

MODULE I

Syllabus: Introduction to circuit variables and circuit elements. Review of Kirchhoff's Laws. Independent and dependent sources. Source Transformation.

Network topology, Network graphs, Trees, Incidence Matrix, Tie-set matrix and cut-set matrix.

Solution Methods applied to dc and phasor circuits: Mesh and node analysis of network containing independent and dependent sources.

Introduction to circuit Variables and circuit elements :-

Network: It is an interconnection of elements or electronic components and there is a source provides power to the network and there is a load that dissipates power.

Two types of elements

1) passive

↓
Dissipates power in the form of heat.
eg: Resistor, capacitor, inductor

2) Active

Eg: All semiconductor devices, current and voltage source etc.

Network contain only passive elements is known as passive network. Similarly network contain only active elements is known as

active network.

Circuit Elements

1. Voltage: potential difference in electrical terminology is known as voltage and is denoted either by V or v . It is expressed in terms of energy (W) per unit charge (Q)

$$\text{i.e. } V = \frac{W}{Q} \quad \text{or} \quad v = \frac{dW}{dq}$$

2. current: The movement of e^- s from one end of the material to the other end constitutes an electric current denoted by I or i

$$I = \frac{Q}{t}$$

$$\text{or } i = \frac{dq}{dt}$$

3. Power and Energy

Energy is the capacity for doing work

Power is the rate of change of energy and is denoted by either P or p . If certain amount of energy is used over a certain length of time

$$\text{then power } (p) = \frac{\text{energy}}{\text{time}} = \frac{W}{t}$$

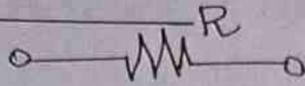
$$P = \frac{dW}{dt}$$

We can also write

$$P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt}$$

$$= \frac{dW}{dt} = v \times i = v_i W$$

4. Resistance ρ



According to Ohm's law the current is directly proportional to the voltage and inversely proportional to the total circuit resistance $I = V/R$; $i = V/R$

We can write the above equation in terms of charge as follows

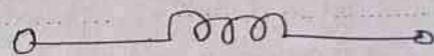
$$V = R \frac{dq}{dt} \quad \text{or} \quad i = \frac{V}{R} = G V$$

$$P = V i = (iR) i = i^2 R$$

Energy lost in a resistance in time t is given by

$$W = \int_0^t P dt = P t = i^2 R t = \frac{V^2}{R} t$$

5. Inductance ρ



A change in current produces change in the electromagnetic field which induces a voltage across the coil corresponds according to Faraday's law of electromagnetic induction. Unit henry

The current voltage relation

$$V = L \frac{di}{dt}$$

We can rewrite the above equation

$$di = \frac{1}{L} V dt$$

Integrating both sides

$$\int_0^t di = \frac{1}{L} \int_0^t V dt$$

$$i(t) - i(0) = \frac{1}{L} \int_0^t v dt$$

$$i(t) = \frac{1}{L} \int_0^t v dt + i(0)$$

The power absorbed by inductor is

$$P = vi = Li \frac{di}{dt} \text{ watts}$$

Energy stored by the inductor is

$$W = \int_0^t P dt$$

$$= \int_0^t Li \frac{di}{dt} dt = \frac{Li^2}{2}$$

4/116

6 Capacitance: The amount of charge per unit voltage that is capacitor can store is its capacitance denoted by C. unit farad

$$C = \frac{Q}{V} = C = Q/V$$

in terms of current

$$i = C \frac{dv}{dt} \quad \left(\because i = \frac{dq}{dt} \right)$$

$$dv = \frac{1}{C} i dt$$

Integrating Both sides

$$\int_0^t dv = \frac{1}{C} \int_0^t i dt$$

$$v(t) - v(0) = \frac{1}{C} \int_0^t i dt$$

$$V(t) = \frac{1}{C} \int_0^t i dt + v(0)$$

power absorbed by the capacitor

$$P = vi = v_c \frac{dv}{dt}$$

energy stored

$$W = \int_0^t P dt = \int_0^t v_c \frac{dv}{dt} dt$$

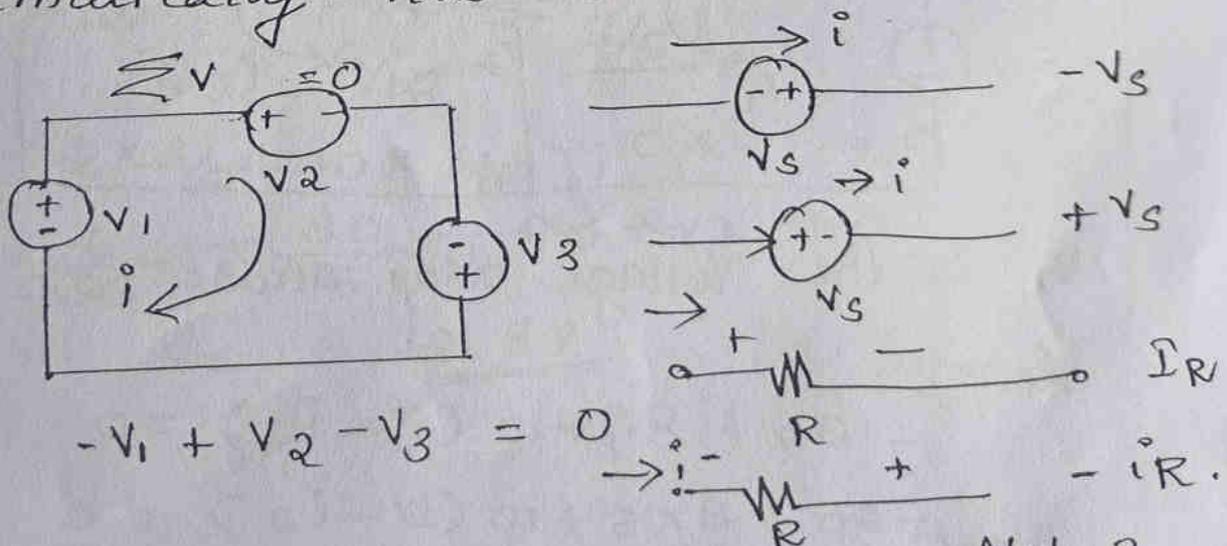
$$W = \frac{1}{2} CV^2$$

Review of Kirchhoff's law

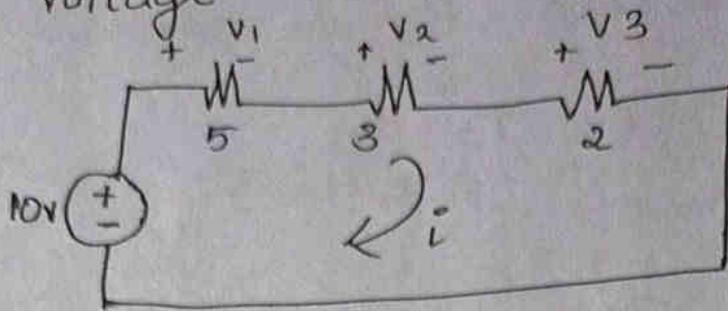
a) Kirchoff's Voltage Law : This law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time.

Kirchoff's Voltage Law can be explained by the light of conservation of energy. The

Mathematically KVL can be written as



Qn Find the current through the network and Voltage across each resistor



$$-10 + 5i_1 + 3i_1 + 2i_1 = 0$$

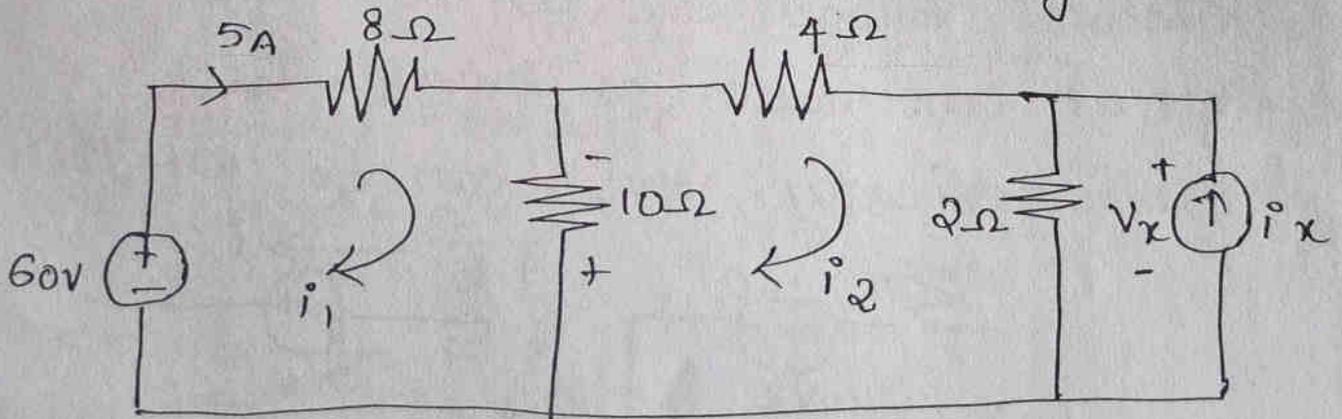
$$10 = 10i_1$$

$$i_1 = \frac{10}{10} = \underline{\underline{1A}}$$

$$10 = 5 \times 1 + 3 \times 1 + 2 \times 1 = 10$$

$$V_1 = \underline{\underline{5V}}; V_2 = \underline{\underline{3V}}; V_3 = \underline{\underline{2V}}$$

2) Determine V_x in the following circuit



V_x is the Voltage drop across 2Ω Resistor

loop I :

$$-60 + 8i_1 + 10(i_1 - i_2) = 0$$

$$-60 + 8 \times 5 + 10(5 - i_2) = 0$$

$$-60 + 40 + 50 - 10i_2 = 0$$

$$-20 + 50 - 10i_2 = 0$$

$$30 = 10i_2$$

$$i_2 = \frac{30}{10} = \underline{3A}$$

$$\text{Loop II} : 4i_2 + 10(i_2 - i_1) + 2i_2 = 0$$

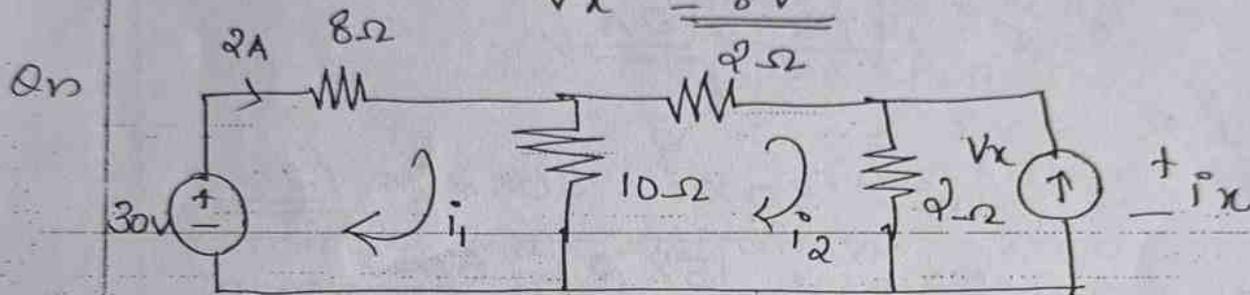
$$10i_2 - 10i_1 + 4i_2 + v_x = 0$$

$$10 \times 3 - 10 \times 5 + 4 \times 3 + v_x = 0$$

$$30 - 50 + 12 + v_x = 0$$

$$-20 + 12 + v_x = 0$$

$$v_x = \underline{8V}$$



$$\text{Loop I} : -30 + 8 \times 2 + 10(2 - i_2) = 0$$

$$-30 + 16 + 20 - 10i_2 = 0$$

$$-10 + 16 - 10i_2 = 0$$

$$6 = 10i_2$$

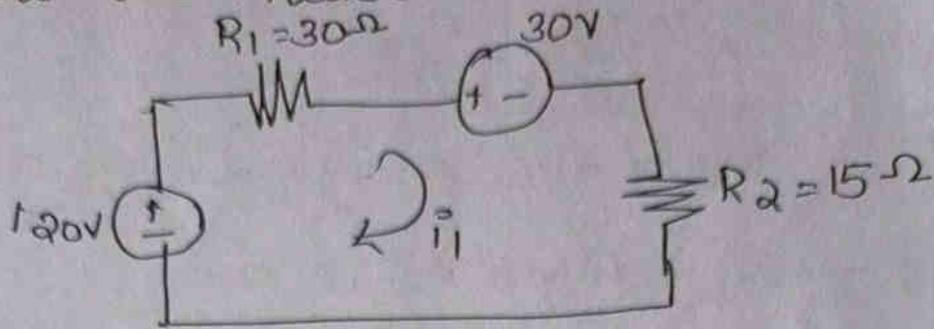
$$i_2 = \frac{6}{10} = \underline{\frac{3}{5}A}$$

$$\text{Loop II} : 2 \times \frac{3}{5} + 10 \times \frac{3}{5} - 10 \times 2 = 0$$

$$= \frac{6}{5} + \frac{30}{5} - 20 + v_x = 0$$

$$v_x = \underline{12.8V}$$

Q Following circuit compute the power in the two resistors



$$-120 + 30i_1 + 30 + 15i_1 = 0$$

$$-90 + 45i_1 = 0$$

$$45i_1 = 90$$

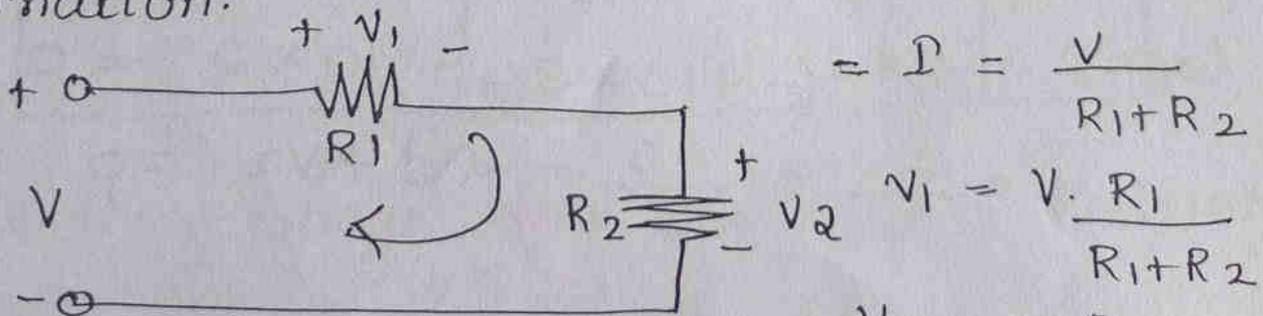
$$i_1 = \frac{90}{45} = \underline{\underline{2A}}$$

$$P = I \cdot R = V \cdot I =$$

$$P \rightarrow R_1 = 120 = 30 \times i_1 = 30 \times 2^2 = \underline{\underline{120W}}$$

$$P_{R_2} = 15 \times i_1^2 = 15 \times 2 = \underline{\underline{60W}}$$

Voltage Division Rule is used to express the voltage across one of the several series resistors in terms of voltage across combination.



$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = V \cdot \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \cdot \frac{R_2}{R_1 + R_2}$$

N/A

2) KIRCHOFF'S CURRENT LAW (KCL) NODAL LAW

Based on conservation of charge

At any instant of time, the algebraic sum of current at a node is zero

$$\text{Mathematically } \sum_{\text{node } n} i(t) = 0$$

If the current entering a node are assigned positive sign, then the currents leaving the node will be assigned negative sign or vice versa.

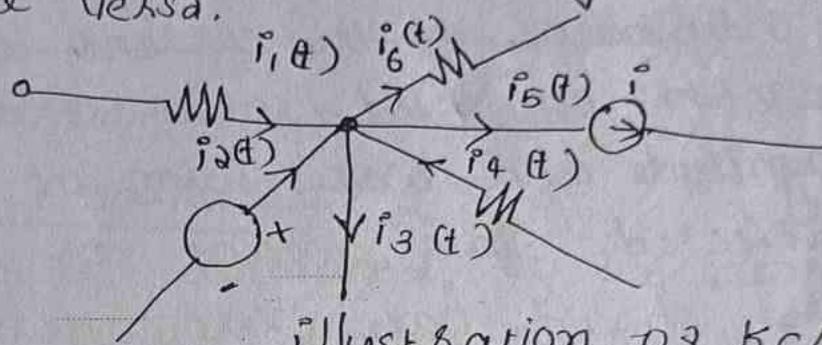


Illustration of KCL

Consider a portion of some network as shown in figure. Current $i_1(t)$, $i_2(t)$ and $i_4(t)$ are entering the node n . Hence they are assigned positive sign. Current $i_3(t)$, $i_5(t)$, $i_6(t)$ are leaving n and hence they are assigned negative sign. Applying KCL at node n

$$i_1(t) + i_2(t) + i_4(t) - i_3(t) - i_5(t) - i_6(t) = 0$$

$$i_1(t) + i_2(t) + i_4(t) = i_3(t) + i_5(t) + i_6(t)$$

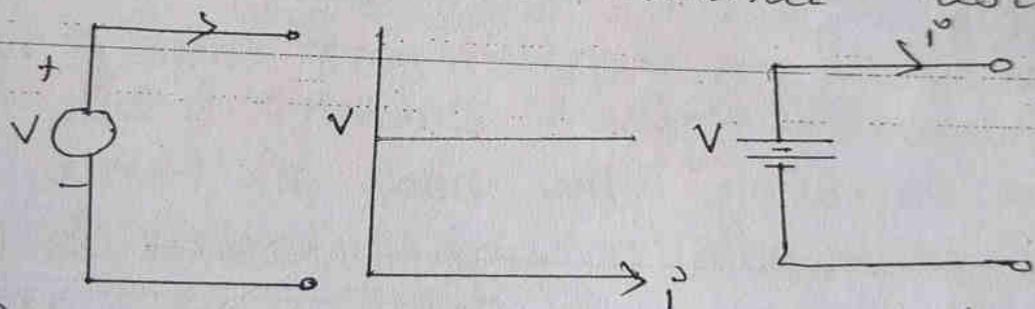
Sum of the incoming = sum of the outgoing current

Thus, an alternative form of KCL can be stated as follows: "At any instant of time, the sum of all the currents flowing into a node is equal to the sum of all the current leaving the same node."

Independent and Dependent Sources:

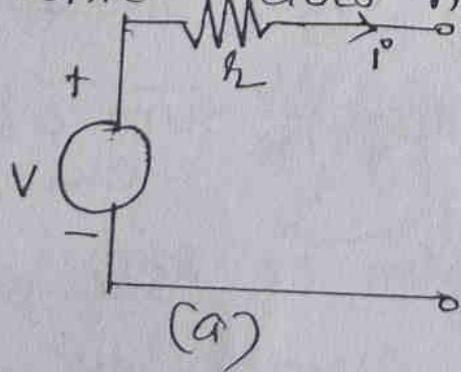
Independent Sources: Mainly there are two types of sources (i) voltage source (ii) current source. Again these are classified as ideal or independent and dependent sources.

Ideal voltage source: An ideal voltage source is a two terminal device whose terminal voltage is independent of the current drawn by the networks connected to its terminals. Both the magnitude and wave form of voltage remain unaffected. This means an ideal voltage source should have zero internal resistance.

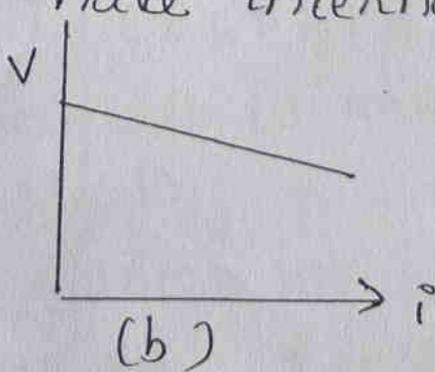


(a) Ideal voltage source (ii) $V-i$ chara (iii) dc source.

However in actual practice there is no voltage source which does not have internal resistance.

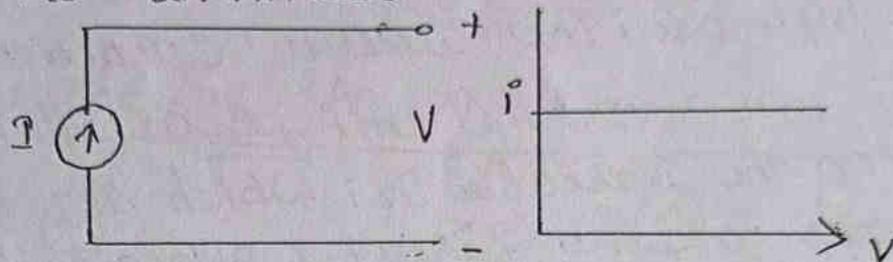


(a) voltage source



(b) $V-i$ characteristic

Ideal current source :- It is a two terminal device which delivers a constant current to the network connected across its terminals irrespective of the elements of the network i.e. the current is independent of the voltage across its terminals.

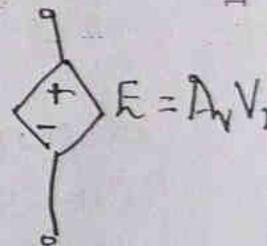


Ideal-current source chara

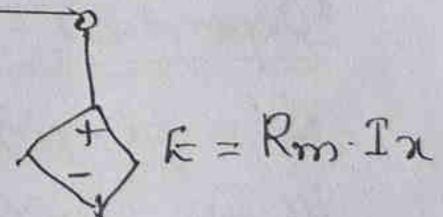
Dependent sources :-

The power developed by the dependent source in a circuit depends on the voltage and current in some part of the same circuit.

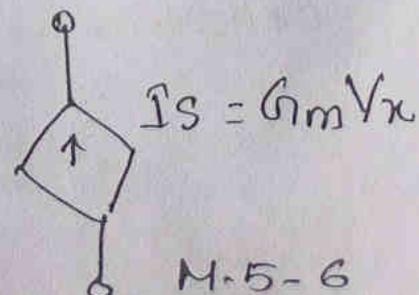
1. Voltage controlled Voltage source
(V C V S)



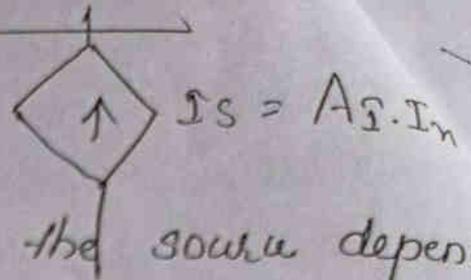
2. Current controlled Voltage source
(C C V S)



3. Voltage controlled current source
(V C C S)



4. Current controlled current source
(CCCS)



In VCVs, the emf E of the source depends on the variable V_x , which is a voltage in some part of the circuit (practically Amp^2)

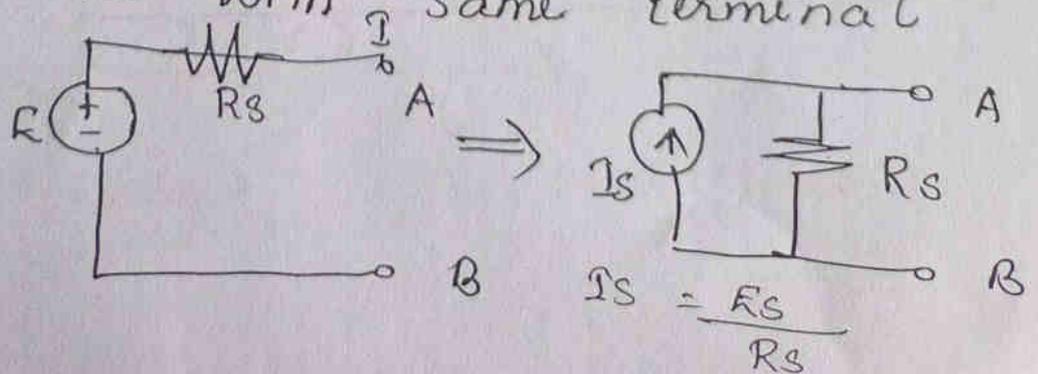
In CCVS, the emf E of the source depends on the variable I_x , which is current in some part of the circuit (practically resistance which converts current to voltage)

In VCCS, the source current I_s depends on the variable V_x , which is a voltage in some part of the circuit (practically conductance which converts voltage to current)

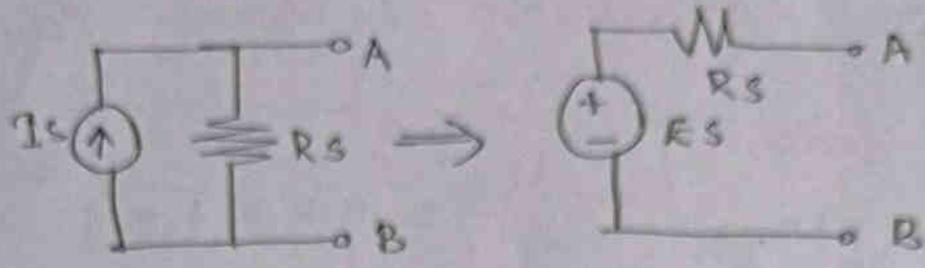
In CCCS the source current I_s depends on the variable I_x , which is a current in some part of the circuit (practically current Amp²).

Dependent Source Transformation:-

The practical voltage source can be converted to an equivalent practical current source and vice versa with same terminal behaviour.

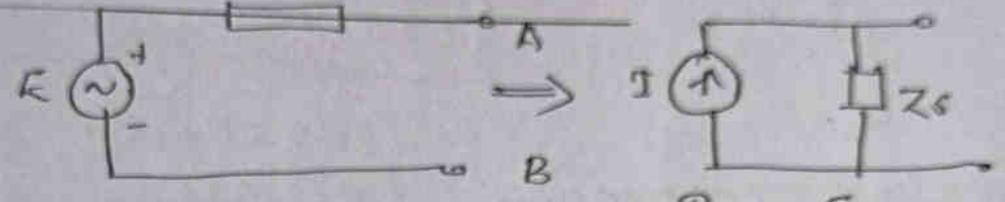


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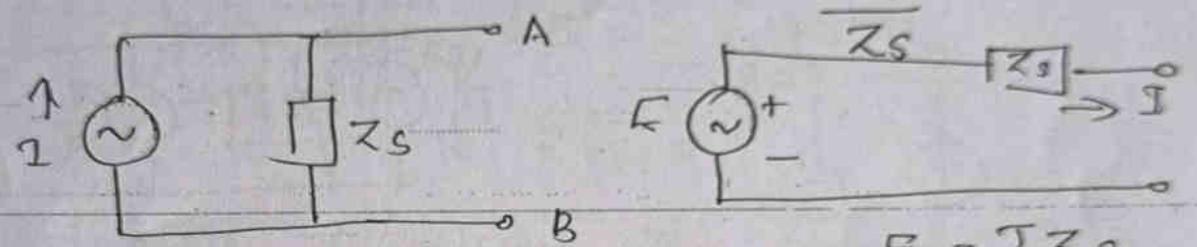


$$E_s = I_s \cdot R_s$$

Conversion of AC sources.

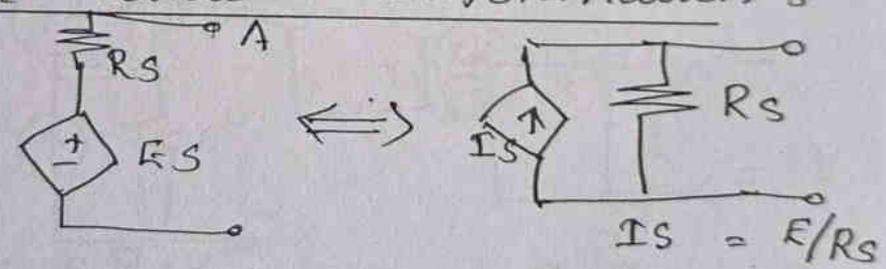


$$I_s = \frac{E_s}{Z_s}$$



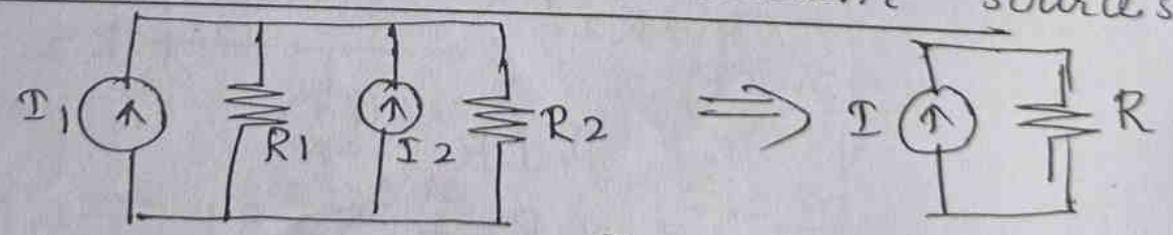
$$E_s = I_s Z_s$$

Dependent Source Transformation :-



$$I_s = E_s / R_s$$

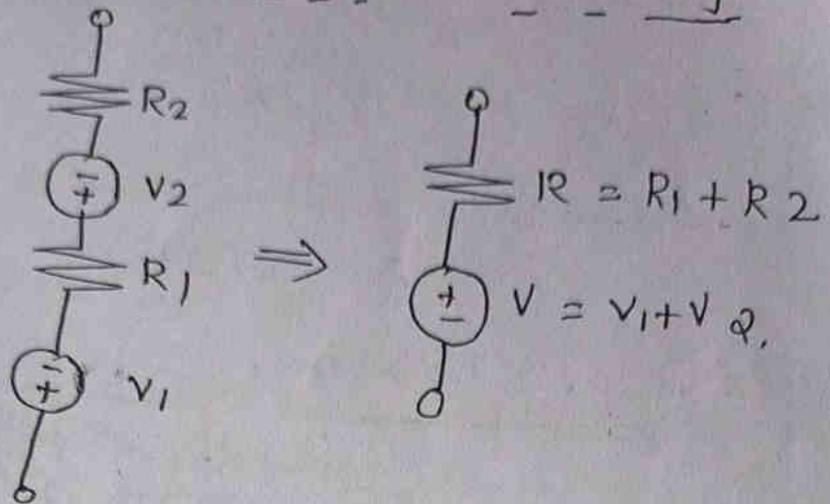
Parallel connection of current sources :



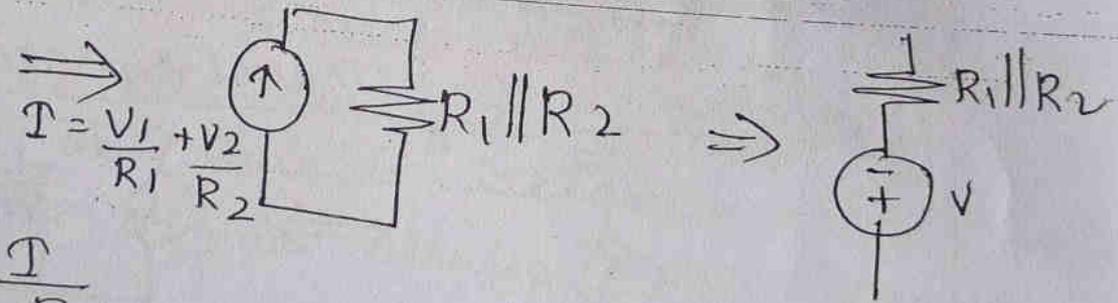
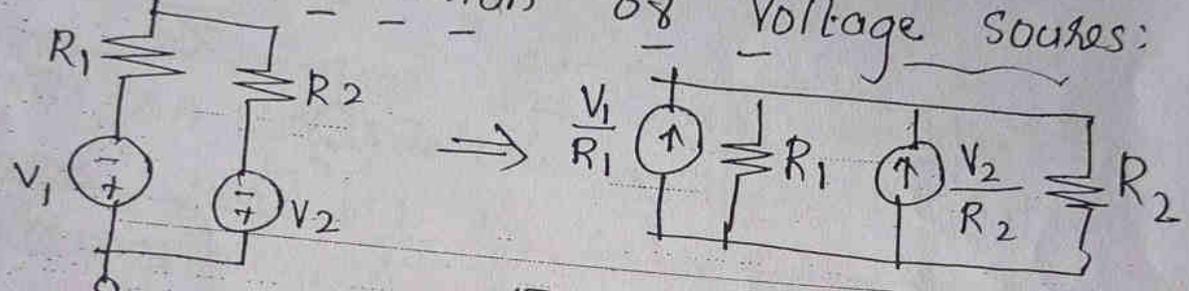
$$I = I_1 + I_2$$

$$R = R_1 \parallel R_2$$

Series Connection of Voltage Sources



Parallel Connection of Voltage Sources:



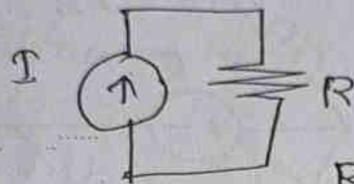
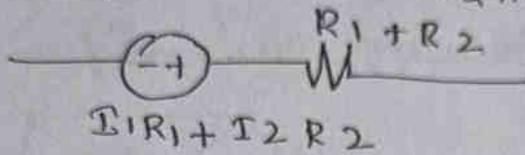
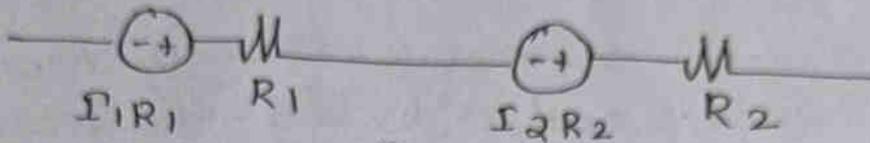
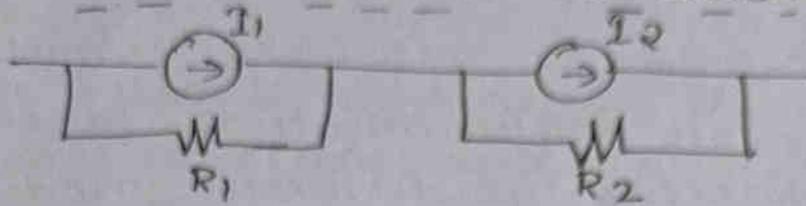
$$V = \frac{I}{R}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad I = \frac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$$

$$V = \frac{I}{R} = \frac{V_1 R_2 + V_2 R_1}{R_1 R_2}$$

14/1116

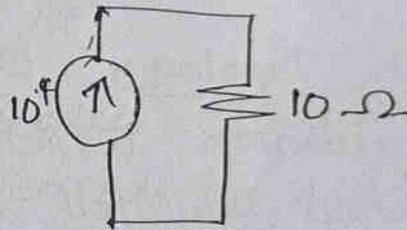
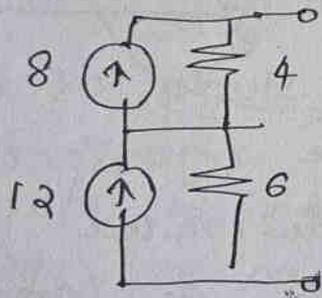
Series Connection of Current Sources



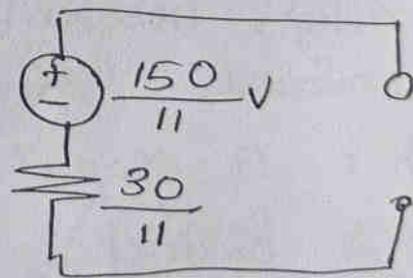
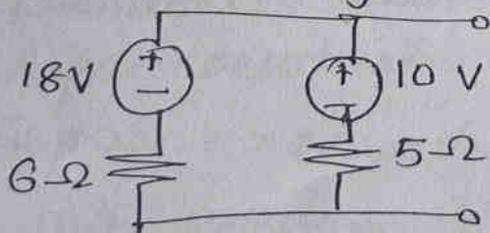
$$I = \frac{I_1 R_1 + I_2 R_2}{R_1 + R_2}$$

$$R = R_1 + R_2$$

Convert the following ckt into a single c/s



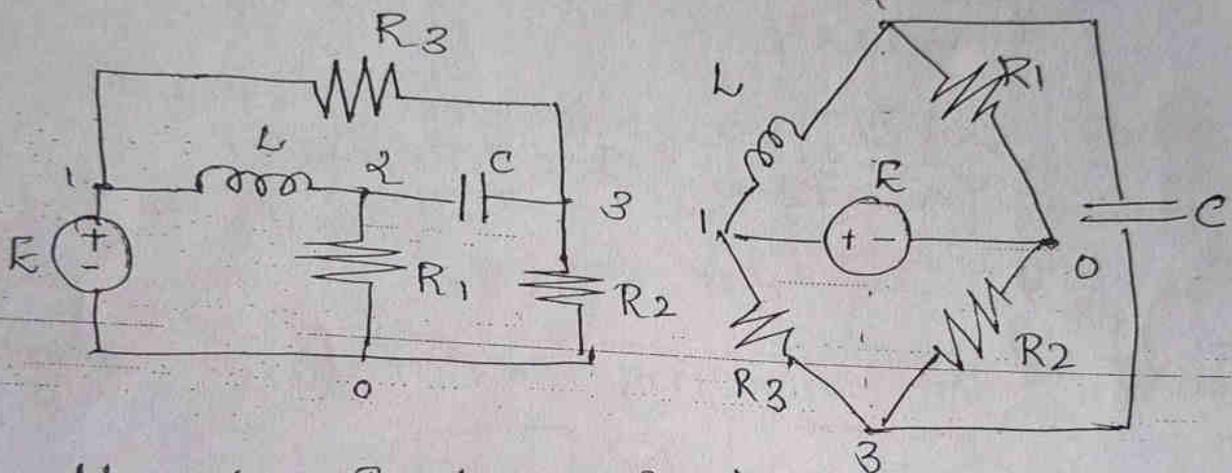
obtain a single v.s



NETWORK TOPOLOGY

Network topology is network geometry. A network is an interconnection of elements in various branches at different nodes.

A circuit or network can be drawn in different shape and sizes by maintaining the relationship between the nodes and branches.

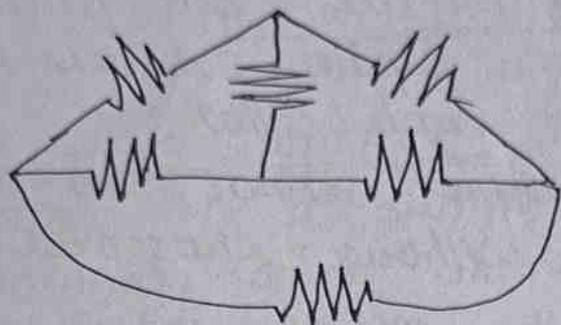


Network Topology is the study of the properties of the network which are unaffected when it is stretched, twisted or distorted the size and shape of the network. As long as the relationship between the nodes & branches are maintained the circuit response will be same.

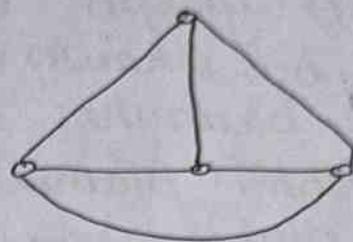
Graph: A graph of a network consist of nodes & branches of the network. In network branches will have elements but in a graph this branches are drawn in lines.

When arrows are placed in a graph then it will be called Oriented Graph

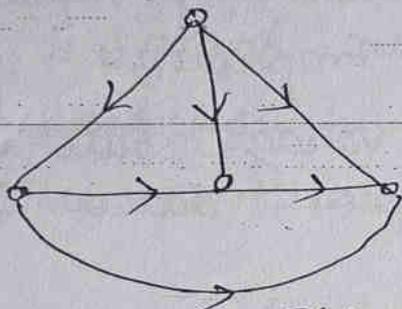
→ The arrows indicate the direction of branch current and polarity of branch voltage.



Network



Graph of a network



Oriented graph.

A branch is represented by a line segment connecting a pair of nodes in the graph of a network.

→ A node is a terminal of a branch, which is represented by a point. Nodes are the end point of branches.

A node and a branch are incident if the node is a terminal of the branch.

→ The no. of branches incident at a node of a graph indicates the degree of the node

→ A sequence of branch traversed while going from one node to another node is called a path.

planar and Non planar graph :-

→ A graph is said to be planar if it can be drawn on a plane surface that no two branches cross each other.

→ Non planar graph cannot be drawn on a plane surface without crossover.

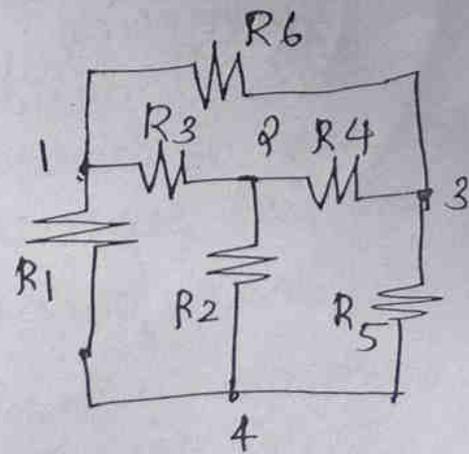
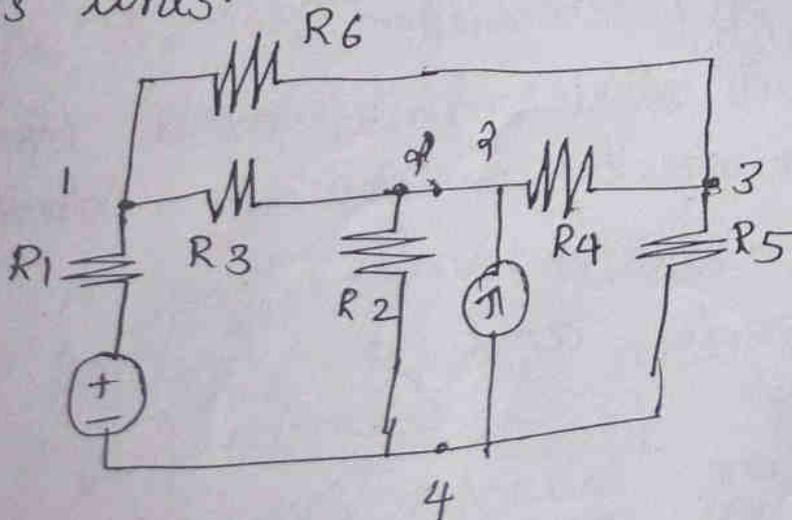
Steps to draw the graph of a circuit

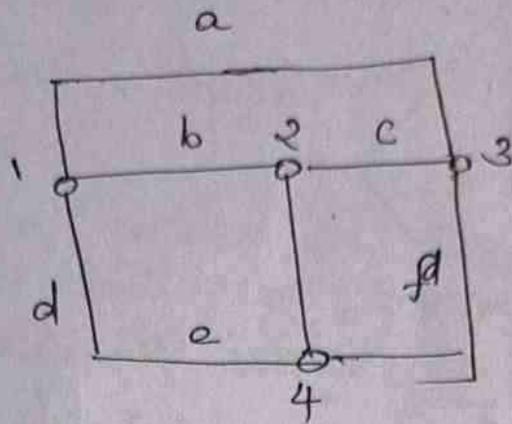
1. Redraw the circuit by replacing the sources by their internal impedances.

The ideal voltage source is replaced by s.c and ideal current source is replaced by ∞ .

2. Represent nodes of the network as small circles and the elements connected between the nodes as lines.

Eg:





TREE, LINK COTREE

TREE is a subgraph which is obtained by removing some branches such that the subgraph includes all the nodes of the original graph, but does not have any closed path. A Tree of a graph with N nodes have the following properties:

1. The Tree contains all the nodes of the graph
2. The Tree contains $N-1$ branches.
3. The Tree does not have a closed path.

The branches removed from the tree are called Links or chords. Number of closed path in a graph is equal to number of links. By removing a link from a graph, one closed path can be eliminated.

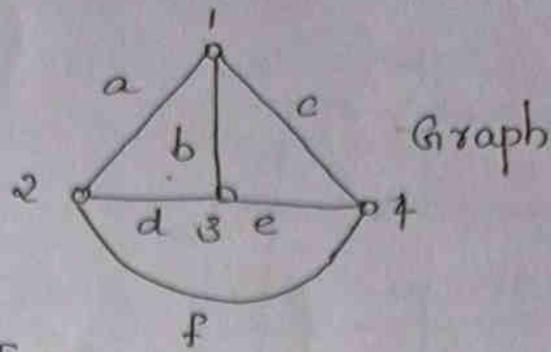
Co-Tree is a complement of a tree

Branch of a Tree \rightarrow Twigs
 Branch of a CoTree \rightarrow Links

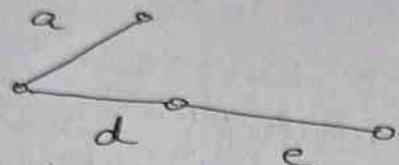
M-1-10

19/116

Egs-

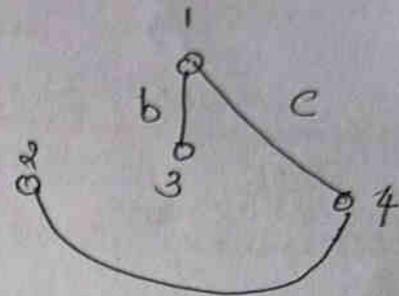


a) TREE



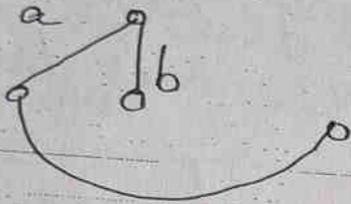
Twigs a, d, e

CO-TREE

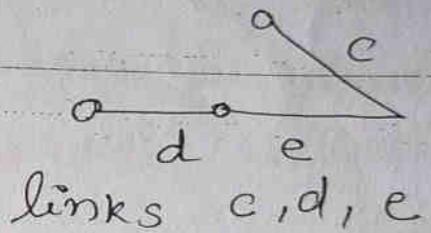


f links b, c, f

b)



Twigs a, b, f



links c, d, e

20/116

Incidence Matrix :- Incident matrix tabulate the geometric features of a graph as two dimensional array. The incidence matrix can be constructed only for oriented graph, in which an arrow is placed in each branch, each column of the incidence matrix provide information regarding the connections of one oriented branch to the various nodes of the graph. The numbers 0, +1 and -1 are used to represent the

connection of the branch to a node.

The procedure for construction of incidence matrix is

1. Mark the nodes of the graph by numerals 1, 2, 3 etc. and the branches of the graph by lower case letters: a, b, c, d etc.
2. Prepare a table in such a way that the branches are listed in columns and nodes in rows.
3. At the intersection of a row and column, write the incidence of the branch to the node by putting '0' or '+1' or '-1' as explained.

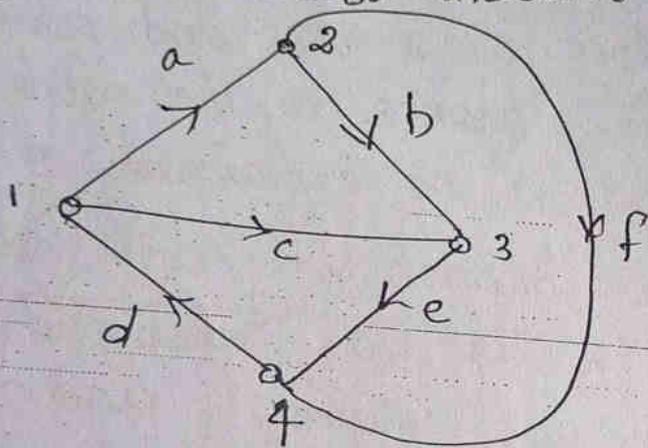
Consider a branch - k and node 1

- 1) If the branch - k is not connected to node 1 then an intersection of column - k and node 1 enters 0.
- 2) If the branch k is connected to node 1 and the arrow in the branch is toward node-1 then at the intersection of column k and row-1 enter "-1"
- 3) If the branch k is connected to node 1 and arrow in the branch away from node-1 then the intersection of column k and row 1 enter "+1"

The incident matrix with inlets of all nodes is called complete incident matrix. Incident matrix with one row eliminated is called Reduced incident matrix or simply incident matrix.

Size of complete incident matrix is $N \times B$

Size of reduced incident matrix is $(N-1) \times B$



Nodes	Branches					
	a	b	c	d	e	f
1	+1	0	+1	-1	0	0
2	-1	+1	0	0	0	+1
3	0	-1	-1	0	+1	0
4	0	0	0	+1	-1	-1

Complete Incidence Matrix

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

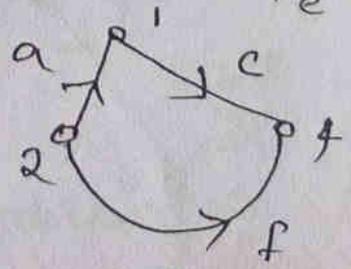
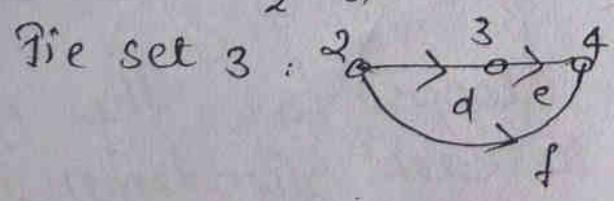
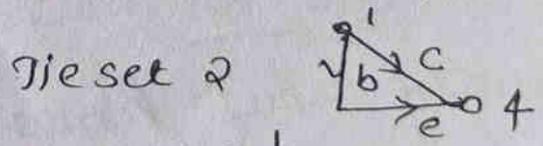
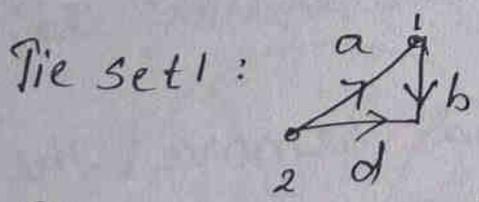
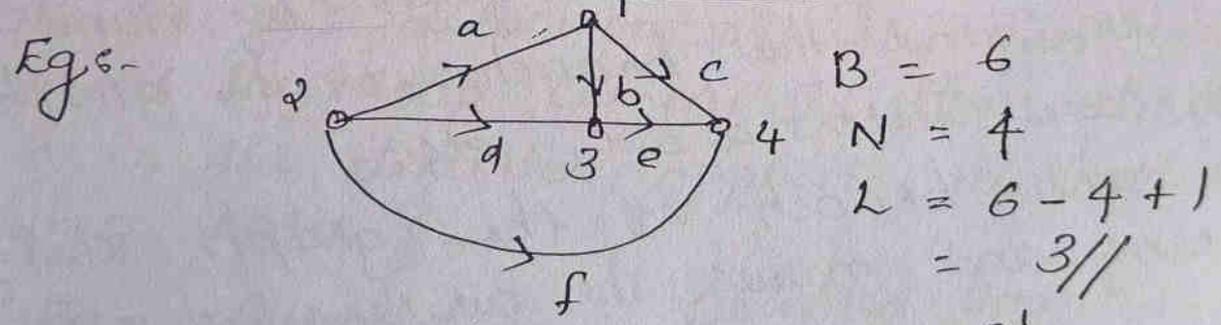
Reduced Incidence Matrix

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \end{bmatrix}$$

TIE SET : Tie set is a set of branches that forms a closed path in a graph such that the closed path contain one link and remainder are tree branches. The closed path is also known as loop.

No. of Loop = no. of Links = No. of Tie set.

If a graph has B Branches & N nodes then
no. of links $L = B - N + 1$



Tie Set Matrix

Current attached to each branch is called Branch current. All Branch currents are independent. Number of independent currents in a graph is equal to number of links (loops) i.e. loop currents are independent currents and Branch currents depends on loop currents.

The relation between the loop current and branch currents can be summarized in the form of a matrix called Tie set matrix. The procedure for constructing Tie set matrix is given below

1. Mark the nodes of the graph by numerals 1, 2, 3 etc:- and the branches of the graph by lower case letters a, b, c, etc:-
2. To each branch of the graph assign a current and names the current as I_a, I_b, I_c, I_d etc:-
3. Identify the links and remove the links form a tree.
4. Draw the tie-sets (loops) of the graph
5. Assign a current to each fundamental circuit (loop) such that the direction of loop current is same as that of branch current

corresponding to the link in that loop denoted as I_1, I_2, I_3 etc:-

6. prepare a table as

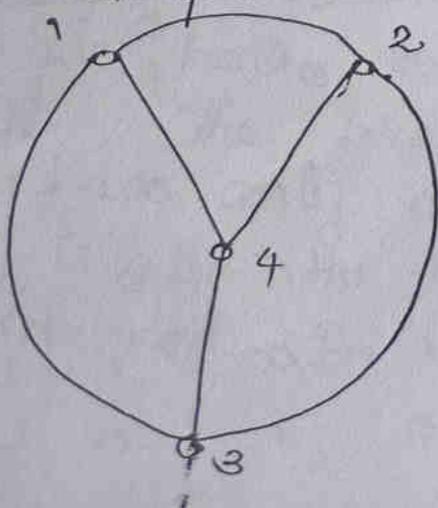
Loop current	Branches
I_1	a b e d e f
I_2	
I_3	

7. At the intersection of a row and column the relation between the loop & branch current is represented as 0, +1, or -1 as explained below.

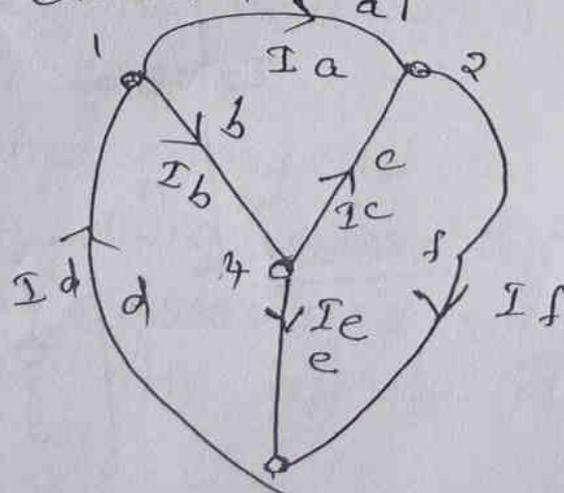
consider a branch k and loop current I_i

- (i) If the loop current I_i does not flow through branch k then enter '0'
- (ii) If the loop current I_i flows through branch k in this direction or I_k then enter +1
- (iii) If the loop current I_i flows through branch k in the direction opposite to that of I_k , then enter '-1'

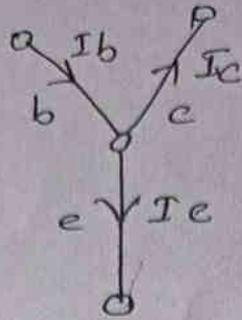
Ex:- Graph



Oriented Graph

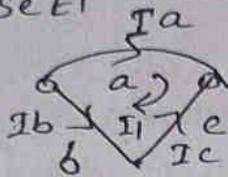


Tree of the graph

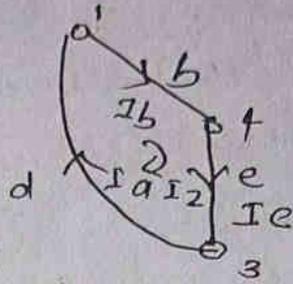


Link: a, d, f
 Twig: b, c, e

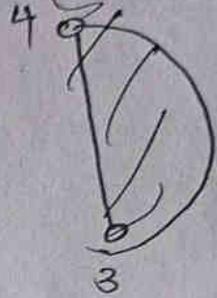
Tie set 1



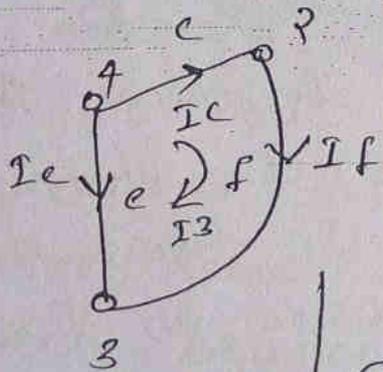
Tie set 2



Tie set 3



Tie set 3



a b c d e f

I_1	1	-1	-1	0	0	0
I_2	0	1	0	1	1	0
I_3	0	0	1	0	-1	1

Tie set Matrix

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

CUT SET : A cutset is a set of branches whose removal cuts the connected graph into two parts.

Each branch of a cutset has one of its terminals connected to a node in one part and its other end connected to a node in another part. Each cutset contains one twig and the remaining branches links

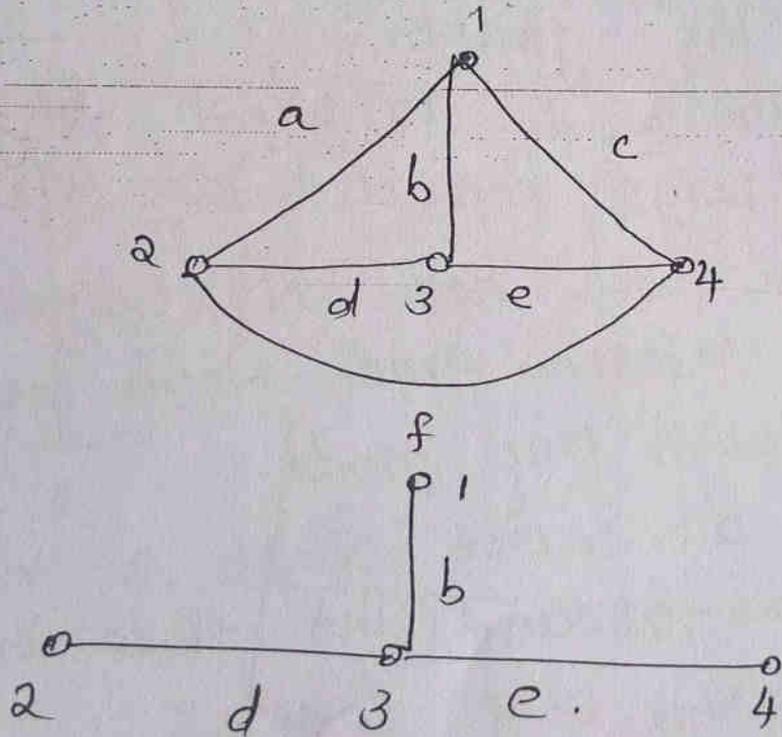
General procedure for formation of cutset can be obtained as

1. From a tree of the graph. The tree will have $N-1$ twigs, where N is the number of nodes in the graph.
2. Select the node to which largest no. of branches (twigs) connected as reference node.
3. The tree will have $N-1$ twigs and $N-1$ nodes except the reference node. Hence associate one twig with one node.
4. To form a cutset select a node and cut the branches around the node in this graph. Now we have two cases
Case (i) : The branches connected to the selected node has only one twig connected to it which is also the twig associated with the selected node. In this case all the $(N-1)$

branches connected to this node for
cut set.

Case ii: The branches connected to the selected node has more than one twig connected to it in this case cut the graph in two parts such that in one part minimum portion of graph is included along with the selected node. one short circuits the extra twig some more branches may be connected to the selected node. Now the cut set is given by the branches connected to the selected node after short circuiting the extra twig

Ex:-

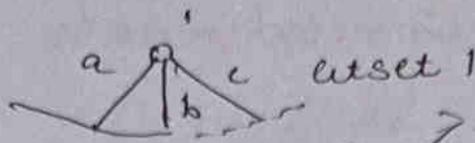


Reference node: node 3

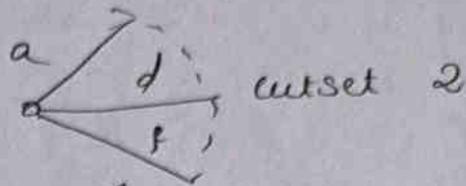
28/116

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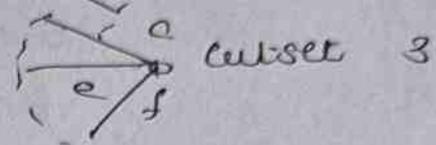
Consider node 1



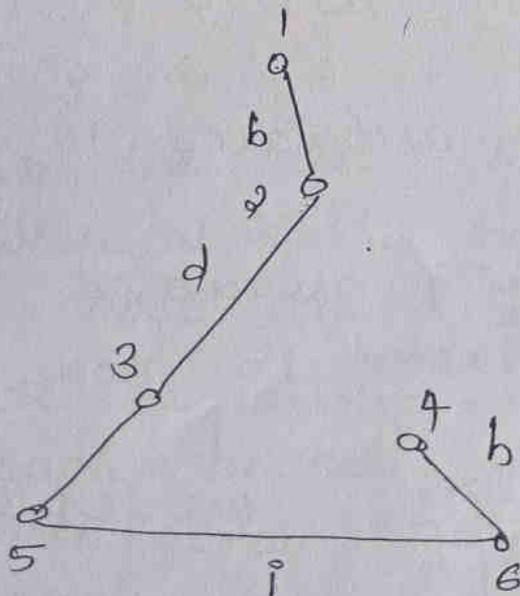
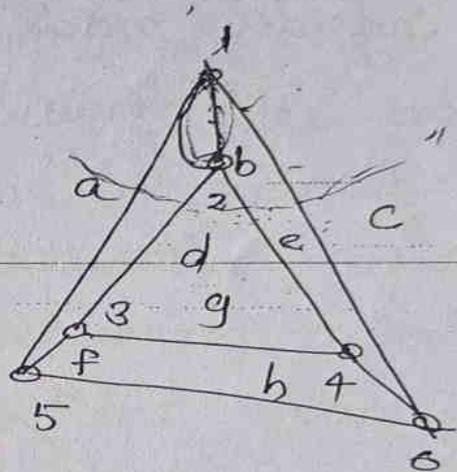
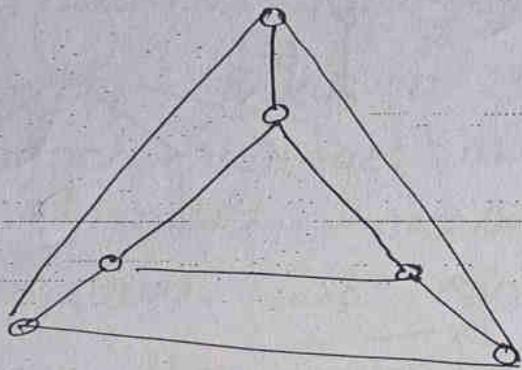
consider node 2



Consider node 4



Twig of the graph is 3 so there is only 3 cutsets

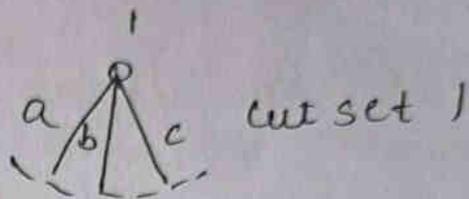


M-1-15

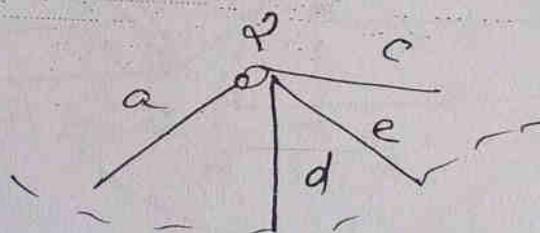
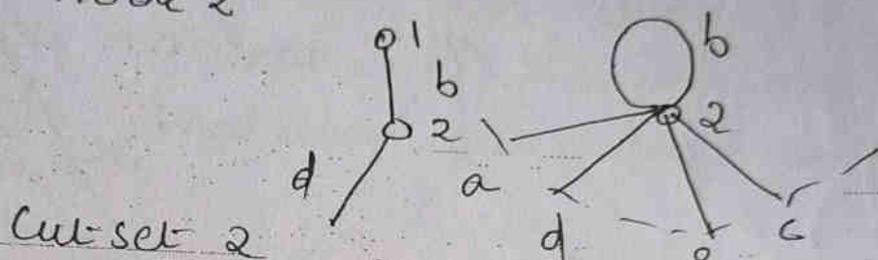
Node 6 is considered as reference node

Twigs	connected node
b	1
d	2
f	3
i	5
h	4

1) consider node 1



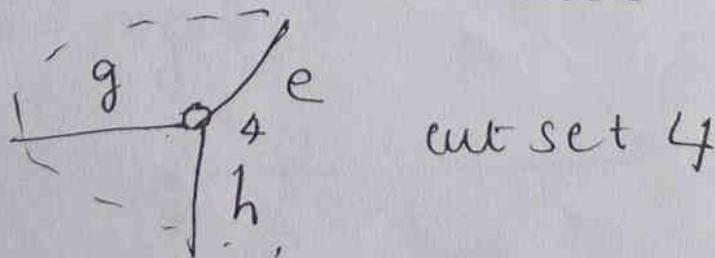
2) consider node 2



Consider node 3

Twigs are d & f associated to node 3 is f so twig d

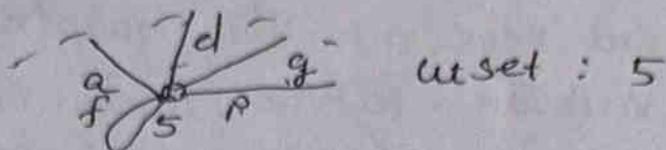
Consider node 4 : Twig associated with node in the Twigs connected to node 4 is h



do consider node 5

Twig associated with node 5 is P

Twig connected to node 5 is i, f



Cut set Matrix :

In a graph if there is B branches then there will be B no: of Voltage Variables called branch voltage.

The Relation between the node vltg and branch voltages can be summarized in the form of matrix called cut set matrix ³⁻¹¹⁻¹⁶

The procedure for constructing cut set matrix is

1. Mark the nodes of the graph by numerals 1, 2, 3 etc. and branches of the graph by a, b, c
2. Draw the tree, The tree will have $N-1$ twigs, where N is the no: of nodes in the graph. chose one node as reference node & associate remaining $N-1$ nodes with $N-1$ twigs.
3. To each branch of the graph assign an orientation (or an arrow) which is the reference for polarity of branch voltage

For the twig always assign the orientation as away from the node associated with it. (iii) Cutset Opposite

4. Draw the cut sets of the graph.
5. Assign a voltage to the node in each cutset. Let the node voltage be denoted as V_1, V_2, V_3 etc.
6. Prepare a table as shown below

Node Voltage	Branch			
	a	b	c	d ...
V_1				
V_2				
V_3				
...				

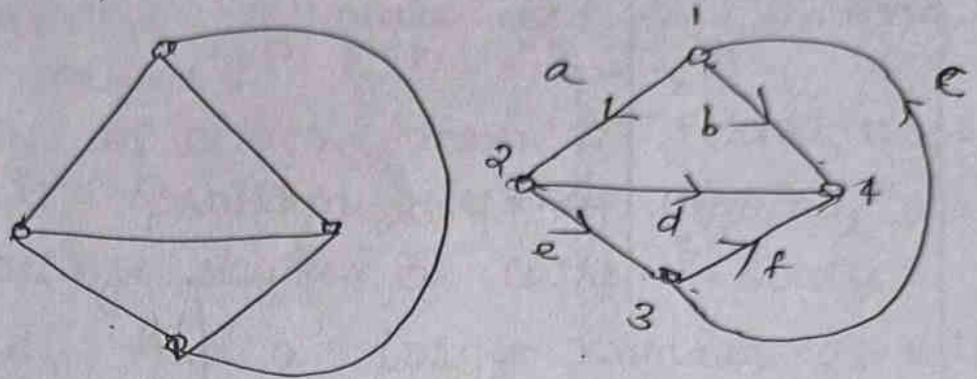
7. At the intersection of a row & column the relation between the node and branch voltage is represented as 0, +1, or -1 as explained below.

8. Consider a branch $-k$ and cutset 1

- (i) If the branch k is not included in the cutset then enter 0 at the intersection of column $-k$ and row 1
- (ii) If the branch $-k$ is included in the cut-set and orientation of branch $-k$ is same as that of twig in the cut-set then enter +1 at the intersection of column $-k$ and row 1

(iii) If the branch k is included in the cutset and orientation of branch k is opposite to that of a twig in the cutset then enter -1 ,

Eg:-



$B = 6$
 $N = 4$

Links = $6 - 4 + 1 = 3$

There will be a, e, c

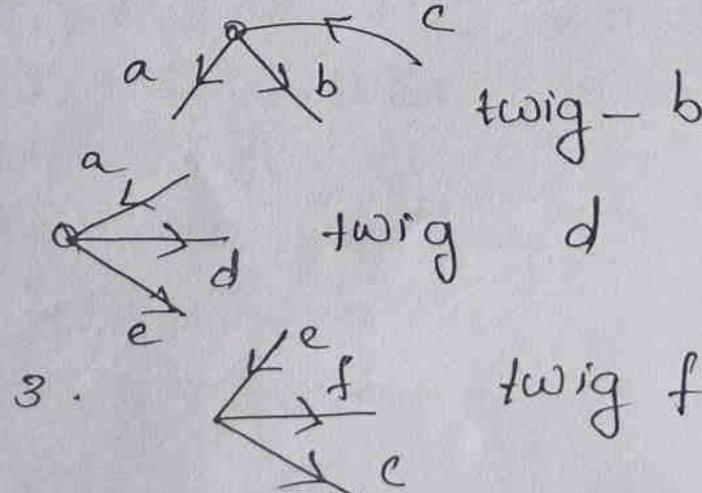
Twigs are b, d, f

Reference node = 4

Twig associated with each node

- 1 → b
- 2 → d
- 3 → f

3 cut sets



	a	b	c	d	e	f
V_1	+1	+1	-1	0	0	0
V_2	-1	0	0	+1	+1	0
V_3	0	0	+1	0	-1	+1

Cutset Matrix $Q =$
$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

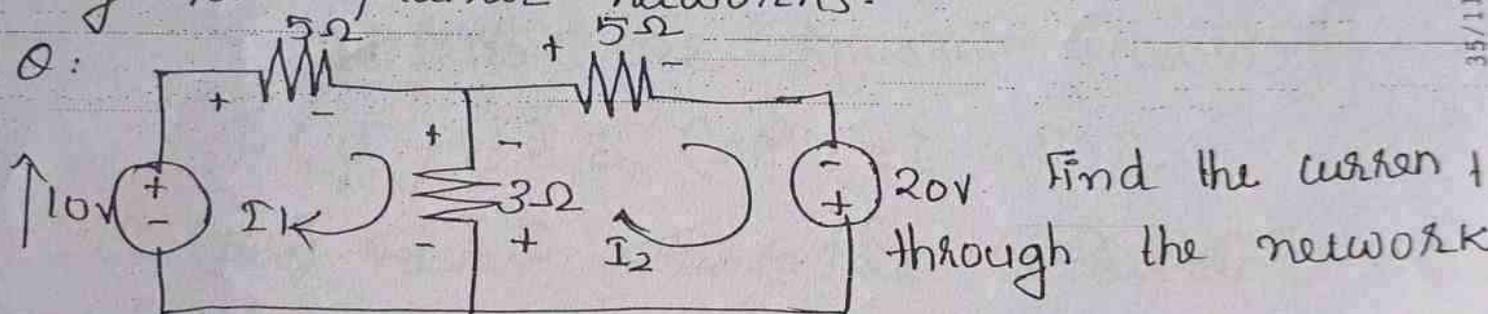
Solution Methods applied to dc and phasor circuits :-

Mesh Analysis :- Mesh and Nodal analysis are two basic important techniques used in finding solution for a network.

→ The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources.

→ If a network has a large number of voltage sources, it is useful to use mesh analysis; as this

analysis requires that all the sources in a circuit be voltage sources. Mesh analysis is applicable only for planar networks.



$$\text{Loop I: } -10 + 5I_1 + 3(I_1 - I_2) = 0$$

$$= -10 + 5I_1 + 3I_1 - 3I_2 = 0$$

$$= -10 + 8I_1 - 3I_2 = 0$$

$$8I_1 - 3I_2 = 10 \quad \text{--- (1)}$$

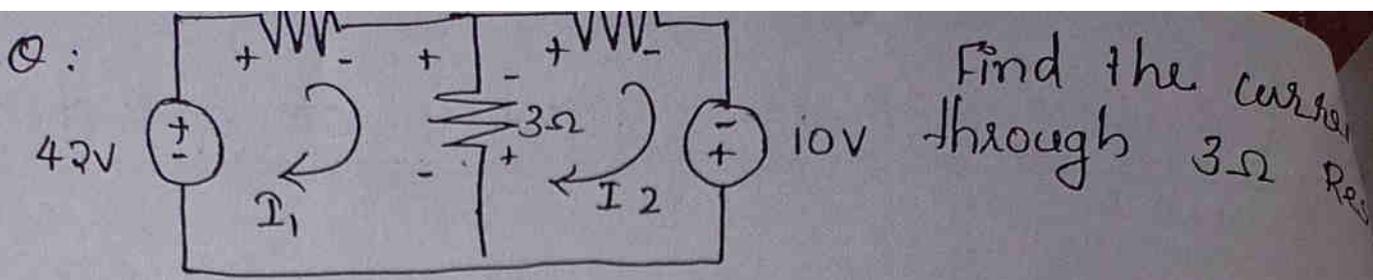
$$\text{Loop II: } 3(I_2 - I_1) + 5I_2 + 20 = 0$$

$$= +3I_2 - 3I_1 + 5I_2 + 20 = 0$$

$$= 8I_2 - 3I_1 = -20 \quad \text{--- (2)}$$

$$I_1 = 2.545 \text{ A} \parallel$$

$$I_2 = 3.45 \text{ A} \parallel$$



$$-42 + 6I_1 + 3(I_1 - I_2) = 0$$

$$-10 + 4I_2 + 3(I_2 - I_1) = 0$$

$$6I_1 + 3I_1 - 3I_2 = 42$$

$$4I_2 + 3I_2 - 3I_1 = 10$$

$$9I_1 - 3I_2 = 42 \quad \text{--- (1)}$$

$$-3I_1 + 7I_2 = 10 \quad \text{--- (2)}$$

$$I_1 = 6A //$$

$$I_2 = 4A //$$

Current through 3Ω Resistor

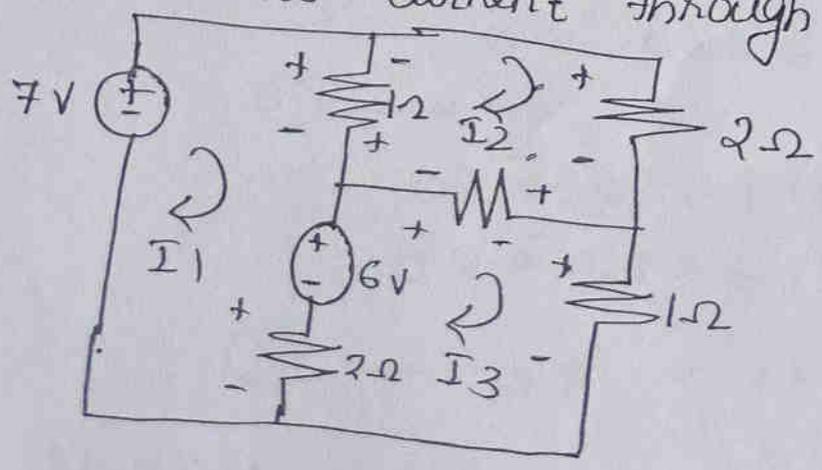
$$3(I_1 - I_2) \text{ (or)} 3(I_2 - I_1)$$

~~$3(6 - 4) = 6$~~ $\Rightarrow I_1 - I_2 \text{ (or)} I_2 - I_1$

$$\Rightarrow \underline{\underline{6 - 4 = 2 // \text{ (or) } 4 - 6 = -2}}$$

-ve not valid

Q Find the current through the network



$$-7 + (I_1 - I_2) + 6 + 2(I_1 - I_3) = 0$$

$$-7 + I_1 - I_2 + 6 + 2I_1 - 2I_3 = 0$$

$$3I_1 - I_2 - 2I_3 = 1 \quad \text{--- (1)}$$

$$I_2 - I_1 + 2I_2 + 3(I_2 - I_3) = 0$$

$$3I_2 - I_1 + 3I_2 - 3I_3 = 0$$

$$6I_2 - I_1 - 3I_3 = 0 \quad \text{--- (2)}$$

$$3(I_3 - I_2) + I_3 + 2(I_3 - I_1) - 6 = 0$$

$$3I_3 - 3I_2 + I_3 + 2I_3 - 2I_1 - 6 = 0$$

$$6I_3 - 3I_2 - 2I_1 = 6 \quad \text{--- (3)}$$

Solving the above 3 equations

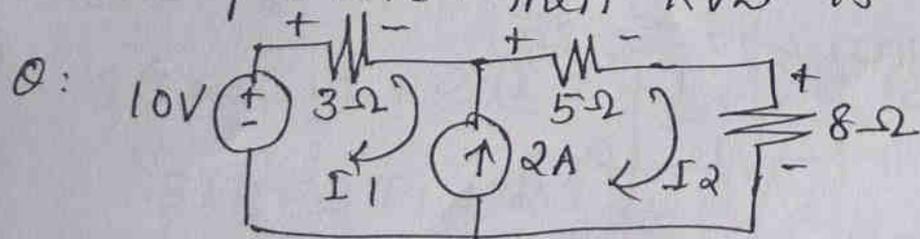
$$I_1 = 3A //$$

$$I_2 = 2A //$$

$$I_3 = 3A //$$

Super Mesh :-

Two meshes that have a current source as common element is called super mesh. We thus reduce the meshes by one for each current source present. Then KVL is applied.



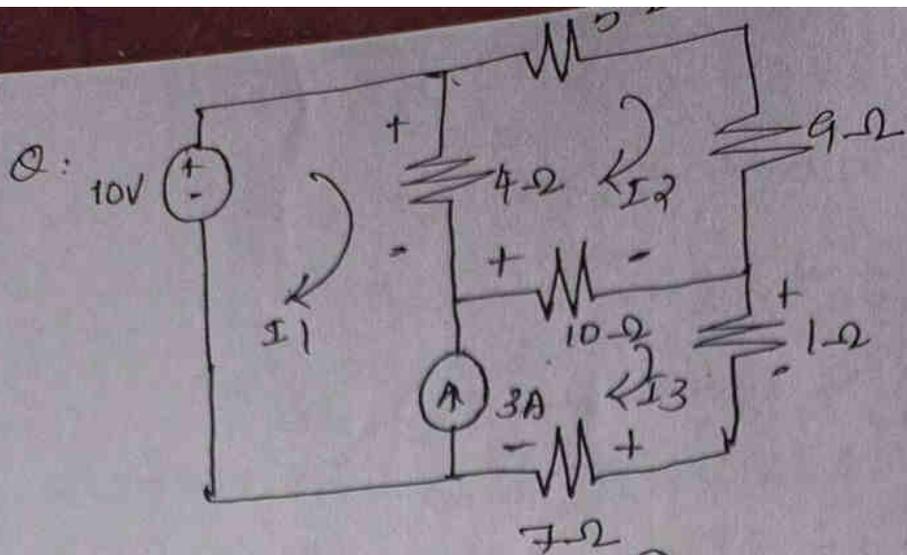
$$-10 + 3I_1 + 5I_2 + 8I_2 = 0$$

$$I_2 - I_1 = 2A \quad \text{--- (1)}$$

$$13I_2 + 3I_1 = 10$$

$$I_1 = -1A //; \quad I_2 = 1A //$$

M-1-19



$$I_3 - I_1 = 3A \quad \text{--- (1)}$$

$$-10 + 4(I_1 - I_2) + 10(I_3 - I_2) + I_3 + 7I_3 = 0$$

$$-10 + 4I_1 - 4I_2 + 10I_3 - 10I_2 + I_3 + 7I_3 = 0$$

$$-10 + 4I_1 - 14I_2 + 18I_3 = 0$$

$$4I_1 - 14I_2 + 18I_3 = 10 \quad \text{--- (2)}$$

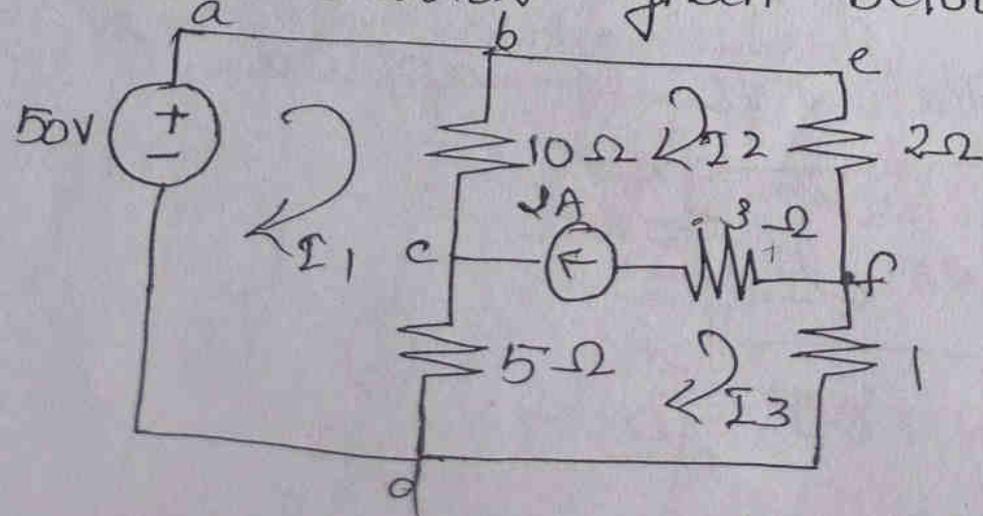
$$4(I_2 - I_1) + 5I_2 + 9I_2 + 10(I_2 - I_3) = 0$$

$$4I_2 - 4I_1 + 5I_2 + 9I_2 + 10I_2 - 10I_3 = 0$$

$$-4I_1 + 28I_2 - 10I_3 = 0 \quad \text{--- (3)}$$

$$I_1 = \underline{\underline{-1.93}} ; \quad I_2 = \underline{\underline{0.105}} \quad I_3 = \underline{\underline{1.066}}$$

Q: Determine the current in the 5Ω resistor in the network given below.



$$-50 + 10(I_1 - I_2) + 5(I_1 - I_3) = 0$$

$$-50 + 10I_1 - 10I_2 + 5I_1 - 5I_3 = 0$$

$$15I_1 - 10I_2 - 5I_3 = 50 \quad \text{--- (1)}$$

$$I_2 - I_3 = 2A \quad \text{--- (2)}$$

iind & iiird Meshes, we can form a supermesh

$$10(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) + I_3 + 5(I_3 - I_1) = 0$$

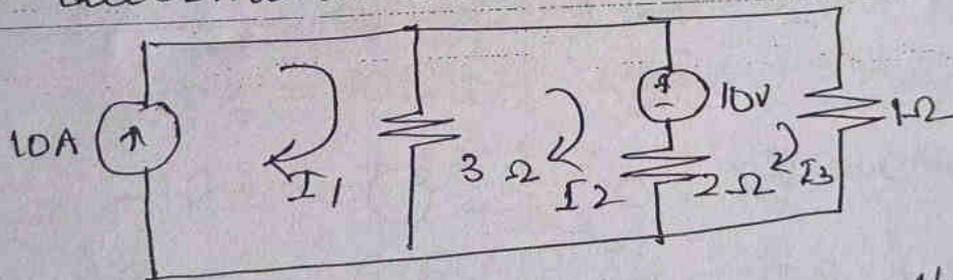
$$10I_2 - 10I_1 + 2I_2 + 3I_2 - 3I_3 + I_3 + 5I_3 - 5I_1 = 0$$

$$-15I_1 + 12I_2 + 6I_3 = 0 \quad \text{--- (3)}$$

Solving (1) and (2) & (3)

$$I_1 = 19.99A, I_2 = 17.33A, I_3 = \underline{15.33A}$$

Q: Write the mesh equations for the circuit and determine the currents I_1 , I_2 & I_3 .



The current source lies on the perimeter of the circuit, and the first mesh is ignored from iind Mesh:

$$3(I_2 - I_1) + 2(I_2 - I_3) + 10 = 0$$

$$3I_2 - 3I_1 + 2I_2 - 2I_3 = -10$$

$$-3I_1 + 5I_2 - 2I_3 = -10 \quad \text{--- (1)}$$

iiird

$$I_3 + 2(I_3 - I_2) = 10$$

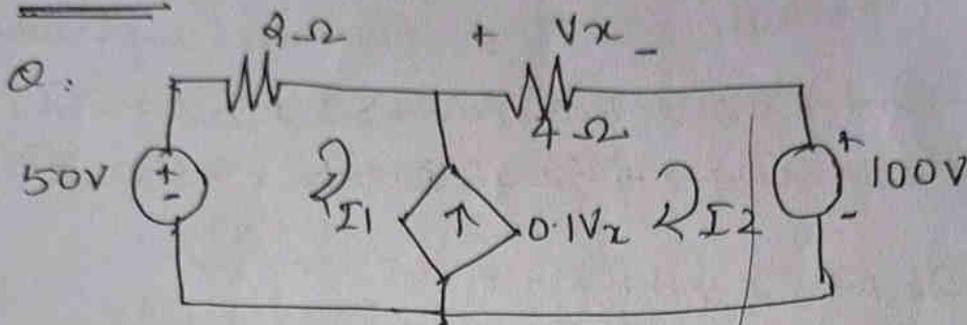
$$-2I_2 + 3I_3 = 10 \quad \text{--- (2)}$$

IM-1-20

From first Mesh $I_1 = 10 \text{ A}$. — (3)

$$I_1 = 10 \text{ A}, I_2 = 7.27 \text{ A}, I_3 = 8.18 \text{ A}$$

Mesh analysis of circuit with dependent sources



$$V_x = 4I_2 \quad \text{--- (1)}$$

$$I_2 - I_1 = 0.1V_x$$

$$I_2 - I_1 = 0.1 \times 0.4 \times I_2$$

$$I_2 - I_1 = 0.04I_2$$

$$I_2 - 0.04I_2 = I_1$$

$$I_1 - 0.6I_2 = 0 \quad \text{--- (1)}$$

$$-50 + 2I_1 + 4I_2 + 100 = 0$$

$$2I_1 + 4I_2 = -50 \quad \text{(1)}$$

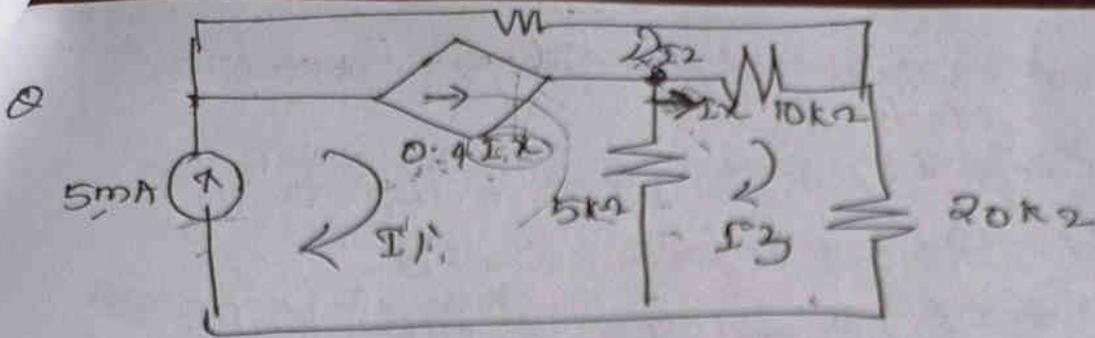
$$I_1 - 0.6I_2 = 0 \quad \text{--- (2)}$$

$$I_1 = \underline{\underline{5.76 \text{ A}}}$$

$$I_2 = \underline{\underline{-9.61 \text{ A}}}$$

$$I_2 = -5.46 \text{ A}$$

$$I_1 = -8.10 \text{ A}$$



$$I_1 = 5 \text{ mA}$$

$$I_x = I_3 - I_2$$

$$0.4(I_3 - I_2) = I_1 - I_2$$

$$10(I_3 - I_2) + 20I_3 + 5(I_3 - I_1) = 0$$

$$10I_3 - 10I_2 + 20I_3 + 5I_3 - 5I_1 = 0$$

$$10I_3 - 10I_2 + 20I_3 + 5I_3 - 5 \times 5 \times 10^{-3} = 0$$

$$35I_3 - 10I_2 = 25 \times 10^{-3}$$

$$0.6I_2 + 0.4I_3 = 5 \times 10^{-3} \quad \text{--- (1)}$$

$$10I_3 - 10I_2 + 20I_3 + 5I_3 - 5I_1 = 0$$

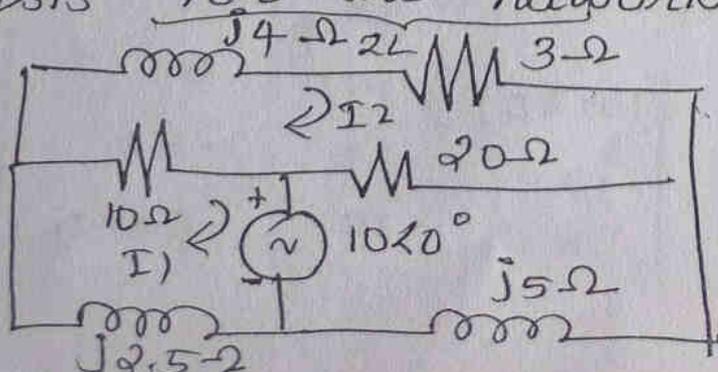
$$-10I_2 + 25I_3 = 25 \text{ mA} \quad \text{--- (2)}$$

$$I_2 = \underline{\underline{6.6 \times 10^{-3} \text{ A}}}$$

$$I_3 = \underline{\underline{2.6 \times 10^{-3} \text{ A}}}$$

Mesh Analysis for phasor circuit :-

Find the current through Z_L using mesh analysis for the network.



M-1-21

by using write down Mesh equations

Mesh 1:

$$10(I_1 - I_2) + I_1 j2.5 + 10 = 0$$

$$10I_1 - 10I_2 + I_1 j2.5 = -10$$

$$(10 + 2.5j)I_1 - 10I_2 = -10 \quad \text{--- (1)}$$

Mesh 2:

$$10(I_2 - I_1) + 4jI_2 + 3I_2 + 20(I_2 - I_3) = 0$$

$$10I_2 - 10I_1 + 4jI_2 + 3I_2 + 20I_2 - 20I_3 = 0$$

$$(33 + 4j)I_2 - 10I_1 - 20I_3 = 0$$

$$-10I_1 + (33 + 4j)I_2 - 20I_3 = 0 \quad \text{--- (2)}$$

Mesh III

$$20(I_3 - I_2) + 5jI_3 - 10 = 0$$

$$20I_3 - 20I_2 + 5jI_3 = 10$$

$$(20 + 5j)I_3 - 20I_2 = 10$$

$$-20I_2 + (20 + 5j)I_3 = 10 \quad \text{--- (3)}$$

$$\begin{bmatrix} (10 + 2.5j) & -10 & 0 \\ -10 & (33 + 4j) & -20 \\ 0 & -20 & (20 + 5j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

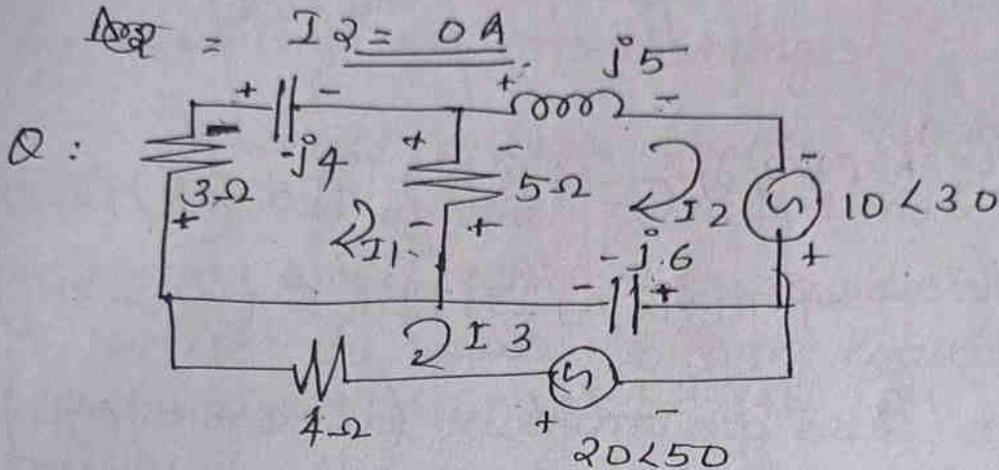
$$= \frac{\Delta}{\Delta} \begin{vmatrix} (10 + 2.5j) & -10 & 0 \\ -10 & 0 & -20 \\ 0 & 10 & (20 + 5j) \end{vmatrix} \Bigg| \begin{vmatrix} -10 \\ 0 \\ 10 \end{vmatrix}$$

$$\Delta = (10 + 2.5j)(0 + 200) - 10(-200 - 50j) + 0$$

$$= 2000 + 500j - 2000 - 500j$$

$$= \underline{\underline{0}}$$

$$I_2 = \underline{\underline{0A}}$$



Mesh I

$$-3I_1 + 4jI_1 + 5(I_1 - I_2) = 0$$

$$-3I_1 + 4jI_1 + 5I_1 - 5I_2 = 0$$

$$-8I_1 + 4jI_1 - 5I_2 = 0$$

$$(-8 + 4j)I_1 - 5I_2 = 0 \quad \text{--- (1)}$$

Mesh II

$$5jI_2 + 5(I_2 - I_1) + 6j(I_2 - I_3) - 10\angle 30^\circ = 0$$

$$5I_2 + 5I_2 + 5jI_2 + 6jI_2 - 6jI_3 - 10\angle 30^\circ = 0$$

$$-5I_1 + (5 - j)I_2 + 6jI_3 = 10\angle 30^\circ \quad \text{--- (2)}$$

Mesh III

$$6j(I_3 - I_2) + 20\angle 50^\circ - 4I_3 = 0$$

$$+ 6jI_3 - 6jI_2 + 20\angle 50^\circ - 4I_3 = 0$$

$$6jI_2 + (4 - 6j)I_3 = 20\angle 50^\circ \quad \text{--- (3)}$$

$$\begin{bmatrix} (8-4j) & -5 & 0 \\ -5 & 5-j & -6j \\ 0 & 6j & 4-6j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \angle 30^\circ \\ 20 \angle 150^\circ \end{bmatrix}$$

$$\begin{aligned} \Delta &= (8-4j) \left((5-j)(4-6j) - (6j)(-6j) \right) - 5 \left((-5)(4-6j) - 0 \right) \\ &= (8-4j) \left((20 - 30j - 4j - 6) - (36) \right) + 5 \left(-20 + 30j \right) \\ &= (8-4j) \left(14 - 34j \right) - (36) + \left(-100 + 150j \right) \\ &= (8-4j) \left(-22 - 34j \right) + \left(-100 + 150j \right) \end{aligned}$$

$$\Delta_1 = -136 - 4 - 545 - 6j$$

$$\Delta_2 = -37.44 + 470.72j$$

$$\Delta_3 =$$

$$I_1 = \frac{\Delta_1}{\Delta} = \underline{\underline{2.42 + 0.92j}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\underline{-1.68 - 1.38j}}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \underline{\underline{4.7 + 1.83j}}$$

44/116

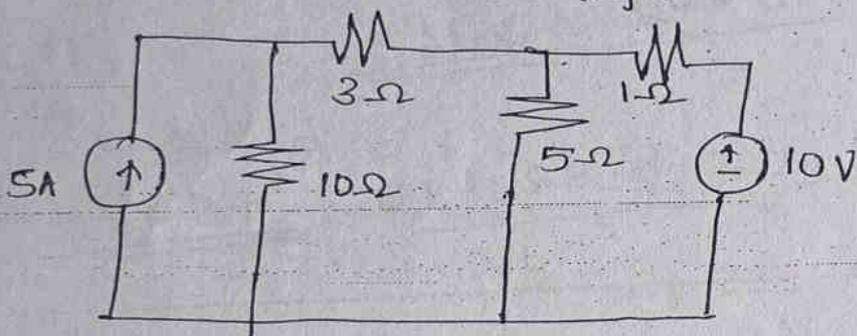
LOCAL ANALYSIS
in general, in a
mode is
than

NODAL ANALYSIS :

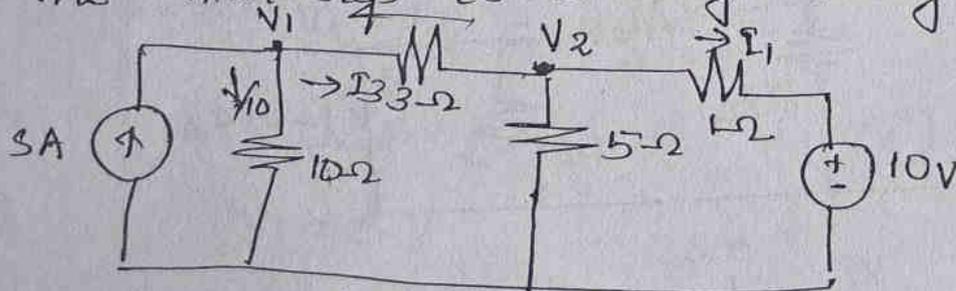
in general, in an N -node circuit, one of the nodes is chosen as reference or datum node then it is possible to write $N-1$ nodal equations by assuming $N-1$ node voltages.

The node voltage is the voltage of a given node with respect to one particular node called the reference node, which we assume at zero potential.

Q: Write the node voltage equations and determine the currents in each branch for the network shown in fig.



→ The first step is to assign voltages at each node



Applying Kirchhoff's current law at the node 1

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 0.433V_1 - 0.333V_2 = 5$$

— (1)

$$\text{OR } V_1 \left[\frac{1}{10} + \frac{1}{3} \right] - V_2 \left[\frac{1}{3} \right] = 5 \quad \text{--- (1)}$$

Apply Kirchhoff's current law at the node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

M-1-23

$$0 \text{ A} - V_1 \left[\frac{1}{3} \right] + V_2 \left[\frac{1}{3} + \frac{1}{5} + 1 \right] = 10 \quad \text{--- (2)}$$

$$-V_1 0.333 + 1.533 V_2 = 10 \quad \text{--- (2)}$$

$$V_1 = \underline{\underline{19.796 \text{ V}}}$$

$$V_2 = \underline{\underline{10.823 \text{ V}}}$$

$$I_{10} = \frac{V_1}{10} = \frac{19.796}{10} = \underline{\underline{1.9796 \text{ A}}}$$

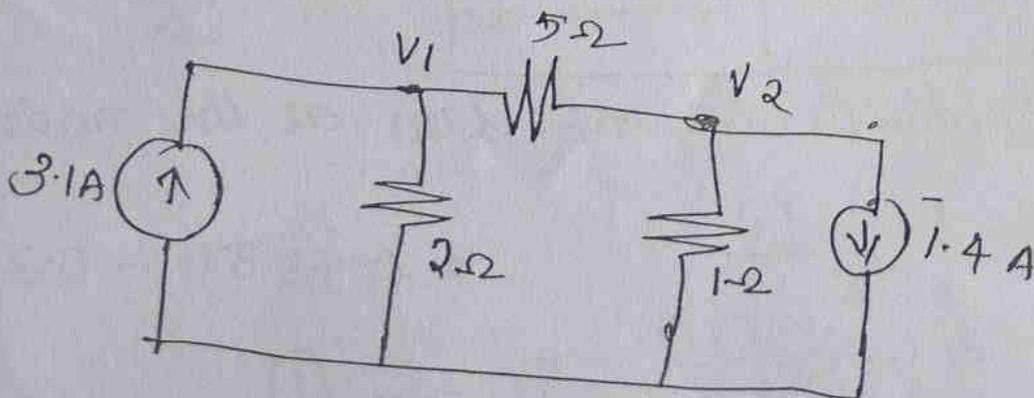
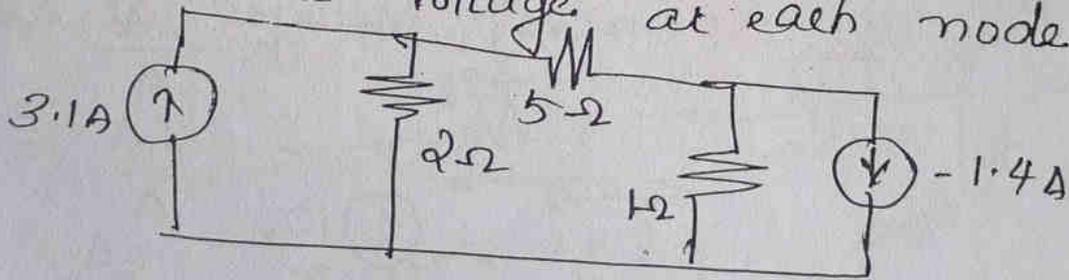
$$I_3 = \frac{V_1 - V_2}{3} = \frac{19.796 - 10.823}{3} = \underline{\underline{2.991 \text{ A}}}$$

$$I_5 = \frac{V_2}{5} = \frac{10.823}{5} = \underline{\underline{2.16 \text{ A}}}$$

$$I_1 = \frac{V_2 - 10}{1} = \frac{10.823 - 10}{1} = \underline{\underline{0.823 \text{ A}}}$$

46/116

Q: Find the Voltage at each node



$$\text{Node 1: } 3.1 = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$\text{node 2: } \frac{v_2 - v_1}{5} + \frac{v_2 - 1.4}{1} = 0$$

$$3.1 = v_1 \left[\frac{1}{2} + \frac{1}{5} \right] - v_2 \left[\frac{1}{5} \right] \quad \text{--- (1)}$$

$$= v_1 \left[\frac{1}{5} \right] + v_2 \left[\frac{1}{5} + 1 \right] = 1.4 \quad \text{--- (2)}$$

$$3.1 = 0.7v_1 - 0.2v_2 \quad \text{--- (1)}$$

$$-0.2v_1 + 1.2v_2 = 1.4 \quad \text{--- (2)}$$

v) =

$$\text{Node 1} \Rightarrow 3.1 = \frac{v_1 - v_2}{5} + \frac{v_1}{2}$$

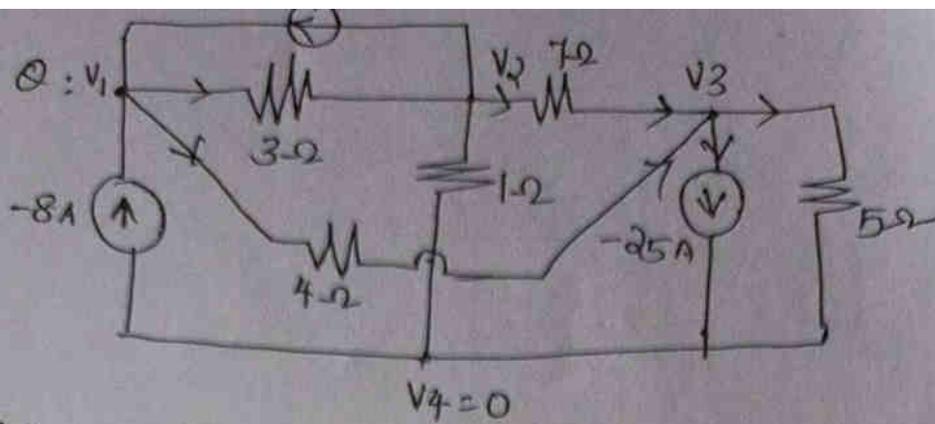
$$7v_1 - 2v_2 = 31 \quad \text{--- (1)}$$

$$\text{node 2} \Rightarrow \frac{v_1 - v_2}{5} = v_2 - 1.4$$

$$v_1 - 6v_2 = -7 \quad \text{--- (2)}$$

$$v_1 = 5V$$

$$v_2 = 2V$$



At node 1:

$$-8 - 3 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4}$$

$$-11 = \frac{V_1}{3} - \frac{V_2}{3} + \frac{V_1}{4} - \frac{V_3}{4}$$

$$12 \times -11 = 4V_1 - 4V_2 + 3V_1 - 3V_3$$

$$-132 = 7V_1 - 4V_2 - 3V_3 \quad \text{--- (1)}$$

At node 2:

$$-3 + \frac{V_2 - V_3}{7} + V_2 + V_2 \frac{-V_1}{3}$$

$$-21 + V_2 - V_3 + 7V_2 = \frac{V_1 - V_2}{3}$$

$$-63 + 3V_2 - 3V_3 + 21V_2 = 7V_1 - 7V_2$$

$$\Rightarrow -7V_1 + 31V_2 - 3V_3 = 63 \quad \text{--- (2)}$$

At node 3:

$$\frac{V_2 - V_3}{7} + \frac{V_3 - V_1}{4} = -25 + \frac{V_3}{5}$$

$$V_2 = V_3$$

$$5(4V_2 - 4V_3 + 7V_1 - 7V_3) = (-125 + V_3) \times 28$$

$$35V_1 + 20V_2 - 83V_3 = -3500 \quad \text{--- (3)}$$

48/116

$V_1 = 5.413V$
 $V_2 = 7.1V$
 V_3

$$V_1 = \underline{\underline{5.413V}}$$

$$V_2 = \underline{\underline{7.736V}}$$

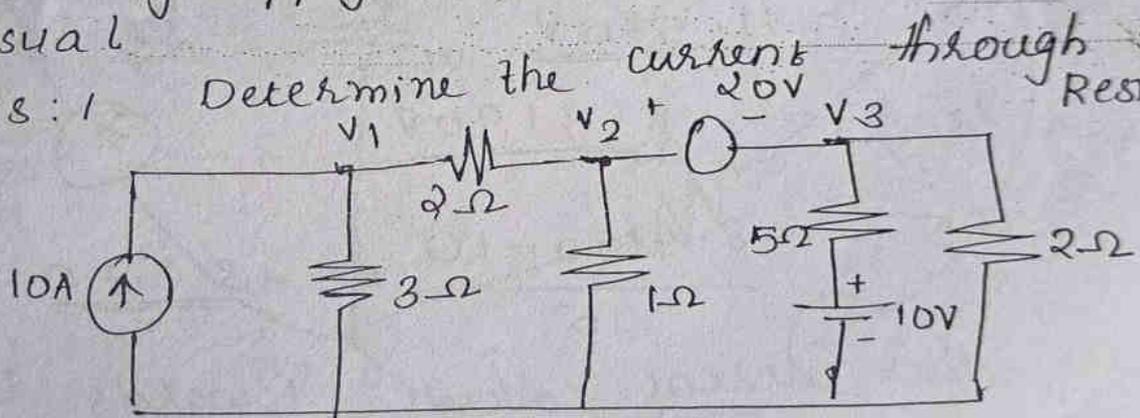
$$V_3 = \underline{\underline{46.315V}}$$

Super Node Analysis:

Suppose any of the branches in the network has a voltage source; then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique.

→ In this method the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual.

Problems: 1 Determine the current through 5Ω Resistor



Model:

$$\frac{V_1}{3} + \frac{V_1 - V_2}{2} = 10A$$

$$V_1 \left[\frac{1}{3} + \frac{1}{2} \right] - V_2 \left[\frac{1}{2} \right] = 10A$$

$$0.833V_1 - 0.5V_2 - 10 = 0 \quad \text{--- (1)}$$

M-1-25

At node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-V_1 \left[\frac{1}{2} \right] + V_2 \left[\frac{1}{2} + 1 \right] + V_3 \left[\frac{1}{5} + \frac{1}{2} \right] = 2$$

$$-0.5V_1 + 1.5V_2 + 0.7V_3 - 2 = 0 \quad (2)$$

The node 2 & 3 are the super nodes

$$V_2 - V_3 = 20 \quad (3)$$

Current in the 5Ω resistor

$$I_5 = \frac{V_3 - 10}{5} \Rightarrow$$

$$V_1 = \underline{18.956 \text{ V}}$$

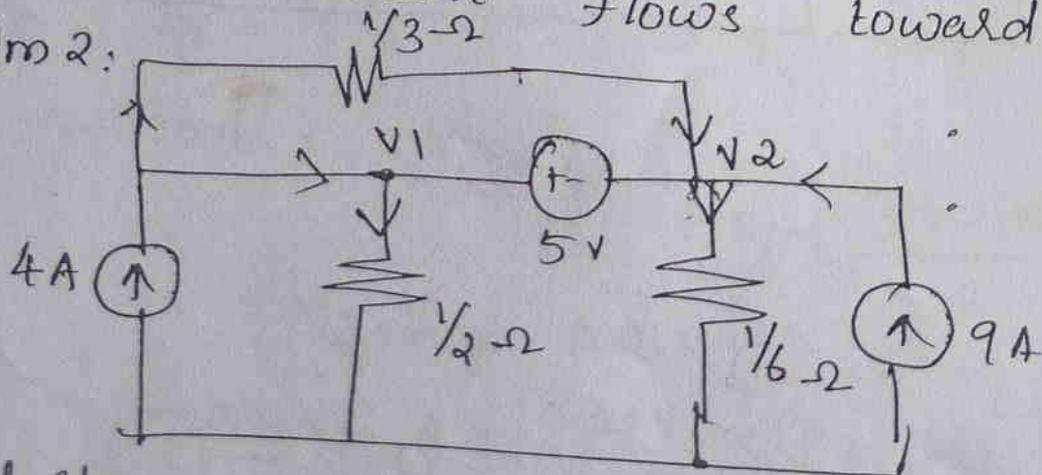
$$V_2 = \underline{11.580 \text{ V}}$$

$$V_3 = \underline{-8.4190 \text{ V}}$$

$$I_5 = \frac{-8.4190 - 10}{5} = \underline{-3.68 \text{ A}}$$

i.e. the current flows towards the network

pb1m 2:



Find the voltage across $V_3 = 0$
the current source

Super node equation

$$v_1 - v_2 = 5V \quad \text{--- (1)}$$

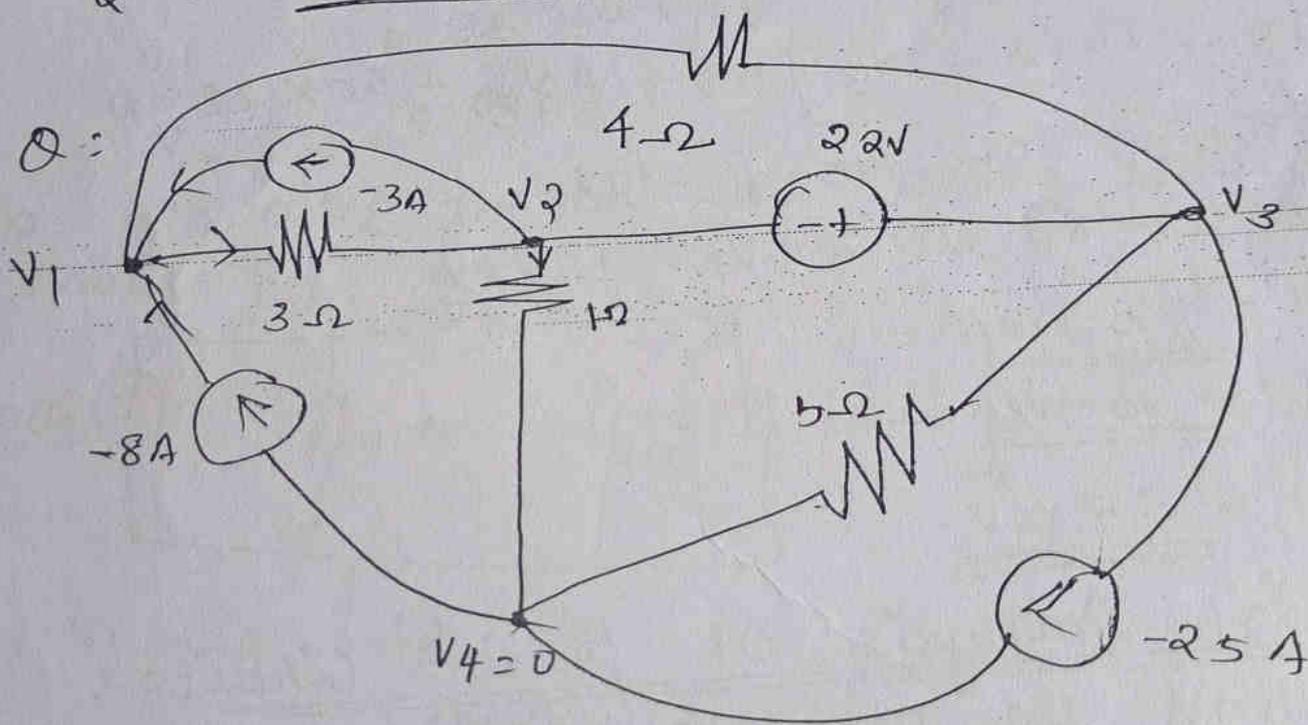
$$4A - \frac{v_1}{\frac{1}{2}} + 9 - \frac{v_2}{\frac{1}{6}} + \frac{v_1 - v_2}{\frac{1}{3}} + \frac{v_1 - v_2}{\frac{1}{3}} = 0$$

$$13 - \frac{v_1}{\frac{1}{2}} - \frac{v_2}{\frac{1}{6}} = 0$$

$$-2v_1 - 6v_2 = -13 \quad \text{--- (2)}$$

$$v_1 = \underline{5.375V}$$

$$v_2 = \underline{0.375V}$$



Node 1: $\frac{v_1 - v_3}{4} + 3 + 8 + \frac{v_1 - v_2}{3} = 0$

$$3v_1 - 3v_3 + 12 \times 3 + 12 \times 8 + 4v_1 - 4v_2 = 0$$

$$3v_1 - 3v_3 + 132 + 4v_1 - 4v_2 = 0$$

$$7v_1 - 4v_2 - 3v_3 = -132 \quad \text{--- (1)}$$

$$v_2 - v_3 = -22V$$

Super node equation

$$\frac{v_1 - v_2}{3} + 3 - v_2 + \frac{v_1 - v_3}{4} - \frac{v_3}{5} + 25 = 0$$

$$\left[\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} \right] - v_2 - \frac{v_3}{5} + 28 = 0$$

$$\left[\frac{4v_1 - 4v_2 + 3v_1 - 3v_3}{12} \right] - v_2 - \frac{v_3}{5} + 28 = 0$$

$$\left[\frac{7v_1 - 4v_2 - 3v_3 - 12v_2}{12} \right] - \frac{v_3}{5} + 28 = 0$$

$$\left[7v_1 - 16v_2 - 3v_3 \right] 5 - 12v_3 + 60 \times 28 = 0$$

$$35v_1 - 80v_2 - 15v_3 - 12v_3 = -1680$$

$$35v_1 - 80v_2 - 27v_3 = -1680$$

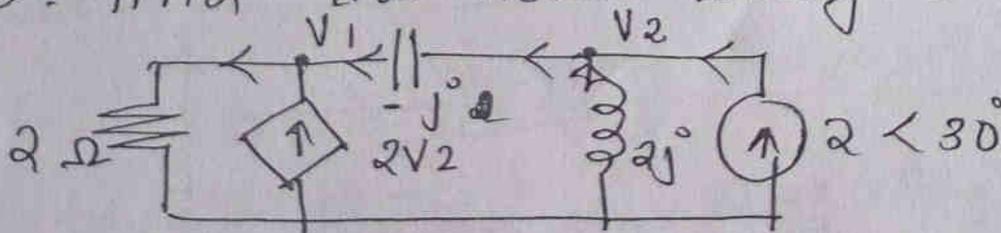
$$v_1 = \underline{1.071V}$$

$$v_2 = \underline{10.5V}$$

$$v_3 = \underline{32.5V}$$

Nodal Analysis of phasor circuits:

Q: Find the node voltages



Consider node 1:

$$2v_2 + \frac{v_1 - v_2}{2} = \frac{v_1}{2}$$

$$2(2jv_2 + v_2 - v_1) - v_1j$$

$$4jV_2 + 2V_2 - 2V_1 = -V_1j$$

$$V_1(-2 + j) + V_2(4j + 2) = 0 \quad \text{--- (1)}$$

consider node 2,

$$\frac{V_2 - V_1}{-j} + \frac{V_2}{2j} = 2 \angle 30^\circ$$

$$1.73 + 1j = -\frac{2V_2 + 2V_1 + V_2}{2j}$$

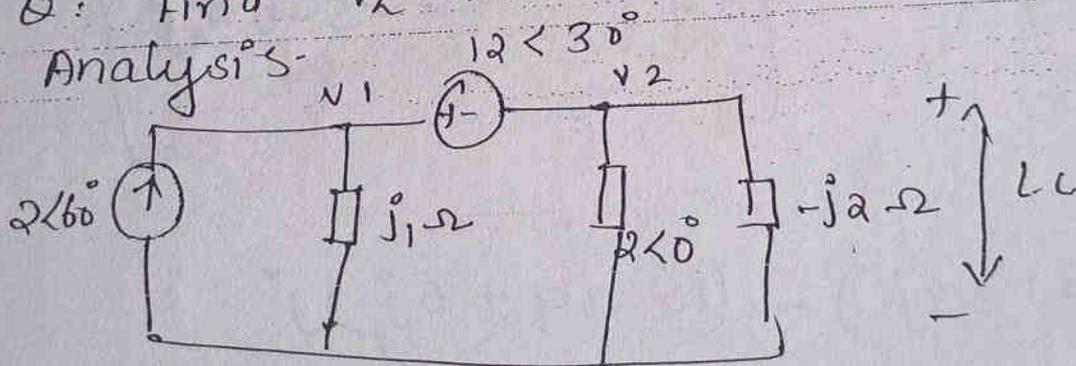
$$-2 + 3.46j = 2V_1 - V_2 \quad \text{--- (2)}$$

$$V_1 = 0.36 + 0.05j$$

$$V_2 = 0.037 + 0.05j$$

Q: Find V_L in the ckt using nodal

Analysis:



$$V_1 - V_2 = 12 \angle 30^\circ$$

$$V_1 - V_2 = 12 \cos 30^\circ + 12j \sin 30^\circ$$

$$V_1 - V_2 = 10.39 + 6j \quad \text{--- (1)}$$

$$2 \angle 60^\circ - \frac{V_1}{j} - \frac{V_2}{2} - \frac{V_2}{-2j} = 0$$

$$1 + 1.73j + V_1j - \frac{V_2}{2} - \frac{V_2j}{2}$$

$$V_1j - (1/2 + j/2)V_2 = 1 + 1.73j$$

$$jV_1 - (0.5 + 0.5j)V_2 = 1 + 1.73j \quad \text{--- (2)}$$

$$\begin{bmatrix} 1 & -1 \\ j & -0.5 - 0.5j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10.39 + 6j \\ 1 + 1.73j \end{bmatrix} \quad \frac{\Delta_2}{\Delta}$$

$$V_2 = \frac{\Delta_2}{\Delta} \begin{bmatrix} 1 & 10.39 + 6j \\ j & 1 + 1.73j \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ j & -0.5 - 0.5j \end{bmatrix}$$

$$\Delta_2 = (1 + 1.73j) - (10.39 + 6j)j$$

$$= (1 + 1.73j) - (10.39j - 6)$$

$$= -1 - 1.73j - 10.39j + 6$$

$$= \underline{\underline{5 - 12.12j}}$$

$$\Delta = (-0.5 - 0.5j) - (-j)$$

$$= -0.5 - 0.5j + j$$

$$= \underline{\underline{-0.5 + 0.5j}}$$

$$\frac{\Delta z}{\Delta} = \frac{5 - 12.12j}{-0.5 + 0.5j} = \frac{13.1 \angle -67.58^\circ}{0.707 \angle 135^\circ}$$

$$= \underline{\underline{-47.42 \angle 18.5^\circ}}$$

$$-12.12 + 7.12j$$

end

$\frac{R_1}{5\%}$