text{ mm}$	...TB. 12.14/Pg	DHB
For $\alpha = 20^\circ$ Stub teeth and steel - steel combination		2.36	
② $e = 0.01 \text{ mm}$	$C = 118.7 \text{ N/mm}$	...TB. 12.12/Pg	DHB

$$\therefore \text{at } v = 41.89 \text{ m/s, } C = \frac{0.0127 \times 118.7}{0.01} = 150.75 \text{ N/mm}$$

$$\therefore \text{Eq. (iii) yields, } F_d = 358 + \frac{(20.67 \times 41.89) \times \sqrt{(150.75 \times 19 \times \cos^2 45) + 358}}{(20.67 \times 41.89) + \sqrt{(150.75 \times 19 \times \cos^2 45) + 358}}$$

$$F_d = 1564.84 \text{ N.}$$

d. To find dynamic tooth load ( $F_w$ )

$$\text{We know that, } F_w = \frac{d_1 Q K}{\cos^2 \beta} \quad [\text{Refer Eq. 12.26(c)}]$$

- Diameter of pinion,  $d_1 = 60 \text{ mm}$
- Ratio factor,  $Q = \frac{2d_2}{d_2 + d_1} \quad [\text{Refer Eq. 12.26(c)}]$

$$= \frac{2 \times 320}{320 + 80}$$

$$Q = 1.6$$

- Here BHN is not given. Since both the gears are made of steel, assuming  $E = 2.1 \times 10^5 \text{ MPa}$ , we have

$$K = \frac{\sigma_e^2 \sin \alpha_e}{1.4} \left[ \frac{1}{E_1} + \frac{1}{E_2} \right] \quad [\text{Refer Eq. 12.26(c)}]$$

Since all

$$\begin{aligned}\tan \alpha_1 &= \tan \alpha \cos \beta \\&= \tan 20 \times \cos 45\end{aligned}$$

$$\alpha_1 = 14.43 \text{ deg}$$

$$K = \frac{618^2 \times \sin(14.43)}{1.4} \left[ \frac{1}{21 \times 10^3} + \frac{1}{21 \times 10^5} \right]$$

$$K = 0.6474$$

$$\text{Eq. (iv) yields } F_u = \frac{80 \times 19 \times 1.6 \times 0.6474}{\cos^2 45}$$

$$F_u = 3149 \text{ N}$$

Since  $F_u > F_d$ , the material is safe against wear.

13. The following data refers to the design of a helical gear drive:

- i. Power transmitted 34 kW at 2800 rpm of pinion
- ii. Speed ratio 4.5, number of teeth on pinion 18
- iii. Helix angle 45°, pressure angle  $\alpha = 20^\circ$  stub
- iv. Material for both pinion and gear is medium carbon steel whose allowable stress may be taken as 230 MPa.
- v. Pinion diameter is limited to 125 mm.

Determine the axial thrust on the shaft and check the gears for dynamic and wear loads.

Solution:  $P = 34 \text{ kW}$ , pinion speed,  $N_1 = 2800 \text{ rpm}$ ,  $i = 4.5$ ,  $z_1 = 18$  teeth, helix angle,  $\beta = 25^\circ$ ,  $\alpha = 20^\circ$ -stub teeth, pitch diameter of pinion,  $d_1 = 125 \text{ mm}$ ,  $\sigma_{u1} = \sigma_{u2} = 230 \text{ MPa}$ .

We know that,

$$i = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

$$4.5 = \frac{d_2}{d_1} = \frac{d_2}{125}$$

$$d_2 = 562.5 \text{ mm}$$

$$i z_2 = 4.5 \times 18 = 81 \text{ teeth}$$

Identifying weaker part (pinion/gear):

We know that strength factor =  $\sigma_u Y$

Since both pinion and gear are made of same material, the pinion is weaker, i.e.,  $\sigma_{u1} Y_1 < \sigma_{u2} Y_2$

Hence design is based on pinion.

- Virtual number of teeth,  $z_v = \frac{z}{\cos^3 \beta} = [211, \text{eq 12.22(a)}]$

$$z_v = \frac{z_1}{\cos^3 \beta} = \frac{18}{\cos^3 25}$$

$$z_v = 24.17$$

- And  $z_{v2} = i z_v = 24.17 \times 4.5 = 108.81$

a. To find module

$$\text{Since the diameter is known, we have } m_a = \left[ \frac{F_u C_w}{\sigma_u C_v K_f} \right]^{\frac{1}{2}} [ \text{page 214, eq 12.24(b)} ]$$

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.125 \times 2800}{60} = 18.33 \text{ m/s}$$

$$\text{For } 10 < v < 20 / \text{ms}, C_v = \frac{15.25}{15.25 + v} [ \text{Eq 14, 12.25(c)} ]$$

$$= \frac{15.25}{15.25 + 18.33}$$

$$C_v = 0.4541$$

$$F_t = \frac{1000 P C_v}{v} \quad (\text{205, eq 12.7(e)})$$

Assuming moderate shocks and 8 - 10 hours service, service factor,  $C_s = 1.5$  ...Tb. 12.8/Pg DHB 235

$$F_t = \frac{1000 \times 34 \times 1.5}{18.33} \quad (\text{Eq 12.24(b)})$$

• Form factor, Here  $Y = \pi Y_1$  [page 214, Below] 12.24(b)

$$= \pi \left[ 0.175 - \frac{0.95}{z_1} \right] [ \text{204, eq 12.5(e)} ]$$

$$= \pi \left[ 0.175 - \frac{0.95}{24.17} \right]$$

$$Y = 0.4263$$

$$k = b/m_a = 15$$

$$C_p = 1.15$$

$$\therefore \text{Eq. (i) yields } m_a = \sqrt{\frac{2782.32 \times 1.15}{230 \times 0.4541 \times 15 \times 0.4263}} \quad (24)$$

$$m_a = 2.19 \text{ mm}$$

Therefore, standard module,  $m_s = 2.5 \text{ mm}$  ...Tb. 12.2/Pg DHB

$$b = 15m_a = 15 \times 2.5 = 37.5 \text{ mm}$$

• Face width, Check for  $b$ :

$$\text{i.e. } b_{\min} = \frac{(1.15) \pi m_a}{\sin \beta} [ \text{213, eq 12.23(L)} ]$$

$$= \frac{(1.15) \times \pi \times 2.5}{\sin 25}$$

$$b_{\min} = 21.37 \text{ mm}$$

Since  $b > b_{\min}$ , the calculated value of  $b$  is safe.

• Check for  $\sigma_{u1}$ :

We know that,

$$F_t = \frac{\sigma_u C_v b Y m_a}{C_p} [ \text{214, 12.24(a)} ]$$

$$2782.32 = \frac{\sigma_u \times 0.4541 \times 37.5 \times 0.4263 \times 2.5}{1.15}$$

# Page ④

$\sigma_{\text{in}} = 176.31 \text{ MPa} < 230 \text{ MPa}$  ... hence safe.  
Since calculated values are < permissible values, the assumed values are satisfactory.

b. To find face width (b)  $b = 37.5 \text{ mm}$

c. To find dynamic tooth load ( $F_d$ )

$$\text{We know that, } F_d = F_t + \frac{K_3 v (C_b \cos^2 \beta + F_t) \cos \beta}{K_3 v + \sqrt{(C_b \cos^2 \beta + F_t)}}$$

here  $K_3 = 20.67 \text{ kN}$

⑥ $v > 15 \text{ m/s}$	Error, $e = 0.0230 \text{ mm}$	... Tb. 12.14 / Pg 191, DHB
For $\alpha = 20^\circ$ Stub teeth and steel - steel combination		
⑦ $e = 0.02 \text{ mm}$	$C = 237.3 \text{ N/mm}$	... Tb. 12.12 / Pg 190, DHB
$e = 0.0230 \text{ mm}$	$C = ?$	

$$\text{at } v = 18.33 \text{ m/s, } C = \frac{0.0230 \times 237.3}{0.02}$$

∴ Eq. (ii) yields...

$$F_d = 2782.32 + \frac{[20.67 \times 18.33] \times [(272.9 \times 37.5 \times \cos^2 25) + 2782.32]}{[20.67 \times 18.33] + \sqrt{[(272.9 \times 37.5 \times \cos^2 25) + 2782.32]}}$$

$$F_d = 10.71 \text{ kN}$$

d. To find dynamic tooth load ( $F_o$ )

$$\text{We know that, } F_o = \frac{d_1 e Q K}{\cos^2 \beta} \quad (214, 12.26(c))$$

• Diameter of pinion,  $d_1 = 125 \text{ mm}$

$$\bullet \text{ Ratio factor, } Q = \frac{2d_2}{d_2 + d_1} \quad (\text{page Below}, 214)$$

$$= \frac{2 \times 562.5}{562.5 + 125}$$

$$Q = 1.64$$

203, 12.15(d)

• Surface endurance limit,  $\sigma_{\text{es}} = [2.75 \times (BHN) - 70]$

$$(BHN) = \frac{BHN_1 + BHN_2}{2}$$

Referring to Tb 1.18 for medium carbon steel having  
 $\sigma_{\text{es}} = 218 \text{ MPa}$ , we have  $BHN = 120$

$$(BHN) = \frac{120 + 120}{2}$$

" 120

$$\sigma_{\text{m}} = [2.75 \times (120) - 20]$$

$$\sigma_{\text{m}} = 260 \text{ MPa}$$

- If gear  $N/N_1$  is not even, since both the gears are made of steel, assuming

$$k = 2.1 \times 10^6 \text{ MPa}, \text{ we have}$$

$$K = \frac{\Omega^2 d_1 \sin \beta}{1.4} \left[ \frac{1}{L_1} + \frac{1}{L_2} \right] \quad (\text{Eq 214, Below eq 12.26 (c)})$$

$$\begin{aligned} \tan \alpha_w &= \tan \alpha \cos \beta \\ &= \tan 20^\circ \cos 25^\circ \end{aligned}$$

$$d_1 = 18.26''$$

$$K = \frac{2d_1^2 \times \sin (18.26)}{1.4} \left[ \frac{1}{2.1 \times 10^6} + \frac{1}{2.1 \times 10^6} \right]$$

$$K = 0.1441$$

$$\therefore \text{Eq. (ii) yields, } F_{\text{w}} = \frac{125 \times 37.5 \times 1.64 \times 0.1441}{\cos^2 25}$$

$$F_{\text{w}} = 1448.65 \text{ N}$$

Since,  $F_{\text{w}} < F_{\text{w}1}$ , the pinion is subjected to rapid wear and hence has to be surface hardened to higher  $H/H_1$ .

i.e.,  $F_{\text{w}} \geq F_{\text{w}1}$

$$\frac{d_1^2 M Q K}{\cos^2 \beta} \geq 10.71 \times 10^6$$

$$\frac{125 \times 37.5 \times 1.64 \times k}{\cos^2 25} \geq 10.71 \times 10^6$$

$$k \geq 1.144$$

Therefore for steel - steel combination, having  $\alpha = 20^\circ$  and  $K \geq 1.788$ , we have

$$\begin{aligned} BHN_1 &= BH/N_1 = 300 \\ k &= \text{Axial or thrust force, } F_1 = F_1 \tan \beta \\ &= 2762.32 \times \tan 25^\circ \\ F_1 &= 1297.42 \text{ N.} \end{aligned}$$

15. A compressor running at 350 rpm is driven by a 120 kW motor running at 1400 rpm. The center distance is 400 mm and helix angle is  $25^\circ$ . The motor pinion is made of forged steel and the driven gear is cast steel design the gear using 20° FDI system.

Solution: motor/pinion speed (driver),  $N_1 = 1400$  rpm, compressor speed (driven),  $N_2 = 350$  rpm,  $P = 120$  kW, center distance = 400 mm,  $\beta = 25^\circ$ ,  $\alpha = 20^\circ$  FDI. Design:  $m_m$ ,  $b$ ,  $F_d$ ,  $F_{\text{w}}$ ,  $F_{\text{m}}$

$$\text{We know that, } i = \frac{N_1}{N_2} = \frac{d_1}{d_2} = \frac{z_2}{z_1}$$

$$i = \frac{N_1}{N_2} = \frac{1400}{350} = 4$$

$$\text{Also, center distance} = \frac{d_1 + d_2}{2}$$

$$400 = \frac{d_1 + d_2}{2}$$

$$d_1 + 4d_1 = 800$$

$$\begin{aligned} d_1 &= 160 \text{ mm and} \\ d_2 &= 640 \text{ mm.} \end{aligned}$$

#### Material properties

Pinion: forged steel:  $\sigma_{\text{u}} = 220 \text{ MPa, BHN}_1 = 200$

Gear: Cast steel:  $\sigma_{\text{u}} = 193.2 \text{ MPa, BHN}_2 = 250$

Assume  $z_1 = 20$

$$z_2 = 4z_1 = 4 \times 20 = 80$$

$$\therefore \text{Virtual number of teeth, } z_v = \frac{z}{\cos^3 \beta} \quad [\text{Eq 211, eq 12.22 (c)}]$$

$$z_v \geq 1.144$$

$$z_v = \frac{z_1}{\cos^3 \beta} = \frac{20}{\cos^3 25}$$

$$z_v = 26.86 = 27$$

$$z_2 = i z_1 = 27 \times 4 = 108$$

## page 6

Design of weaker part (pinion/gear)

$$Y = G_d Y$$

$$\text{For } \alpha = 20^\circ \text{ FDF}, Y_r = [0.154 - 0.912] / 2 = 0.1202 \quad (204, 12.5(d))$$

$$Y_1 = \left( 0.154 - \frac{0.912}{Z_{e1}} \right) = 0.154 - 0.912 / 27 = 0.1202$$

$$Y_2 = \left( 0.154 - \frac{0.912}{Z_{e2}} \right) = 0.154 - 0.912 / 108 = 0.1456$$

For pinion  $\sigma_{d1} Y_1 = 220 \times 0.1202 = 26.44 \text{ MPa}$

For Gear  $\sigma_{d2} Y_2 = 193.21 \times 0.1456 = 28.21 \text{ MPa}$

$\sigma_{d1} Y_1$  is less, Design based on pinion

(a) Module (mm)

$$M_0 = \left( \frac{F_t C_W}{G_d P_k Y} \right)^{\frac{1}{2}} [914, 12.24(b)]$$

$$\text{tohere } \vartheta = \pi d_1 N_1 = \pi \times 0.16 \times 1400 / 60 = 11.72 \text{ rad/s}$$

$$\cdot \text{For } 10 < V < 20 \text{ rad/s} \Rightarrow C_V = \frac{15.25}{15.25 + V} = \frac{15.25}{15.25 + 11.72} = 0.5654$$

$$\cdot F_b = \frac{1000 P c_s}{V^2} \quad (205, 12.7(a))$$

$$= \frac{1000 \times 150 \times 1.5}{11.72^2} = 1536 \text{ kN}$$

$$\cdot Y = \pi Y_1 = \pi \times 0.1202 = 0.3776$$

$\cdot K = b / M_0 = 15 \text{ (Assume)}, C_W = 1.15 \text{ (Table 12.21, 24)}$

$$M_0 = \frac{(15.36 \times 10^3 \times 1.15)}{220 \times 0.5654 \times 15 \times 0.3776} = \frac{5 \text{ mm}}{K_3 - 0.0167}$$

Standard Module  $m = 5 \text{ mm}$

No of teeth on pinion  $Z_{1-d}, \cos \beta / m_m$   $(12.19(a), 21)$

$$Z_1 = \frac{160 \times \cos 95}{5} = \frac{29 \text{ teeth}}{\cos 95} , Z_2 = i Z_1 = 29 \times 4 = 116$$

$$\cdot Z_{e1} = \frac{Z_1}{\cos \beta} = \frac{29}{\cos 32.5} = \frac{29}{\cos 32.5} = 38.9 = 39$$

$$\cdot Z_{e2} = i Z_{e1} = 4 \times 39 = 156$$

$$\cdot b = 15m_0 = 15 \times 5 = 75 \text{ mm}$$

$$\cdot \text{check } b \quad b_{min} = \frac{1.15 \pi m}{\sin \beta} = \frac{1.15 \pi \times 5}{\sin 32.5} = 42.74 \text{ mm} , b > b_{min}, \text{ calculated value of } b \text{ safe}$$

$$b_{min} = \frac{1.15 \pi \times 5}{\sin 32.5} = \frac{1.15 \pi \times 5}{\sin 32.5} = 42.74 \text{ mm} , b > b_{min}, \text{ calculated value of } b \text{ safe}$$

• check Module

$$F_b = \frac{G_d C_b Y_{min}}{C_W} [214, 12.24(a)]$$

$$Y = \pi Y_1 = \pi \left[ 0.154 - \frac{0.912}{27} \right] = \pi \left[ 0.154 - \frac{0.912}{27} \right] = 0.1104$$

$$15.36 \times 10^3 = \frac{G_d \times 0.5654 \times 75 \times 0.4104 \times 52}{1.15}$$

$$6d_1 = 208 \text{ mm} < 220 \text{ mm} , M_0 = 5 \text{ mm Safe}$$

(b) Force on teeth  $b = 75 \text{ mm}$

(c) Dynamic load ( $F_d$ )

$$F_d = \frac{F_t + k_g \sqrt{(C_b \cos^2 \beta + F_b) C_W \beta}}{K_3 \sqrt{1 + \sqrt{C_b \cos^2 \beta + F_b}}} \quad (214, 12.26(c))$$

$$K_3 = 0.0167$$

$\textcircled{O} \nu > 10 \text{ m/s}$	Error, $a = 0.0386 \text{ mm}$	Tb. 12.14/Pg, DHB
For $\alpha = 20^\circ$ FDL and steel - steel combination	C = 343.4 N/mm	Tb. 12.12/Pg, DHB
$\textcircled{O} c = 0.03 \text{ mm}$	C = ?	
$c = 0.0386 \text{ mm}$		

$$\text{at } v = 11.72 \text{ m/s, } C = \frac{0.0386 \times 343.4}{0.03} = 441.84 \text{ N/mm}$$

$\therefore$  Eq. (iii) yields...

$$F_d = \frac{(20.67 \times 11.72) \times [(441.84 \times 75 \times \cos^2 25) + 15.36 \times 10^3] \times \cos 25}{(20.67 \times 11.72) + \sqrt{[(441.84 \times 75 \times \cos^2 25) + 15.36 \times 10^3]}}$$

$$F_d = 36.2 \times 10^3 \text{ N}$$

#### d. To find endurance strength ( $F_s$ )

We know that,

$$\begin{aligned} \textcircled{O} \text{ BHN}_1 &= 200, \sigma_{\text{en}} = 345 \text{ MPa} & \text{Tb. 12.15/Pg, DHB} \\ F_s &= 345 \times 75 \times 0.4104 \times 5 & \downarrow \\ F_s &= 53.1 \times 10^3 \text{ N} & \text{Q28} \end{aligned}$$

Since  $F_s > F_d$ , the material is safe against static tooth load.

#### e. To find dynamic tooth load ( $F_w$ )

We know that,

$$F_w = \frac{d_1 b O K}{\cos^2 \beta} \quad [\text{Q14, 12.26(c)}]$$

\* Diameter of pinion  $d_1 = 450 \text{ mm}$

\* Ratio factor,

$$Q = \frac{2d_2}{d_2 + d_1}$$

$$= \frac{2 \times 640}{640 + 160} \quad [\text{Below eq 12.26(c), 214}]$$

$$Q = 1.6$$

\* Since given BHN combination is not available in Tb 12.16/Pg 193, DHB, assuming  $E = 2.1 \times 10^5 \text{ MPa}$ , we have

$$[\text{Q14, Below eq 12.26(c)}]$$

$$\text{Load stress factor, } K = \frac{\sigma_{\text{es}}^2 \sin \alpha}{1.4} \left[ \frac{1}{E_1} + \frac{1}{E_2} \right] \quad [\text{Q11, 12.22(b)}]$$

$$\tan \alpha_e = \tan \alpha \cos \beta$$

$$= \tan 20 \times \cos (25)$$

$$\alpha_e = 18.26 \text{ deg}$$

$$K = \frac{(345)^2 \times \sin (18.26)}{1.4} \left[ \frac{1}{2.1 \times 10^5} + \frac{1}{2.1 \times 10^5} \right]$$

$$K = 0.2537$$

$$\text{Eq. (iv) yields... } F_w = \frac{160 \times 75 \times 1.6 \times 0.2537}{\cos^2 25}$$

$$F_w = 5.9 \times 10^3 \text{ N}$$

Since  $F_{w1} < F_r$ , the pinion is subjected to rapid wear and hence has to be surface hardened to higher BHN.  
i.e.,  $F_w \geq F_r$

$$\frac{d_b Q K}{\cos^2 \beta} \geq 36.2 \times 10^3$$

$$\frac{160 \times 75 \times 1.6 \times K}{\cos^2 25} \geq 36.2 \times 10^3$$

$$K \geq 1.548$$

Therefore for steel - steel combination, having  $20^\circ$  and  $K \geq 1.548$ , we have  $\uparrow$

$$BHN_1 = 350 \text{ and } BHN_2 = 300$$

$$TB, 12.16/P_B, DMB$$

#### 4.44 PROBLEMS BASED ON UNKNOWN DIAMETER

16. A helical cast steel gear with  $30^\circ$  helix angle has to transmit  $35 \text{ kW}$  at  $1500 \text{ rpm}$ . If the gear has  $24$  teeth, determine the necessary module, pitch diameter and face width for  $20^\circ$  full depth teeth. The static stress for cast steel may be taken as  $56 \text{ MPa}$ . The width of the face may be taken as  $3$  times the normal pitch. What would be the end thrust on the gear? The tooth factor for  $20^\circ$  full depth involute gears may be taken as  $0.154 - \frac{0.912}{Z_e}$ , where  $Z_e$  is the equivalent number of teeth.

[IITJU - June 2009 - 16 Marks; May/June 2010 - 15 Marks]

Solution: helix angle,  $\beta = 30^\circ$ ,  $P = 35 \text{ kW}$ , pinion speed,  $N_1 = 1500 \text{ rpm}$ ,  $Z_1 = 24$  teeth,  $\alpha = 30^\circ$ . FDI teeth,  $b = 3p_n$ ,  $\sigma_{s1} = \sigma_{s2} = 56 \text{ MPa}$ ,  $y = 0.154 - \frac{0.912}{Z_e}$

a.  $m_a = ?$    b.  $b = ?$    c. End thrust,  $F_t = ?$

Identifying weaker part (pinion/gear)

We know that strength factor =  $\sigma_d/y$

Since both pinion and gear are made of same material, the pinion is weaker, i.e.,  $\sigma_{d1}/y_1 < \sigma_{d2}/y_2$   
Hence design is based on pinion.

- Virtual number of teeth,  $Z_v = \frac{z}{\cos^3 \beta} [211, 12.220]$

$$Z_{v1} = \frac{z_1}{\cos^3 \beta} = \frac{24}{\cos^3 30}$$

$$z_1 = 37$$

#### a. To find module ( $m_a$ )

Since the diameter is unknown, we have

$$m_a = \left[ \frac{2M_1 C_u \cos \beta}{\sigma_d C_d k Y Z} \right]$$

$$P = \frac{2\pi N_1 M_1}{60}$$

$$\frac{1}{3} [214, 12.24(b)]$$

$$35 \times 10^3 = \frac{2\pi \times 1500 \times M_t}{60}$$

$$M_t = 222.82 \text{ N-m} = 222.82 \times 10^3 \text{ N-mm}$$

$$\text{but, } k = b/m_n$$

i.e.

$$b = 3p_n$$

$$\text{use } \sigma_c = \tau_m = \pi m_n [211, 12.19(a)]$$

... (data)

$$k = \frac{b}{m_n} = 3\sqrt{\pi} m_n$$

$$k = \frac{b}{m_n} = \frac{3\sqrt{\pi} m_n}{m_n}$$



$$\boxed{k = 3\pi}$$

• Assume,

$$\begin{aligned} &\text{Assume,} \\ &\text{Form factor,} \end{aligned}$$

$$C_v = 1.15$$

..

$$\text{..Tb. 12.21/Tg 24, DHB}$$

..(data)

$$\begin{aligned} &= \pi \left[ 0.154 - \frac{0.912}{z_{41}} \right] \\ &= \pi \left[ 0.154 - \frac{0.912}{37} \right] \end{aligned}$$

$$\therefore \text{Eq. (6) yields... } m_n = \frac{2 \times 222.82 \times 10^3 \times 1.15 \times \cos 30}{56 \times 0.5 \times 3\pi \times 0.4064 \times 24}$$

(here z - weaker member)

$$m_n = 5.57$$

Therefore, standard module  $m_s = 6 \text{ mm}$

$$\begin{aligned} &\text{..Tb. 12.2/Tg 33q, DHB} \\ &\therefore z_1 = 24 \end{aligned}$$

$$\therefore \text{PCD of pinion, } d_1 = \frac{m_s z_1}{\cos \beta} = \frac{6 \times 24}{\cos 30} = 166.27 \text{ mm} = 168 \text{ mm} [211, 12.19(a)(c)]$$

- Face width,
- b =  $3p_n = 3(\pi m_n) = 3 \times \pi \times 6 = 56.55 \text{ mm} = 57 \text{ mm}$
- b. Check for b:

$$\begin{aligned} \text{i.e. } b_{\min} &= \frac{(1.15)\pi m_n}{\sin \beta} [213, 12.23(c)] \\ &= \frac{(1.15) \times \pi \times 6}{\sin 30} \end{aligned}$$

Since  $b > b_{\min}$ , the calculated value of is safe.

- Since  $C_v$  was assumed, Check for  $\sigma_{d1}$ . We know that  $F_t = \frac{\sigma_{d1} C_v b m_n}{C_w} [214, 12.24(a)]$

$$\therefore v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.168 \times 1500}{60} = 13.19 \text{ m/s}$$

$$\therefore C_v = \frac{15.25}{15.25 + v} [214, 12.25(c)]$$

• For  $10 < v < 20 \text{ m/s.}$

$$\begin{aligned} F_t &= \frac{2 M_t}{d} \\ &= \frac{2 \times 222.82 \times 10^3}{168} \quad (\text{here } d - \text{weaker member}) \\ F_t &= 2652.62 \text{ N} \end{aligned}$$

$$\begin{aligned} \sigma_{d1} &= \frac{F_t \tan \beta}{2552.62 \times \tan 30} \\ &= 1531.5 \text{ N.} \end{aligned}$$

Since calculated values are < permissible values, the assumed values are satisfactory.

### c. Axial or thrust force

$$\begin{aligned} F_a &= F_t \tan \beta \quad [212, \text{ Fig 12.7}] \text{ Read Page 258} \\ &= 2552.62 \times \tan 30 \\ F_a &= 1531.5 \text{ N.} \end{aligned}$$

• 17. A helical gear is required to transmit 35 kW at 1500 rpm. The helix angle is  $30^\circ$  and  $20^\circ$  FDI system is used. Both gears are made of same material having design stress of 56 MPa and BHN = 200. The starting torque is 20% greater than the mean torque. The drive is to be safe for continuous operation. The speed reduction is 3:1. Specify the details of the drive. Use 24 teeth on pinion.

Solution: P = 35 kW, pinion speed,  $N_1 = 1500 \text{ rpm}$ , helix angle,  $\beta = 30^\circ$ ,  $\alpha = 20^\circ$ , FDI teeth,  $\sigma_{d1} = \sigma_{d2} = 56 \text{ MPa}$ , BHN<sub>1</sub> = BHN<sub>2</sub> = 200,  $M_{t\max} = 1.2 M_t$ , i = 3:1, service - continuous operation,  $z_1 = 24$  teeth.

### Identifying weaker part (pinion/gear)

We know that strength factor =  $\sigma_d / \sigma_y$ . Since both pinion and gear are made of same material, the pinion is weaker, i.e.  $\sigma_{d1} / \sigma_y < \sigma_{d2} / \sigma_y$ .

Hence design is based on pinion.

$$\begin{aligned} \text{• Virtual number of teeth, } z_r &= \frac{z}{\cos^3 \beta} [211, 12.22(a)] \\ z_{41} &= \frac{z_1}{\cos^3 \beta} = \frac{24}{\cos^3 30} \\ z_1 &= 37 \end{aligned}$$

a. To find module ( $m_n$ )

$$\left[ \frac{2M_{t,\max} C_e \cos \beta}{\sigma_y C_v k Y Z} \right] \frac{Y}{Z} \left[ P214, 12^{\circ}24^{\circ}(b) \right]$$

Since the diameter is unknown,  $m_n = \frac{(1.15) \times \pi \times 6}{\sin 29}$

$b_{min} = 43.35 \text{ mm}$

But  $M_{t,\max} = 1.2 M_t$

$$P = \frac{2\pi N_1 M_t}{60}$$

$$35 \times 10^3 = \frac{2\pi \times 1500 \times M_t}{60}$$

$$M_t = 222.82 \text{ N-m} = 222.82 \times 10^3 \text{ N-mm}$$

$$M_{t,\max} = 1.2 \times 222.82 \times 10^3$$

$$M_{t,\max} = 267.38 \times 10^3 \text{ N-mm}$$

• Assume

$$C_v = 0.5$$

• For continuous operation (data),  $C_w = 1.15$

• Form factor,  $Y = \pi y_1$   $\left[ 214, \text{below gear } [2.24(b)] \right]$

$$= \pi \left[ 0.154 - \frac{0.912}{z_1} \right] \left[ 204, \text{below } [2.24(c)] \right]$$

$$= \pi \left[ 0.154 - \frac{0.912}{37} \right]$$

$$Y = 0.4064$$

$$\therefore \text{Eq. (i) yields... } m_n = \frac{2 \times 267.38 \times 10^3 \times 1.15 \times \cos 30}{56 \times 0.5 \times 15 \times 0.4064 \times 24}$$

$$m_n = 5.06 \text{ mm}$$

Therefore, standard module,  $m_n = 6 \text{ mm}$

$$\therefore \text{PCD of pinion, } d_1 = \frac{m_n z_1}{\cos \beta} = \frac{6 \times 24}{\cos 30} = 166.27 \text{ mm} = 168 \text{ mm } \left( 214, 12^{\circ}19^{\circ}(b) \right)$$

We know that,  $i = \frac{d_2}{d_1}$

$$3 = \frac{d_2}{168}$$

PCD of gear,  $d_2 = 504 \text{ mm}$

• Face width,

Check for  $b$ :

$$\text{i.e. } b_{min} = \frac{(1.15) \times m_n}{\sin \beta} \left[ 213, 12^{\circ}23^{\circ}(b) \right]$$

$$\text{at } v = 13.19 \text{ m/s, } C = \frac{0.0330 \times 3434}{0.03} = 377.63 \text{ N/mm}$$

Since  $b > b_{min}$ , the calculated value of  $b$  is safe.

• Check for  $\sigma_{yt}$ :

We know that,  $F_t = \frac{\sigma_y C_v Y m_n}{C_e}$

$$P = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.168 \times 1500}{60} = 13.19 \text{ m/s}$$

$$\bullet \text{ For } 10 < v < 20 \text{ m/s, } C_v = \frac{15.25}{15.25 + v}$$

$$= \frac{15.25}{15.25 + 13.19}$$

$$C_e = 0.536$$

$$F_t = \frac{2M_{t,\max}}{d} \times C_s \left[ 206, 12^{\circ}8^{\circ}(a) \right]$$

$$\bullet \text{ For continuous operation, assuming light shocks, } C_s = 1.5 \left[ \text{TB, 12.8/Pg, DHB} \right]$$

$$= \frac{2 \times 267.38 \times 10^3}{168} \times 1.5$$

$$F_t = 4774.64 \text{ N}$$

$$\therefore \text{Eq. (ii) yields... } 4774.64 = \frac{0.41 \times 0.536 \times 90 \times 0.4064 \times 6}{1.15}$$

$$\sigma_{yt} = 46.67 \text{ MPa} < 56 \text{ MPa}$$

...hence safe.

b. To find face width (b)

b = 90 mm

c. To find dynamic tooth load ( $F_d$ )

$$F_d = F_t + \frac{K_3 p (C_b \cos^2 \beta + F_t) \cos \beta}{K_3 p + \sqrt{C_b \cos^2 \beta + F_t}}$$

here  $K_3 = 20.67$

$\left[ \text{Prop. 214, } 12^{\circ}26^{\circ}(a) \right]$

Wk 237

$\left[ \text{Prop. 214, } 12^{\circ}14^{\circ}/\text{Pg, DHB} \right]$

$\left[ \text{Prop. 214, } 12^{\circ}26^{\circ}(a) \right]$

$$\text{at } v = 13.19 \text{ m/s, } C = \frac{0.0330 \times 3434}{0.03} = 377.63 \text{ N/mm}$$

$$F_d = 4774.64 + \frac{(20.67 \times 13.19) \times [(377.63 \times 90 \times \cos^2 30) + 4774.64]}{(20.67 \times 13.29) + [(377.63 \times 90 \times \cos^2 30) + 4774.64]}$$

$$F_d = 20.78 \times 10^3 \text{ N}$$

d. To find endurance strength ( $F_e$ )

We know that,

$$\text{@ BHN}_1 = 200, \sigma_{en} = 345 \text{ MPa}$$

$$F_e = 345 \times 90 \times 0.4064 \times 6$$

$$F_e = 75.71 \text{ kN}$$

Since  $F_d > F_e$ , the material is safe against static tooth load.

e. To find dynamic tooth load ( $F_w$ )

We know that,

$$F_w = \sigma_d b Y m_r$$

$$\bullet \text{Diameter of pinion, } d_1 = 168 \text{ mm}$$

• Ratio factor,

$$Q = \frac{2d_2}{d_1 + d_2}$$

$$= \frac{2 \times 504}{504 + 168}$$

$$Q = 1.5$$

• For  $\alpha = 20^\circ$  and steel - steel combination having  $\text{BHN}_1 = \text{BHN}_2 = 200$

$$K = 0.539 \quad \text{... Eq. 214, Below eq. 12.26 (c)}$$

$$F_w = \frac{168 \times 90 \times 1.5 \times 0.539}{\cos^2 30}$$

∴ Eq. (e) yields...

$$F_w = 16.3 \text{ kN}$$

Since  $F_w < F_d$ , the pinion is subjected to rapid wear and hence has to be surface hardened to higher BHN.

i.e.  $F_w \geq F_d$

$$\frac{d b Q K}{\cos^3 \beta} \geq 20.78 \times 10^3$$

$$\frac{168 \times 90 \times 1.5 \times K}{\cos^2 30} \geq 20.78 \times 10^3$$

$$K \geq 0.687$$

Therefore, for steel - steel combination, having  $\alpha = 20^\circ$  and  $K \geq 0.687$ , we have

$$\text{BHN}_1 = 250 \text{ and BHN}_2 = 200 \quad \text{... Eq. 214, Below}$$

16. Design a helical gear pair to transmit a power of 15 kW from a shaft rotating at 1000 rpm to another shaft to be run at 360 rpm. Assume involute profile with a pressure angle of  $20^\circ$ . The material of the pinion is forged steel SAE 1030 whose  $\sigma_u = 172.375 \text{ MPa}$  and the material for gear is cast steel 0.2% C untreated with  $\sigma_u$

= 137.4 MPa. The gears operate under a condition of medium shocks for a period of 10 hours per day. Check for dynamic load, if load factor  $C = 560 \text{ N/mm}$  and also for wear load.

[VTU - Dec. 2010 - 16 Marks]

Solution:  $P = 15 \text{ kW}$ , pinion speed,  $N_1 = 1000 \text{ rpm}$ , driven speed,  $N_2 = 360 \text{ rpm}$ , helix angle,  $\alpha = 20^\circ$ , FDI teeth,  $\sigma_{en} = 172.375$ ,  $\sigma_u = 137.4 \text{ MPa}$ , service - medium shock, continuous operation 10 h per day,  $C = 580 \text{ N/mm}$ .

$$i = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

$$i = \frac{1000}{360} = 2.78$$

We know that,

$$\dots \text{Eq. 214, Below eq. 12.16 (f), DHB}$$

Identifying weaker part (pinion/gear)

We know that strength factor  $\sigma_u/Y$

• Assume

• Assume

$z_1 = 20$

$z_2 = iz_1 = 2.78 \times 20 = 55.6 \approx 56$

and

$$z_1 = \frac{z_1}{\cos^3 \beta} = \frac{20}{\cos^3 25}$$

$$z_2 = iz_1 = 27 \times 2.78 = 75$$

$$\bullet \text{Virtual number of teeth, } z_v = \frac{z}{\cos^3 \beta} \quad \text{[Page 211, Eq. 12.22(a)]}$$

$$\bullet \text{For } \alpha = 20^\circ \text{ FDI teeth, } z_v = 0.154 - \frac{0.912}{27} \quad \text{[Page 204, Eq. 12.57(d)]}$$

$$\text{i.e. } y_1 = 0.154 - \frac{0.912}{27} \text{ or } y_1 = 0.154 - \frac{0.912}{27} = 0.1202$$

$$y_2 = 0.154 - \frac{0.912}{27} \text{ or } y_2 = 0.154 - \frac{0.912}{75} = 0.1418$$

$$\bullet \text{for pinion, } \sigma_d \cdot y_1 = 172.375 \times 0.1202 = 20.72 \text{ MPa}$$

$$\bullet \text{for gear, } \sigma_d \cdot y_2 = 137.4 \times 0.1418 = 19.48 \text{ MPa}$$

Since  $\sigma_d \cdot y_2$  is less, design is based on gear.

a. To find module ( $m_r$ )

$$\text{Since the diameter is unknown, we have } m_r = \sqrt{\frac{2 M C_u \cos \beta}{\sigma_d C_s K Y}} \quad \text{[Page 214, Eq. 12.24(b)]}$$

$$P = \frac{2 \pi N_1 M_1}{60} \quad \text{... Eq. 214, Below}$$

$$15 \times 10^3 = \frac{2 \pi \times 360 \times M_1}{60}$$

- Assume,  $M_t = 397.88 \text{ N-m} = 397.88 \times 10^3 \text{ N-mm}$
- Assume,  $k = M/m_n = 15$
- For continuous operation (data),  $C_o = 1.15$
- Form factor,  $Y = \pi Y_c / \pi Y_{eq} = 12.21 / 12.54 = 0.9455$

$$Y = 0.9455$$

$$\text{Eq. (i) yields... } m_n = \frac{2 \times 397.88 \times 10^3 \times 1.15 \times \cos 25}{137.34 \times 0.5 \times 15 \times 0.4455 \times 56}$$

Therefore standard module,  $m_n = 4 \text{ mm}$   
 $\text{PCD of gear, } d_2 = 15m_n = 15 \times 4 = 60 \text{ mm}$

$$\left[ \text{page 211, eq 12.23(a)} \right]$$

$$\text{Face width, } b = 15m_n = 15 \times 4 = 60 \text{ mm}$$

Check for  $b$ :

$$\text{i.e., } b_{min} = \frac{(1.15) m_n}{\sin \beta} = \frac{(1.15) \pi \times 4}{\sin (25)} = 34.2 \text{ mm}$$

Since  $b > b_{min}$ , the calculated value of  $b$  is safe.

Check for  $\sigma_x$ :

$$F_t = \frac{\sigma_x C_o b Y_m}{C_w}$$

$$v = \frac{\pi d_2 N_2}{60} = \frac{\pi \times 0.250 \times 360}{60} = 4.71 \text{ m/s}$$

$$\text{For } v < 5 \text{ m/s, } C_v = \frac{4.58}{4.58 + v} = \frac{4.58}{4.58 + 4.71} = 0.4928$$

$$\left[ \text{page 214, eq 12.24(a)} \right]$$

We know that,

$$F_t = \frac{2M_t}{d} \times C_v$$

For medium shocks and operating 10 h per day,  $C_v = 1.5$

$$= \frac{2 \times 397.88 \times 10^3}{250} \times 1.5$$

$$F_t = 4774.65 \text{ N}$$

$$\therefore \text{Eq. (ii) yields... } 4774.56 = \frac{\sigma_{eq} \times 0.4928 \times 60 \times 0.4455 \times 4}{1.15}$$

$$\sigma_{eq} = 104.21 \text{ MPa} < 137.34 \text{ MPa}$$

...hence safe.

- To find face width (b)
- b = 60 mm
- c. To find dynamic tooth load ( $F_d$ )

$\left[ \text{page 214, eq 12.26(a)} \right]$

We know that,

$$F_d = F_t + \frac{K_d v (C_b \cos^2 \beta + F_t) \cos \beta}{K_b v + \sqrt{C_b \cos^2 \beta + F_t}}$$

here  $K_d = 20.67$

$$C = 580 \text{ N/mm}$$

$$F_d = 4774.56 + \frac{(20.67 \times 4.71) \times [(580 \times 60 \times \cos^2 25) + 4774.56] \times \cos 25}{(20.67 \times 4.71) + \sqrt{[580 \times 60 \times \cos^2 25] + 4774.56}}$$

$$F_d = 4774.56 \text{ N}$$

d. To find endurance strength ( $F_e$ )  
 We know that,

- For forged steel-pinion,  $BHN_1 = 150$  and gear-cast steel 0.2% C untreated,  $BHN_1 = 180$
- Surface endurance limit,  $\sigma_{es} = [2.75 \times (BHN) - 70]$

$$(BHN) = \frac{BHN_1 + BHN_2}{2}$$

$\left[ \text{page 214, eq 12.26(b)} \right]$

We know that,

$$F_e = \sigma_{es} b Y_m$$

$$\therefore \begin{aligned} &\text{For forged steel-pinion, } BHN_1 = 150 \text{ and} \\ &\text{gear-cast steel 0.2% C untreated, } BHN_1 = 180 \\ &\text{Surface endurance limit, } \sigma_{es} = [2.75 \times (BHN) - 70] \\ &(BHN) = \frac{BHN_1 + BHN_2}{2} \end{aligned}$$

Since  $F_d > F_e$ , the material is safe against static tooth load.

e. To find dynamic tooth load ( $F_w$ )  
 We know that,

$$F_w = \frac{q_b Q K}{\cos^2 \beta}$$

- Diameter of pinion,  $d_1 = 90 \text{ mm}$
- Ratio factor,  $Q = \frac{2d_2}{d_1 + d_2}$

$$\therefore \begin{aligned} &\text{For medium shocks and operating 10 h per day, } C_v = 1.5 \\ &\text{...Tb, 12.8/Pg, DHB} \\ &= \frac{2 \times 250}{250 + 90} \\ &= 250 \text{ N} \end{aligned}$$

$$Q = 1.47$$