

MODULE IV

Syllabus : Network functions for the single port and two ports. Properties of driving point and transfer function, poles and zeros of network functions. Significance of poles and zeros.

Time domain response from pole zero plot, Impulse Response. Network functions in the sinusoidal steady state, Magnitude and phase response.

The concept of transform impedance and transform admittance, further more, a function relating currents or voltages at different part of the network called a transfer function, is found to be mathematically similar to the transform impedance function. These functions are called network functions.

Concept of Complex frequency -

The solution of differential equation for the networks is of the form

$$x(t) = k e^{s_n t} \quad \text{--- (1)}$$

$x(t)$ may be a voltage $V(t)$ or a current $i(t)$

Generally $x(t)$ is a function of time. And s_n is a complex number, which may be expressed as

$$S_n = \sigma_n + j\omega_n$$

$\omega_n \rightarrow$ imaginary part

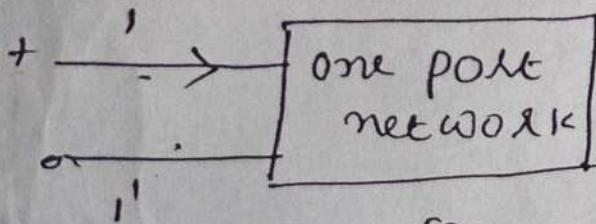
$S_n \rightarrow$ Angular frequency

The radiation frequency may be expressed as

$$\omega_n = 2\pi f_n = \frac{2\pi}{T}$$

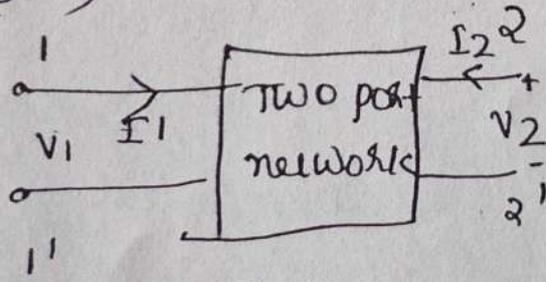
$S_n = \sigma_n + j\omega_n$ is defined as the complex frequency. The real part of the complex frequency is neper frequency, corresponds to exponential decay or exponential increase (depending on the sign) and the imaginary part of the complex frequency is the radian frequency (real frequency) corresponds to oscillation.

TERMINAL PAIRS (OR) PORTS



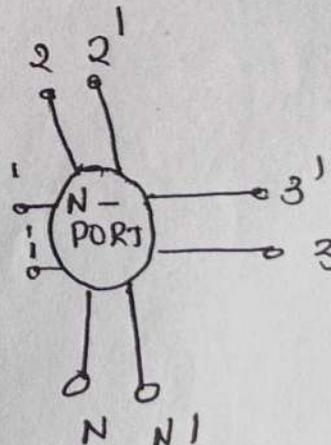
(a)

one port



(b)

two port



(c)

N-port

Fig (b) is our study of interest

Here 1-1' is assumed to be connected to the driving force (as an input) and port 2-2' is connected to a load (as an output)

NETWORK FUNCTIONS

Transform impedance: It is defined as the ratio of voltage transform to current transform

Thus we write $Z(s) = \frac{V(s)}{I(s)}$

Transform Admittance is defined as the ratio of current transform to voltage transform

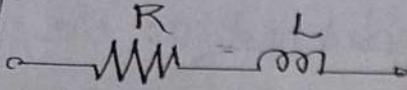
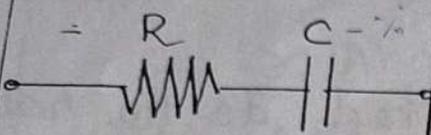
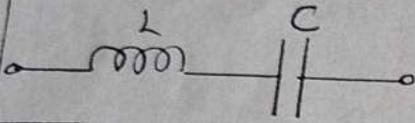
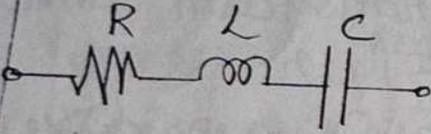
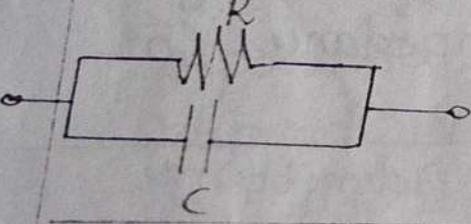
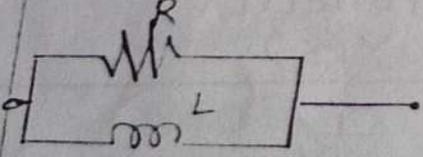
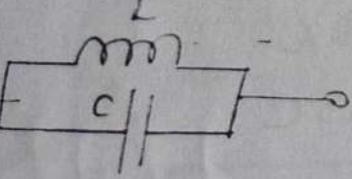
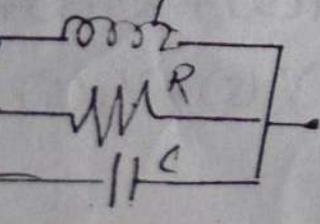
$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)}$$

The transform impedance and transform admittance must relate to the source port 1-1' or 2-2'. Transform impedance (or admittance) at a given port is called driving point impedance (or admittance)

SL No	Elements	Impedance Function $Z(s)$	Admittance Function $Y(s)$
1	Resistance	R	$\frac{1}{R}$
2	Inductance (L)	sL	$\frac{1}{sL}$
3	capacitance (C)	$\frac{1}{sC}$	sC

Note: For series circuit $Z(s) = Z_1(s) + Z_2(s) + Z_3(s)$

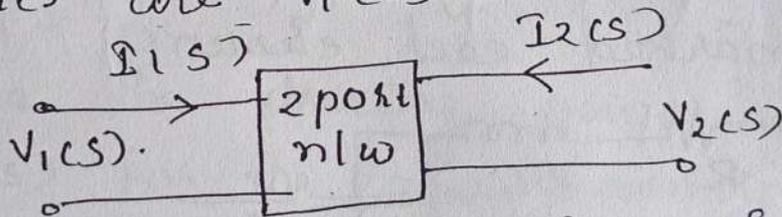
Parallel circuit $Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$

Network	Impedance function	Admittance
	$Z(s) = R + sL$	$\frac{1}{R + sL}$
	$Z(s) = R + \frac{1}{sC}$ $= \frac{RCs + 1}{Cs}$	$\frac{sC}{sRC + 1}$
	$\frac{s^2 LC + 1}{sC}$	$\frac{sC}{s^2 LC + 1}$
	$\frac{sRC + s^2 LC + 1}{sC}$	$\frac{sC}{sRC + s^2 LC + 1}$
	$\frac{sRL}{sL + R}$	$\frac{sL + R}{sRL}$
	$\frac{R}{sRC + 1}$	$\frac{sRC + 1}{R}$
	$\frac{sL}{s^2 LC + 1}$	$\frac{s^2 LC + 1}{sL}$
	$\frac{sRL}{s^2 RLC + sL + R}$	$\frac{s^2 RLC + sL + R}{sRL}$

THE TRANSFER FUNCTION

Transfer function is used to describe networks which have at least two ports. In general, the transfer function relates the transform of a quantity at one port to the transform of another quantity at another port. Thus transfer functions have the following possible forms:

Transfer function is defined as the ratio of an o/p quantity to an i/p quantity. The o/p quantities are $I_2(s)$ and $V_2(s)$. & the i/p quantities are $V_1(s)$ and $I_1(s)$.



Four basic transfer functions for the 2 port n/w they are

- (i) Transfer impedance function $Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$
- (ii) Transfer Admittance function $Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$
- (iii) Voltage Transfer function $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$
- (iv) Current Transfer function; $\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$

The ratio of an input quantity to an o/p quantity is termed as the inverse transfer function.

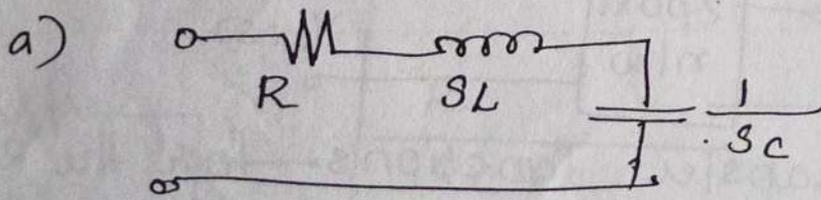
$\frac{V_1(s)}{I_2(s)} = Z_{12}(s)$; The inverse transfer impedance function.

$\frac{I_1(s)}{V_2(s)} = Y_{12}(s)$; The inverse transfer admittance function.

$\frac{V_1(s)}{V_2(s)} = G_{12}(s)$; The inverse voltage transfer function.

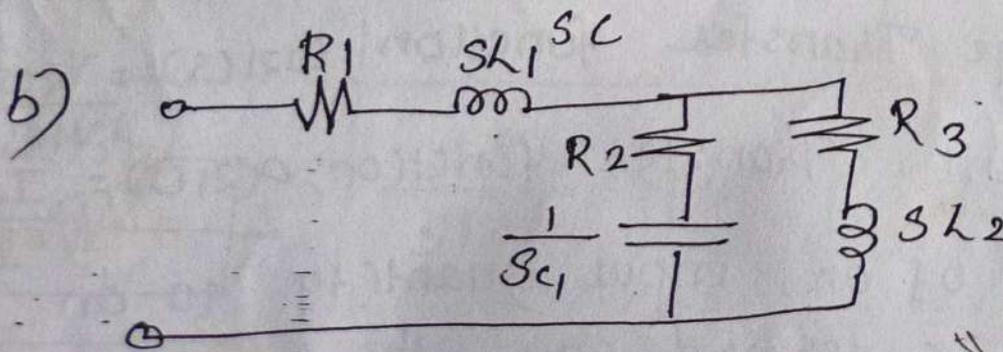
$\frac{I_1(s)}{I_2(s)} = \alpha_{12}(s)$ the inverse current transfer function.

Q: Obtain the driving point impedance function or transform impedance for the network shown in figure, in which transform marked each element



$$Z(s) = R + sL + \frac{1}{sC}$$

$$= R + sL + \frac{1}{sC}$$



$$sL_2 + R_3 \parallel \frac{R_2 sC_2 + 1}{sC_2}$$

$$\frac{sC_2}{1 + R_2 sC_2} + \frac{1}{R_3 + sL_2}$$

$$sL_2 + R_3 \parallel \frac{R_2 + 1/sC_2}{sC_2}$$

$$sL_2 sC_2 + 1 + R_2 sC_2$$

$$\frac{(1 + R_2 sC_2)(R_3 + sL_2)}{sC_2}$$

$$sL_2 + R_3 \parallel \frac{R_2 + 1/sC_2}{sC_2}$$

Peda

$$\Rightarrow R_1 + sL_1 + \left[CR_2 + \frac{1}{sC_1} \right] \parallel (R_3 + sL_2)$$

$$= R_1 + sL_1 + \left(R_2 + \frac{1}{sC_1} \right) (R_3 + sL_2)$$

$$= \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$$

$$\frac{\beta_2 s^2 + \beta_1 s + \beta_0}{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}$$

$$\beta_2 = L_2 \cdot C$$

$$\beta_1 = (R_2 + R_3) C_1$$

$$\beta_0 = 1$$

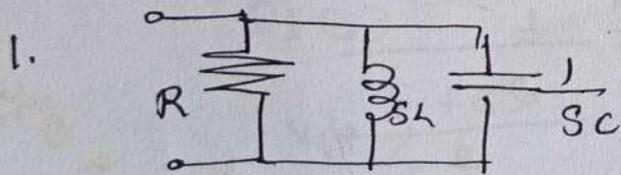
$$\alpha_3 = L_1 L_2 C_1$$

$$\alpha_2 = \left[(R_2 + R_3) L_1 + (R_2 + R_1) L_2 \right] C_1$$

$$\alpha_1 = (L_1 + L_2) + (R_1 R_2 + R_2 R_3 + R_3 R_1) C_1$$

$$\alpha_0 = R_1 + R_3$$

Q: Determine the driving point Admittance function $Y(s)$ for the n/w shown in figure



$$Y(s) = \frac{1}{R} + \frac{1}{sL} + sC$$

$$= \frac{sL + R + RCLs^2}{RLS}$$

$$RLS$$

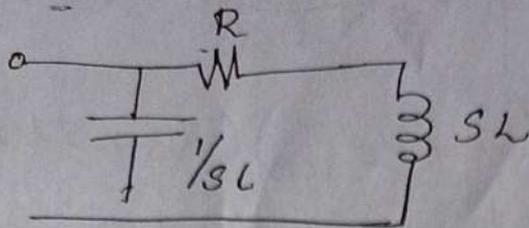
$$sC + \frac{1}{sL}$$

$$\frac{s^2 CL + 1}{sL}$$

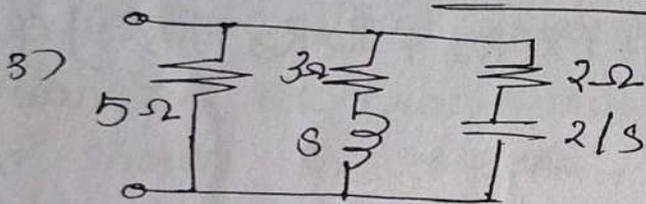
$$\frac{sL}{s^2 CL + 1}$$

$$\frac{s^2 CL + 1}{sL} + \frac{1}{R}$$

$$\frac{R s^2 CL + R + sL}{RLS}$$



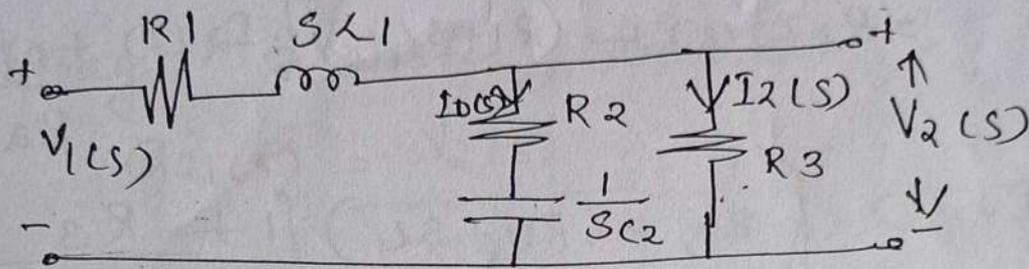
$$\begin{aligned}
 Y(s) &= sC + \frac{1}{R + sL} \\
 &= \frac{sC(R + sL) + 1}{(R + sL)} \\
 &= \frac{Ls^2C + Rsc + 1}{R + sL}
 \end{aligned}$$



$$\begin{aligned}
 Y(s) &= \frac{1}{5} + \frac{1}{3 + s} + \frac{1}{2 + \frac{2}{s}} \\
 &= \frac{1}{5} + \frac{1}{3 + s} + \frac{1}{\frac{2s + 2}{s}} \\
 &= \frac{1}{5} + \frac{1}{3 + s} + \frac{s}{2s + 2} \\
 &= \frac{5(3 + s) + (3 + s)(2 + 2s) + 5(2 + 2s)}{5(3 + s)(2 + 2s)} \\
 &= \frac{15 + 5s + 6 + 2s + 6s + 2s^2 + 10 + 10s}{5(3 + s)(2 + 2s)}
 \end{aligned}$$

$$= \frac{7s^2 + 33s + 16}{10s^2 + 40s + 20}$$

4. For the circuit shown in fig. Determine the network transfer function.



Network Transfer function = $\frac{V_2(s)}{V_1(s)}$.

$$V_2(s) = R_3 \cdot I_2(s)$$

$$= \left(R_2 + \frac{1}{sC_2} \right) I_0(s)$$

$$I_0(s) = \frac{V_2(s)}{R_2 + \frac{1}{sC_2}}$$

$$I_1(s) = I_2(s) + I_0(s)$$

$$V_1(s) = V_2(s) + (R_1 + sL_1) I_1(s)$$

Transfer Impedance function

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{\left(R_2 + \frac{1}{sC_2} \right) I_0(s)}{I}$$

Admittance function $Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$.

$$\begin{aligned}
 V_1(s) &= V_2(s) + (R_1 + sL_1) I_1(s) \\
 &= R_3 I_2(s) + (R_1 + sL_1) [I_2(s) + I_0(s)] \\
 &= R_3 I_2(s) + (R_1 + sL_1) \left[I_2(s) + \frac{V_2(s)}{R_2 + \frac{1}{sC_2}} \right]
 \end{aligned}$$

$$= R_3 I_2(s) + (R_1 + sL_1) \left[I_2(s) + \frac{V_2(s)}{R_2 + \frac{1}{sC_2}} \right]$$

$$= I_2(s) \left\{ R_3 + (R_1 + sL_1) \left[1 + \frac{R_3}{R_2 + \frac{1}{sC_2}} \right] \right\}$$

$$= I_2(s) \left[R_3 R_2 C_2 s + R_3 + (R_1 + sL_1) (R_2 C_2 s + 1) + R_3 C_2 s (R_1 + sL_1) \right]$$

$$\frac{(R_2 C_2 s + 1)}{(R_2 C_2 s + 1)}$$

$$I_2 \left[R_2 R_3 C_2 s + R_3 + R_1 R_2 C_2 s + R_1 + R_2 C_2 C_2 s^2 + sL_1 + R_1 R_3 C_2 s + R_3 C_2 L_1 s^2 \right]$$

$$I_2(s) \left[\frac{R_2 C_2 s + 1}{(R_1 + R_3) C_2 L_1 s^2 + (R_2 R_3 C_2 + R_1 R_2 C_2 + L_1 + R_1 R_3) s + R_1 + R_3} \right]$$

$$I_2(s) \frac{(R_2 + R_3) C_2 L_1 s^2 + [(R_1 + R_3) R_2 C_2 + R_1 R_3 C_2 + L_1] s + R_1 + R_3}{R_2 C_2 s + 1}$$

$$R_2 C_2 s + 1$$

$$Y_2(s) = \frac{R_2(s+1)}{L_1 C_2 (R_2 + R_3) s^2 + [C_2 (R_1 R_2 + R_1 R_3 + R_2 R_3) + L_1] s + R_1 + R_3}$$

Voltage Transfer function

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_2(s)}{I_1(s)} \cdot \frac{I_2(s)}{V_1(s)} \cdot \frac{I_1(s)}{I_2(s)}$$

$$= Z_2(s) \cdot Y_2(s) \cdot \frac{I_1(s)}{I_2(s)}$$

$$I_1(s) = I_2(s) + I_0(s)$$

$$I_2(s) + \frac{V_2(s)}{R_2 + \frac{1}{sC_2}} = I_2(s) + \frac{R_3 I_2(s)}{R_2 + \frac{1}{sC_2}}$$

$$\frac{I_1(s)}{I_2(s)} = \left[\frac{1 + R_3}{R_2 + \frac{1}{sC_2}} \right]$$

poles & Zeros of Network functions

obtain following

All network functions of a linear RLC circuit can be expressed as the ratio of two polynomials i.e. $\frac{N(s)}{D(s)}$, where $N(s)$ is the numerator polynomial and $D(s)$ is the denominator polynomial.

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

The polynomial $N(s) = 0$ has n roots, they are called zeros of the network function $T(s)$. The polynomial $D(s) = 0$ has m roots they are called poles of $T(s)$.

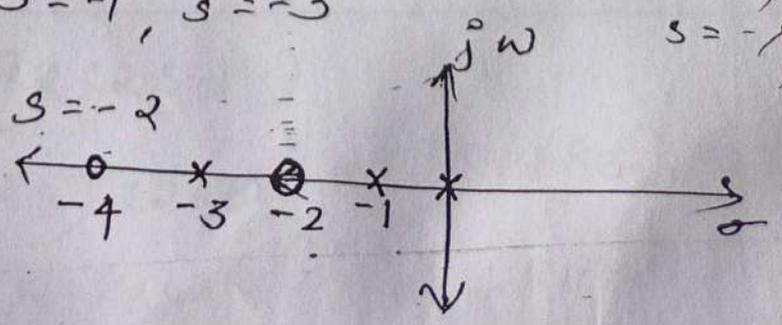
In the complex s -plane, a pole is denoted by a small cross (\times) and a zero by a small circle (\circ)

Q. Obtain the pole zero location for the function

$$T(s) = \frac{(2s+4)(s+4)}{s(s+1)(s+3)}$$

poles ; $s = 0, s = -1, s = -3$

Zeros ; $s = -4, s = -2$

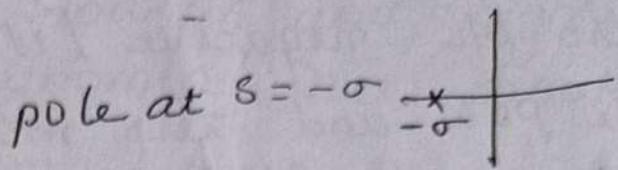


$$\begin{aligned} 2s+4 \\ 2s &= -4 \\ s &= -2 \end{aligned}$$

obtain the pole-zero location for the following function

(i) $f_1(t) = e^{-\sigma t}$

$F_1(s) = \frac{1}{s + \sigma}$

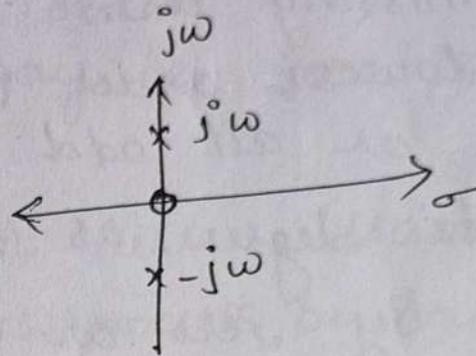


(ii) $f_2(t) = \cos \omega t$

$F_2(s) = \frac{s}{s^2 + \omega^2}$

pole $s^2 = -\omega^2$
 $s = \pm j\omega$

Zero $s = 0$



(iii) $f_3(t) = e^{-\sigma t} \cos \omega t$

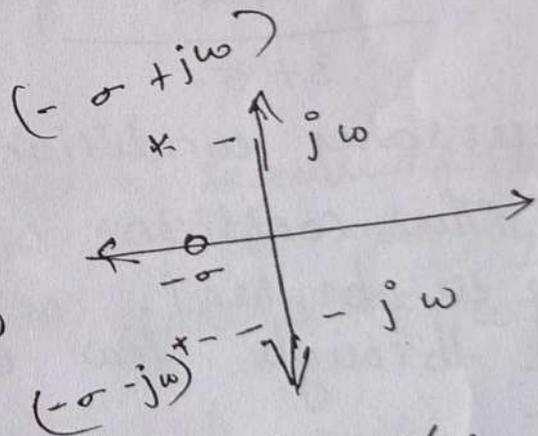
$= \frac{s + \sigma}{(s + \sigma)^2 + \omega^2}$

Zero
~~pole~~ : $s = -\sigma$

$(s + \sigma)^2 + \omega^2 = 0$

$s + \sigma = \pm j\omega$

$s = -\sigma + j\omega$



Restrictions on Location of poles and zeros in driving point functions.

(a) The coefficient of the polynomial $N(s)$ and $D(s)$ of the n/w in HCS must be real & +ve

(b) poles and zeros, if complex or imaginary must occur in conjugate pairs

(c) The real part of all poles & zeros must be zero or negative [if the real part ≥ 0 , then the pole and zero must be simple]

(d) The polynomial $N(s)$ or $D(s)$ cannot have any missing terms between those of highest and lowest order values under all even order or all odd order terms are missing

(e) The degree of $N(s)$ and $D(s)$ may differ by zero or one only

(f) - The lowest degree in $N(s)$ and $D(s)$ may differ in degree ~~at~~ by at the most one.

eg:
$$\frac{s^4 - s^3 + 2s^2}{s+5} \quad \text{(ii)} \quad \frac{s^4 + s^3 + 1}{s^2 + 2s^2 - 2s + 10}$$

Necessary condition for transfer function.

(i) The coefficient of polynomial $N(s)$ and $D(s)$ are to be real. coefficient of $D(s)$ must be +ve through few coefficients of $N(s)$ may be

-ve

(ii) The poles and zeros, if complex or imaginary must be in conjugate.

(iii) The real part of the pole must be -ve or zero. If the real part is zero, the pole must be simple.

(iv) The polynomial $N(s)$ may have missing terms between the lowest and highest degree. However polynomial $D(s)$ should not have any missing term between the highest & lowest degree unless all even or all odd times are missing

(v) Degree of $N(s)$ may be zero, independent of degree of $D(s)$

(vi) For the voltage or current transfer ratio the max degree of $N(s)$ must be equal the degree of $D(s)$

(vii) For the transfer impedance and admittance the maximum degree of $N(s)$ must equal the degree of $D(s) + 1$.

STABILITY

The most important requirement considered while designing a system is that the system must be stable. Unstable systems are considered to be useless.

A system is said to be stable if its Output (response) cannot be made to increase indefinitely by the application of bounded i/p.

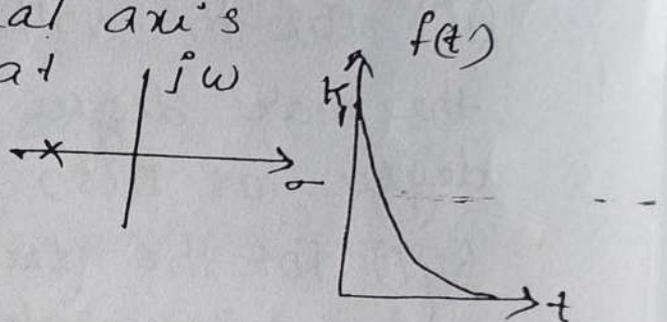
Relationship btwn pole position & Stability

The necessary & sufficient condition for a system to be stable is that the poles of the transfer function $T(s) = \frac{N(s)}{D(s)}$ (i.e. roots of $D(s) = 0$) lie on the left half of the s-plane i.e. if there is any pole in the RHS of s-plane implies that the s/m is unstable

(i) poles on -ve Real axis

$$F(s) = \frac{k_1}{s+a} ; f(t) = k_1 \cdot e^{-at}$$

\Rightarrow stable s/m

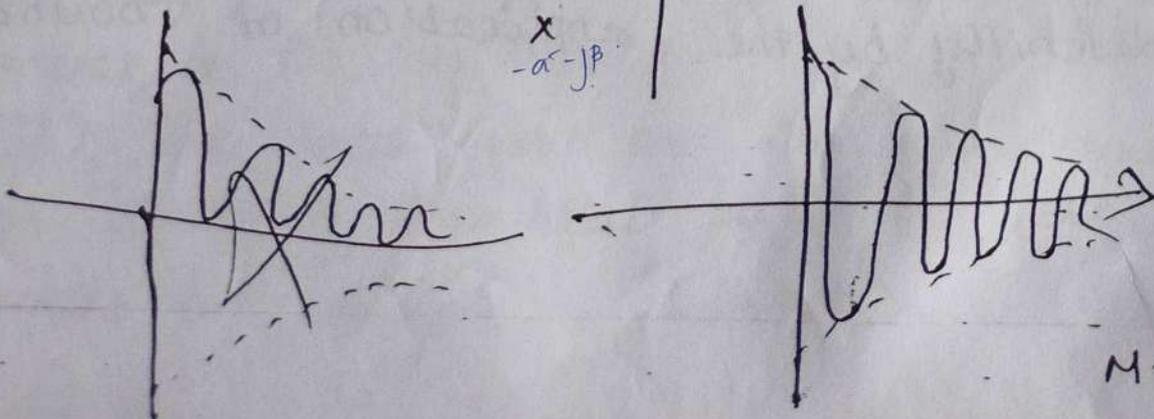
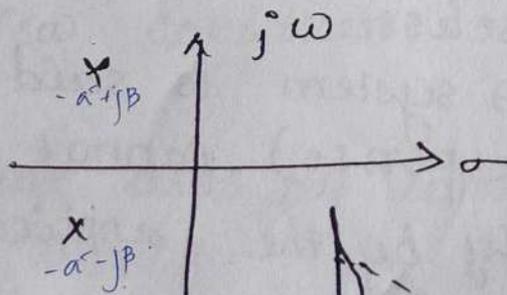


(ii) complex pole in LHS of s-plane

$$s = -\alpha + j\beta$$

$$F(s) = \frac{k_1}{s + \alpha - j\beta} + \frac{k_1}{s + \alpha + j\beta}$$

$$f(t) = \mathcal{L}^{-1} \left[\frac{2k_1 (s + \alpha)}{(s + \alpha)^2 + \beta^2} \right] = 2k_1 e^{-\alpha t} \cos \beta t$$

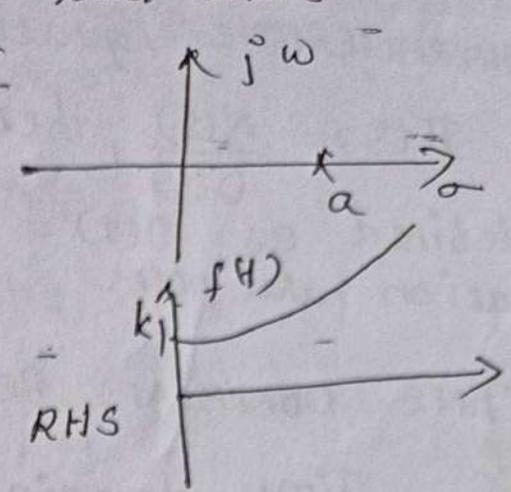


stability

(ii) - poles on positive real axis

at
 $F(s) = \frac{k_1}{s-a}$; $f(t) = k_1 e^{-at}$

system is unstable.



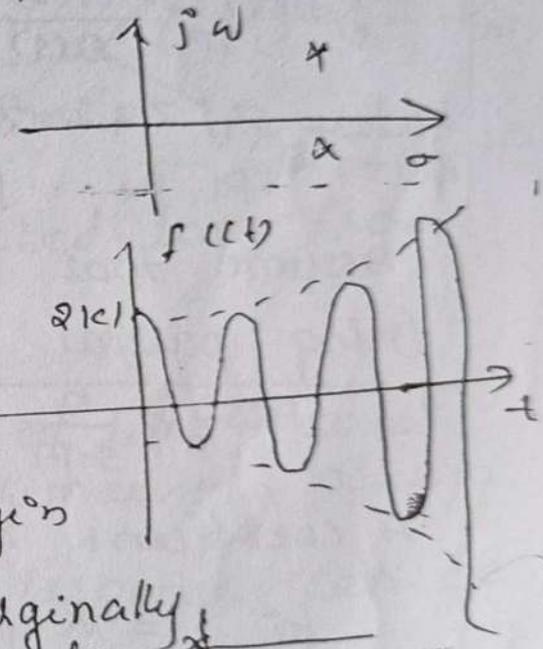
(iv) complex pole in the RHS of s-plane

poles = $\alpha \pm j\beta$; $F(s) = \frac{k_1}{s-\alpha-j\beta} + \frac{k_1}{s-\alpha+j\beta}$

$f(t) = \mathcal{L}^{-1} \left[\frac{2k_1(s-\alpha)}{(s-\alpha)^2 + \beta^2} \right]$

$= 2k_1 e^{\alpha t} \cos \beta t$

unstable sm

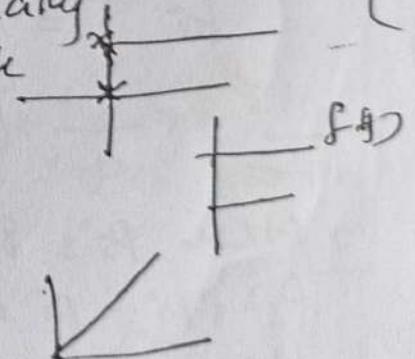


(v) pole at origin and multiple poles at origin

$F(s) = \frac{k_1}{s}$; $f(t) = k_1$ Marginally stable

OR $f(s) = \frac{k}{s^2}$

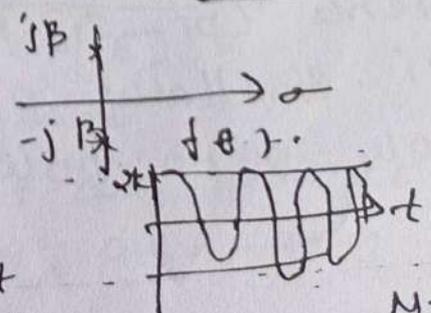
$f(t) = kt$ unstable



(vi) poles on j*omega Axis

$F(s) = \frac{k_1}{s-j\beta} + \frac{k_1}{s+j\beta}$
 $= \frac{k_1(s+j\beta) + k_1(s-j\beta)}{s^2 + \beta^2}$

$= \frac{2k_1 s}{s^2 + \beta^2} = 2k_1 \cos \beta t$



CHARACTERISTIC Equation

If $T(s) = \frac{N(s)}{D(s)}$, then characteristic equation is defined as $D(s) = 0$. Roots of characteristic equation are poles.

TIME DOMAIN RESPONSE - FROM POLE-ZERO PLOT

Time domain response can be obtained by poles-zero plot of a network function is given by

$$H(s) = \frac{N(s)}{D(s)} = k \frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-p_1)(s-p_2)\dots(s-p_m)}$$

Where $z_1, z_2, \dots, z_n \Rightarrow$ zero's

$p_1, p_2, \dots, p_m \Rightarrow$ poles of the fn $H(s)$

Assume that poles and zeros are distinct.

Using partial fraction expansion

$$H(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \dots + \frac{k_m}{s-p_m}$$

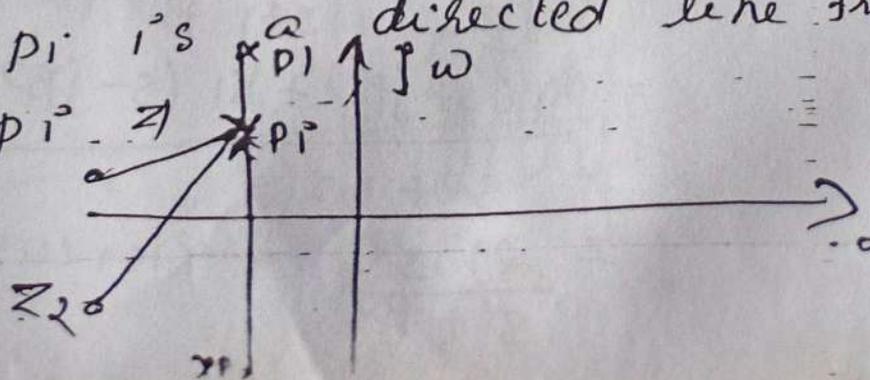
A coefficient k_i can be found as

$$k_i = \frac{k (p_i - z_1)(p_i - z_2)\dots(p_i - z_n)}{(p_i - p_1)(p_i - p_2)\dots(p_i - p_{i-1})(p_i - p_{i+1})\dots(p_i - p_m)}$$

Here p_i 's & z_i 's are complex number's, so $p_i - z_i$

or $p_i - p_i$ will also be a complex number

Hence $(p_i - z_1)$ is a directed line from z_1 to p_i . Similarly $(p_i - p_1)$ is a directed line from pole p_1 to pole p_i .



Each of these lines have a magnitude and phase (polar form)

$$\text{Let } p_i - z_1 = k_{1i} \angle \alpha_{1i}^\circ$$

$$p_i - z_2 = k_{2i} \angle \alpha_{2i}^\circ$$

⋮

$$\text{and } p_i - p_1 = R_{1i} \angle \beta_{1i}^\circ$$

$$p_i - p_2 = R_{2i} \angle \beta_{2i}^\circ$$

⋮

$$K_i = K \cdot \frac{k_{1i} \cdot k_{2i} \dots k_{ni}}{R_{1i} \cdot R_{2i} \dots R_{mi}} \angle (\alpha_{1i} + \alpha_{2i} + \dots + \alpha_{ni}) - (\beta_{1i} + \beta_{2i} + \dots + \beta_{mi})$$

K_i = product of all directed lines from zeros to p_i

product of all directed lines from all other imaginary remaining poles to p_i

Now the time domain response can be obtained by

$$h(t) = \mathcal{L}^{-1}[H(s)] = \mathcal{L}^{-1} \left[\frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \dots + \frac{k_m}{s-p_m} \right]$$

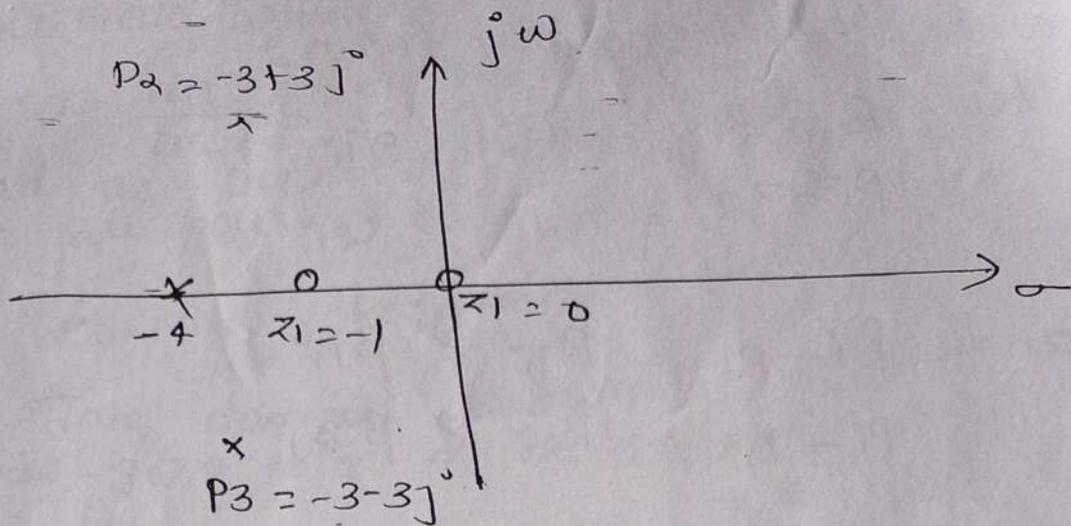
Q: If $H(s) = \frac{s(s+1)}{(s+4)(s^2+6s+18)}$ find $h(t)$

using pole zero diagram

Here all poles & zeros are distinct.

$$z_1 = 0, z_2 = -1, p_1 = -4, p_2 = -3+3j$$

$$p_3 = -3-3j$$



$$K_1 = \frac{(p_1 - z_1)(p_2 - z_2)}{(p_1 - p_2)(p_1 - p_3)} = \frac{(-4 - 0)(-4 - -1)}{[(-4 - (-3 + 3j))][(-4 - (-3 - 3j))]}$$

$$= \frac{-4 \times -3}{(-1 + 3j)(-1 - 3j)} = \frac{12}{1 + 9} = \frac{12}{10} = 1.2$$

$$K_2 = \frac{(-3 + 3j - 0)(-3 + 3j + 1)}{(-3 + 3j + 4)(-3 + 3j - (-3 - 3j))}$$

$$K_2 = \frac{(-3 + 3j)(-2 + 3j)}{(1 + 3j)(6j)}$$

$$= \frac{6 + 9j^2 - 6j + 9j^2}{6j + 18j^2} = \frac{-3 - 15j}{-18 + 6j}$$

$$= \frac{1 + 5j}{6 - 2j} = \frac{(1 + 5j)(6 + 2j)}{(6 - 2j)(6 + 2j)}$$

$$= \frac{6 + 2j + 30j + 10j^2}{36 - 4j^2} = \frac{-4 + 32j}{40}$$

$$= \frac{-1 + 8j}{10}$$

$$k_3 = k_2^* = \frac{-1 - 8j}{10}$$

$$H(s) = \frac{1.2}{s-p_1} + \frac{1/10(-1-8j)}{s-p_3} + \frac{1/10(-1+8j)}{s-p_2}$$

$$= 1.2 e^{p_1 t} + \frac{1}{10}(-1-8j) e^{p_3 t} + \frac{1}{10}(-1+8j) e^{p_2 t}$$

$$= 1.2 e^{-4t} + \frac{1}{10}(-1-8j) e^{(-3-3j)t} + \frac{1}{10}(-1+8j) e^{(-3+3j)t}$$

$$= 1.2 e^{-4t} + \frac{1}{10} e^{-3t} \left[(-1-8j) e^{-3jt} + (-1+8j) e^{3jt} \right]$$

$$= 1.2 e^{-4t} + \frac{1}{10} e^{-3t} \left[e^{-3jt} - e^{3jt} - 8j e^{-3jt} + 8j e^{3jt} \right]$$

$$= 1.2 e^{-4t} + \frac{1}{10} e^{-3t} \left[-2 \cos 3t - 8 \times 2 \sin 3t \right]$$

$$= 1.2 e^{-4t} - \frac{1}{15} e^{-3t} \left[\cos 3t + 8 \sin 3t \right]$$

NETWORK FUNCTIONS IN SINUSOIDAL STEADY STATE MAGNITUDE & PHASE RESPONSE

The response given by the system when the input frequency ω is changed over a certain range is called frequency response of the s/m.

Frequency response can be obtained by expressing given term $H(s)$ in frequency domain.

The frequency domain transfer function is obtained by replacing the complex frequency variable s by $j\omega$. It is denoted as

$$H(j\omega) = H(s) \Big|_{j\omega = s}$$

such frequency domain transfer function is expressed in polar form as

$$H(j\omega) = |H(j\omega)| \angle H(j\omega) \\ = M_R \angle \theta_R$$

where $M_R \rightarrow$ Resultant magnitude which is a fn of ω

$\theta_R \rightarrow$ Resultant phase angle, which is a fn of ω

$$\text{Eg: } H(s) = \frac{20}{(s+1)(s+3)} = \frac{20}{(j\omega+1)(j\omega+3)}$$

$$M_R = \frac{20}{\sqrt{(1+\omega^2)} \sqrt{9+\omega^2}} = \frac{20}{\sqrt{(1+\omega^2)(9+\omega^2)}}$$

$$\theta_R = \underline{\underline{0 - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}}}$$

~~20~~
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