

MODULE 5

Syllabus: Parameters of two port networks:

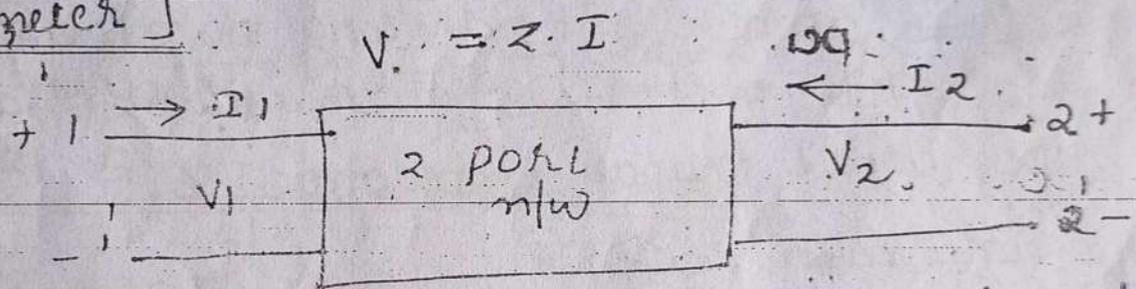
Impedance; Admittance, Transmission and Hybrid Parameters. Inter relationship among parameters in

Series and parallel connection of two port network

Reciprocal and symmetrical two port network

Characteristic impedance, Image impedance and propagation constant.

Impedance parameter [open circuit Impedance Parameter]



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

V_1 & $V_2 \rightarrow$ dependent variables
 I_1 & $I_2 \rightarrow$ independent variables

Z_{11} , Z_{12} , Z_{21} , Z_{22} are called network functions and are called impedance parameter.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Make $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2 = 0}$$

Make $I_1 = 0$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2 = 0}$$

M-5-1

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} \quad \& \quad Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

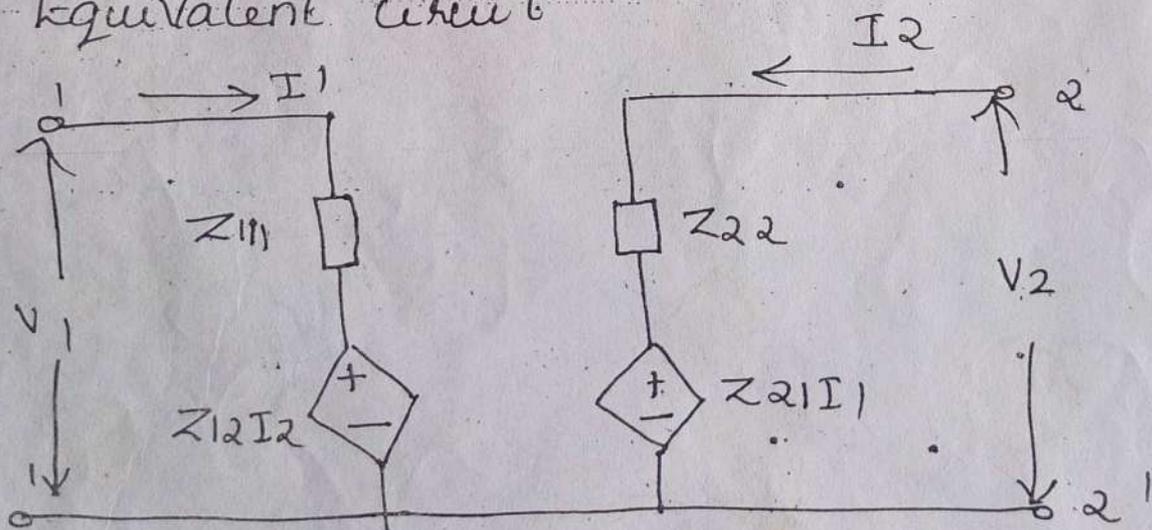
$Z_{11} \Rightarrow$ Driving point impedance at port 1-1' with port 2-2' open circuited. It is called open circuit input impedance.

$Z_{21} \Rightarrow$ Transfer impedance at 1-1' with port 2-2' open circuited. It is called the open circuit forward transfer impedance.

$Z_{12} \Rightarrow$ Transfer impedance at 2-2' with port 1-1' open circuited. It is called the open circuit reverse transfer impedance.

$Z_{22} \Rightarrow$ open circuit driving point impedance at port 2-2' with port 1-1' open circuited. It is also called the open circuit output impedance.

Equivalent circuit

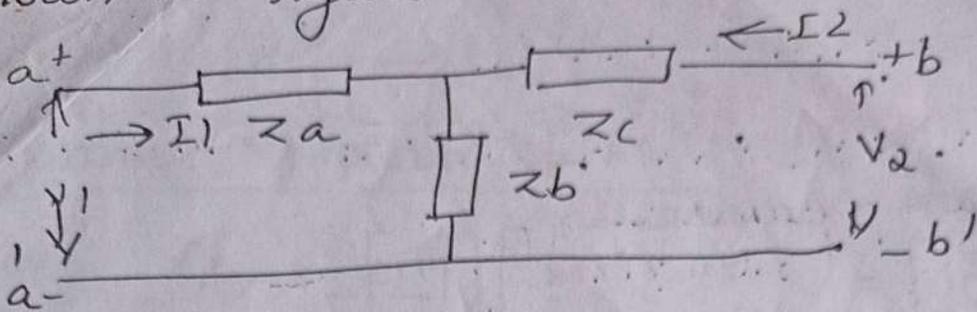


If the network under study is reciprocal/bilateral, then in accordance with the reciprocity principle

$$\frac{V_2}{I_1} \Big|_{I_2=0} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$\boxed{Z_{12} = Z_{21}}$$

Q: Find the z-parameter for the circuit shown in figure.



Sln: $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_1 = (Z_a + Z_b) I_1 + Z_b I_2$

$V_2 = Z_b I_1 + (Z_b + Z_c) I_2$

$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \underline{(Z_a + Z_b)}$

$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \underline{Z_b}$ *

$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \underline{Z_b}$ *

$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = \underline{(Z_b + Z_c)}$

$Z_{12} = Z_{21} \Rightarrow$ satisfy the principle of reciprocity

ADMITTANCE PARAMETER (Y) [short circuit admittance parameter]: Expressing two port currents in terms of two port voltages I_1 & $I_2 \Rightarrow$ dependent variables
 V_1 & $V_2 \Rightarrow$ Independent variables.

$(I_1, I_2) = f(V_1, V_2)$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{cases} [I] \\ [V] \end{cases} = [Y] \cdot [V]$$

$Y_{11}, Y_{12}, Y_{21}, Y_{22} \Rightarrow$ network functions, also admittance parameters

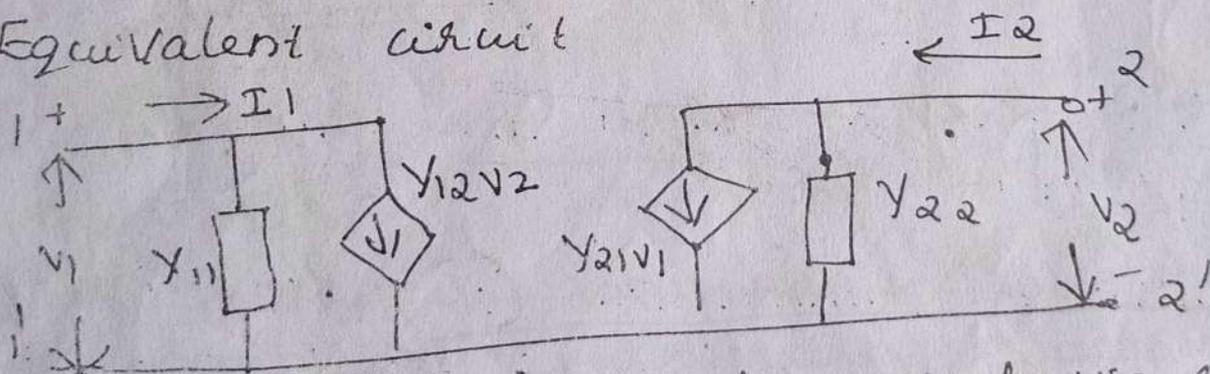
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$; driving point admittance at port 1-1' with 2-2' short circuited. It is also called the short circuit input admittance.

$Y_{21} = \frac{I_2}{V_2} \Big|_{V_1=0}$; Transfer admittance at port 2-2' with port 1-1' short circuited; it is also called short circuit reverse transfer admittance.

$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$; short circuit driving point admittance at port 2-2' with port 1-1' short ckt. It is also called short circuit output admittance.

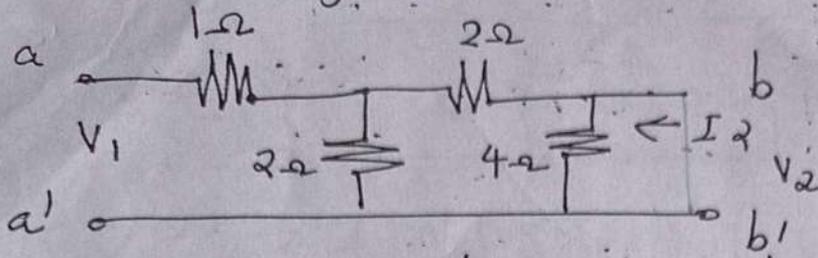
Equivalent circuit



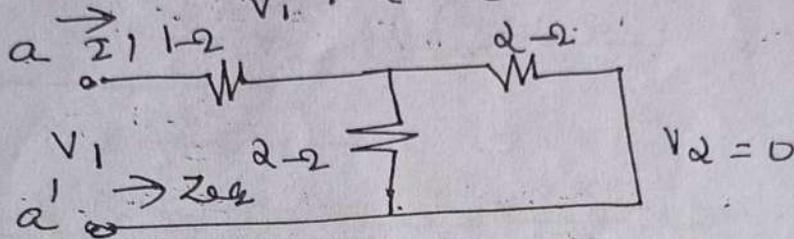
If the network under study is reciprocal or bilateral, then $\frac{I_1}{V_2} \Big|_{V_1=0} = \frac{I_2}{V_1} \Big|_{V_2=0}$

$$[Y] = [Z]^{-1} \quad \boxed{Y_{12} = Y_{21}}$$

Q. Find the Y parameter for the network shown in figure



Soln: $Y_{11} = \frac{I_1}{V_1} \Big|_{V_2 = 0}$



$$V_1 = Z_{eq} I_1 \quad ; \quad Z_{eq} = 2\Omega$$

$$Y_{11} = \frac{I_1}{V_1} = \underline{\underline{\frac{1}{2}}}$$

To find Y_{21}

$$3I_1 + 2I_2 = V_1 \quad \text{--- (a)}$$

$$2I_1 + 4I_2 = 0 \quad \text{--- (b)}$$

$$2I_1 = -4I_2$$

$$I_1 = -2I_2 \quad \text{--- (c)}$$

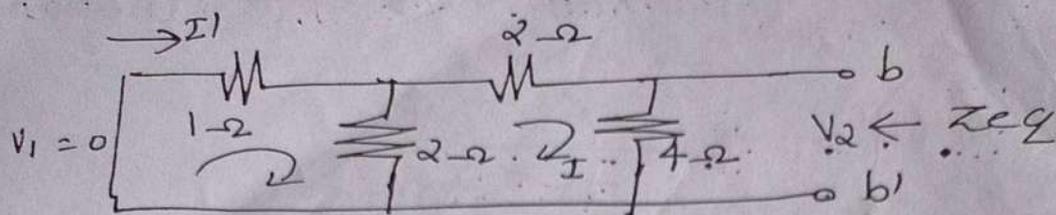
Substitute (c) in (a)

$$-6I_2 + 2I_2 = V_1$$

$$-4I_2 = V_1$$

$$\frac{2I_2}{V_1} = \underline{\underline{-\frac{1}{4}}}$$

With port a-a' short circuited



$$\begin{aligned}
 Z_{eq} &= \left\{ (1 \parallel 2) + 2 \right\} \parallel 4 \\
 &= \left(\frac{2}{3} + 2 \right) \parallel 4 = \frac{\frac{8}{3} \cdot 4}{\frac{8}{3} + 4} = \frac{\frac{32}{3}}{\frac{20}{3}} \\
 &= \frac{32 \times 3}{3 \cdot 20} = \frac{32}{20} = 8/5 //
 \end{aligned}$$

$$I_2 Z_{eq} = v_2 \Rightarrow v_2 = \frac{I_2}{v_2} = \frac{1}{Z_{eq}} = \underline{\underline{5/8}}$$

$$3I_1 - 2I_2 = 0 \quad \text{--- (d)}$$

$$-2I_1 + 8I_2 + 4I_2 = 0 \quad \text{--- (e)}$$

$$4I_2 + 4I_1 = v_2 \quad \text{--- (f)}$$

$$3I_1 = 2I_2$$

$$I_1 = \frac{2}{3} I_2 \quad \text{--- (g)}$$

substitute (g) in (e)

$$-2I_1 + 8 \cdot \frac{2}{3} I_2 + 4I_2 = 0$$

$$-2I_1 + 12I_2 + 4I_2 = 0$$

$$10I_2 + 4I_2 = 0$$

$$10I_2 = -4I_2$$

$$I_2 = -5/2 I_1 \quad \text{--- (h)}$$

substituting (b) in (f)

$$4 \times \frac{-5}{2} I_1 + 4 \cdot \frac{3}{2} I_1 = V_2$$

$$= -10 I_1 + 6 I_1 = V_2$$

$$= -4 I_1 = V_2$$

$$Y_{12} = \frac{I_1}{V_2} = \underline{\underline{\frac{-1}{4}}}$$

$$I_1 = 0.5 V_1 - 0.25 V_2$$

$$I_2 = -0.25 V_1 + 0.625 V_2$$

Transmission (ABCD) or chain parameters

The input variables V_1 & I_1 at port 1-1 usually called the sending end, are expressed in terms of the output variables V_2 and I_2 at port 2-2', called the receiving end. The transmission parameters provide a direct relationship between input and output

Expressing one port variable in terms of other port variable $(V_1, I_1) = f(V_2, I_2)$

$$V_1 = A V_2 - B I_2$$

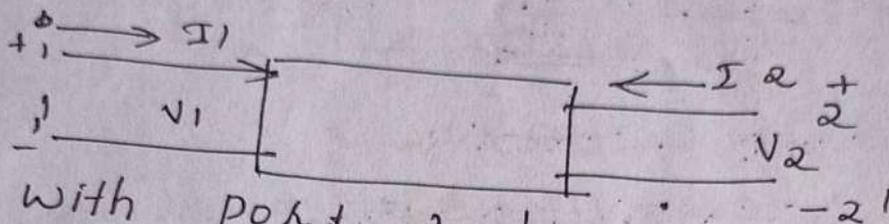
$$I_1 = C V_2 - D I_2$$

The negative sign is used with I_2 and not for the parameter B and D . Both the port currents I_1 and $-I_2$ are directed, to the right, i.e. with a negative sign (the current at port 2-2') which leaves the port is called designated as positive)

The parameters A, B, C, D are called the transmission parameters.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

↓ Transmission Matrix



• with port 2-2' open

$$A = \frac{V_1}{V_2} \Big|_{I_2=0}; \quad C = \frac{I_1}{V_2} \Big|_{I_2=0}$$

$g_{21} = \frac{1}{A} = \frac{V_2}{V_1} \Big|_{I_2=0}$; open circuit voltage gain
a dimensionless parameter.

$$\frac{1}{C} = \frac{V_2}{I_1} \Big|_{I_2=0} = Z_{21}; \text{ Open circuit transfer impedance}$$

With port 2-2' short circuited, $V_2 = 0$, applying voltage V_1 at port 1-1'

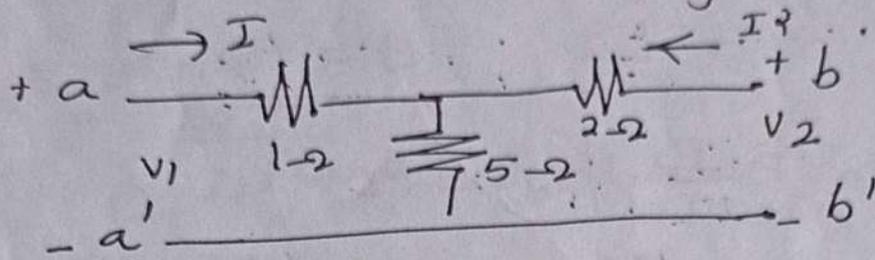
$$-B = \frac{V_1}{I_2} \Big|_{V_2=0} \quad \text{and} \quad -D = \frac{I_1}{I_2} \Big|_{V_2=0}$$

$-\frac{1}{B} = \frac{I_2}{V_1} = Y_{21}$; short circuit transfer admittance

$$-\frac{1}{D} = \frac{I_2}{I_1} \Big|_{V_2=0} = \alpha_{21} \Big|_{V_2=0}$$

• Short circuit current gain:

Q: Find the Transmission (general circuit) parameters for the circuit shown in figure



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$b-b'$ open

$$A = \frac{V_1}{V_2} \Big|_{I_2 = 0}$$

$$V_1 = 6I_1; \quad V_2 = 5I_1$$

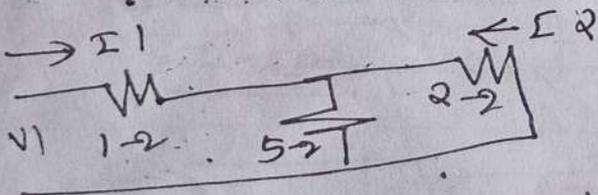
$$A = 6/5 //$$

$$C = \frac{I_1}{V_2} \Big|_{I_2 = 0}$$

$$= \frac{6 \frac{V_2}{5}}{V_2} = \frac{6}{5} //$$

$b-b'$ short circuited

(i.e. $V_2 = 0$)



$$6I_1 + 5I_2 = V_1$$

$$5I_1 + 7I_2 = 0$$

$$D I_1 = -\frac{7}{5} I_2$$

$$-\frac{17}{5} I_2 = V_1$$

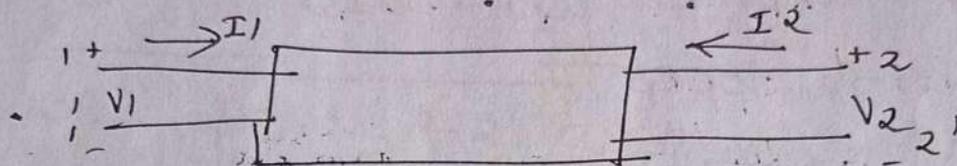
$$-B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$= -\frac{17}{5} ; B = \underline{\underline{17/5}}$$

$$-D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

$$= -7/5 ; D = \underline{\underline{7/5}}$$

* Inverse Transmission ($A' B' c' D'$) parameters



$$V_2 = A' V_1 - B' I_1$$

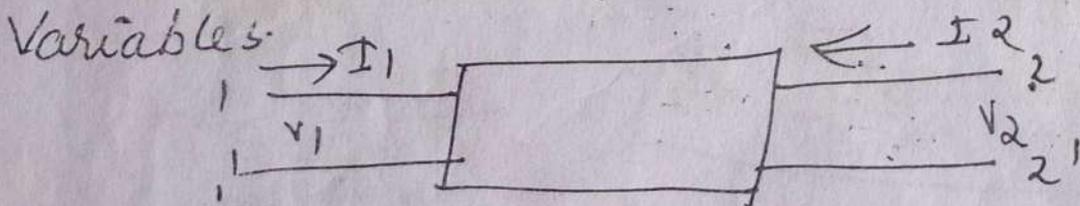
$$I_2 = c' V_1 - D' I_1$$

$$A' = \left. \frac{V_2}{V_1} \right|_{I_1=0} ; c' = \left. \frac{I_2}{V_1} \right|_{I_1=0}$$

$$B' = - \left. \frac{V_2}{I_1} \right|_{V_1=0} ; D' = - \left. \frac{I_2}{I_1} \right|_{V_1=0}$$

Hybrid (h) parameter: Find extensive use in transistor circuits.

in these parameter voltage of one port and the current of other port are taken as the independent variables.



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

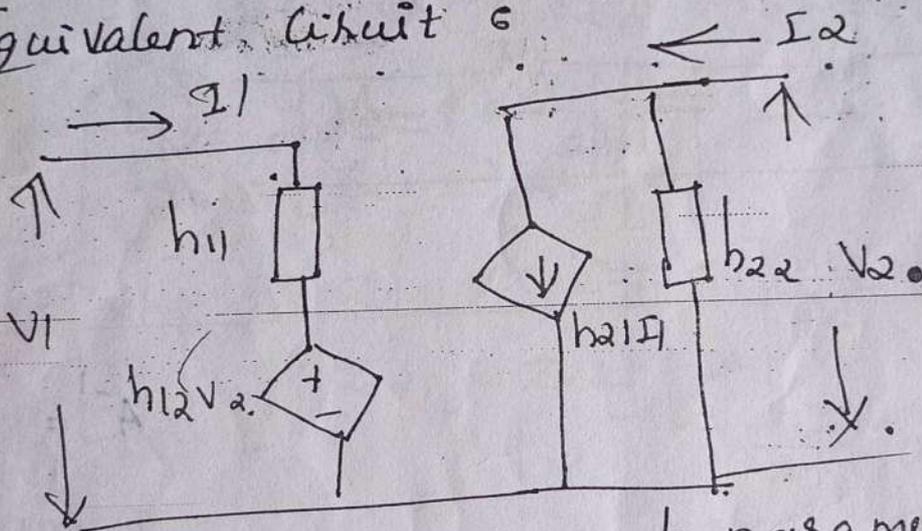
$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} \text{ Short circuit input impedance } \left(\frac{1}{Y_{11}} \right)$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} \text{ Short circuit forward current gain } \left(\frac{Y_{21}}{Y_{11}} \right)$$

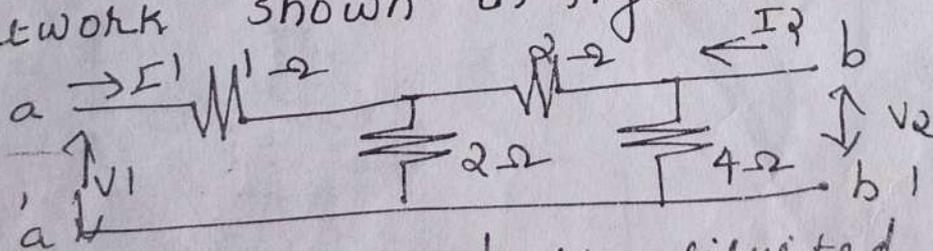
$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} \text{ Open circuit reverse voltage gain } \left(\frac{Z_{12}}{Z_{22}} \right)$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} \text{ Open circuit admittance } \left(\frac{1}{Z_{22}} \right)$$

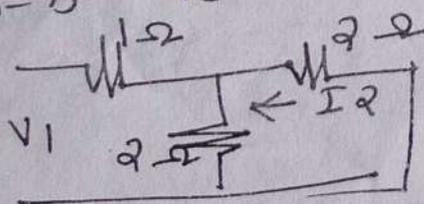
Equivalent circuit



Find the transmission h parameter of the network shown in figure



sol: $b-b'$ is short circuited



$$V_1 = I_1 \cdot 2 \Omega$$

$$V_1 = 2 I_1$$

$$V_1 = I_1 Z_{eq}$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = \underline{\underline{2\Omega}} \quad V_1 = 2I_1$$

$$3I_1 + 2I_2 = V_1$$

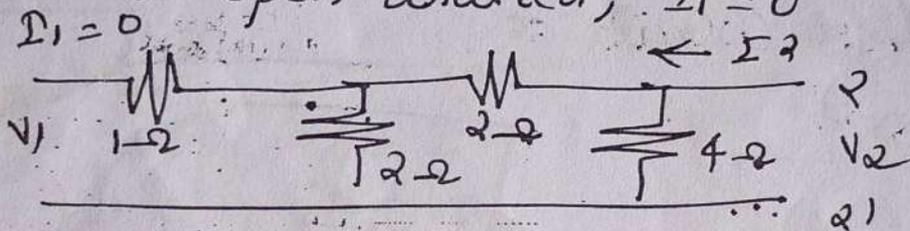
$$2I_1 + 4I_2 = 0$$

$$2I_1 = -4I_2$$

$$I_1 = -2I_2$$

$$h_{21} = \frac{I_2}{I_1} = \underline{\underline{-\frac{1}{2}}}$$

If $a-a'$ is open circuited; $I_1 = 0$



$$h_{12} = \frac{V_1}{V_2} =$$

$$h_{22} = \frac{I_3}{V_2} =$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\text{adj } A = \text{cofactor}(A)$$

Interrelationship among parameter sets

If we want to express α -parameters in terms of β -parameters, we have to write β parameters equation and then by algebraic manipulation rewrite the equations as needed for α -parameters

Z parameter in terms of other parameters

(i) Z parameter in terms of Y -parameter

$$Y \text{ parameter } (I_1, I_2) = f(V_1, V_2)$$

$$Z \text{ " } (V_1, V_2) = f(I_1, I_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{--- (1)}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{--- (2)}$$

From (1) $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

ie $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$

$$\frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}; \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}; \quad Z_{21} = \frac{-Y_{21}}{\Delta Y}$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

(ii) Z parameter in terms of T parameter.
T parameter or ABCD parameters.

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$V_2 = \frac{1}{C} [I_1 + DI_2] = \frac{I_1}{C} + \frac{D}{C} I_2$$

$$V_1 = A \left[\frac{I_1}{C} + \frac{D}{C} I_2 \right] - BI_2$$

$$V_1 = \frac{A}{C} I_1 + \left[\frac{AD}{C} - B \right] I_2 \quad \text{--- (3)}$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{--- (4)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (5)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (6)}$$

Comparing (3) with (5) & (4) with (6)

$$Z_{11} = \frac{A}{C} \quad Z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C}$$

$$Z_{21} = \frac{1}{C} \quad Z_{22} = D/C$$

(iii) Z parameter in terms of h parameters

h parameter $(V_1, I_2) = f(I_1, V_2)$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

$$V_2 = \frac{(I_2 - h_{21} I_1)}{h_{22}}$$

$$V_2 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Substitute V_2 in ①

$$\begin{aligned} V_1 &= h_{11} I_1 + h_{12} \left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right] \\ &= \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}} \right] I_1 + \frac{h_{12}}{h_{22}} I_2 \\ &= \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \end{aligned}$$

$$V_1 = \frac{\Delta H}{h_{22}} I_1 - \frac{h_{12}}{h_{22}} I_2 \quad \text{--- ③}$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{--- ④}$$

By comparison

$$\begin{aligned} Z_{11} &= \frac{\Delta H}{h_{22}} & Z_{12} &= \frac{+h_{12}}{h_{22}} \\ Z_{21} &= \frac{-h_{21}}{h_{22}} & Z_{22} &= \frac{1}{h_{22}} \end{aligned}$$

Y parameter in terms of other parameters

1) Y-parameter in terms of Z-parameter

$$[V] = [Z] [I] \quad \text{--- ①}$$

$$[I] = [Y] [V] \quad \text{--- ②}$$

$$\text{from ① } [I] = \frac{[V]}{[Z]} = Z^{-1} [V]$$

$$\Rightarrow [Y] = [Z]^{-1}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta z} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$y_{11} = \frac{z_{22}}{\Delta z}, \quad y_{12} = \frac{-z_{12}}{\Delta z}, \quad y_{21} = \frac{-z_{21}}{\Delta z}, \quad y_{22} = \frac{z_{11}}{\Delta z}$$

2) Y parameter in terms of T-parameter

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

From (1) $BI_2 = AV_2 - V_1$

$$I_2 = \frac{1}{B} V_1 + \frac{A}{B} V_2 \quad \text{--- (3)}$$

Substitute I_2 in (2)

$$I_1 = CV_2 - D \left[\frac{1}{B} V_1 + \frac{A}{B} V_2 \right]$$

$$I_1 = \frac{D}{B} V_1 + \left[\frac{BC - AD}{B} \right] V_2 \quad \text{--- (4)}$$

$$y_{11} = \frac{D}{B}, \quad y_{12} = -\frac{DT}{B}, \quad y_{21} = \frac{1}{B}, \quad y_{22} = \frac{A}{B}$$

3) Y parameter in terms of h parameter

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

from (1) $I_1 = \frac{1}{h_{11}} [V_1 - h_{12} V_2]$

$$I_2 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \quad \text{--- (3)}$$

$$Y_{11} = \frac{1}{h_{11}} \quad Y_{12} = \frac{-h_{12}}{h_{11}}$$

Substitute (3) in (2)

$$I_2 = h_{21} \left[\frac{1}{h_{11}} v_1 - \frac{h_{12} v_2}{h_{11}} \right] + h_{22} v_2$$

$$= \frac{h_{21}}{h_{11}} v_1 + \left[-\frac{h_{12} h_{21}}{h_{11}} + h_{22} \right] v_2$$

$$= \frac{h_{21}}{h_{11}} v_1 + \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} \right] v_2$$

$$I_2 = \frac{h_{21}}{h_{11}} v_1 + \frac{\Delta h}{h_{11}} v_2 \quad (4)$$

From 4

$$Y_{21} = \frac{h_{21}}{h_{11}} \quad Y_{22} = \frac{\Delta h}{h_{11}}$$

T parameter in terms of other parameter

(1) T parameter in terms of Z parameter

$$v_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$

$$v_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

from (2)

$$I_1 = \frac{1}{Z_{21}} [v_2 - Z_{22} I_2]$$

$$I_1 = \frac{1}{Z_{21}} v_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad (3)$$

from (3)

$$\boxed{C = \frac{1}{Z_{21}} \quad D = \frac{Z_{22}}{Z_{21}}}$$

substitute (3) in (1)

$$V_1 = Z_{11} \left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22} I_2}{Z_{21}} \right] + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 + \left[Z_{12} - \frac{Z_{11} Z_{22}}{Z_{21}} \right] I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 + \left[\frac{Z_{21} Z_{12} - Z_{11} Z_{22}}{Z_{21}} \right] I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right] I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2 \quad \text{--- (4)}$$

from (4)

$$\boxed{A = \frac{Z_{11}}{Z_{21}} \quad B = \frac{\Delta Z}{Z_{21}}}$$

2) T parameter in terms of Y parameter

T-parameter

$$V_1 = A V_2 - B I_2 \quad \text{--- (1)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (2)}$$

Y-parameter

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (4)}$$

From (4)

$$V_1 = \frac{1}{Y_{21}} [I_2 - Y_{22} V_2]$$

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2$$

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \quad (5)$$

sub (5) in (3)

$$I_1 = Y_{11} \left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \right] + Y_{12} V_2$$

$$= -\frac{Y_{11} Y_{22}}{Y_{21}} V_2 + Y_{12} V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$= \left[\frac{-Y_{11} Y_{22} + Y_{12} Y_{21}}{Y_{21}} \right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$= \left[\frac{-Y_{11} Y_{22} + Y_{12} Y_{21}}{Y_{21}} \right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$= -\left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{21}} \right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = \frac{-\Delta Y}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 \quad (6)$$

Comparing (5) & (6) with (1) & (2)

$$A = -\frac{Y_{22}}{Y_{21}} \quad B = \frac{1}{Y_{21}}$$

$$C = -\frac{\Delta Y}{Y_{21}} \quad D = -\frac{Y_{11}}{Y_{21}}$$

3) T-parameter in terms of h-parameter

$$T \text{ parameter: } V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

$$h \text{ parameter } V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- (3)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- (4)}$$

from (4)

$$I_1 = \frac{1}{h_{21}} [I_2 - h_{22}V_2]$$

$$I_1 = \frac{-h_{22}V_2 + I_2}{h_{21}} \quad \text{--- (5)}$$

sub (5) in (3)

$$V_1 = h_{11} \left[\frac{-h_{22}V_2 + I_2}{h_{21}} \right] + h_{12}V_2$$

$$= \left[\frac{-h_{11}h_{22} + h_{12}h_{21}}{h_{21}} \right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$= \left[\frac{-h_{11}h_{22} + h_{12}h_{21}}{h_{21}} \right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$V_1 = - \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}} \right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

$$V_1 = -\frac{\Delta h}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2 \quad \text{--- (6)}$$

Comparing equation (6) & (5) with (1) & (2)

$$h_{11} = \frac{\Delta z}{z_{22}} \quad h_{12} = \frac{z_{12}}{z_{22}}$$

$$h_{21} = \frac{-z_{21}}{z_{22}} \quad h_{22} = \frac{1}{z_{22}}$$

2) h-parameter in terms of Y parameter

h-parameter

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

Y parameter

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (4)}$$

From (3) $V_1 = \frac{1}{Y_{11}} [I_1 - Y_{12} V_2]$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad \text{--- (5)}$$

sub equation (5) in (4)

$$I_2 = Y_{21} \left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \right] + Y_{22} V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + \left[-\frac{Y_{12} Y_{21}}{Y_{11}} + Y_{22} \right] V_2$$

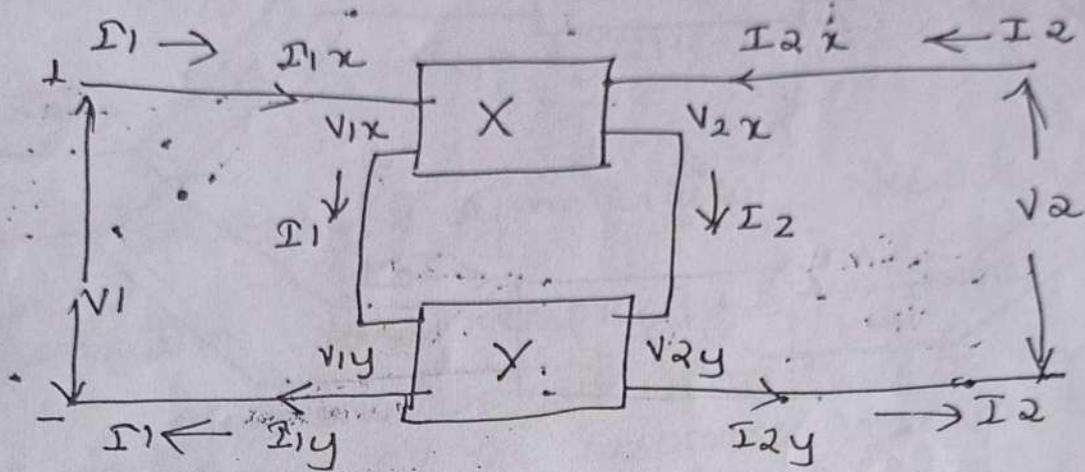
$$= \frac{Y_{21}}{Y_{11}} I_1 + \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}} \right] V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2 \quad \text{--- (6)}$$

Compare equations (5) & (6) with (1) & (2)

Series & parallel connection of 2 port networks

Case I: Series connection of two-port n/w



$$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x}$$

$$V_{2x} = Z_{21x} I_{1x} + Z_{22x} I_{2x}$$

$$V_{1y} = Z_{11y} I_{1y} + Z_{12y} I_{2y}$$

$$V_{2y} = Z_{21y} I_{1y} + Z_{22y} I_{2y}$$

$$I_1 = I_{1x} = I_{1y}; \quad I_2 = I_{2x} = I_{2y}$$

$$V_1 = V_{1x} + V_{1y}; \quad V_2 = V_{2x} + V_{2y}$$

$$V_1 = (Z_{11x} + Z_{11y}) I_1 + (Z_{12x} + Z_{12y}) I_2$$

$$V_2 = (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2$$

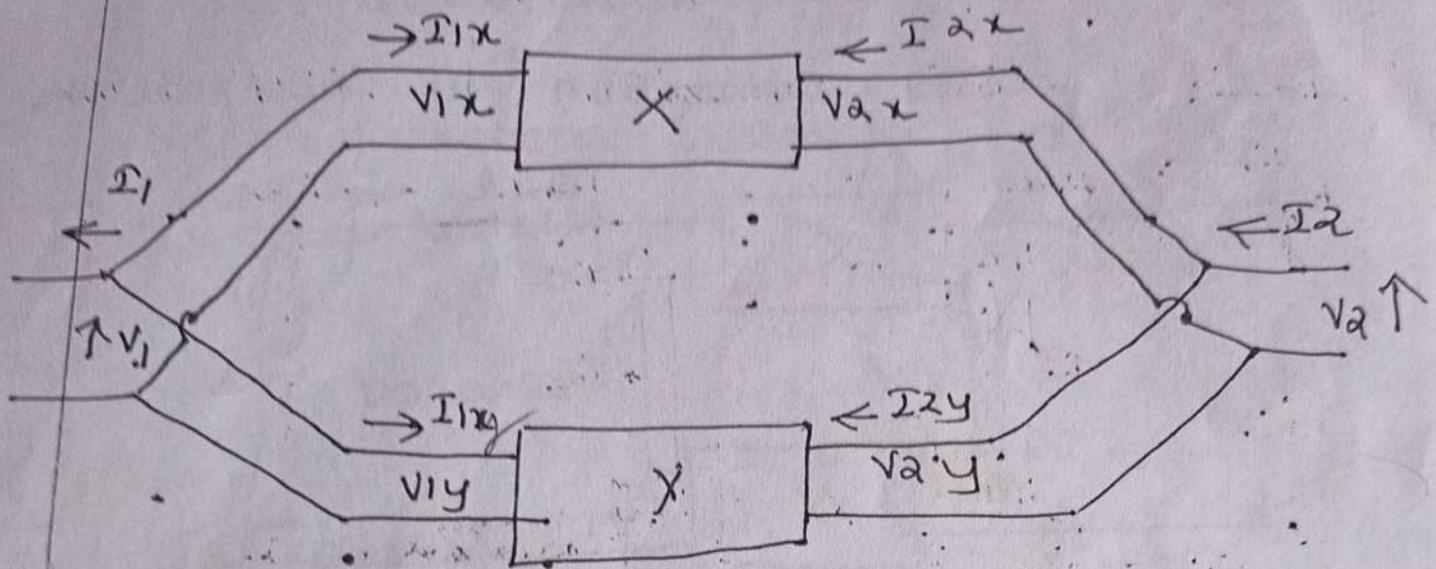
$$\therefore V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = Z_{11x} + Z_{11y}; \quad Z_{12} = Z_{12x} + Z_{12y}$$

$$Z_{21} = Z_{21x} + Z_{21y}; \quad Z_{22} = Z_{22x} + Z_{22y}$$

Case II: parallel connection of Twoport Networks



$$I_{1x} = Y_{11x} v_{1x} + Y_{12x} v_{2x}$$

$$I_{2x} = Y_{21x} v_{1x} + Y_{22x} v_{2x}$$

$$I_{1y} = Y_{11y} v_{1y} + Y_{12y} v_{2y}$$

$$I_{2y} = Y_{21y} v_{1y} + Y_{22y} v_{2y}$$

$$v_1 = v_{1x} = v_{1y} ; v_2 = v_{2x} = v_{2y}$$

$$I_1 = I_{1x} + I_{1y} ; I_2 = I_{2x} + I_{2y}$$

$$I_1 = Y_{11x} v_{1x} + Y_{12x} v_{2x} + Y_{11y} v_{1y} + Y_{12y} v_{2y}$$

$$= (Y_{11x} + Y_{11y}) v_1 + (Y_{12x} + Y_{12y}) v_2$$

$$I_2 = Y_{21x} v_{1x} + Y_{22x} v_{2x} + Y_{21y} v_{1y} + Y_{22y} v_{2y}$$

$$= (Y_{21x} + Y_{21y}) v_1 + (Y_{22x} + Y_{22y}) v_2$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = Y_{11x} + Y_{11y}; \quad Y_{12} = Y_{12x} + Y_{12y}$$

$$Y_{21} = Y_{21x} + Y_{21y}; \quad Y_{22} = Y_{22x} + Y_{22y}$$

Reciprocal & symmetrical Networks

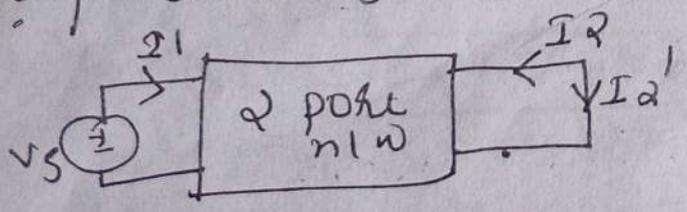
Condition for Reciprocity & symmetry.

A network is termed to be reciprocal if the ratio of the response variable to the excitation variable remains identical even if the position of response and excitation in the network are interchanged.

A two port network is said to be symmetrical if the input and output ports can be interchanged without altering the port voltages and currents.

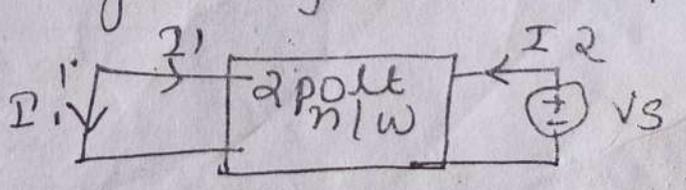
Reciprocity in Z parameter representation

* Network containing resistors, inductors and capacitors are generally reciprocal. Networks that additionally have dependent sources are generally non reciprocal.



$$V_1 = V_S \quad I_1 = I_1;$$

$$V_2 = 0 \quad I_2 = -I_2'$$



$$V_2 = V_S \quad I_2 = I_2$$

$$I_1 = -I_1'$$

$$V_1 = 0$$

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(1) Reciprocity in Z parameter.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

from fig (a) $V_1 = V_S$; $I_1 = I_1$; $V_2 = 0$; $I_2 = -I_2'$

$$\text{Therefore } V_S = Z_{11} I_1 + Z_{12} I_2'$$

$$0 = Z_{21} I_1 + Z_{22} I_2'$$

$$\text{Hence } I_1 = \frac{Z_{22} I_2'}{Z_{21}}$$

$$V_S = Z_{11} \frac{Z_{22} I_2'}{Z_{21}} + Z_{12} I_2'$$

$$I_2' = \left(\frac{Z_{11} Z_{22} - Z_{21} Z_{12}}{Z_{21}} \right) = V_S$$

$$I_2 = \frac{Z_{21} V_S}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

From fig b.

$$V_2 = V_S; V_1 = 0; I_1 = -I_1'; I_2 = I_2'$$

$$0 = -Z_{11} I_1' + Z_{12} I_2'$$

$$V_S = -Z_{21} I_1' + Z_{22} I_2'$$

$$I_2 = \frac{Z_{11} I_1'}{Z_{12}}$$

$$V_S = -Z_{21} I_1' + \frac{Z_{22} \cdot Z_{11} I_1'}{Z_{12}}$$

$$I_1' = \frac{Z_{12} \cdot V_S}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

If the two port n/w is reciprocal

$$I_1' = I_2'$$

$$\frac{Z_{21} \cdot V_S}{Z_{11}Z_{22} - Z_{21}Z_{12}} = \frac{Z_{12} V_S}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

$$\boxed{Z_{21} = Z_{12}}$$

2) In terms of Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--- (2)}$$

from fig a $V_1 = V_S$; $V_2 = 0$; $I_1 = I_1$; $I_2 = -I_2'$

$$I_1 = Y_{11}V_S \quad \text{--- (3)}$$

$$-I_2' = Y_{21}V_S \quad \text{--- (4)}$$

from fig b $V_2 = V_S$; $V_1 = 0$; $I_1 = -I_1'$; $I_2 = I_2$

$$-I_1' = Y_{12}V_S \quad \text{--- (5)}$$

$$I_2 = Y_{22}V_S \quad \text{--- (6)}$$

For reciprocity condition

$$I_1' = I_2'$$

Equating equations (4) & (5)

$$\boxed{Y_{21} = Y_{12}}$$

3) In terms of T parameter

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

From (a)

$$V_S = -B(-I_2') \quad \text{--- (3)}$$

$$I_1 = DI_2' \quad \text{--- (4)}$$

from (b)

$$0 = AV_S - BI_2' \quad \text{--- (5)}$$

$$-I_2' = CV_S - DI_2' \quad \text{--- (6)}$$

From (5) $I_2' = \frac{A}{B} V_S$

in (6)

$$-I_2' = CV_S - \frac{DA}{B} V_S$$

$$I_2' = -\left(\frac{BC - AD}{B}\right) V_S$$

$$I_2' = \left(\frac{AD - BC}{B}\right) V_S$$

From (3)

$$I_2' = \frac{V_S}{B}$$

For reciprocity $I_2' = I_1'$

$$\frac{V_S}{B} = \left(\frac{AD - BC}{B}\right) V_S$$

$AD - BC = 1$	$A \quad B$	$= 1$
$\Delta_T = 1$	$C \quad D$	

$A = D$ \therefore Symmetrical

4) In terms of h parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- (2)}$$

from fig (a)

$$V_S = h_{11}I_1 \quad \text{--- (3)}$$

$$-I_2' = h_{21}I_1 \quad \text{--- (4)}$$

from fig (b)

$$0 = h_{11}(-I_1') + h_{12}V_S \quad \text{--- (5)}$$

$$I_2 = h_{21}(-I_1') + h_{22}V_S \quad \text{--- (6)}$$

from (5)

$$h_{12}V_S = h_{11}I_1'$$

$$I_1' = \frac{h_{12}}{h_{11}} V_S \quad \text{--- (7)}$$

from (4) & (3)

$$-I_2' = h_{21} \cdot \frac{V_S}{h_{11}}$$

$$I_2' = -\frac{h_{21}}{h_{11}} V_S \quad \text{--- (8)}$$

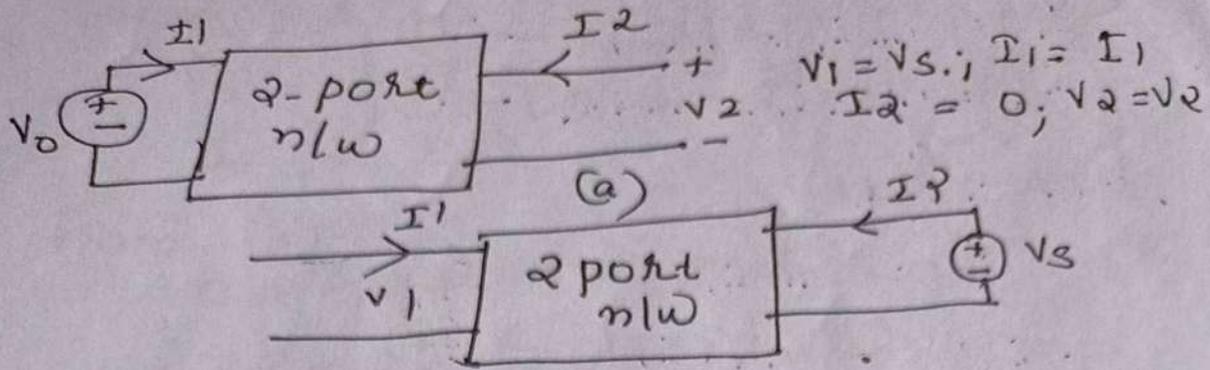
For Reciprocity $I_1' = I_2'$

$$\frac{h_{12}}{h_{11}} V_S = -\frac{h_{21}}{h_{11}} V_S$$

$$\boxed{h_{12} = -h_{21}}$$

$\Delta h = 1$
symmetrical

Condition For symmetry



$$V_2 = V_s \quad V_1 = V_s \quad I_1 = 0 \quad I_2 = I_2$$

Condition for symmetry

$$\frac{V_s}{I_1} \Big|_{I_2=0} = \frac{V_s}{I_2} \Big|_{I_1=0}$$

(1) In terms of Z parameter

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

From fig (a)

$$V_s = Z_{11} I_1$$

$$V_2 = Z_{21} I_1$$

$$\frac{V_s}{I_1} \Big|_{I_2=0} = Z_{11} \quad \text{--- (3)}$$

From fig b $V_1 = Z_{12} I_2$

$$V_s = Z_{22} I_2$$

$$\frac{V_s}{I_2} \Big|_{I_1=0} = Z_{22} \quad \text{--- (4)}$$

Condition for symmetry (3) = (4)

$$\boxed{Z_{11} = Z_{22}}$$

2) In terms of Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--- (2)}$$

from fig (a)

$$I_1 = Y_{11}V_S + Y_{12}V_2$$

$$0 = Y_{21}V_S + Y_{22}V_2$$

$$V_2 = \frac{-Y_{21}V_S}{Y_{22}}$$

$$I_1 = Y_{11}V_S - \frac{Y_{12}Y_{21}V_S}{Y_{22}}$$

$$I_1 = \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}} \right] V_S$$

$$\frac{V_S}{I_1} \Big|_{I_2=0} = \frac{Y_{22}}{\Delta Y} \quad \text{--- (3)}$$

from fig b

$$0 = Y_{11}V_1 + Y_{12}V_S$$

$$I_2 = Y_{21}V_1 + Y_{22}V_S$$

$$V_1 = \frac{-Y_{12}V_S}{Y_{11}}$$

$$I_2 = \frac{-Y_{21}Y_{12}V_S}{Y_{11}} + Y_{22}V_S$$

$$I_2 = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}} V_S$$

$$\frac{V_S}{I_2} \Big|_{I_1=0} = \frac{Y_{11}}{\Delta Y} \quad \text{--- (4)}$$

Applying condition for symmetry

$$\textcircled{3} = \textcircled{4}$$

$$\frac{Y_{22}}{\Delta Y} = \frac{Y_{11}}{\Delta Y}$$

$$\boxed{Y_{22} = Y_{11}}$$

3) in terms of T parameters

$$V_1 = AV_2 - BI_2 \quad \textcircled{1}$$

$$I_1 = CV_2 - DI_2 \quad \textcircled{2}$$

from fig (a)

$$V_S = AV_2$$

$$I_1 = CV_2$$

$$\frac{V_S}{I_1} \Big|_{I_2=0} = \frac{A}{C} \quad \textcircled{3}$$

from fig (b)

$$V_1 = AV_S - BI_2$$

$$0 = CV_S - DI_2$$

$$DI_2 = CV_S$$

$$\frac{V_S}{I_2} = \frac{D}{C} \quad \textcircled{4}$$

Condition for symmetry $\textcircled{3} = \textcircled{4}$

$$\frac{A}{C} = \frac{D}{C}$$

$$\boxed{A = D}$$