MODULE 3

III	Gears- classification, Gear nomenclature, Tooth profiles, Materials of gears, Law of gearing (review only), virtual or formative number of teeth, gear tooth failures, Beam strength, Lewis equation, Buckingham's equation for dynamic load, wear load, endurance strength of tooth, surface durability, heat dissipation – lubrication of gears – Merits and demerits of each type of gears.	3	15%
	Design of spur gear	3	

INTRODUCTION

- ***Gears are defined as toothed wheels that transmit power and motion from one shaft to another due to the successive engagement of teeth.***
- Hence, gear drives are also called **positive drives.**
- In any pair of gears, the smaller one is called **pinion** and the larger one is called **gear,** immaterial of which is driving the other.
- When pinion is the driver, it results in step down drive in which the output speed decreases and the torque increases.
- On the other hand, when the gear is the driver, it results in step up drive in which the output speed increases and the torque decreases.

Advantages of Gear drives

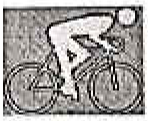
1. It is a positive drive and the velocity ratio remains constant.
2. The dive is compact, since the centre distance between the shafts are relatively small.
3. It can transmit more power.
4. The efficiency of gear drive is very high (99% in case of spur gears)
5. In the gear box, shifting of gears is possible. Hence, velocity ratio can be changed over a wide range

Disadvantages of gear drives

1. Gear drives are costly.
2. Maintenance cost is high.
3. Manufacturing processes for gears are complicated.
4. Gear drives require careful attention.
5. Accurate alignment of shafts is required.

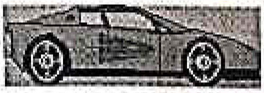
APPLICATIONS OF GEARS

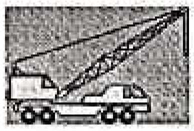
Importance



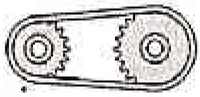
Modern Multi Speed Bicycles

•Automobiles

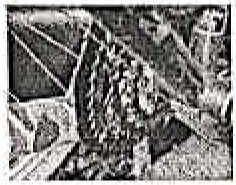




•Construction Equipment



Timing Gears



Recreation (fishing)

Efficient Transportation (sprocket system)

CLASSIFICATION OF GEARS

1. According to the position of axes shafts:

a. parallel b. intersecting c. non-parallel and non-intersecting.

2. According to peripheral velocity of gears:

A.Low velocity b. medium velocity c. high velocity gears.

3. According to the type of gearing:

a. External b. internal c. rack and pinion.

4. According to the position of the teeth:

a. straight b. inclined c. curved.

CLASSIFICATION OF GEARS ACCORDING TO THE POSITION OF AXIS OF SHAFT

- **Parallel**

1. Spur gear

2. Helical gear

3. Rack and pinion

- **Intersecting**

1. Bevel gear

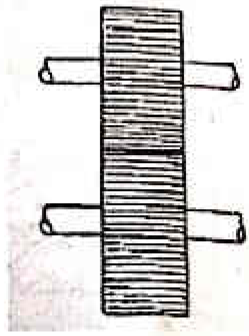
- **Non intersecting & non parallel**

1. Worm &worm gear

1. <u>Spur gear</u>

- Teeth cut parallel to axis of shaft
- Transmit power between parallel shaft

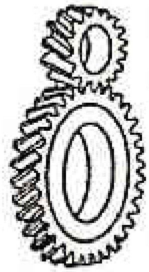
SPUR GEARS



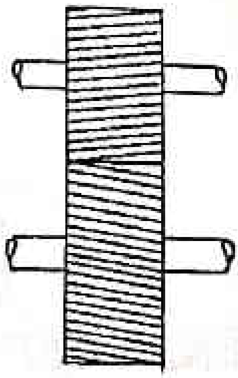
SPUR

<u>2. Helical gear</u>

- Teeth cut on periphery are helical screw form
- A helical tooth is thus inclined at an angle to axis of shaft
- Helical gear imposed radial and trust load on shaft
- Double helical gear used to eliminate end trust load
- These gears are also called herringbone gear.

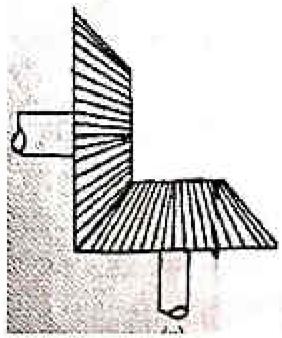


HELICAL GEARS



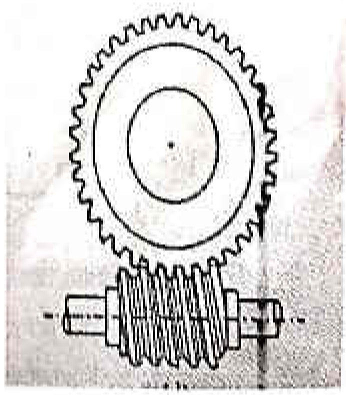
<u>3. Bevel gear</u>

- Bevel gears are used for shafts whose axes intersect each other at right angles.
- However, the angle need not be precisely 90°.
- It can be slightly, less than 90° or slightly greater than 90°.
- The tooth of the bevel gears can be cut straight or spiral. Bevel gears impose both radial and thrust loads on the shafts.



<u>4. Worm and worm wheel</u>

- Power transmitted between non intersecting shaft at right angle each other
- Worm-threaded screw and toothed wheel
- Mainly used for high speed reduction ratio.
- The worm is 'in the form of a threaded screw, which meshes with a wheel.
- The worm imposes high thrust load, while the worm wheel imposes high radial load on the shafts.



MERITS AND DEMERITS OF EACH TYPE OF GEAR

SPUR GEAR

(a) Merits of Spur gears

- Spur gears are simple in design, hence it is easy to manufacture and install them.
- It is highly suitable for compact structures.
- Spur gears are more efficient when compared it to the other gears of same size
- Spur gear teeth are parallel to its axis. Hence, spur gear train does not produce axial thrust. Therefore, the gear shafts can be mounted easily using ball bearings.
- Spur gears are less expensive when compared to other gears

(b) De merits of Spur gears

- Spur gears produce significant noise at high speeds.
- Tooth engagements in spur gears are not gradual when compared to other gears.
- The loads in spur gears are transmitted over fewer teeth only .This makes their design weaker when compared to other gears
- t is not suitable for applications requiring heavy load. because, spur gears take significant stresses during 'operation, Which makes them vulnerable to wear and tear
- Spur gears are not suitable for transmitting power between 1 nonparallel shafts

HELICAL GEAR

(a) Merits of helical gears

- Helical gears can be used for transferring power between 'non parallel shafts.
- At any given time their load is distributed over several teeth i..e, minimum two or three teeth of each gear are always in contact with other gears This results in lesser 1wear and makes them suitable for higher load applications
- During engagement, the teeth engage a little at a time instead of the entire face of each tooth at once This allows' for a smoother and quieter operation
- For same tooth size (module) and equivalent width, helical gears can handle more load than spur gears because the helical gear teeth are positioned diagonally.

(b) De merits of helical gears

- Designing and Manufacturing costs of helical gears are more when compared to the spur gears.
- Design of helical gears is too complicated when compared to the spur gears
- While meshing helical gear make a sliding contact between two gears. This leads to the generation of more heat, which wills eventually results in the power loss and decrease in efficiency. , reducing the sliding fliction, additives are added to the lubricants.
- Also, during meshing, helical gear develops an unwanted thrust in the axial direction. This leads to a decrease in gear efficiency. Therefore, special thrust bearings need to be used for reducing the axial thrust in helical gears

BEVEL GEAR

(a) Merits of bevel gears

- The operating angles of bevel gears can be changed. This characteristic of bevel gears makes them flexible in their operation.
- Efficiency of bevel gears is quite high when compared to the worm gears.
- Due to the rolling action of bevel gears, the sliding friction in bevel gearing mechanism is lower.

(b) De Merits of bevel gears

- To get the maximum efficiency, the bevel gears should be precisely positioned and assembled with the respective shaft
- It has a limited gear ratio, so more gear are required in a gear train to achieve a high total gear ratio
- These gears are not suitable for high-speed reduction
- Bevel gears produce significant noise at high speeds

WORM GEAR

(a) Merits of worm gears

- Worm gears are silent in operation
- Higher speed reduction ratio up to 300:1 is possible in case of worm gear

- Worm and worm gears have self locking characteristics, which restricts their reverse motion. This characteristic makes them suitable for applications like elevators, hoisting , machines etc
- Worm gears occupy lesser space
- These gears can be used for reducing speed and increasing torque.

(b) De Merits of worm gears

- Worm gear materials are expensive.
- Worm gears have high power losses
- Worm gears generate a lot of heat during their operation
- Efficiency of worm gear very low compared to other gears

GEAR NOMENCLATURE

Page 202 of design date book

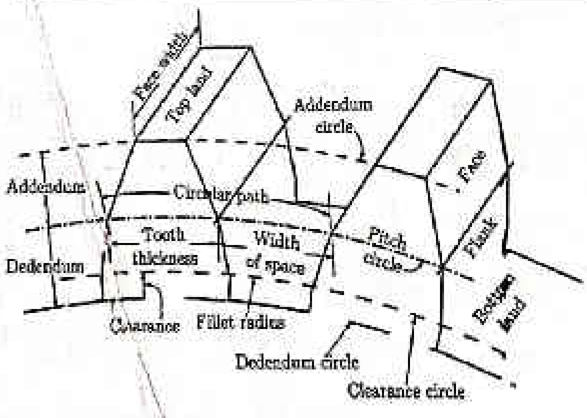


Fig. 12.1(a): *Nomenclature of Spur-gear Teeth*

1. Pitch Circle: It is an imaginary circle which allows pure rolling action without slip

2. Pitch Circle diameter (PCD) (d): It is the diameter of the pitch circle. The size of the gear is usually specified by pitch circle diameter (d).

3. Pitch point: It is a common point of contact between two pitch circles.

4. Base circle: It is an imaginary circle used in involutes gearing from which the involutes curve of the tooth profile is generated.

5. Pressure angle or angle of obliquity (α): It is the angle between the common normal to two gear teeth at the point of contact and the common tangent to the pitch circles at the pitch point. The standard pressure angles are $14\frac{1^0}{2}$ *and* 20^0

6. Circular Pitch (p): It is the distance measured along the pitch circle between two similar points on adjacent teeth.

$$p = \frac{\pi d}{z}$$ **Page 203 ,e q 12.1 (a)**

7. **Diametric pitch ( ):** It is the ratio of the number of teeth to the pitch circle diameter.

$$p_d = \frac{z}{d}$$

$$p\, p_d = \pi$$

Page 203 ,e q 12.1 (b),12.1 (c)

8. ***Module (m):*** it is reciprocal of diametral pitch or ratio between pitch diameter and number of teeth

$$m = \frac{d}{z} = \frac{1}{p_d}$$

Page 203 ,e q 12.1 (d)

9. **Gear ratio (i):** it is the ratio of number of teeth on gear to the number of teeth on pinion

$$i = \frac{Z_2}{Z_1} = \frac{N_1}{N_2}$$

Z_1 = No. of teeth on pinion
Z_2 = No. of teeth on gear
N_1 = Speed of pinion
N_2 = Speed of gear

10. Trasnsmiision ratio: It is the ratio of angular speed of first driving gear to angular speed of last driven gear train

11. **Addendum:** The radial distance between the pitch circle and the top of the teeth. The imaginary circle drawn through the top of the teeth, which is concentric with the pitch circle is called addendum circle.

12. **Dedendum:** The radial distance between the bottom of the tooth to pitch circle. The imaginary circle drawn through the bottom of the teeth, which is concentric with the pitch circle is called dedendum circle.

13. **Centre Distance:** The distance between centres of two gears. It is calculated as the half of the sum of pitch diameters of two gears.

$$\text{Centre distance, } a = \frac{d_1 + d_2}{2} = \frac{m\,(Z_1 + Z_2)}{2}$$

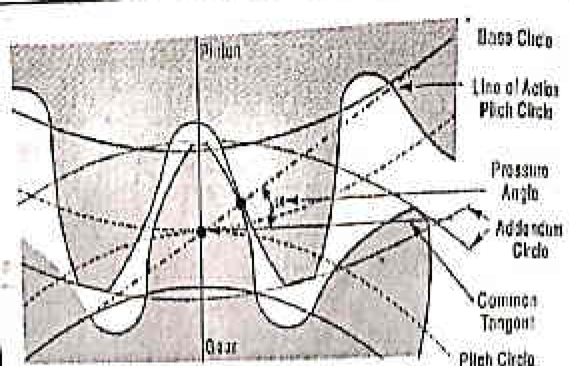


Fig. 3.8

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14. Clearance: The distance between the top of a tooth and the bottom of the tooth in a meshing gear. An imaginary circle passing through the top of the meshing gear is known as clearance circle.

15. Working Depth: The depth to which a tooth extends into the space between teeth on the mating gear is known as working depth. It is also defined as the radial distance from the addendum circle to the clearance circle. It is the sum of the addendum of two meshing gears.

16. Total Depth: It is the radial distance between the addendum and dedendum circles of a gear. It is equal to the sum of addendum and dedendum.

17. Face: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

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18. Face Width: The width of the tooth measured parallel to the gear axis.

19. Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth.

20. Top land: It is the surface on top of the tooth.

21. Bottom land: The bottom surface of the tooth space.

22. Tooth space: The space between successive teeth.

23. Backlash: The difference between the tooth thickness of one gear and the tooth space of the mating gear.

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29. Arc of Approach: The arc of approach is the arc of the pitch circle through which a tooth moves from the beginning of contact to the point of contact at the pitch point.

30. Arc of Recess: The arc of recess is the arc of the pitch circle through which a tooth moves from the point of contact at the pitch point until the contact ends.

31. Length of Path of Contact: It is the sum of the arc of approach and arc of recess.

32. Contact ratio: The maximum number of teeth in contact at any time. It is the ratio of the length of path of contact on pitch circle divided by the circular pitch. Gears are generally designed to have a contact ratio larger than 1.2. because the inaccuracies in mounting the gears might increase the possibility of impact between meshing gears, which in turn increases the noise level.

TOOTH PROFILES

1. Involutes teeth

- An Involutes of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping.

Advantages of Involutes tooth

- The centre distance for a pair of involutes gears can be varied within limit without changing the velocity ratio.
- In involutes gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement remains constant.
- Due to this condition drive is smooth and also the wear of gears is less
- The face and flank of involutes teeth are generated by a single curve
- Therefore, the manufacturing of **involutes teeth are easy**

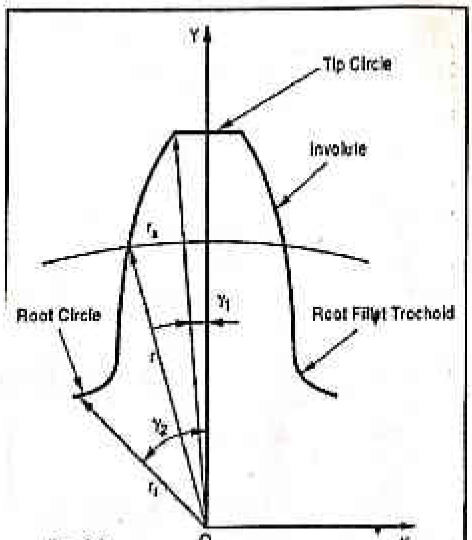


Fig. 3.9

Disadvantages of Involutes Tooth

- The main disadvantage of the involutes teeth is that the interference occurs with pinions having smaller number of teeth"
- This may be avoided by altering the heights of addendum and dedendum of the mating teeth.
- Interference: When the addendum of the gear enters the non-involute addendum' of the mating gear, it is known as Interference

Interference may be prevented,

- (By providing a undercut as shown in Fig. Undercutting in pinion can seriously weaken the tooth. However, when the gear meshes with the undercut pinion, no interference occurs.
- If the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.
- (OR) The point of contact between the two teeth is always on the involutes profiles of both the teeth.

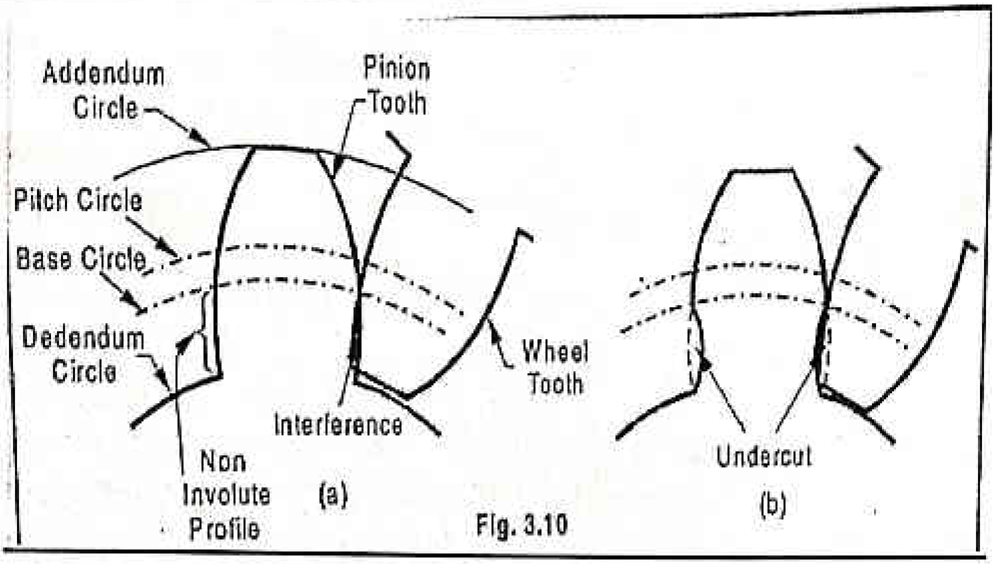


Fig. 3.10

2. CYCLOIDAL TEETH

- Cycloid is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line
- Epicycloids are the curve traced by a point on the circumference of a circle, when the circle rolls without slipping on the outside of a fixed circle.
- A hypocycloid is the curve traced by a point on the circumference of a circle, when the circle rolls without slipping on the inside of a fixed circle.

Advantages of Cycloidal Tooth

- The cycloidal teeth have wider flanks, therefore for the same pitch, cycloidal gears are stronger.
- Cycloidal gears have less wear, because the contact takes place between a convex flank and concave surface.
- In cycloidal gears, the interference does not occur at all

Cycloidal tooth

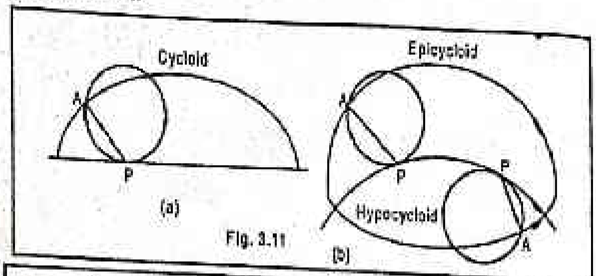
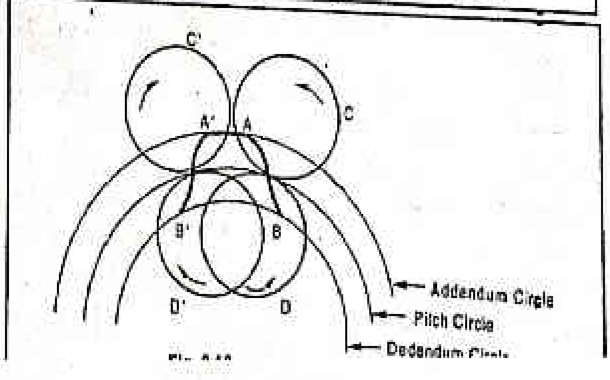


Fig. 3.11



MATERIALS FOR GEARS

The gears are manufactured from both metallic and non-metallic materials. Some of the materials are given here.

PAGE 238,241,247,246

Metallic materials

1. Cast Iron

2. Steel

3. Bronze, etc.

Non-metallic materials

1. Wood

2. Rawhide

3. Compressed paper

4. Synthetic resins like nylon etc.

Selection of gear materials

The gear materials should have following properties.

1. High tensile strength to prevent failure against static loads.

2. High endurance strength to withstand dynamic loads.

3. Low co-efficient of friction:

4. Good manufacturability

TOOTH FAILURES

There are two basic modes of gear tooth failure

1. Breakage of the tooth due to the static and dynamic loads

2. Surface destruction or tooth wear.

- The breakage of the tooth can be avoided by adjusting the parameters in the gear design, such as module and face width.
- The various types of gear tooth wear are as follows:

1. Abrasive wear: The dust particles in the lubricant, dirt, rust, metallic debris can scratch the tooth surface. It can be prevented by providing oil filters, increasing surface hardness and use of high viscosity oil.

2. Corrosive wear: The corrosion of the tooth surface is caused by extreme pressure of additives present in the lubricating oil and due to external contamination.

- It can be prevented by providing complete enclosure of gears from external contamination.

3. Initial pitting: It is caused due to errors in the tooth profile, surface Irregularities and misalignment. It can be prevented by precision machining of gears, adjusting the correct alignment of gears and reducing the dynamic loads.

4.Destructive pitting:

- It is a surface fatigue failure which occurs when the load on gear tooth exceeds the surface endurance strength of material.
- It can be prevented by designing the gears so that the wear strength of the gear tooth is more than the sum of static and dynamic loads.

5. Scoring:

- This is due to excessive surface pressure, high surface speed and inadequate supply of lubricant.
- Due to the breakdown of the oil film, excessive frictional heat leads to overheating of the meshing teeth.
- Scoring can be avoided by selecting the parameters, such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits.

LAW OF GEARING

To maintain a constant velocity ratio for all positions of the gears, the profile of the teeth must be designed in such a way that the common normal to the tooth profile at the point of contact must always pass through the pitch point, which is the tangency of their pitch circles. This is known as the fundamental law of gearing.

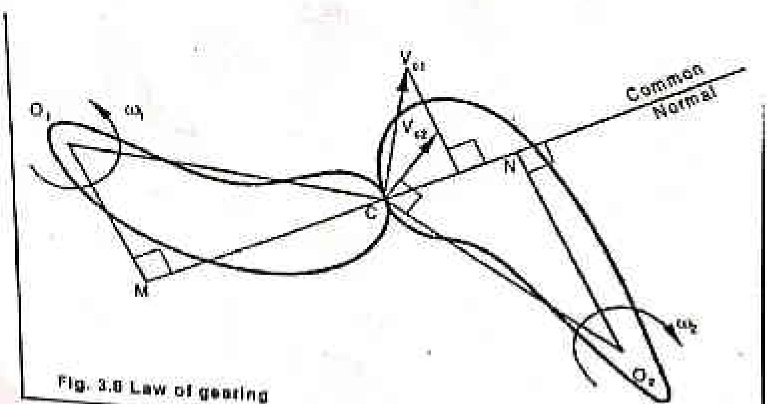


Fig. 3.8 Law of gearing

VIRTUAL OF FORMATIVE NUMBER OF TEETH

- In figure plane normal to gear teeth intersect the pitch cylinder to form an ellipse
- The gear tooth profile generated in the plane using the radius of curvature of ellipse would be a spur gear having the same properties as helical gear.

The radius of curvature of an ellipse is $r_c = \dfrac{d}{\cos^2\beta}$ ...(Eq. 4.23)

The virtual number of teeth on equivalent spur gear in normal plane is called the virtual or formative or equivalent number of teeth.

i.e.

$$z_r = P_n \times 2r_c$$

$$= \frac{z}{d\cos\beta} \times \frac{2d}{2\cos^2\beta}$$

$$z_r = \frac{z}{\cos^3\beta}$$ ...(Eq. 4.24) 12.52a/ Pg 169, DHB

Where, z = Actual number of teeth on a helical gear

β = helix angle.

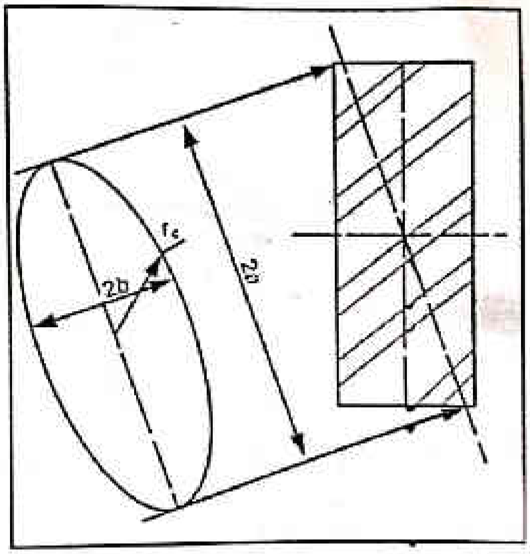


Fig. 4.3: Virtual number of teeth

Equation 12.22(a) in page 211

LUBRICATION OF GEARS

Lubrication is very essential to the proper functioning of gears in a gearbox. The purpose of lubrication is as follows.

1. To reduce the friction between the sliding faces of the gear teeth

2. To prevent the gear against various damages such as wear, pitting, scoring, scuffing, corrosion, rust etc.

3. To remove the heat generation during gear operation.

- Lubricant is also used for lubricating the various bearing inside the gearbox. Adequate lubrication will always result in high reliability, low maintenance, and a long equipment life span

METHOD OF LUBRICATION

3.14.1 Methods of Lubrication

S.No.	Lubrication Methods	Description
1.	Grease Lubrication	• Grease lubrication is highly used in very low speed applications where operations are intermittent.
		• It is suitable for both open and closed gearbox systems.
		• In this system, providing proper quantity of lubrication is important because, with grease, using too little lubrication results in friction loss and too much lubrication results in power loss.
		• The major drawback of using grease lubrication is that the lubricant has no cooling effect.
2.	Splalch Lubrication	• Splash lubrication is suitable for medium speed applications.
		• It is normally used in a closed gearbox system.
		• In this system, the gears are enclosed in a box and dipped in the bath of mineral oil.
		• In addition, provisions are made to ensure that the gear teeth are not fully immersed in oil. This is because, when the teeth are fully immersed, a condition known as churning occurs.
		• The term churning in lubrication of gears (or) lubricant churning refers to one of the common factors that affect the performance of both gear and bearings.
		• Churning occurs when the gears or bearings must work harder to push through the excess lubricant. This action will contribute to the power loss in the system.
3.	Spray Lubrication	• Spray lubrication is suitable for high-speed applicatins.
		• In this system, the lubricant is sprayed through the nozzle. This nozzle is carefully engineered to ensure that the oil reaches the contacting surfaces and bearings.
		• However, this method is not always effective, as high centrifugal forces of the gears affect the direction of the oil spray.

Heat Dissipation

- Normally, when gears operating under conditions of high speed, the heat developed from the frictional losses may become appreciable.
- In such cases, additional lubrication is necessary to avoid frictional losses. However, this additional lubrication should be directed only on the gear blanks but not into the mesh of the teeth.
- This is because, the excessive lubrication at the tooth mesh will create more heat due to the rapid expulsion from the mesh
- heat developed from the frictional losses may become appreciable. In such cases, additional lubrication is necessary to avoid frictional losses.
- However, this additional lubrication should be directed only on the gear blanks but not into the mesh of the teeth.
- This is because, the excessive lubrication at the tooth mesh will create more heat due to the rapid expulsion from the mesh

$$\Delta T = \frac{0.01\, WV}{hA}$$

Where,

W – Load on tooth in N

V – Pitch line velocity or rubbing velocty in m/s

h – Convective heat transfer coefficient in W/m^2 °C

A – Area of exposed surface of gearbox in m^2

The amount of heat generated Q (in J/s) is given by

$$Q = \mu F_n V$$

Where

μ – Coefficient of friction

F_n – Force normal to the tooth surface (N)

V – Pitch line velocity or rubbing velocity in m/s

Note: For worm gears, the amount of heat generated is given as $Q = \dfrac{\mu F_n V}{\cos\gamma}$ where γ is the lead angle.

4.9 DESIGN CONSIDERATIONS FOR GEAR DRIVE

In designing the gear drive, the following data/specifications are usually given:

- The power to be transmitted.
- The speed of driving gear.
- The speed of driven gear or the velocity ratio and
- The center distance.

The following requirements have to be met by the gear drive:

- The gear teeth should have sufficient strength so that they do not fail under static/dynamic loading during normal operating conditions.
- The gear teeth should have good wear characteristics.
- The drive should be compact.
- Lubrication of gears should be satisfactory.
- Alignment of gears and deflections of shafts must be considered since the performance is affected.
- The drive should be properly aligned.

4.6 SYSTEM –OR- PRESSURE ANGLES OF GEAR TOOTH

1. 14 ½° composite system	12	minimum no. of teeth.
2. 14 ½° full depth involute system	32	
3. 20° full depth involute system	18	
4. 20° stub involute system	14	

1. **14 ½° composite system:** is used for general purpose gears. It is the strongest but has no interchangeability. The tooth profile is a cycloidal curve at the top and bottom while the middle portion is a involute curve. This system is based on a 12 tooth pinion as the smallest that will give satisfactory tooth action. The teeth are produced by hobs or milling cutters.
2. **14 ½° full depth involute system:** This system is used with gear hobs for spur and helical gears. The minimum number of teeth is 32 with pinion as the smallest for full involute action with a rack.
3. **20° full depth involute system:** This system has the same proportion as that of 14 ½° full depth involute system, except that the pressure angle has been increased to 20° to avoid interference and undercutting. An 18 tooth pinion is the smallest for full involute action with a rack.
4. **20° stub involute system:** This was developed to produce a strong tooth so as to take heavy loads free from undercutting. The tooth is made shorter than the full depth tooth. This is extensively used in automotive industry because of its ruggedness.

DESIGN METHODOLOGY FOR SPUR GEAR

Method I : If pitch diameter known

$$\boxed{F_t = \sigma_d C_v b Y m}$$ → page 204, eq 12.5(a)

Where $k = b/m$

Sub: eq ⇒ $F_t = \sigma_d C_v (km) Y m = \sigma_d C_v k Y m^2$

$$\boxed{m = \sqrt{\frac{F_t}{\sigma_d C_v k Y}}}$$ page 204 (eq 12.5(b)]

Method 2 If pitch diameter unknown

Torque = F x distance

$M_t = F_t \times d/2$

$F_t = \frac{2M_t}{d}$ [page 206, eq 12-8(a)]

$\sigma_d C_v k Y m^2 = \frac{2M_t}{d}$ | $m = \frac{d}{z}$ $(d = mz)$

$\sigma_d C_v k Y m^2 = \frac{2M_t}{mz}$

$m^3 = \frac{2M_t}{\sigma_d C_v k Y z} \Rightarrow m = \left[\frac{2M_t}{\sigma_d C_v k Y z}\right]^{1/3}$

{page 204, eq 12.5(b)]

when dia known ⇒ design for smallest pitch dia
when dia unknown ⇒ design for largest no: of teeth

Spur Gear Design Problems

① A pair of straight teeth spur gear is transmit 20kW the pinion rotates at 300rpm. The velocity ratio is 3:1. The allowable static stress for pinion and gear materials are 120mpa and 100mpa respectively. The pinion has 15 teeth and is face width 10 times the module.

Assume $C_v = \frac{3}{3+v}$ Determine (a) Module

(b) Face width

(c) pitch circle diameter, from standard point of strength only

① ⇒ pinion

② ⇒ Gear

Solution

$P = 20kW$

speed of pinion $N_1 = 300rpm$, No: of teeth on pinion $Z_1 = 15$, face width $= 10m$

$\sigma_{dp} = 120mpa$, $\sigma_{dG} = 100mpa$

Speed ratio $i = \frac{Z_2}{Z_1} = \frac{N_1}{N_2} = 3$ ⇒ $Z_2 = 3Z_1 = 3 \times 15 = \underline{45 \text{ teeth}}$

{Step 1} Identify weak part (pinion/Gear)

Strength Factor $= \sigma_d y$

where $y = \left(0.154 - \frac{0.912}{z}\right)$ for 20° involute teeth [page 204, eq 12.5 (d)]

$y_{pinion} = 0.154 - \frac{0.912}{Z_1} = 0.154 - \frac{0.912}{15} = \underline{0.0932}$

$y_{gear} = 0.154 - \frac{0.912}{Z_2} = 0.154 - \frac{0.912}{45} = \underline{0.1337}$

For pinion $\sigma_{dp} \times y_p = 120 \times 0.0932 = \underline{11.184}\,mpa$, For gear $\sigma_{dG} \times y_G = 100 \times 0.1337 = \underline{13.37}\,mpa$

Here $\sigma_{dp} y_p$ is Less, Design Based on pinion

Step 2 (a) To find Module (m)

Here diameter unknown Module $m = \left\{\frac{2M_t}{\sigma_d C_v k Y z}\right\}^{1/3}$ [page 204 eq 12.5(b)]

- where $k = \frac{b}{m} = \frac{10\cancel{m}}{\cancel{m}} = \underline{10}$
- Form Factor $Y = \pi y = \pi y_p = \underline{\pi \times 0.0932}$

$d = mz$ [page 203 12.1(c)]

- Velocity $V = \frac{\pi d N}{60} = \frac{\pi d_p N_1}{60} = \frac{\pi (m Z_1) N_1}{60} = \frac{\pi \times m \times 15 \times 300}{60}$

$v = 235.5m$ (m/s) $= .235m$ (mm/sec)

$C_v = \frac{3}{3+v} = \frac{3}{3+0.235m}$

- $P = \frac{2\pi N M_t}{60} = \frac{2\pi N_1 M_t}{60}$ ⇒ $20 \times 10^3 = \frac{2\pi \times 300 \times M_t}{60}$ ⇒ $M_t = 636.942\,Nm$
$= 636.942 \times 10^3\,Nmm$

Sub value in Eq 12.5(b)

$$m = \left\{\frac{2M_t}{\sigma_{dp} \times C_v \times k \, y \, z_1}\right\}^{1/3} = m = \left\{\frac{2 \times 636.62 \times 10^3}{120 \times \frac{3}{3+0.235m} \times 10 \times 0.0932\pi \times 15}\right\}^{1/3}$$

$$m^3 = \frac{1273240}{\frac{15802.992}{3+0.235m}} \Rightarrow m^3 = \frac{1273240}{1} \times \frac{3+0.235m}{15802.992}$$

$$\Rightarrow m^3 = (3+0.235m)\, 80.569$$

$$\Rightarrow m^3 = 241.709 + 18.933 \Rightarrow \boxed{m^3 - 18.92m - 241.59 = 0}$$

$m = 7.23mm \Rightarrow$ Standard module $= 8$ [page 229 Table 12.2]

(b) Face width $b = 10m = 10 \times 8 = 80mm$

(c) pitch circle diameter $d = mz$ [page 203, eq 12.1(c)]

$$d_p = mz_1 = 8 \times 15 = 120mm$$

$$d_g = mz_2 = 8 \times 45 = 360mm$$

(2) A pair of spur gears has to transmit 20KW from a shaft rotating at 1000 rpm to parallel shaft which is rotate at 310rpm. Number of teeth on pinion is 31 with 20° full depth involute teeth form. The material for pinion is SAE 1040 untreated with allowable static stress 206.81mpa and material for the gear is cast steel 0.20% C untreated with allowable static stress 137.34mpa. Determine module & facewidth of gear. Also find dynamic tooth load on gear. Take service factor 1.5

Solution

pinion (1) → driver G.D

Gear (2) → driven $P = 20KW = 20 \times 10^3 W$, $N_1 = 1000 rpm$, $N_2 = 310 rpm$, $z_1 = 31$, $\alpha = 20°$ FDI

$\sigma d_{pinion} = 206.81 Mpa$, $\sigma d_{gear} = 137.34 mpa$, $C_s = 1.5$

We know $i = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$

$i = \frac{1000}{310} = 3.23 \Rightarrow z_2 = iz_1 = 3.23 \times 31 = 100$ teeth

Step 1 Identify weaker part (Gear/pinion)

For $\alpha = 20°$ full depth involute $\Rightarrow y = 0.154 - \frac{0.912}{z}$ (page 204 eq 12.5(d))

$$y_{pinion} = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{31} = 0.1246$$

$$y_{Gear} = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{100} = 0.1449$$

For p

For pinion $\sigma_{d\,pinion} \times y_{pinion} = 206.81 \times .1246 = \underline{25.77\text{ Mpa}}$

$\sigma_{d\,gear} \times y_{gear} = 137.34 \times 0.1449 = \underline{19.90\text{ Mpa}}$

$\sigma_{d\,gear} \times y_{gear}$ is Less, Design based on gear

Step 2 To find Module (m)

Here diameter unknown $m = \left\{ \dfrac{2M_t}{\sigma_d C_v K Y z} \right\}^{\frac{1}{3}}$ [page 204 eq 12.5(b)]

where • $P = \dfrac{2\pi N M_t}{60} \Rightarrow P = \dfrac{2\pi N_2 M_t}{60} \Rightarrow 20 \times 10^3 = \dfrac{2\pi \times 310 \times M_t}{60}$

$M_t = \underline{616.08\text{ Nm}} = \underline{616.08 \times 10^3\text{ Nmm}}$

- Assume $K = \dfrac{b}{m} = 10$
- Assume $C_v = 0.5$
- Form factor $Y = \pi y = \pi y_2 = \underline{\pi \times 0.1449}$

Sub value in eq 12.5(b)

$$m = \frac{2M_t}{\sigma_{d\,gear} C_v K Y z_2} \Rightarrow m = \left\{ \frac{2 \times 616.08 \times 10^3}{137.34 \times 0.5 \times 10 \times 0.1449 \times \pi \times 100} \right\}^{1/3} = \underline{3.4\text{ mm}}$$

page 229 (Table 12.2) Standard module $m = 3.5$

Trial I

Here C_v Assumed, check $\sigma_{d\,gear}$

Lewis eq $\boxed{F_t = \pi \sigma_d C_v b y m}$ → page 204, eq 12.5(a)

where $v = \dfrac{\pi d N}{60} = \dfrac{\pi d_2 N_2}{60} = \dfrac{\pi \times 350 \times 310}{60}$

$= 5681\text{ mm/s} = \underline{5.681\text{ m/s}}$

$d = mz$ [203 12.1(e)]

$d = m z_2$

$= 3.5 \times 100 = 350$

- For $v < 8\text{ m/s}$ $C_v = \dfrac{3.05}{3.05 + v}$ [page 205, eq 12.6(a)]

$= \dfrac{3.05}{3.05 + 5.681} = \underline{0.349}$

$b = 10m$

$= 10 \times 3.5$

$= \underline{35}$

- $F_t = 2M_t/d$ [page 206, eq 12.8(a)]

$= 2M_t/d \times C_s = \dfrac{2 \times 616.08 \times 10^3}{350} \times 1.5 = 5280.68\text{ N}$

Sub: eq 12.5(a)

$5280.68 = \pi \times \sigma_{d\,gear} \times 0.349 \times 35 \times 0.1449 \times 3.5$

$\sigma_{d\,gear} = 271.34\text{ mpa} > 137.374\text{ mpa}$

So it is not satisfactory

Change Module