

## MODULE II

### NETWORK THEOREMS & LAPLACE TRANSFORM

NETWORK Theorems for BOTH DC & PHASOR  
Circuits :-

Syllabus :- Thevenin's theorem, Norton's theorem, Super position theorem, Reciprocity theorem, Millman's theorem, Maximum power Transfer Theorem.

1. THEVENIN'S THEOREM :- The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network.

It tells us that, it is possible to replace everything except the load resistance by an independent voltage source in series with a resistance. The response measured at the load resistance will be unchanged.

"A linear active bilateral network can be replaced by an equivalent voltage source that is known as Thevenin's voltage and is denoted by  $V_{th}$  or  $V_{oc}$  in series with an equivalent impedance i.e. Thevenin's impedance  $R_{th}$ .

Steps to Find Thevenin's equivalent ckt

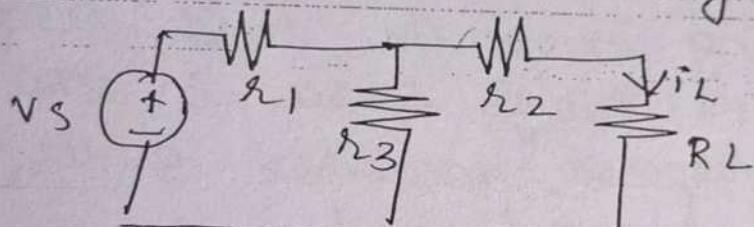
- 1) Remove the load resistance and find the open ckt voltage  $V_{oc}$  across the open circuited terminals.

M-2-1

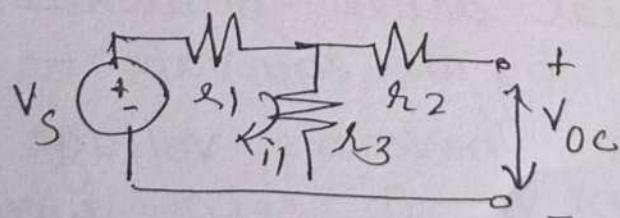
- 2) Deactivate all the independent sources,  
 i.e replace all independent voltage sources by a short ckt and current source by open circuit to find the internal resistance  $R_{th}$ .
- 3) obtain the thevenins equivalent ckt by placing  $V_{th}$  in series with  $R_{th}$ .
- 4) Reconnect the load resistance across the load terminal so that load current.

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Consider the circuit given below.



→ Remove Load resistance.



$$-V_s + i^1 r_1 + r_3 i_1 = 0$$

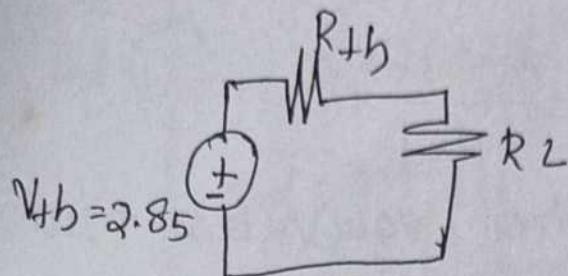
$$(r_1 + r_3) i_1 = V_s$$

$$i_1 = \frac{V_s}{r_1 + r_3}$$

$$V_{oc} = i_1 \times r_3 = \frac{V_s}{r_1 + r_3} \cdot r_3$$

$$\Rightarrow \frac{5 \times 9}{5+2} + 3$$

$$= \frac{10}{7} + 3 = \frac{31}{7} = \underline{\underline{4.422}}$$



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{2.85}{4.42 + 10} = \underline{\underline{0.197 A}}$$

Network containing dependent sources:  
steps to find  $R_{th}$ .

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1. Find  $V_{oc}$  across the open circuit terminal by any network analysis technique.
2. short circuit the load terminal & find the short circuit current  $i_{sc}$  through the load terminal

$$3. R_{th} = \frac{V_{oc}}{i_{sc}}$$

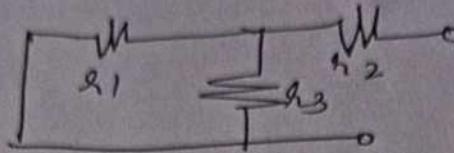
2nd Method:

1. Replace the independent source by its internal resistance.
2. Remove  $R_L$  and apply a dc driving voltage  $V_{dc}$  at the open circuited load terminals. A dc driving current  $i_{dc}$  will flow in the circuit due to  $V_{dc}$

| M-2-2 |

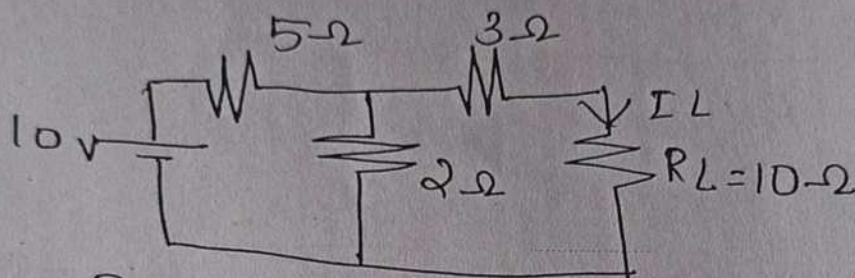
Find  $R_{th}$

short circuit the voltage source.

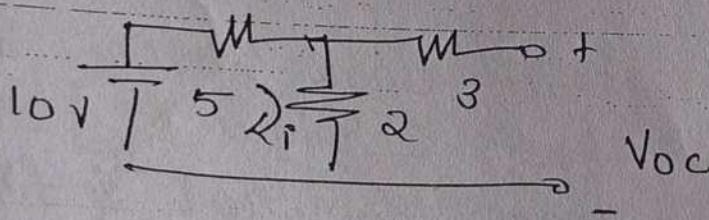


$$R_{th} = R_1 \parallel R_3 + R_2 = \frac{R_1 R_3}{R_1 + R_3} + R_2$$

Ex:- Find the Thevenin's equivalent n/w & load current



→ Remove the Load resistance



$$-10 + 5i + 2i = 0$$

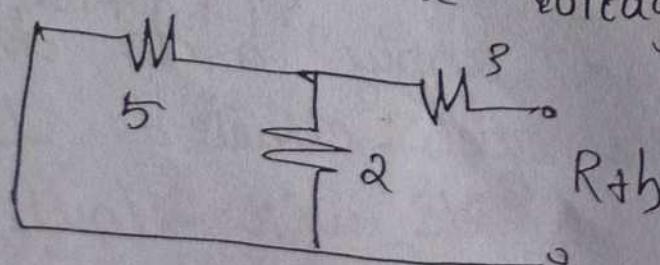
$$-10 + (5+2)i = 0$$

$$7i = 10$$

$$i = \frac{10}{7} = \underline{\underline{1.4281}}$$

$$V_{th} = i \times 2 = 1.4281 \times 2 = \underline{\underline{2.857V}}$$

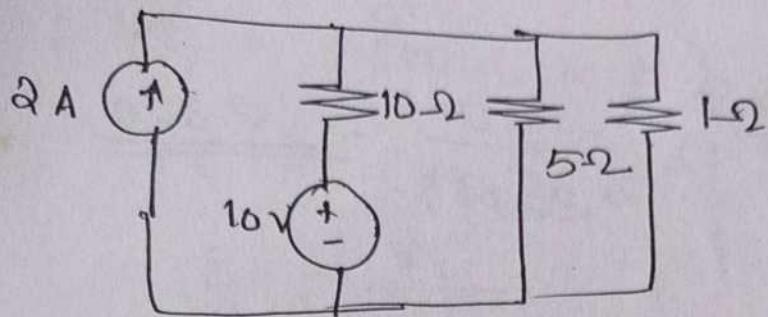
→ short circuit the voltage source.



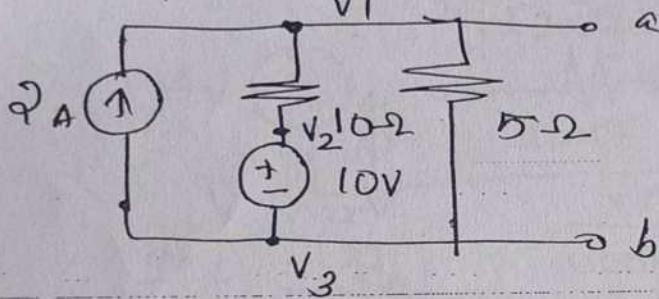
$R_{th} = \underline{\underline{3\Omega}}$   
Q: In the  
resistor

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

Q: In the given nw find the power loss in 1Ω resistor by Thevenins theorem



To find  $V_{oc}$



$$V_2 = 10V$$

$$\varphi = \frac{v_1 - v_2}{10} + \frac{v_1}{5}$$

$$\varphi = v_1 \left[ \frac{1}{10} + \frac{1}{5} \right] - v_2 \left[ \frac{1}{10} \right]$$

$$\varphi = v_1 \left[ \frac{1}{10} + \frac{1}{5} \right] - 10 \left[ \frac{1}{10} \right]$$

$$\varphi = v_1 \left[ \frac{1}{10} + \frac{1}{5} \right] = 1$$

$$3 = 0.3 v_1$$

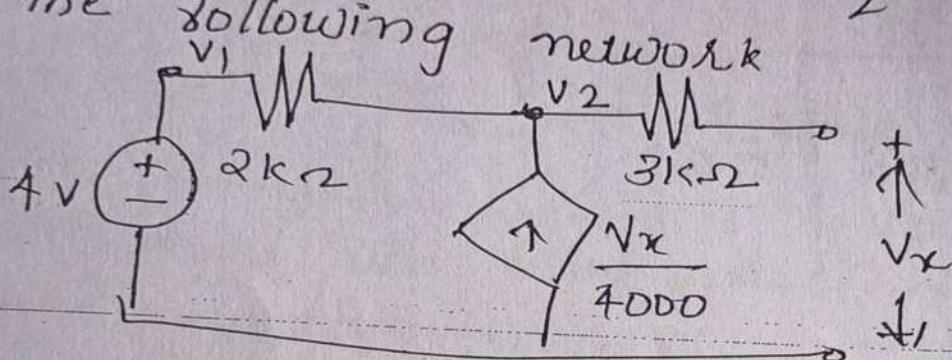
$$v_1 = \frac{10}{0.3} \quad 3/0.3 = \underline{\underline{10V}}$$

To find  $R_{th}$

$$\frac{1}{10} \parallel \frac{1}{5} \stackrel{R_{th}}{\Leftrightarrow} 10 \parallel 5 = \frac{50}{15} = \underline{\underline{3.33}}$$

$$I_L = \frac{10V}{3.33 + 1} = \underline{\underline{2.30A}}$$

Determine the equivalent resistance of the following network



$$V_1 = 4V, V_2 = V_x$$

$$\frac{V_x}{4000} = \frac{V_2 - V_1}{2k\Omega}$$

$$\frac{V_x}{4000} = \frac{V_2 - V_1}{2k\Omega}$$

$$V_x = 4000(V_2 - V_1)$$

$$V_x = \frac{4000(V_2 - V_1)}{2k\Omega}$$

$$V_x = \frac{4000(V_2 - 4)}{2k\Omega}$$

$$V_x =$$

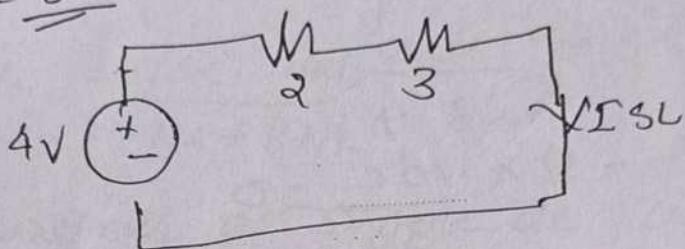
$$\frac{4000(V_x - 4)}{2k}$$

$$2V_x = 4000(V_x - 4)$$

$$V_x = 2V_x - 8$$

$$8 = \underline{\underline{V_x}}$$

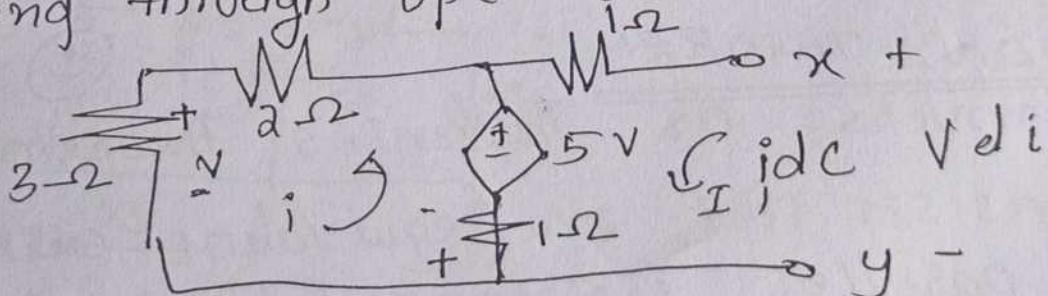
$$V_{OC} = \underline{\underline{8V}}$$



$$I_{SL} = \frac{4}{5} = 0.8 \text{ mA}$$

$$R_{Th} = \frac{V_{OC}}{I_{SL}} = \frac{8}{0.8 \text{ mA}} = 10 \text{ k}\Omega$$

θ: find the internal resistance of the circuit looking through open circuit by terminals



$$\text{Loop 1: } idc + 5V + (idc - ii) = V_{di}$$

$$2idc - ii = V_{di} + 5V$$

$$3idc(1.2 - idc) + 2i_1 = 5V$$

$$V = 3V$$

[M-2-4]

$$2idc - i_1 = vdc + 5 \times 3^\circ$$

$$2idc - 16i_1 = vdc \quad \text{--- (1)}$$

$$6i_1 - idc = 5 \times 3^\circ$$

$$-9i_1 - idc = 0 \quad \text{--- (2)}$$

$$2idc - 16i_1 = vdc$$

$$idc + 9i_1 = 0$$

$$2idc + 18i_1 = 0$$

$$i_1 = \frac{vdc}{34}$$

$$idc - 9 \times \frac{vdc}{34} = 0$$

$$idc = \frac{9}{34} vdc$$

$$\frac{vdc}{idc} = \frac{34}{9}$$

$$R_{th} = \frac{34}{9} \Omega$$

NORTON'S THEOREM :-

→ Converse of Thevenin's theorem.

→ consists of one equivalent current source & a parallel resistance (internal resistance)

Steps :-

1. short circuit the load terminal & find the short circuit current flowing through the short terminal using conventional analysis.

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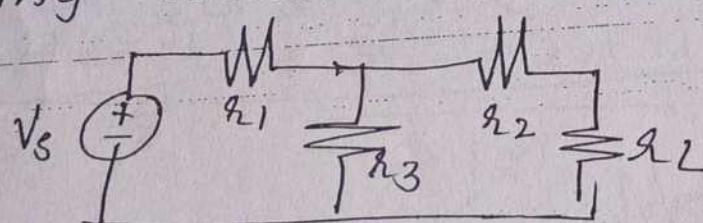
2. Remove the load resistance & find the individual resistance  $R_{int}$  or  $R_{Th}$  of the short circuit by deactivating the independent energy sources.

3. Norton's equivalent circuit can be drawn by keeping  $R_{Th}$  in parallel to  $i_{sc}$ .

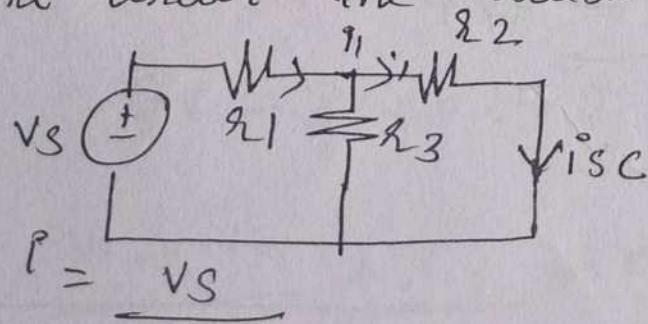
4. Reconnect the load resistance  $R_L$  across the load terminal & current through  $R_L$  can be found out by

$$i_L = \frac{i_{sc} \cdot R_{Th}}{R_L + R_{Th}}$$

Ex:- Consider a simple dc circuit to find out  $i_L$  using Norton's theorem.



short circuit the load terminal.



Req.

$$R_{Req} = R_1 + (R_2 \parallel R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$i^* = \frac{V_s}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

M-2-1

$$i_{SC} = \frac{V_S \cdot R_3}{R_1(R_2 + R_3) + R_2 R_3}$$

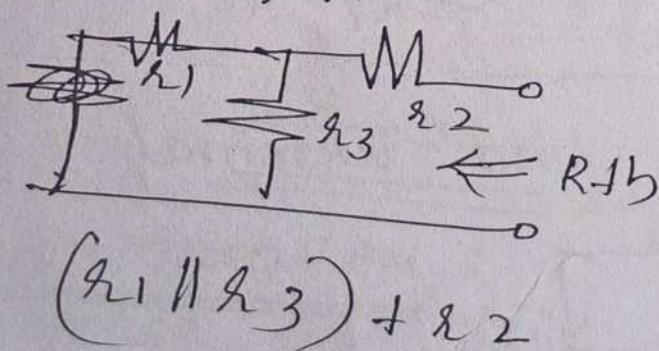
$$i_{SC} = \frac{i^o \times R_3}{R_2 + R_3}$$

$$= \frac{V_S}{\frac{R_1 + (R_2 R_3)}{R_2 + R_3}} \times \frac{R_3}{R_2 + R_3}$$

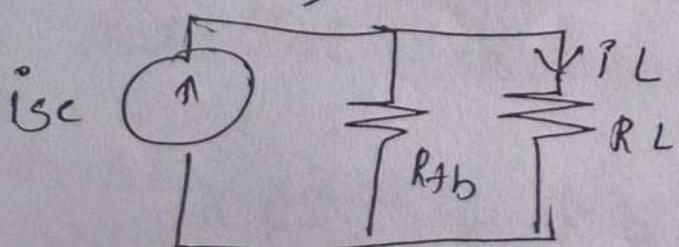
$$i_{SL} = V_S \cdot R_3$$

$$\frac{R_1(R_2 + R_3) + R_2 R_3}{R_2 + R_3}$$

To find  $R_{Th}$ :

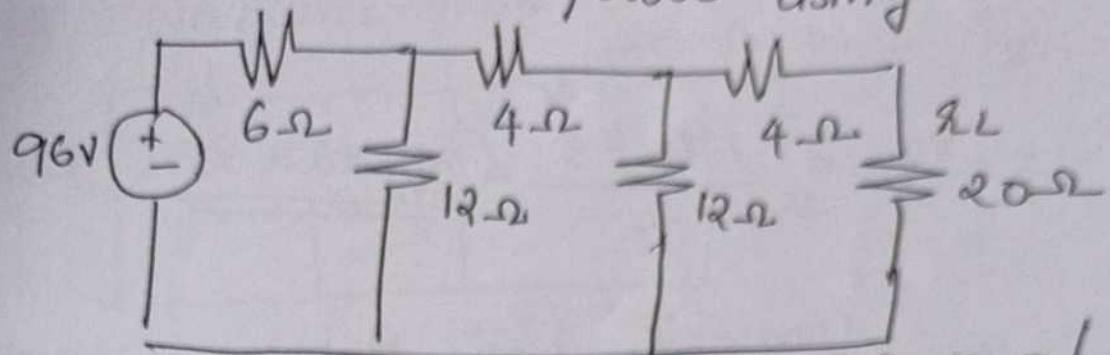


Norton's equivalent:

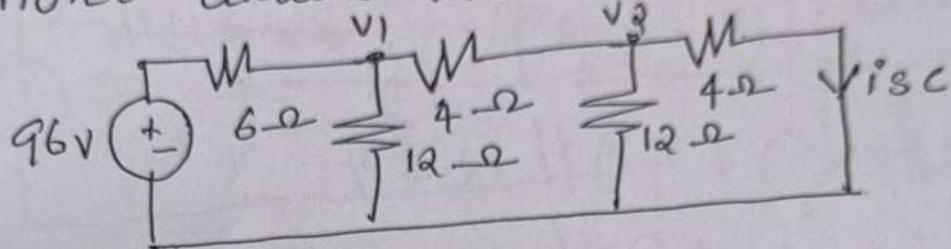


$$i_L = \frac{i_{SC} \cdot R_{Th}}{R_{Th} + R_L}$$

Q: Find the load power using Norton's Theorem



short circuit the load terminal



$$\frac{96 - v_1}{6} + \frac{v_1 - v_2}{4} + \frac{v_1}{12}$$

$$\frac{96 - v_1}{6} = \frac{3v_1 - 3v_2 + v_1}{2 \cdot 12}$$

$$192 - 2v_1 = 3v_1 - 3v_2 + v_1$$

$$6v_1 - 3v_2 = 192 \quad \text{--- (1)}$$

$$\frac{v_1 - v_2}{4} + \frac{v_2}{12} + \frac{v_2}{4}$$

$$\frac{v_1 - v_2}{4} = v_2 + \frac{3v_2}{3 \cdot 12}$$

$$3v_1 - 3v_2 = 4v_2 \quad \text{--- (2)}$$

$$3v_1 - 4v_2 = 0 \quad \text{--- (2)}$$

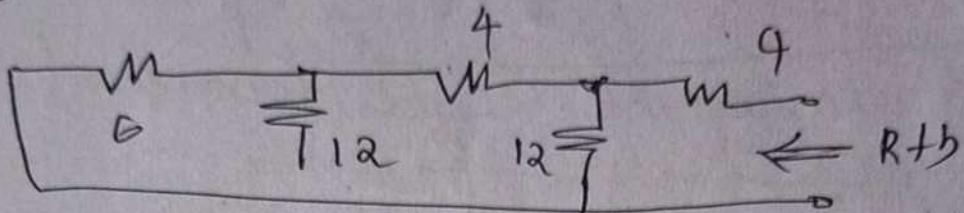
M-2-

$$V_1 = \underline{40 \cdot 7 \Omega}$$

$$V_2 = \underline{\underline{17 \cdot 45}}$$

$$R_{sc} = \frac{V_2}{4} = \frac{17 \cdot 45}{4} = \underline{\underline{4 \cdot 36 \Omega}}$$

R+L



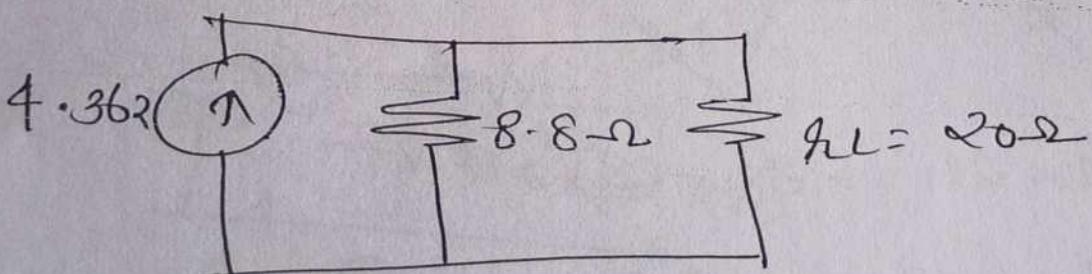
$$(6 \parallel 12) + 4 \parallel 12 + 4$$

$$= (4 + 4) \parallel 12 + 4$$

$$= (8 \parallel 12) + 4$$

$$= \underline{\underline{4 \cdot 8}} + 4$$

$$= \underline{\underline{8 \cdot 8 \Omega}}$$



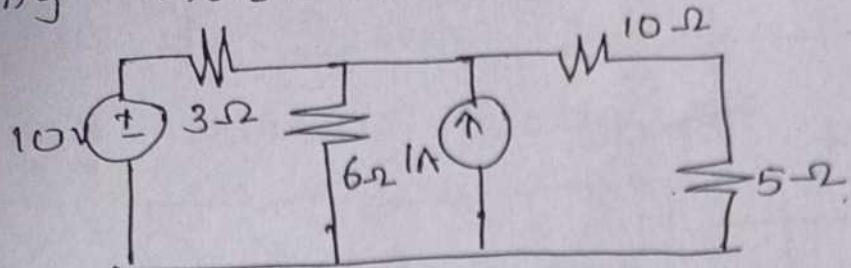
$$i_L = \frac{4 \cdot 36 \times 8 \cdot 8}{8 \cdot 8 + 20}$$

$$= \underline{\underline{1.33}}$$

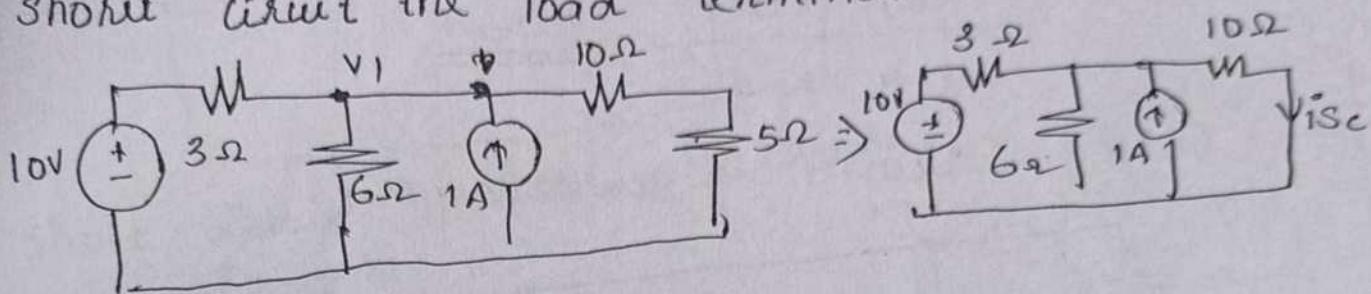
$$P = i_L^2 R_L = \underline{\underline{35 \cdot 49 W}}$$

Q: Fin  
using

: Find out current in a  $5\Omega$  resistor using Norton's theorem.



short circuit the load terminal.



$$\frac{10 - V_1}{3} + \frac{V_1}{G} + \frac{V_1}{10}$$

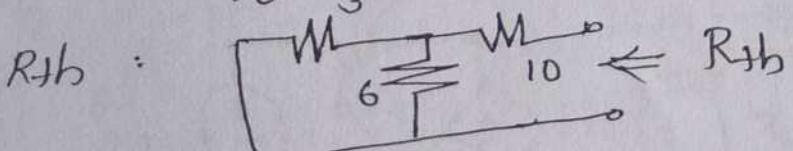
$$\frac{10 - 8V_1 + 3}{3} = 16V_1$$

$$200 - 20V_1 + 60 = 16V_1$$

$$36V_1 = 260$$

$$V_1 = 7.22V$$

$$I_{SC} = \frac{V_1}{10} = \frac{7.22}{10} = 0.722A$$

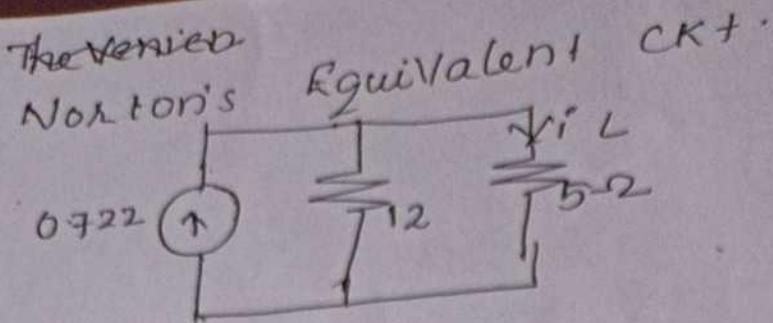


$$(3//6) + 10$$

$$= \frac{3 \times 6}{3+6} + 10 = \frac{18}{9} + 10 = \frac{12}{1} = 12\Omega$$

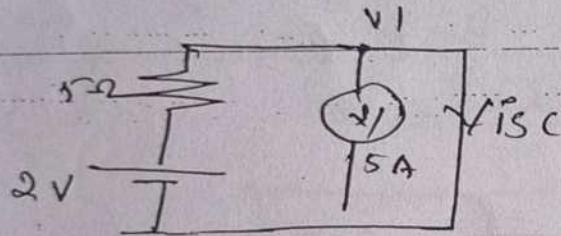
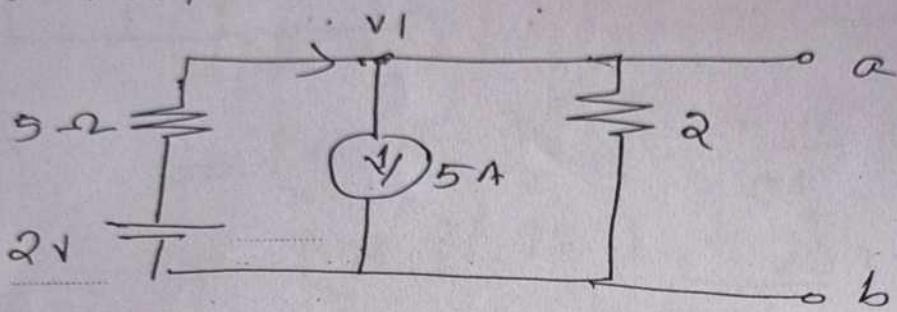
$$= \frac{18}{9} = 2$$

M-Q-



$$i_L = \frac{0.722 \times 12}{12 + 5} = 0.51 \text{ A}$$

Q: Find Norton's Equivalent

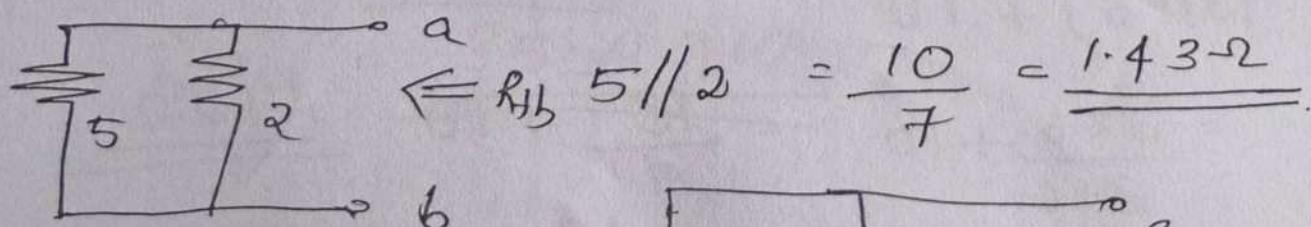


$$\frac{2}{5} = 5 + i_{sc}$$

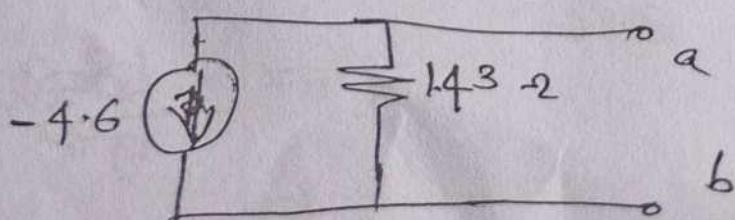
$$i_{sc} = \frac{2}{5} - 5$$

$$= -\underline{4.6}$$

To find  $R_{th}$

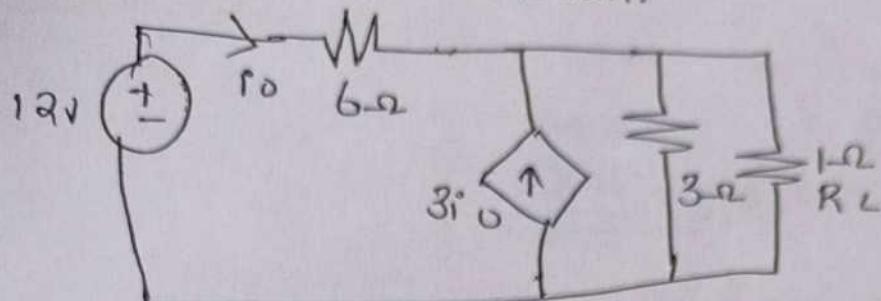


Equivalent

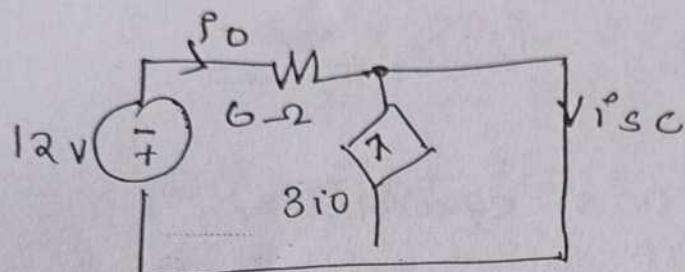


$\theta$  : Circuit with dependent source :-

Find the current through  $R_L$  in the circuit using Norton's theorem.



short circuit the load terminal

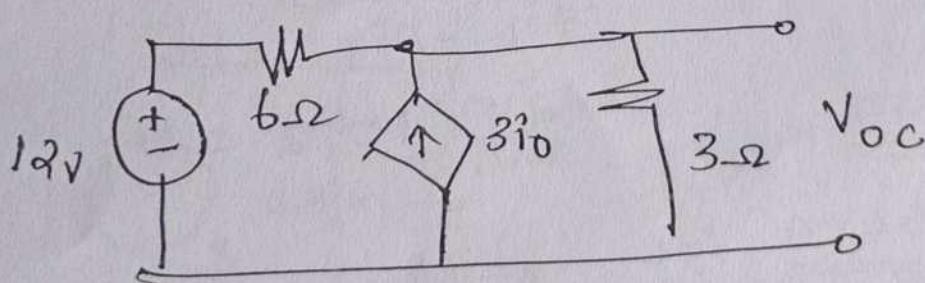


$$i_0 = \frac{12}{6} = 2$$

$$i_{sc} = 3i_0 + i_0 = 4i_0 = 4 \times \frac{12}{6} = 8 \text{ A}$$

To find  $R_{th}$

$$R_{th} = \frac{V_{oc}}{i_{sc}}$$



$$\frac{12 - V}{6} + 3i_0 = -\frac{V}{3}$$

M. 2. 8

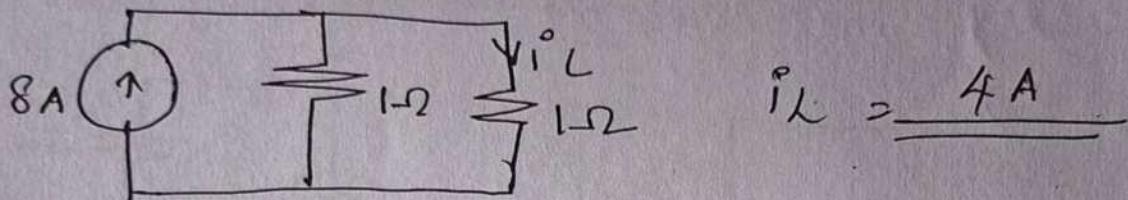
$$V_o = \frac{3V}{3} = 1V$$

$$V_{oc} = 8$$

$$I_{sc} = 8$$

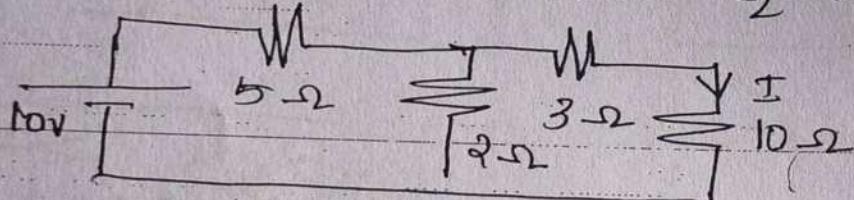
$$R_{th} = \underline{\underline{8/8 = 1\Omega}}$$

Norton's Equivalent

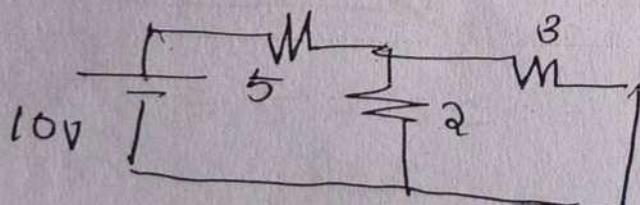


$$i_L = \underline{\underline{4A}}$$

Q. Obtain the Norton's equivalent.



Short circuit the load terminal



$$10 = 5i_1 + 2(i_1 - i_2)$$

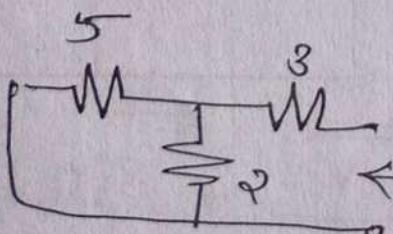
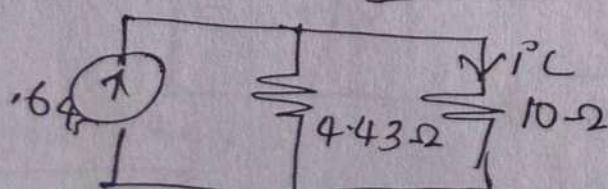
$$2(i_2 - i_1) + 3i_2 = 0$$

$$7i_1 - 2i_2 = 10$$

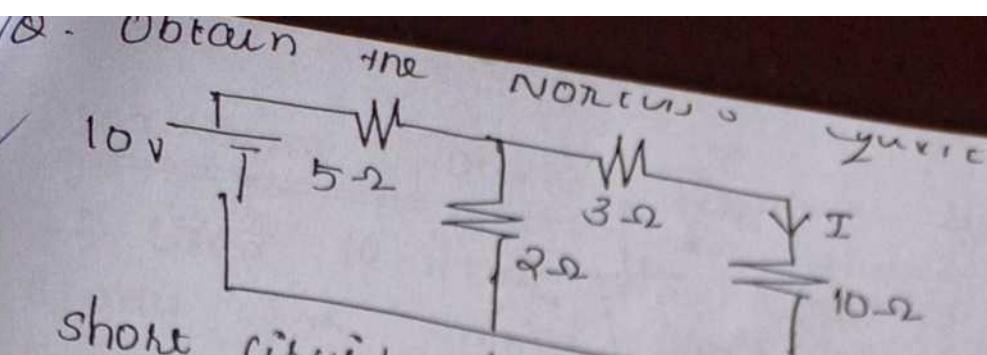
$$-2i_1 + 5i_2 = 0$$

$$i_1 = \underline{\underline{1.61A}}$$

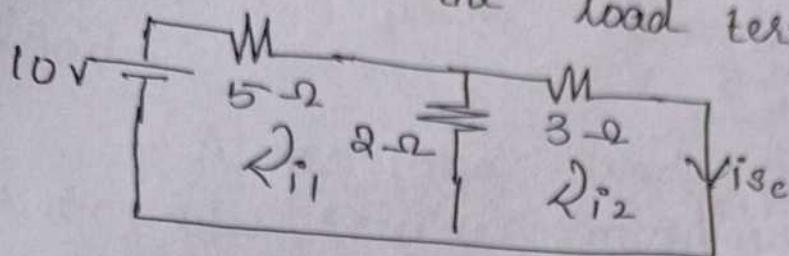
$$i_2 = \underline{\underline{-645A}} \quad i_{sc}$$



$$\begin{aligned} R_{th} &= (5//2) + 3 \\ &= \underline{\underline{1.43 + 3}} \end{aligned}$$



short circuit the load terminal.



$$10 = 5i_1 + 2(i_1 - i_2)$$

$$10 = 5i_1 + 2i_1 - 2i_2$$

$$10 = 7i_1 - 2i_2 \quad \text{--- (1)}$$

$$2(i_2 - i_1) + 3i_2 = 0$$

$$2i_2 - 2i_1 + 3i_2 = 0$$

$$5i_2 - 2i_1 = 0 \quad \text{--- (2)}$$

$$i_1 = \underline{1.61A}$$

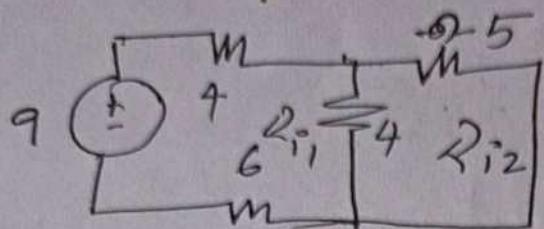
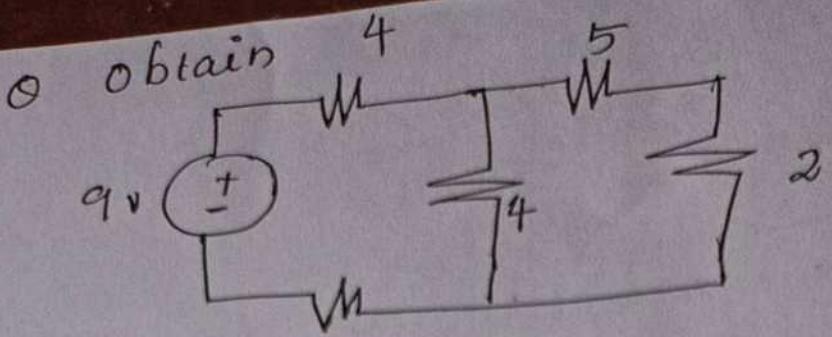
$$i_2 = 0.645A$$

$$i_{SC} = i_2$$

$$R_{th} = \left[ \frac{5}{5+2} \parallel \frac{3}{2} \right] \leftarrow R_{th} = \frac{(5 \parallel 2) + 3}{5+2} = \frac{5 \times 2}{5+2} + 3 = \frac{10}{7} + 3 = \underline{\underline{4.43}}$$

$$i_L = \frac{0.645 \times 4.43}{10 + 4.43} = \underline{\underline{0.198}}$$

M-2-9



$$q = 4i_1 + 6i_1 + 4(i_1 - i_2)$$

$$q = 14i_1 - 4i_2 \quad \text{--- (1)}$$

$$4(i_2 - i_1) + 5i_2 = 0$$

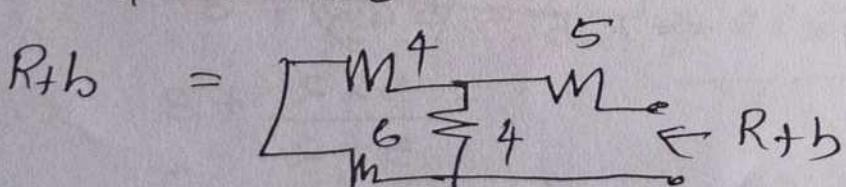
$$\Rightarrow 4i_2 - 4i_1 + 5i_2 = 0$$

$$0 = -4i_1 + 9i_2 \quad \text{--- (2)}$$

$$i_1 = \underline{0.736 \text{ A}}$$

$$i_2 = \underline{0.327 \text{ A}}$$

$$i_2 = i_{SC}$$

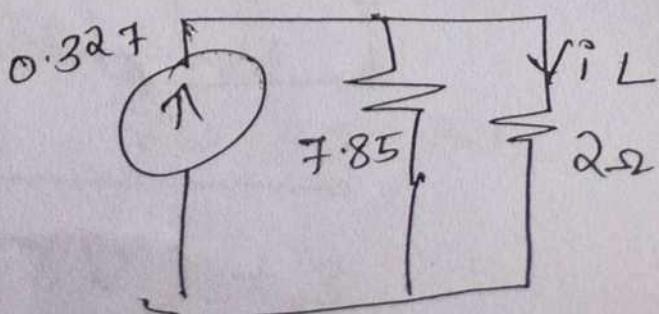


(Ans)

$$(10//4) + 5$$

$$\frac{40}{14} + 5 = \underline{7.857}$$

Equivalent circuit



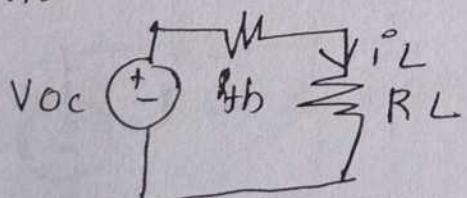
## Maximum Power Transfer Theorem

→ Used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

A resistance load being connected to a dc nw receives maximum power when the load resistance is equal to the internal resistance. i.e. Thevenin's equivalent resistance of the source network seen from the load terminal.

Steps :-

1. Find the Thevenin's Voltage  $V_{oc}$  at the open ckt load terminal
2. Remove the load resistance & find  $R_{th}$
3. As per the maximum power transfer theorem for max. power transfer  $R_L = R_{th}$
4. The maximum power transfer is given by



$$i_L = \frac{V_{oc}}{R_{th} + R_L}$$

for maximum power

$$i_L = \frac{V_{oc}}{2R_{th}}$$

$$R_L = R_{th}$$

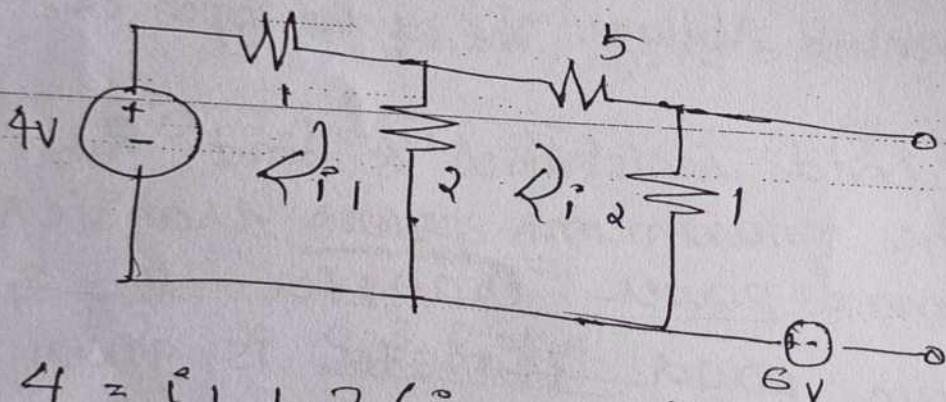
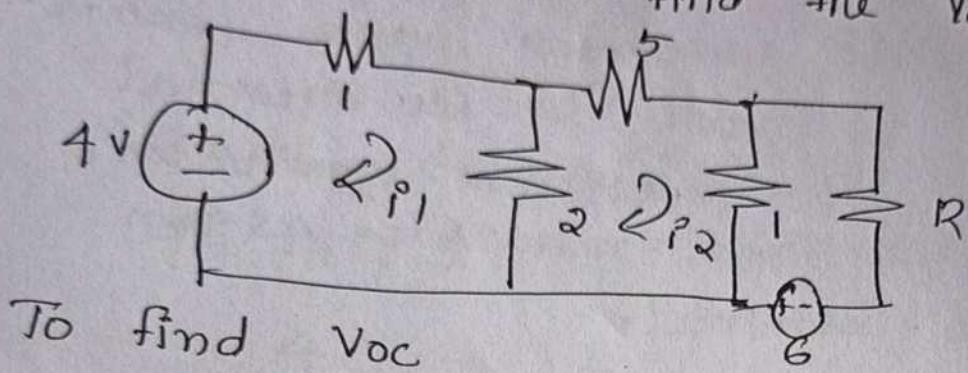
$$P = i_L^2 \cdot R_L$$

$$\begin{aligned} P_{max} &= i_L^2 \cdot R_{th} \\ &= \left[ \frac{V_{oc}}{2R_{th}} \right]^2 \cdot R_{th} \end{aligned}$$

M-2-10

$$P_{\max} = \frac{V_{oc}^2}{4R_{th}}$$

Q: Find the value of  $R_L$  in the ckt such that max. power transfer takes place from source to load. Also find the value of power



$$4 = i_1 + 2(i_1 - i_2)$$

$$4 = i_1 + 2i_1 - 2i_2$$

$$4 = 3i_1 - 2i_2 \quad \text{--- (1)}$$

$$2(i_2 - i_1) + 5i_2 + i_2$$

$$2i_2 - 2i_1 + 6i_2$$

$$-2i_1 + 8i_2 = 0 \quad \text{--- (2)}$$

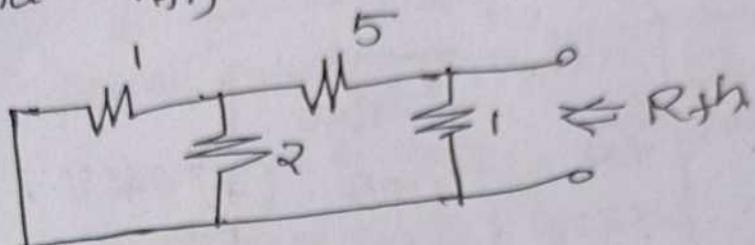
$$i_1 = \underline{\underline{1.6 \text{ A}}}$$

$$i_2 = \underline{\underline{0.4 \text{ A}}}$$

$$V_{OC} = 1.12 = 0.4A + 6V$$

$$= \underline{6.4V}$$

To find  $R_{Th}$



$$\left( (C_2 \parallel 1) + 5 \right) \parallel 1$$

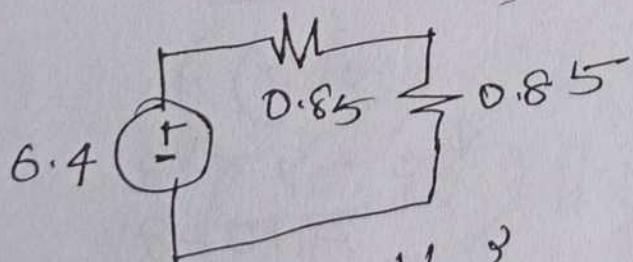
$$= \left( \frac{2 \times 1}{2+1} + 5 \right) \parallel 1$$

$$= \left( \frac{2}{3} + 5 \right) \parallel 1$$

$$= \frac{\frac{17}{3} \times 1}{\frac{17}{3}} = \frac{\frac{17}{3}}{20}$$

$$= \frac{\frac{17}{3} + 1}{\frac{17}{3}} = \frac{3}{20}$$

$$= \frac{\frac{17}{3} \times \frac{3}{20}}{20} = \underline{\underline{0.85}}$$

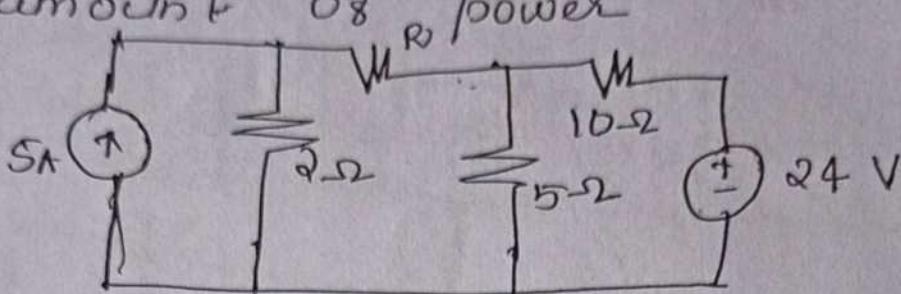


$$\text{power} = \frac{V_{OC}^2}{4R_{Th}} = \frac{6.4^2}{4 \times 0.85}$$

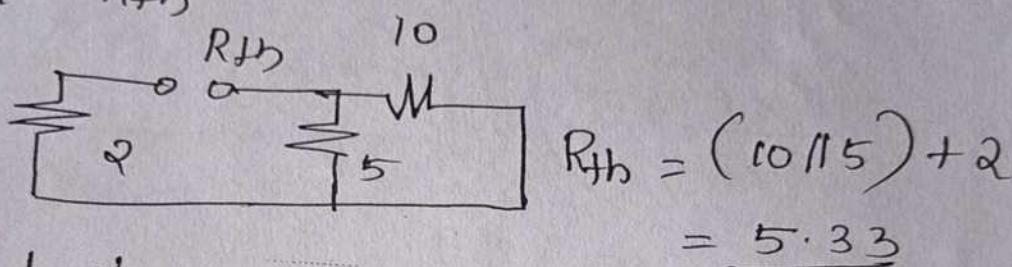
$$= \underline{\underline{12.05W}}$$

M-2-11

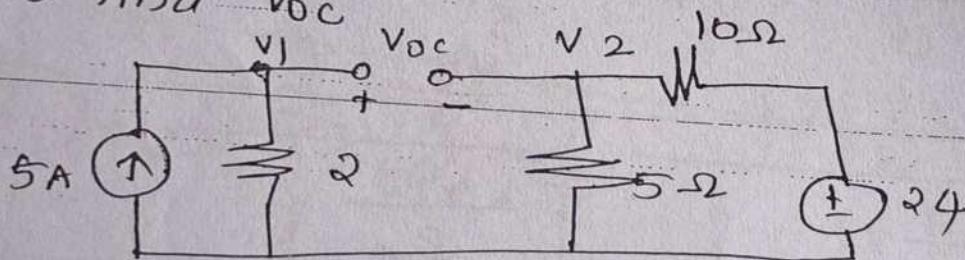
Q: What should be the value of  $R$  such max power transfer takes place from the rest of the circuit to  $R$ ? obtain the amount of power



To Find  $R_{Th}$



To find  $V_{OC}$



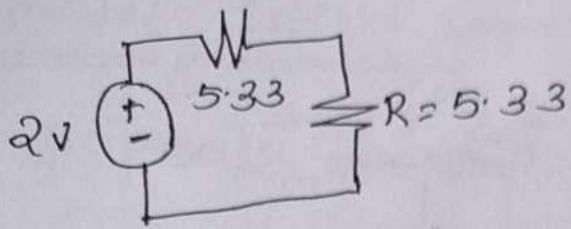
$$V_1 = \frac{V_1}{2} \quad V_1 = \underline{\underline{10V}}$$

$$\frac{24 - V_2}{10} = \frac{V_2}{5} \quad V_{OC} = V_1 - V_2 \\ = 10 - 8$$

$$24 = 3V_2$$

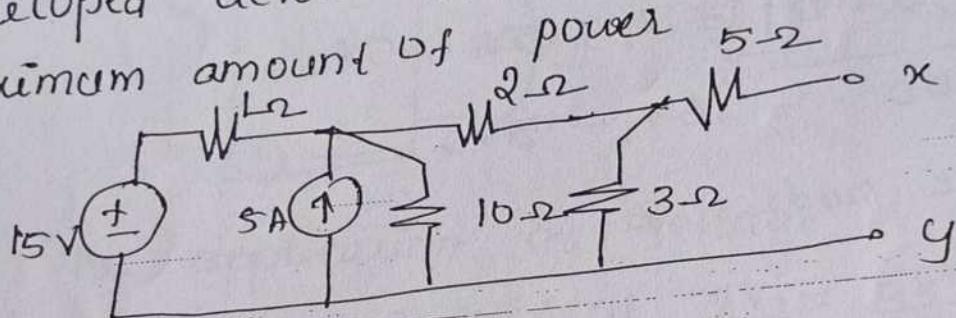
$$V_2 = \underline{\underline{8V}}$$

$$= \underline{\underline{2V}}$$



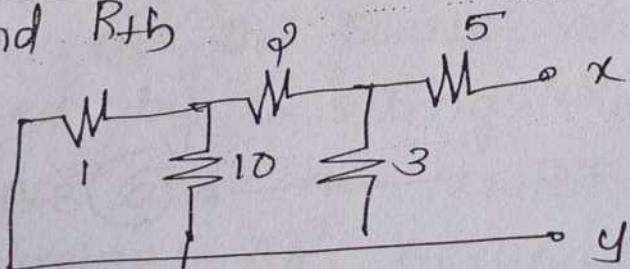
$$\text{Max. power} = \frac{V_{oc}^2}{4 \cdot R_{th}} = \frac{2^2}{5.33 \times 4} = \underline{\underline{0.188 \text{ W}}}$$

Q : Find the resistance across the xy terminals in the circuit such that maximum power is developed across the load terminals. What is the maximum amount of power?



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To Find  $R_{th}$



$$((1 \parallel 10) + 2) \parallel 3 + 5$$

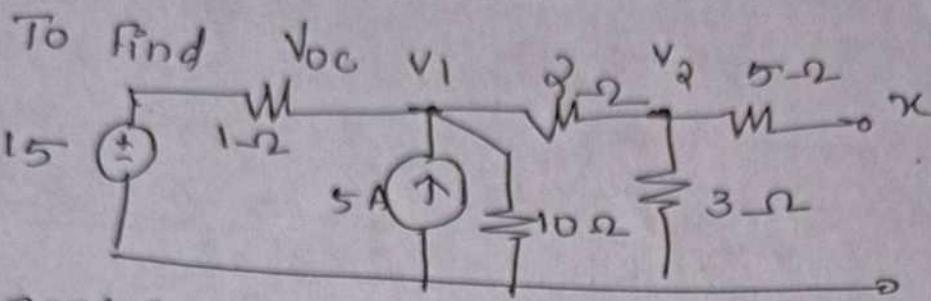
$$\left( \frac{10}{11} + 2 \right) \parallel 3 + 5$$

$\frac{33}{32}$   
 $\frac{32}{65}$

$$\left( \frac{\frac{32}{11} \times 3}{\frac{32}{11} + 3} \right) + 5 \Rightarrow \frac{\frac{96}{11}}{\frac{65}{11}}$$

$$\Rightarrow \left( \frac{96}{11} \times \frac{1}{\frac{65}{11}} \right) + 5 \quad \boxed{M-9-12}$$

$$R_{Th} = \underline{6.48\Omega}$$



Applying KCL at node 1

$$\frac{15 - v_1}{1} + 5 = \frac{v_1}{10} + \frac{v_1 - v_2}{2}$$

$$10(20 - v_1) = v_1 + 5v_1 - 5v_2$$

$$200 = 16v_1 - 5v_2 \quad \text{--- (1)}$$

KCL at node 2

$$\frac{v_1 - v_2}{2} = \frac{v_2}{3}$$

$$3v_1 - 3v_2 = 2v_2$$

$$3v_1 - 5v_2 = 0 \quad \text{--- (2)}$$

$$v_1 = \underline{15.38}$$

$$v_2 = \underline{9.23}$$

$$V_{oc} = v_2 = \underline{\underline{9.23V}}$$

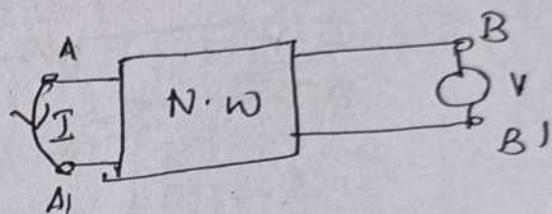
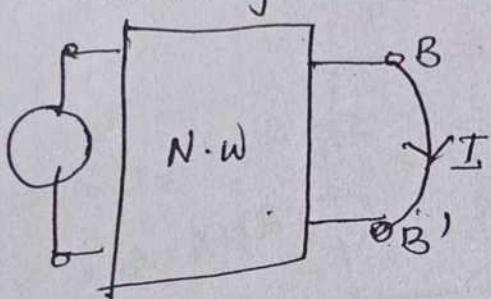
According to max power transfer theorem

$$R_L = R_{Th} = 6.48\Omega$$

$$P_{max} = \frac{V_{oc}^2}{4R_{Th}} = \frac{9.23^2}{4 \times 6.48} = \underline{\underline{3.287W}}$$

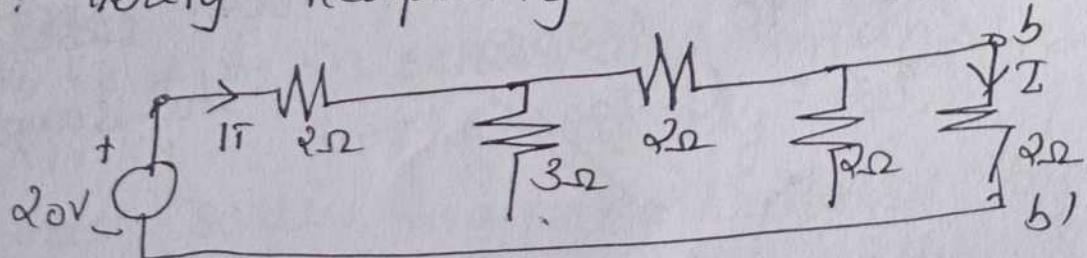
## Reciprocity Theorem

The reciprocity theorem states that in a linear bilateral, single source circuit, the ratio of excitation to response is a constant when the position of excitation and response are interchanged.



The application of voltage  $V$  across  $AA'$  produces a current  $I$  at  $BB'$ . If the position of the source and response are exchanged by connecting the voltage source across  $BB'$  and the resulting current  $I$  will be at terminal  $AA'$ . According to reciprocity theorem, the ratio of input to the response is same in both cases.

Eg: Verify Reciprocity Theorem for the network given



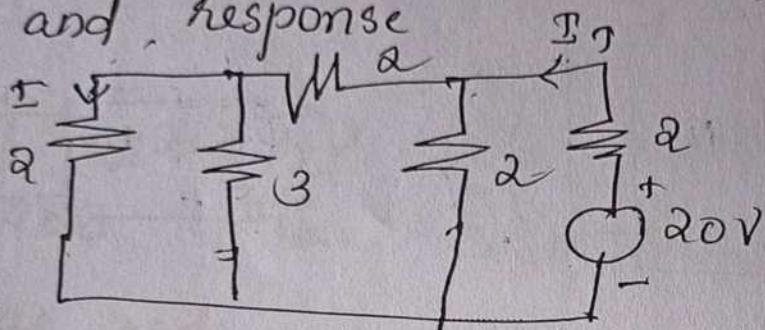
$$\text{Total resistance} = \left( \left( 2 \parallel 2 \right) \parallel 3.5 \right) \Omega$$

$$\text{Total resistance } R_{\text{tot}} = 3.5 \Omega \parallel$$

$$I_T = \frac{20}{3.5} = 5.71 \text{ A}$$

$$I = \underline{1.43}$$

Applying Reciprocity Theorem by interchanging source and response



$$\text{Total Resistance} = \underline{3.23 \Omega}$$

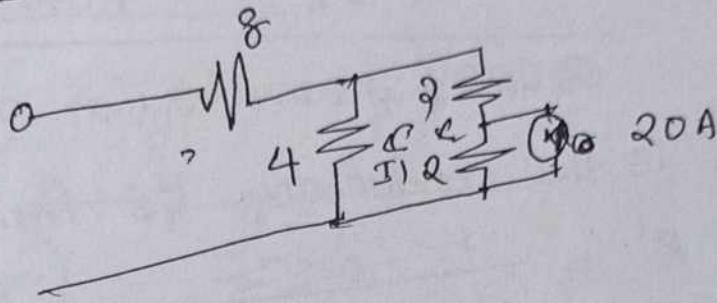
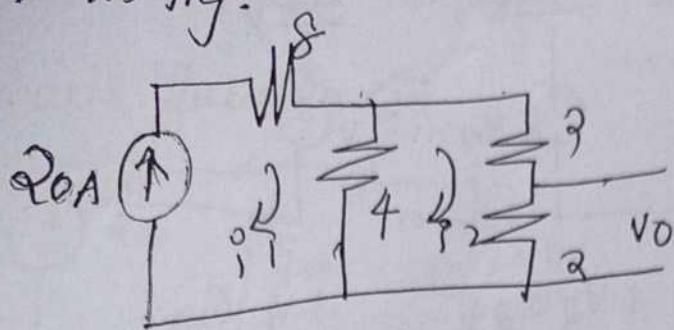
$$I_T = \frac{20}{3.23} = \underline{6.19 \text{ A}}$$

$$I_r = \frac{6.19 \times 2}{5.2} = \underline{2.38}$$

$$I = \frac{2.38 \times 3}{5} = \underline{1.43 \text{ A}}$$

If we compare the results in both cases, the ratio of input to response is the same  
 $\frac{20}{1.43} = 13.99 \text{ A}$

Q: Verify the Reciprocity Theorem for the circuit shown in fig.



$$V_0 = 20V$$

$$I_1 = 20A$$

$$4(I_2 - I_1) + 4I_2 = 0$$

$$2I_2 - I_1 = 0$$

$$I_1 = 20$$

$$2I_2 = 20$$

$$I_2 = 10A$$

$$V_0 = \underline{20V}$$

$$2(I_2 - I_1) + 2I_2 + 4I_2 = 0$$

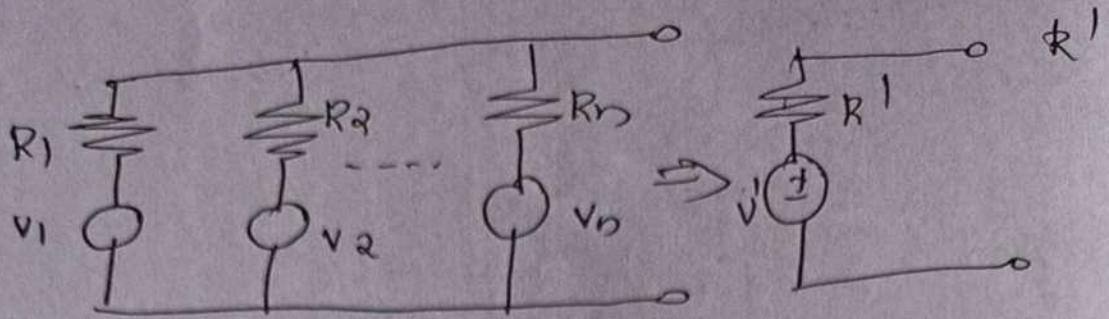
$$V_0 = 5 \times 4 = \underline{20V}$$

$$8I_2 - 2I_1$$

$$I_2 = \frac{I_1}{4} = \underline{5A}$$

MILLMAN'S THEOREM: Millman's theorem states that in any network, if the voltage source  $V_1, V_2, \dots, V_n$  in series with internal resistance  $R_1, R_2, \dots, R_n$  respectively are in parallel, then these sources may be replaced by a single voltage source  $V$  in series with  $R_1$

VM-2-14

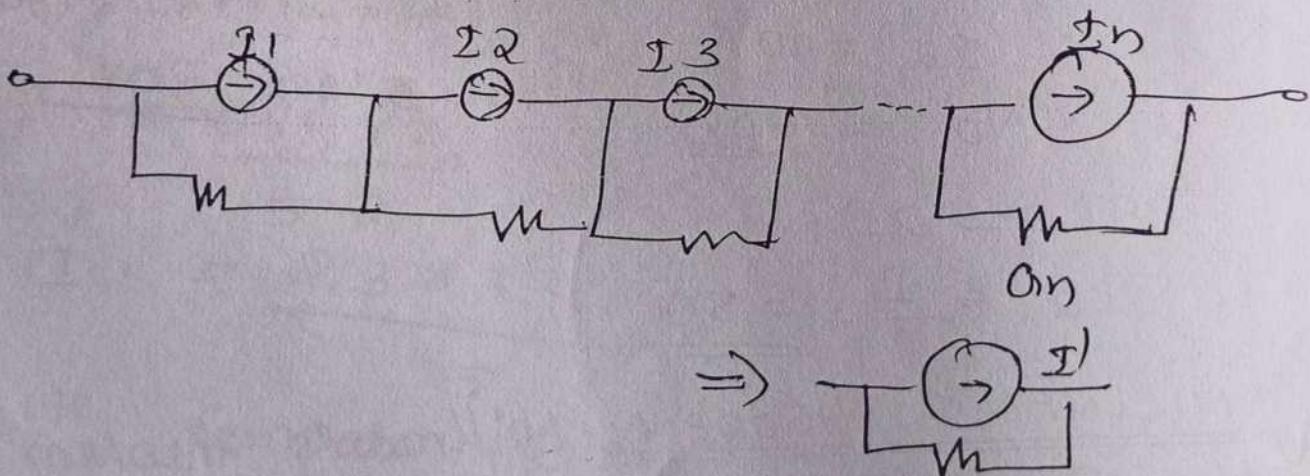


$$\text{where } V' = \frac{v_1 g_1 + v_2 g_2 + \dots + v_n g_n}{g_1 + g_2 + \dots + g_n}$$

where  $g_n$  is the conductance of the  $n^{th}$  branch

and  $R' = \frac{1}{g_1 + g_2 + \dots + g_n}$

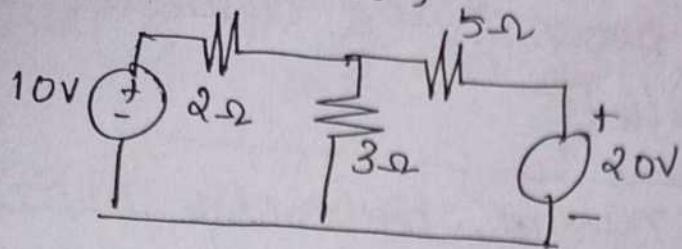
A similar theorem can be stated for  $n$  current sources having internal conductance which can be replaced by a single current source  $I'$  in parallel.



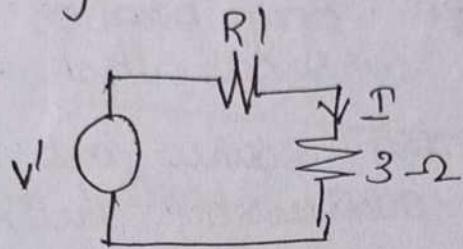
$$I' = \frac{I_1 R_1 + I_2 R_2 + \dots + I_n R_n}{R_1 + R_2 + \dots + R_n} \text{ and}$$

$$G' = \frac{1}{R_1 + R_2 + \dots + R_n}$$

Q calculate the current I shown in fig using  
Nollman's Theorem



According to Nollman's Theorem



$$V' = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} = \frac{10 \times \frac{1}{2} + 20 \times \frac{1}{5}}{\frac{1}{2} + \frac{1}{5}} = \underline{\underline{12.85}}$$

$$R = \frac{1}{G_1 + G_2} = \frac{R_1 R_2}{R_1 + R_2} = \underline{\underline{1.43\Omega}}$$

$$I = \frac{12.85}{3 + 1.43} = \underline{\underline{2.9A}}$$

## Super position Theorem

This theorem is used to solve a n/w where two or more sources are present and connected not in series or in parallel.

"If a number of voltage or current sources are acting simultaneously in a linear n/w the resultant current in any branch is the algebraic sum of the currents that would be produced in it when each source act alone replacing all other independent sources by their internal impedance.

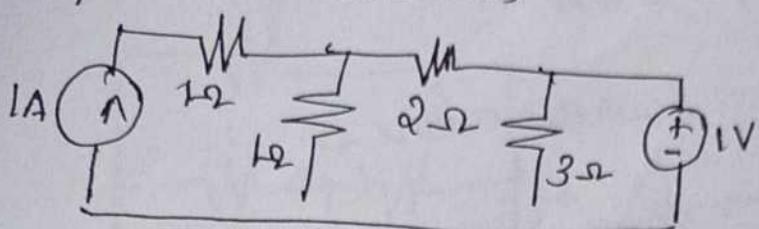
Steps : 1) only one source is considered to act alone. The other sources are replaced by their internal impedance or ideal independent voltage sources a open circled. All dependent source will act normally

2) using any suitable n/w analysing technique, the current through or the voltage across the desired element is found out due to the source under consideration.

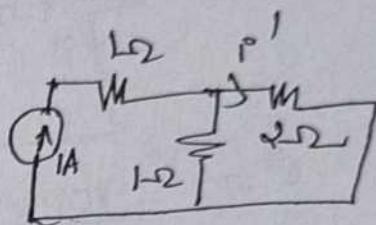
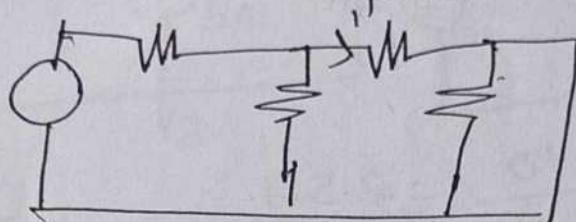
3) The above steps are repeated considering all the independent sources one by one

4) The total response (current or voltage) is obtain by taking its algebraic sum of all the responses

Q: Find the current 'i' in the circuit using super position theorem

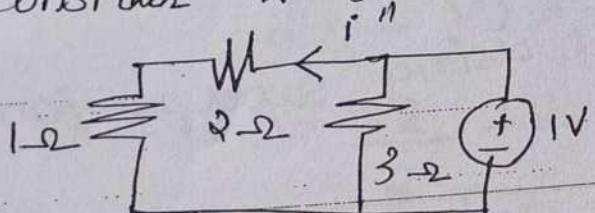


Consider 1A source alone



$$\text{current through } 2\Omega \text{ resistor } i' = \frac{1 \times 1}{3} = \underline{\underline{\frac{1}{3} A}}$$

Consider 1V source:



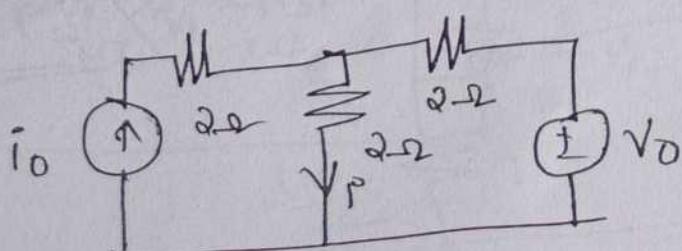
$$\text{Total } = 1.5$$

$$i'' = \frac{1}{1.5} = \underline{\underline{\frac{2}{3} A}}$$

$$i''' = \frac{1}{3} \times \frac{1}{2} = \underline{\underline{\frac{1}{3} A}}$$

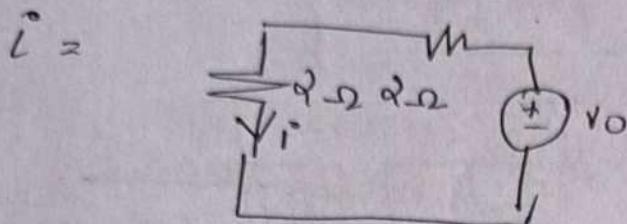
$$i' - i''' = \frac{1}{3} - \frac{1}{3} = \underline{\underline{0 A}}$$

Q: In the given figure, when  $V_0 = 0$ ,  $i = 2A$   
find  $i$  when  $V_0 = 10V$



M-2-16

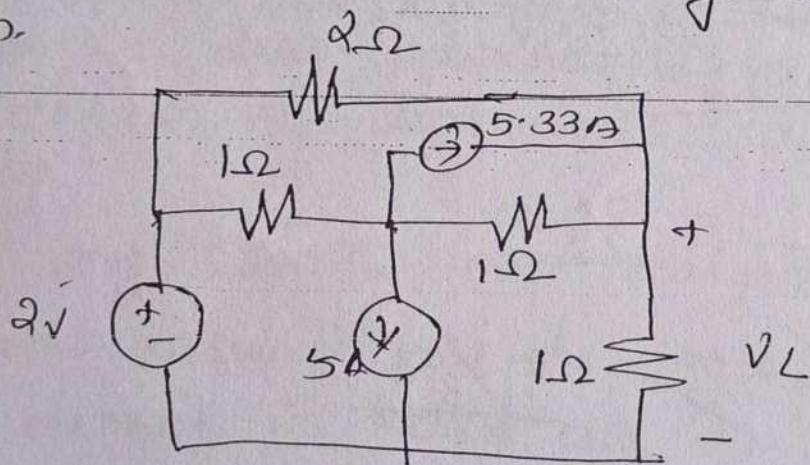
Given  $i^e = 2A$  when current source alone  
is acting ie  $i^e = 2A$   
considering  $v_o$  source alone.



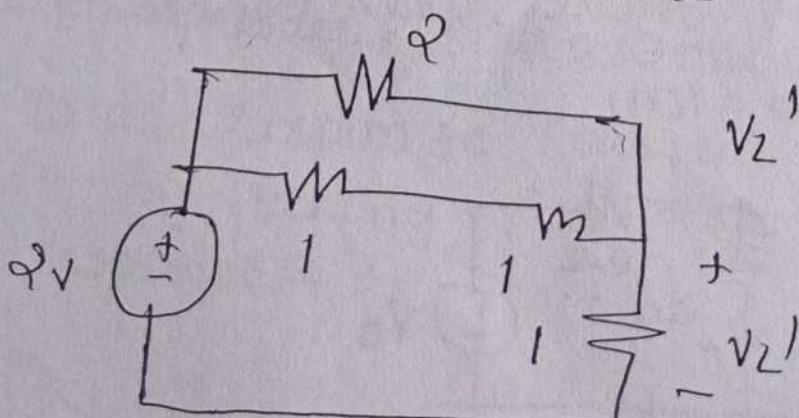
$$i^e = \frac{v_o}{4} = \frac{10}{4} = 2.5A$$

Total current  $i = 2 + 2.5 = 4.5A$

Q. find  $v_L$  in the ckt using superposition theorem.

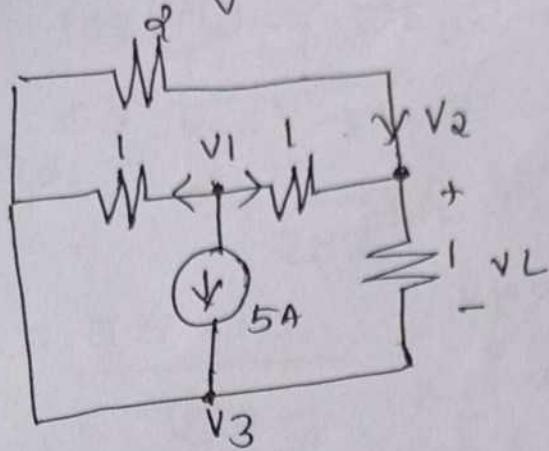


Consider 2V source alone



$$v_L' = \frac{2}{2} = 1$$

Considering 5A source



$$V_3 = 0$$

$$V_1 + 5 + V_1 - V_2 = 0$$

$$2V_1 - V_2 = -5 \quad \text{--- (1)}$$

$$V_1 - V_2 = \frac{V_2 + V_2}{2}$$

$$2V_1 - 2V_2 = 3V_2 \quad \text{--- (2)}$$

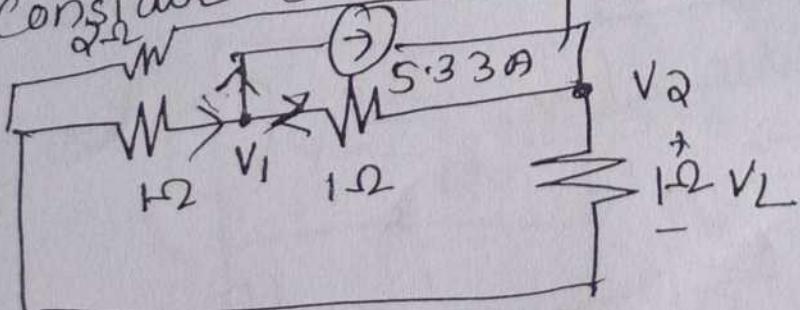
$$2V_1 - 5V_2 = 0$$

$$V_1 = -\underline{\underline{3.125V}}$$

$$V_2 = -\underline{\underline{1.25V}}$$

$$V_L^{(1)} = V_2 = -\underline{\underline{1.25V}}$$

Consider  $5.33\Omega$  source



$$5.33 = \frac{-v_1}{1} + \frac{v_2 - v_1}{1}$$

$$v_2 - 2v_1 = 5.33 \quad \text{--- (1)}$$

$$-\frac{v_2}{2} + 5.33 = v_2 - v_1 + v_2$$

$$5.33 = 2v_2 - v_1 + \frac{v_2}{2}$$

$$= 5v_2 - 2v_1$$

$$-2v_1 + 5v_2 = 10.66 \quad \text{--- (2)}$$

$$v_1 = -1.998$$

$$v_2 = 1.33$$

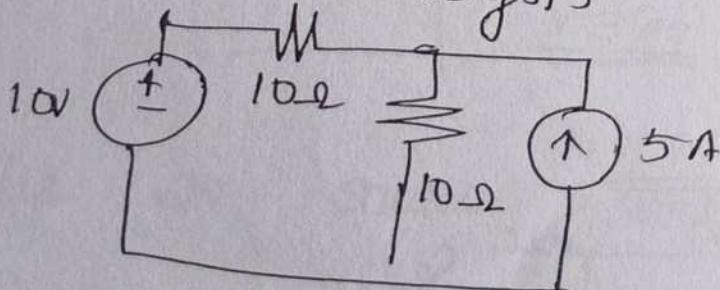
$$v_L''' = v_2 = \underline{\underline{1.33}}$$

Total voltage across  $v_L = v_L' + v_L'' + v_L'''$

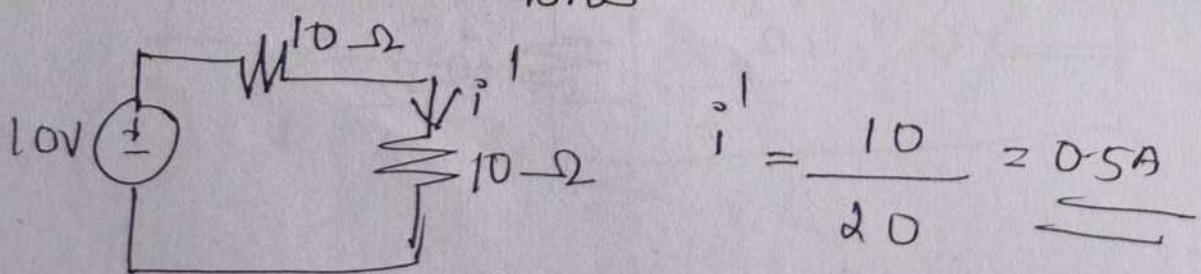
$$= 1 - 1.25 + 1.33$$

$$= \underline{\underline{1.08V}}$$

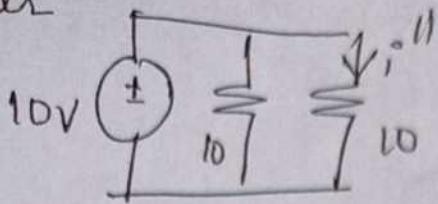
Q: Find  $i$  using superposition without using mesh & nodal analysis



Consider 10V source alone



Consider

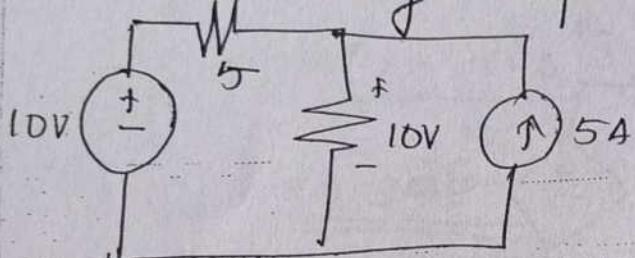


$$i_{\text{total}} = \frac{10}{20} = 0.5 \text{ A}$$

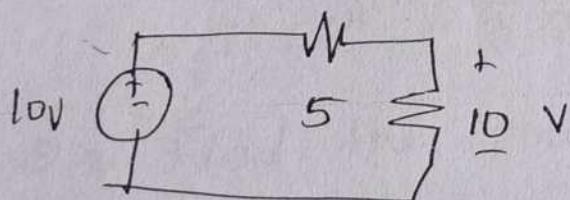
$$i' = \frac{0.5 \times 10}{20} = \underline{\underline{0.25 \text{ A}}}$$

$$i_{\text{total}} = i' + i'' = 0.5 + 0.25 = \underline{\underline{0.75 \text{ A}}}$$

Find V using superposition

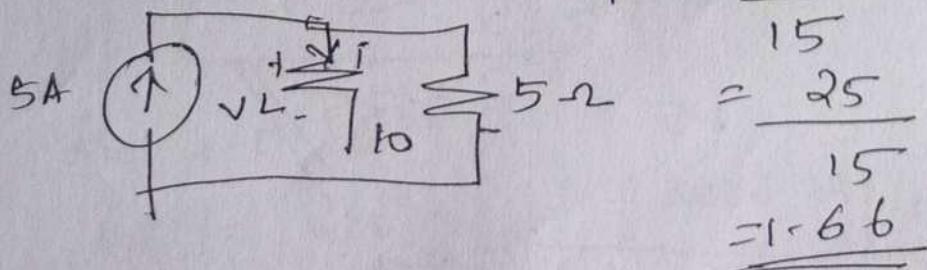


Consider 10V source alone



$$V' = \frac{10 \times 10}{15} = \underline{\underline{6.666 \text{ V}}}$$

Consider 5A source



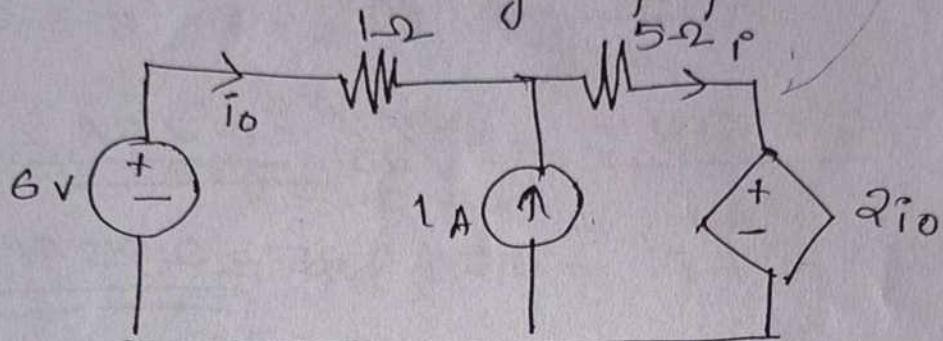
$$i = \frac{5 \times 5}{15} = \frac{25}{15} = 1.66$$

$$V'' = 1.666 \times 10 = \underline{\underline{16.66 \text{ V}}}$$

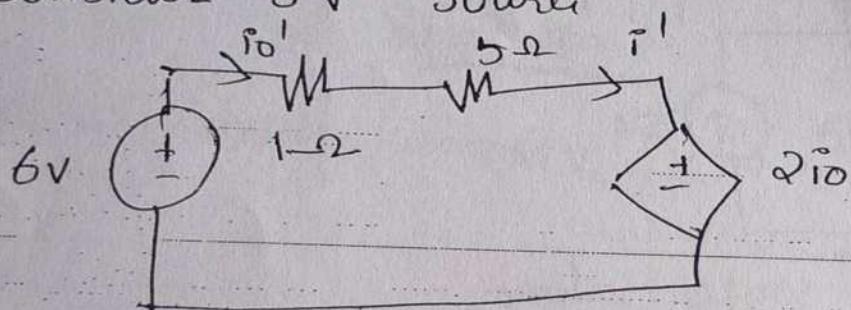
M-2-18

Total voltage across  $10\Omega$  resistor  
 $6.66 + 16.66$   
 $= \underline{23.33V}$

Q: Find  $i_0$  &  $i$  using superposition theorem.



Consider 6V source



$$i_0' = i'$$

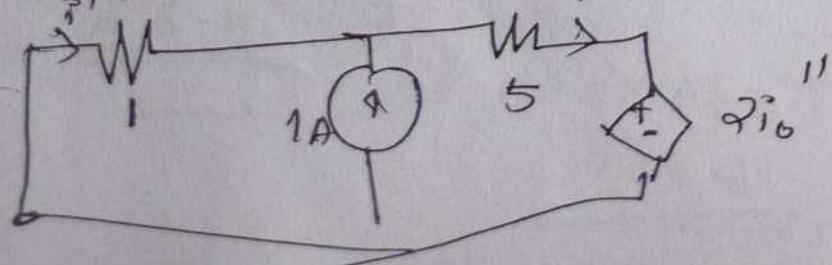
$$-6 + 6i_0' + 2i_0' = 0$$

$$-6 + 8i_0' = 0$$

$$i_0' = 6/8 = 3/4$$

$$i' = 1.5 = 0.75 //$$

Consider 1A source



$$\frac{-v_1}{1} + 1 = v_1 - \underbrace{2i_0''}_{5}$$

$$i_0'' = \frac{-v_1}{1} \quad -v_1 + 1 = v_1 + \frac{2v_1}{5}$$

$$-5v_1 + 5 = 3v_1$$

$$8v_1 = 5$$

$$v_1 = 5/8 = 0.625 //$$

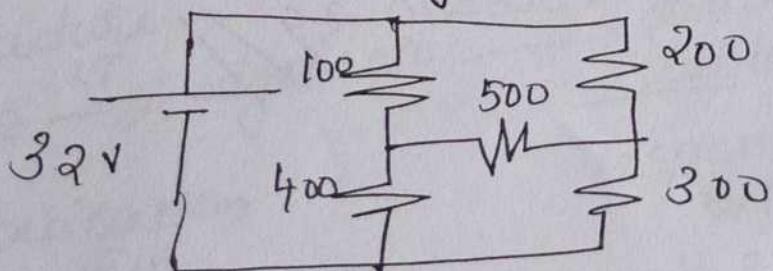
$$i_0'' = -\underline{\underline{0.625}}$$

$$i_1'' = \frac{v_1 - 2i_0''}{5} = \frac{0.625 + 1.25}{5} \\ = \underline{\underline{0.375}}$$

$$i_0 = i_0' + i_0'' = 0.75 + 0.625 = 0.125 A$$

$$i = i_1 + i_1'' = 0.75 + 0.375 = \underline{\underline{1.125 A}}$$

Q: Find the power dissipated in the 50 ohm resistance using Norton's Theorem



$$i_{SC} = 0.1344 A$$

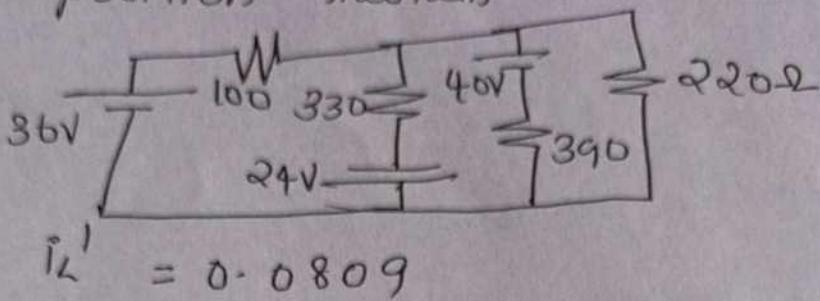
$$R_{Th} = 200 \Omega$$

$$i_L = 0.0387$$

$$P = 0.737$$

M-Q-1

Q: Find the power across the load using super position theorem



$$i_L' = 0.0809$$

$$i_L'' = -0.016$$

$$i_L''' = 0.0237$$

$$i_L = 0.088$$

$$P = \underline{1.7W}$$

Q: A battery has an ~~initial~~ internal resistance of  $0.5\Omega$  & open circuit voltage of 12V. What is the power lost within the battery and the terminal voltage on full load if a  $3\Omega$  resistor is connected across the terminals of the battery

$$I = 3.43A, P_{lost} = 5.8W$$

$$V_{terminal} = 3 \times 3.43 = 10.29 //$$

# Laplace Transform

The Laplace transform changes a function into a new function by using a process that involves integration. The Laplace transform is named after mathematician and astronomer "De Laplace" who used a similar transform called Z-transform in his work on probability theory.

The powerful practical Laplace transform techniques were developed by the English electrical engineer Oliver Heaviside" and were often called Heaviside calculus.

Very powerful method for solving linear differential equation.

$LDE \xrightarrow{LT}$  algebraic equation  
(solving is difficult)  $\xrightarrow{\text{easy to solve}}$

$\rightarrow$  LT will automatically take care of initial cond's  
procedure

$DE \xrightarrow{LT}$  algebraic  $\xrightarrow{\text{solve}}$  solution in laplace domain  $\xrightarrow{ILT \rightarrow}$  solution is time domain  
excitation i/p  $x(t)$  response o/p  $y(t)$

$\Downarrow$   $v(t)$  or  $i(t)$

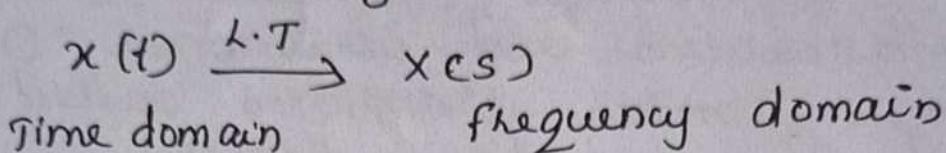
$x(t) \xleftarrow{LT} X(s)$ .

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt - \text{Bilateral Laplace transform}$$

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we need to consider only unilateral transform

$$\therefore X(s) = \int_0^{\infty} x(t) \cdot e^{st} dt.$$



$s$  is defined as

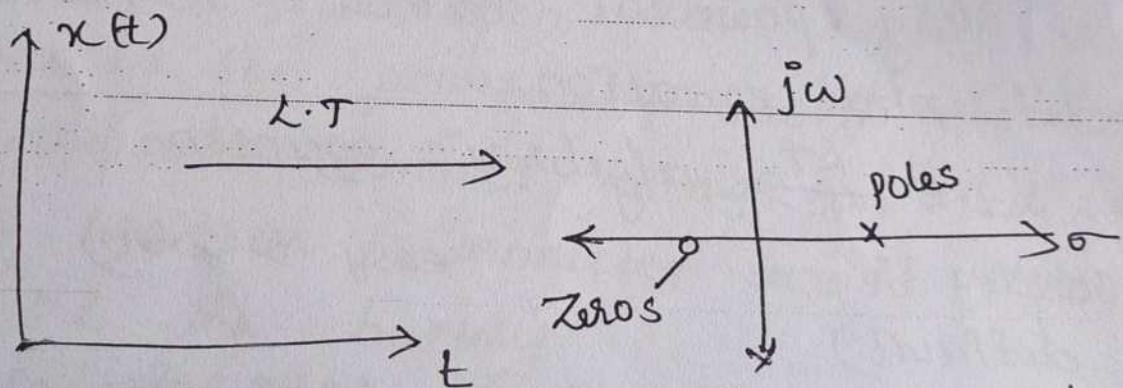
$s \rightarrow$  complex frequency

$$s = \sigma + j\omega$$

$\sigma \rightarrow$  real frequency

$\omega \rightarrow$  angular frequency (rad/s)

$$\sigma, \omega \rightarrow -\infty \text{ to } \infty$$



poles  $\rightarrow$  values of  $s$  for which  $X(s)$  tends to infinity

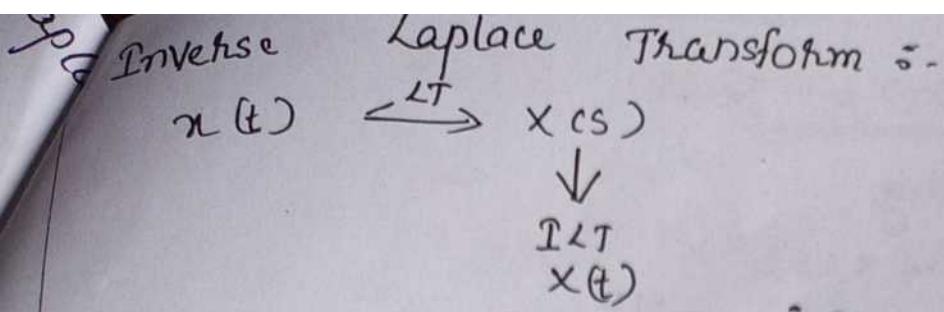
zeros  $\rightarrow$  values of  $s$  for which  $X(s)$  tends to zero

$$\text{eg: if } X(s) = \frac{s}{s^2 - 4}$$

$$\text{poles } s = \pm \sqrt{4} = \pm \sqrt{2}$$

$$\text{zeros } s = 0$$

①



$$x(t) = \frac{1}{2\pi j} \int_{s=\sigma-j\omega}^{s=\sigma+j\omega} X(s) \cdot e^{st} ds$$

Properties of LT

1. Amplitude Scaling

$$\mathcal{L}[x(t)] = X(s)$$

$$\text{then } \mathcal{L}[A(x(t))] = A X(s)$$

Proof:

$$\mathcal{L}[C(x(t))] = \int_0^\infty x(t) \cdot e^{-st} dt = X(s)$$

$$\begin{aligned} \mathcal{L}[A(x(t))] &= \int_0^\infty A(x(t)) \cdot e^{-st} dt = A \int_0^\infty x(t) \cdot e^{-st} dt \\ &= \underline{\underline{A X(s)}} \end{aligned}$$

2. Linearity

$$\text{if } \mathcal{L}[x_1(t)] = X_1(s) \text{ & } \mathcal{L}[x_2(t)] = X_2(s)$$

$$\text{then } \mathcal{L}[A x_1(t) + B x_2(t)] = A X_1(s) + B X_2(s)$$

Proof

$$\begin{aligned} \mathcal{L}[A x_1(t) + B x_2(t)] &= \int_0^\infty [A x_1(t) + B x_2(t)] e^{-st} dt \\ &= \int_0^\infty A x_1(t) \cdot e^{-st} dt + \int_0^\infty B x_2(t) \cdot e^{-st} dt \end{aligned}$$

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$$= A \underline{x_1(s)} + B \underline{x_2(s)}$$

### 3. Time shifting

If  $L[x(t)] = x(s)$

$$\text{then } L[x(t-t_0)] = e^{-st_0} x(s)$$

$$\text{proof : } L[x(t-t_0)] = \int_{t=0}^{t=\infty} x(t-t_0) \cdot e^{-st} dt$$

$$\text{put } p = t - t_0 \quad dp = dt$$

$$t = p + t_0$$

$$\therefore L[x(t-t_0)] = \int_0^{\infty} x(p) \cdot e^{-s(p+t_0)} dp$$

$$= \int_0^{\infty} x(p) \cdot e^{-sp} \cdot e^{-st_0} dp$$

$\underbrace{\phantom{\int_0^{\infty}}}_{x(s)}$

$$= x(s) \cdot e^{-st_0}$$

$$L[x(t-t_0)] = x(s) \cdot e^{-st_0}$$

$$L[e^{at} x(t)] = x(s) \cdot e^{st_0}.$$

### 4. Frequency shifting

If  $L[x(t)] = x(s)$  then

$$L[\bar{e}^{at} x(t)] = x(s+a).$$

Proof

$$L[x(t)] = \int_0^{\infty} x(t) \cdot e^{-st} dt$$

$$L[\bar{e}^{at} x(t)] = \int_0^{\infty} \bar{e}^{at} x(t) \cdot e^{-st} dt$$

(2)

$$\begin{aligned} L[\bar{e}^{at}x(t)] &= \int_0^\infty \bar{e}^{at}x(t) \cdot \bar{e}^{st} dt \\ &= \int_0^\infty x(t) \cdot \bar{e}^{(s+a)t} dt \\ &= \underline{\underline{x(s+a)}} \end{aligned}$$

### 5. Time differentiation

If  $L[x(t)] = X(s)$

then  $L\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0)$

$x(0) = \lim_{t \rightarrow 0} x(t)$  initial value

Proof:  $L[x(t)] = \int_0^\infty x(t) \cdot \bar{e}^{-st} dt$

$$L[\bar{e}^{at}x(t)] = \int_0^\infty \bar{e}^{at}x(t) \cdot \bar{e}^{-st} dt$$

$$= \int_0^\infty x(t) \bar{e}^{(s+a)t} dt$$

$$= \underline{\underline{x(s+a)}}$$

### 5. Time differentiation

If  $L(x(t)) = X(s)$

then  $L\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0)$

$x(0) = \lim_{t \rightarrow 0} x(t)$  initial value

Proof  $L\left[\frac{dx(t)}{dt}\right] = \int_0^\infty \frac{d}{dt}x(t) \cdot \bar{e}^{-st} dt$

$$\begin{aligned}
 &= \left[ e^{-st} x(t) \right]_0^\infty - \int_0^\infty s \cdot e^{-st} \cdot x(t) \cdot dt \\
 &= -x(0) + s \int_0^\infty e^{-st} \cdot x(t) \cdot dt \\
 &= \underline{\underline{s x(s) - x(0)}}
 \end{aligned}$$

### 6. Time integration

If  $L[x(t)] = x(s)$

then  $L \left[ \int_0^t x(t) \cdot dt \right] = \frac{x(s)}{s}$

Proof

$$\begin{aligned}
 L \left[ \int_0^t x(t) \cdot dt \right] &= \int_0^\infty \int_0^t x(t) dt \stackrel{(2)}{=} e^{-st} \cdot dt \\
 &\quad \left[ x(t) \cdot \frac{-e^{-st}}{-s} \right]_0^\infty - \int_0^\infty x(t) \cdot \frac{-e^{-st}}{-s} \\
 &= 0 - 0 + \frac{1}{s} \int_0^\infty x(t) e^{-st} dt \\
 &= \underline{\underline{\frac{x(s)}{s}}}
 \end{aligned}$$

### 7. Frequency differentiation

If  $L[x(t)] = x(s)$

then  $L[t x(t)] = -\frac{d}{ds} x(s)$

Proof:

$$\begin{aligned}
 x(s) &= \int_0^\infty x(t) \cdot e^{-st} dt \\
 \frac{d}{ds} x(s) &\stackrel{(3)}{=} \frac{d}{ds} \int_0^\infty x(t) \cdot e^{-st} dt
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^\infty x(t) \left[ \frac{d}{ds} e^{-st} \right] dt \\
 &= \int_0^\infty -t e^{-st} x(t) dt \\
 &= - \int_0^\infty t x(t) e^{-st} dt \\
 - \frac{d}{ds} x(s) &= \int_0^\infty t x(t) e^{-st} dt \\
 &= L \underbrace{[tx(t)]}_{\longrightarrow}
 \end{aligned}$$

8. Initial Value Theorem:

$$\text{If } L[x(t)] = X(s)$$

According to initial value theorem

$$x(0) = \lim_{s \rightarrow \infty} s x(s)$$

Proof:

$$L \left[ \frac{d}{dt} x(t) \right] = s x(s) - x(0) \quad \text{--- ①}$$

$$L \left[ \frac{d}{dt} x(t) \right] = \int_0^\infty \frac{d}{dt} x(t) \cdot e^{-st} dt$$

Take limit  $s \rightarrow \infty$  on both sides

$$\lim_{s \rightarrow \infty} L \left[ \frac{d}{dt} x(t) \right] = \lim_{s \rightarrow \infty} \int_0^\infty \frac{d}{dt} x(t) \cdot e^{-st} dt$$

$$= \int_0^\infty \frac{d}{dt} x(t) \cdot \lim_{s \rightarrow \infty} e^{-st} dt$$

$$= 0 //$$

②

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① → apply limit  $s \rightarrow \infty$  on both sides

$$\lim_{s \rightarrow \infty} L \left[ \frac{d}{dt} x(t) \right] = \lim_{s \rightarrow \infty} s x(s) - x(0) \quad ③$$

compare ② and ③

$$\lim_{s \rightarrow \infty} s (x(s) - x(0)) = 0$$

$$x(0) = \lim_{s \rightarrow \infty} s x(s)$$

9 Finial value Theorem

$$\text{If } L[x(t)] = x(s)$$

then according to final value theorem

$$\lim_{t \rightarrow \infty} x(t) = x(\infty) = \lim_{s \rightarrow 0} s x(s)$$

Proof

$$L \left[ \frac{d}{dt} x(t) \right] = s x(s) - x(0) \quad ①$$

$$\lim_{s \rightarrow 0} L \left[ \frac{d}{dt} x(t) \right] = \lim_{s \rightarrow 0} s x(s) - x(0) \quad ①$$

$$L \left[ \frac{d}{dt} x(t) \right] = \int_0^{\infty} \frac{d}{dt} x(t) \cdot e^{-st} dt$$

$$\lim_{s \rightarrow 0} L \left[ \frac{d}{dt} x(t) \right] = \lim_{s \rightarrow 0} \int_0^{\infty} \frac{d}{dt} x(t) \cdot e^{-st} dt$$

$$\lim_{s \rightarrow 0} L \left[ \frac{d}{dt} x(t) \right] = \int_0^{\infty} \frac{d}{dt} x(t) \cdot dt = x(t) \quad ②$$

$$\lim_{s \rightarrow 0} L \left[ \frac{d}{dt} x(t) \right] = x(\infty) - x(0) \quad ②$$

Compare ① and ②

$$\lim_{s \rightarrow 0} s x(s) - x(0) = x(\infty) - x(0)$$

$$\lim_{s \rightarrow 0} sX(s) - x(0) = x(\infty) - x(0)$$

$$\boxed{\begin{aligned} \lim_{s \rightarrow 0} sX(s) &= x(\infty) \\ x(\infty) &= \lim_{s \rightarrow 0} sX(s) \end{aligned}}$$

Convolution Theorem :

$$\text{If } L[x_1(t)] = X_1(s) \quad \text{and} \\ L[x_2(t)] = X_2(s) \\ \text{then } L[x_1(t) * x_2(t)] = X_1(s) \cdot X_2(s) \\ x_1(t) * x_2(t) = \int_0^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

$$\text{proof: } L[x_1(t) * x_2(t)] = \int_0^{\infty} x_1(t) * x_2(t) \cdot e^{-st} dt \\ = \int_0^{\infty} \int_0^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau \cdot e^{-st} dt \\ = \int_0^{\infty} x_1(\tau) d\tau \int_0^{\infty} x_2(t-\tau) \cdot e^{-st} dt.$$

$$t-\tau = p; \quad t = p+\tau; \quad dt = dp$$

$$= \int_0^{\infty} x_1(\tau) d\tau \int_0^{\infty} x_2(p) \cdot e^{-s(p+\tau)} dp$$

(5)

M-2-2

$$= \int_0^\infty x_1(\tau) \cdot e^{s\tau} \cdot d\tau \cdot \int_0^\infty x_2(p) \cdot e^{sp} \cdot dp$$

$$= \underline{x_1(s) \cdot x_2(s)}$$

Place Transform of common functions

(i) Impulse  $\delta(t)$

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$X(s) = \int_0^\infty x(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty \delta(t) \cdot e^{-st} \cdot dt$$

$$= 1 \cdot e^{-st} \Big|_{t=0}$$

$$= \underline{\underline{1}}$$

(ii) Unit Step function  $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$X(s) = \int_0^\infty u(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty 1 \cdot e^{-st} \cdot dt = \left[ \frac{e^{-st}}{-s} \right]_0^\infty$$

$$= -\frac{1}{s} \left[ \bar{e} - e \right]$$

$$= \underline{\underline{\frac{1}{s}}}$$

$$\therefore u(t) \xleftrightarrow{LT} \underline{\underline{\frac{1}{s}}}$$

(iii) Ramp function,  $r(t)$

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\therefore x(s) = \int_0^\infty r(t) \cdot e^{-st} dt$$

$$= \int_0^\infty t \cdot e^{-st} dt$$

$$= \left[ t \cdot \frac{-e^{-st}}{-s} + \int_0^\infty \frac{-e^{-st}}{s} dt \right]_0^\infty$$

$$= \left[ \frac{t e^{-st}}{-s} + \frac{e^{-st}}{-s^2} \right]_0^\infty$$

$$= 0 - 0 + \frac{(0-1)}{-s^2}$$

$$= \underline{\underline{\frac{1}{s^2}}}$$

$$\therefore r(t) \xleftrightarrow{LT} \underline{\underline{\frac{1}{s^2}}}$$

(iv) Exponential function  $x(t)$ .

$$x(t) = e^{at}$$

$$x(s) = \int_0^\infty x(t) \cdot e^{-st} dt.$$

$$\begin{aligned}
 &= \int_0^\infty e^{at} \cdot e^{-st} dt \\
 i.e. x(s) &= \int_0^\infty e^{-(s+a)t} dt \\
 &= \left[ \frac{-e^{(s+a)t}}{(s+a)} \right]_0^\infty \\
 &= \frac{-1}{s+a} \left[ -e^{-\infty} - e^0 \right] \\
 &= \frac{-1}{s+a} [-1] = \underline{\underline{\frac{1}{s+a}}}
 \end{aligned}$$

$$\underline{\underline{e^{at}}} \quad \xleftrightarrow{L.T} \quad \underline{\underline{\frac{1}{s+a}}}$$

### (V) sinusoidal function

$$\begin{aligned}
 x(t) &= \sin \omega t \\
 x(s) &= \int_0^\infty x(t) \cdot e^{-st} dt \\
 &= \int_0^\infty \sin \omega t \cdot e^{-st} dt \\
 &= \int_0^\infty \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \cdot e^{-st} dt \\
 &= \frac{1}{2j} \int_0^\infty e^{j\omega t} e^{-st} - e^{-j\omega t} e^{-st} dt
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2j} \left\{ \int_0^\infty e^{(s-j\omega)t} dt - \int_0^\infty e^{(s+j\omega)t} dt \right\} \\
 &= \frac{1}{2j} \left\{ \frac{e^{(s-j\omega)t}}{-s+j\omega} - \frac{e^{(s+j\omega)t}}{-s-j\omega} \right\}_0^\infty \\
 &= \frac{1}{2j} \left\{ \frac{-1}{-s+j\omega} - \frac{-1}{-s-j\omega} \right\} \\
 X(s) &= \frac{1}{2j} \left\{ \frac{1}{(s-j\omega)} - \frac{1}{s+j\omega} \right\} \\
 &= \frac{1}{2j} \left\{ \frac{(s+j\omega) - (s-j\omega)}{(s+j\omega)(s-j\omega)} \right\} \\
 &= \frac{1}{2j} \left\{ \frac{2j\omega}{s^2 + \omega^2} \right\} \\
 &= \frac{\omega}{s^2 + \omega^2}
 \end{aligned}$$

$$\text{Sin}\omega t \xleftrightarrow{L.T} \frac{\omega}{s^2 + \omega^2}$$

(vi) Cosinusoidal function.

$$\begin{aligned}
 x(t) &= \cos \omega t \\
 x(s) &= \int_0^\infty \cos \omega t \cdot e^{-st} dt \\
 &= \int_0^\infty \frac{e^{j\omega t} + e^{-j\omega t}}{2} \cdot e^{-st} dt
 \end{aligned}$$

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$$\begin{aligned}
 &= \frac{1}{2} \int_0^\infty (e^{j\omega t} \cdot e^{-st} + e^{j\omega t} \cdot \bar{e}^{-st}) dt \\
 &= \frac{1}{2} \int_0^\infty e^{(s-j\omega)t} + e^{-(s+j\omega)t} dt \\
 &= \frac{1}{2} \left[ \frac{e^{-(s-j\omega)t}}{-(s-j\omega)} + \frac{e^{-(s+j\omega)t}}{-(s+j\omega)} \right]_0^\infty
 \end{aligned}$$

$$\begin{aligned}
 X(s) &= \frac{1}{2} \left[ \frac{-1}{s-j\omega} + \frac{-1}{-(s+j\omega)} \right] \\
 &= \frac{1}{2} \left[ \frac{1}{s-j\omega} + \frac{1}{s+j\omega} \right] \\
 &= \frac{1}{2} \left[ \frac{s+j\omega + s-j\omega}{(s-j\omega)(s+j\omega)} \right] \\
 &= \cancel{\frac{1}{2}} \left[ \frac{s}{s^2 + \omega^2} \right] \\
 &= \frac{s}{s^2 + \omega^2}
 \end{aligned}$$

$$\cos \omega t \leftrightarrow \frac{s}{s^2 + \omega^2}$$

: (ii) parabolic function

$$x(t) = \begin{cases} t^2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$X(s) = \int_0^\infty t^2 \cdot e^{-st} dt$$

$$\begin{aligned}
 X(s) &= \int_0^\infty t^2 \cdot e^{-st} \cdot dt \\
 &= \left[ t^2 \cdot \frac{-e^{-st}}{-s} \right]_0^\infty - \int_0^\infty 2t \cdot \frac{-e^{-st}}{-s} \cdot dt \\
 &= \frac{2}{s} \int_0^\infty t^2 e^{-st} \cdot dt \\
 &= \frac{2}{s} \left[ \frac{1}{s^2} \right] = \underline{\underline{\frac{2}{s^3}}}
 \end{aligned}$$

$$\therefore t^2 \xleftrightarrow{LT} \underline{\underline{\frac{2}{s^3}}}$$

(viii) Hyperbolic sinusoidal function

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$$\begin{aligned}
 x(t) &= \sinh \omega t \\
 X(s) &= \int_0^\infty \sinh \omega t \cdot e^{-st} \cdot dt \\
 &= \int_0^\infty \frac{e^{\omega t} - e^{-\omega t}}{2} \cdot e^{-st} \cdot dt \\
 &= \frac{1}{2} \int_0^\infty \left( e^{-(s-\omega)t} - e^{-(s+\omega)t} \right) dt \\
 &= \frac{1}{2} \left[ \frac{e^{-(s-\omega)t}}{-(s-\omega)} - \frac{e^{-(s+\omega)t}}{-(s+\omega)} \right]_0^\infty \\
 &= \frac{1}{2} \left[ \frac{0-1}{-(s-\omega)} - \frac{(0-1)}{-(s+\omega)} \right]
 \end{aligned}$$

(?)

M-2-27

$$\begin{aligned}
 &= \frac{1}{2} \left[ \frac{1}{s-\omega} - \frac{1}{s+\omega} \right] \\
 &= \frac{1}{2} \left[ \frac{(s+\omega) - (s-\omega)}{(s-\omega)(s+\omega)} \right] = \frac{1}{2} \left[ \frac{2\omega}{s^2 - \omega^2} \right] \\
 \sinh \omega t &\xleftrightarrow{\text{L.T}} \frac{\omega}{s^2 - \omega^2}
 \end{aligned}$$

(ix) Hyperbolic cosinoidal functions.

$$r = \cosh \omega t$$

$$X(s) = \int_0^\infty \cosh \omega t \cdot e^{-st} dt$$

$$= \int_0^\infty \frac{e^{\omega t} + e^{-\omega t}}{2} \cdot e^{-st} dt$$

$$= \frac{1}{2} \int_0^\infty (e^{-(s-\omega)t} + e^{-(s+\omega)t}) dt$$

$$= \frac{1}{2} \left[ \frac{e^{-(s-\omega)t}}{-(s-\omega)} + \frac{e^{-(s+\omega)t}}{-(s+\omega)} \right]_0^\infty$$

$$= \frac{1}{2} \left[ \frac{-1}{-(s-\omega)} + \frac{-1}{-(s+\omega)} \right]_0^\infty$$

$$= \frac{1}{2} \left[ \frac{1}{s-\omega} + \frac{1}{s+\omega} \right]$$

$$= \frac{1}{2} \left[ \frac{s+\omega + s-\omega}{s^2 - \omega^2} \right] = \frac{s}{s^2 - \omega^2}$$

$$\cosh \omega t \xleftrightarrow{L.T} \frac{s}{s - \omega^2}$$

(\*) Laplace Transform  $\frac{1}{s^n}$

$$x(t) = t^n$$

$$X(s) = \int_{-\infty}^{\infty} e^{-st} \cdot t^n dt$$

$$= \left[ t^n \cdot \frac{e^{-st}}{-s} \right]_0^\infty - \int_0^\infty n \cdot t^{n-1} \cdot \frac{e^{-st}}{-s} dt$$

$$= \frac{n}{s} \int_0^\infty t^{n-1} \cdot \frac{e^{-st}}{-s} dt - \int_0^\infty (n-1) t^{n-2} \cdot \frac{e^{-st}}{-s} dt$$

$$= \frac{n(n-1)}{s^2} \int_0^\infty t^{n-2} \cdot e^{-st} dt$$

$$= \frac{n}{s} \frac{(n-1)}{s} L\left\{ t^{n-2} \right\}$$

$$= \frac{n}{s} \frac{(n-1)}{s} \cdot \frac{n-2}{s} \cdots \frac{2}{s} \cdot L\left\{ t^1 \right\}$$

$$= \frac{n}{s} \cdot \frac{n-1}{s} \cdot \frac{n-2}{s} \cdots \frac{3}{s} \cdot \frac{2}{s} \cdot \frac{1}{s^2}$$

$$= \frac{n}{s} \cdot \frac{n-1}{s} \cdot \frac{n-2}{s} \cdots \frac{3}{s} \cdot \frac{2}{s} \cdot \frac{1}{s} \cdot \frac{1}{s}$$

$$= \frac{n!}{s^n} \cdot \frac{1}{s}$$

$$= \frac{L^n}{s^{n+1}}$$

$$t^n \xleftrightarrow{L.T} \frac{L^n}{s^{n+1}}$$

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M-2-29

Q Find the LT of  $x(t) = e^{-\alpha t} \cos \omega t$  where  $\alpha$  is constant

$$\cos \omega t \xleftrightarrow{LT} \frac{s}{s^2 + \omega^2}$$

$$e^{-\alpha t} \cos \omega t \xleftrightarrow{LT} \frac{s}{s^2 + \omega^2} \quad \left| \begin{array}{l} \text{Frequency shifting} \\ s = s + \alpha \end{array} \right. \quad \text{property}$$

$$= \frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$$

Q Find LT of  $x(t) = 1 - e^{-at}$

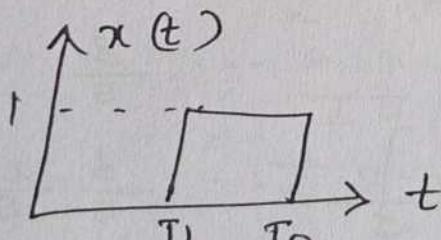
$$x(s) = LT\{1\} - LT\{e^{-at}\}$$

$$= \frac{1}{s} - \frac{1}{s + a}$$

$$= \frac{(s + a) - s}{s(s + a)}$$

$$= \frac{a}{s(s + a)}$$

Q obtain the LT of the pulse given



$$\therefore x(s) = \int_0^\infty x(t) \cdot e^{-st} \cdot dt$$

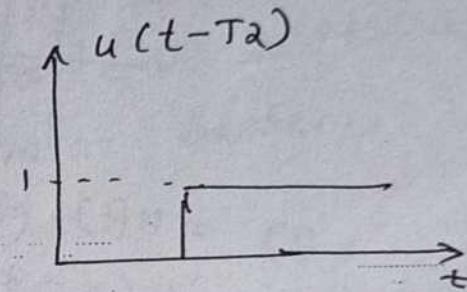
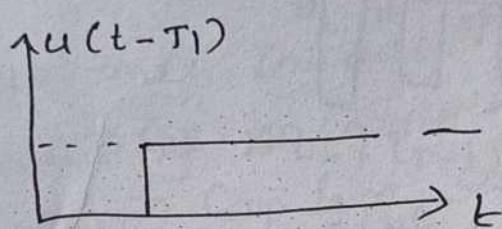
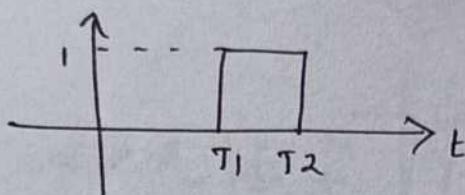
$$= \int_{T_1}^{T_2} 1 \cdot e^{-st} \cdot dt = \left[ \frac{e^{-st}}{-s} \right]_{T_1}^{T_2}$$

$$= \left[ \frac{e^{-sT_2}}{-s} - \frac{e^{-sT_1}}{-s} \right]$$

$$= \frac{-ST_2 - ST_1}{e^{-s} - e}$$

$$= \frac{-ST_1 - ST_2}{e^s - e}$$

$x(t)$  (or)

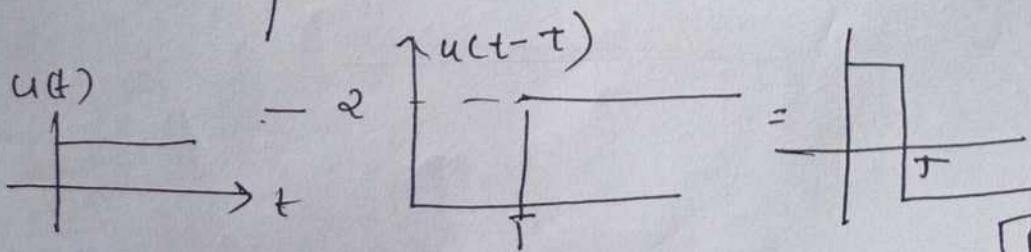
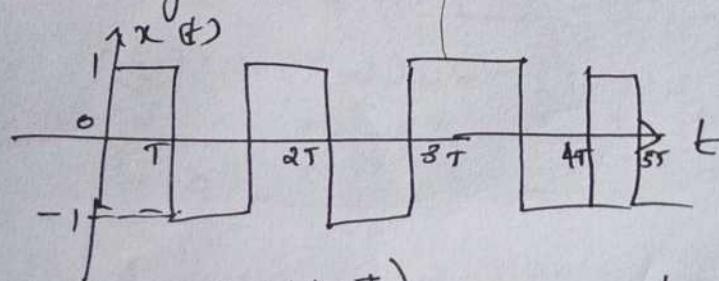


$$x(t) = u(t - T_1) - u(t - T_2)$$

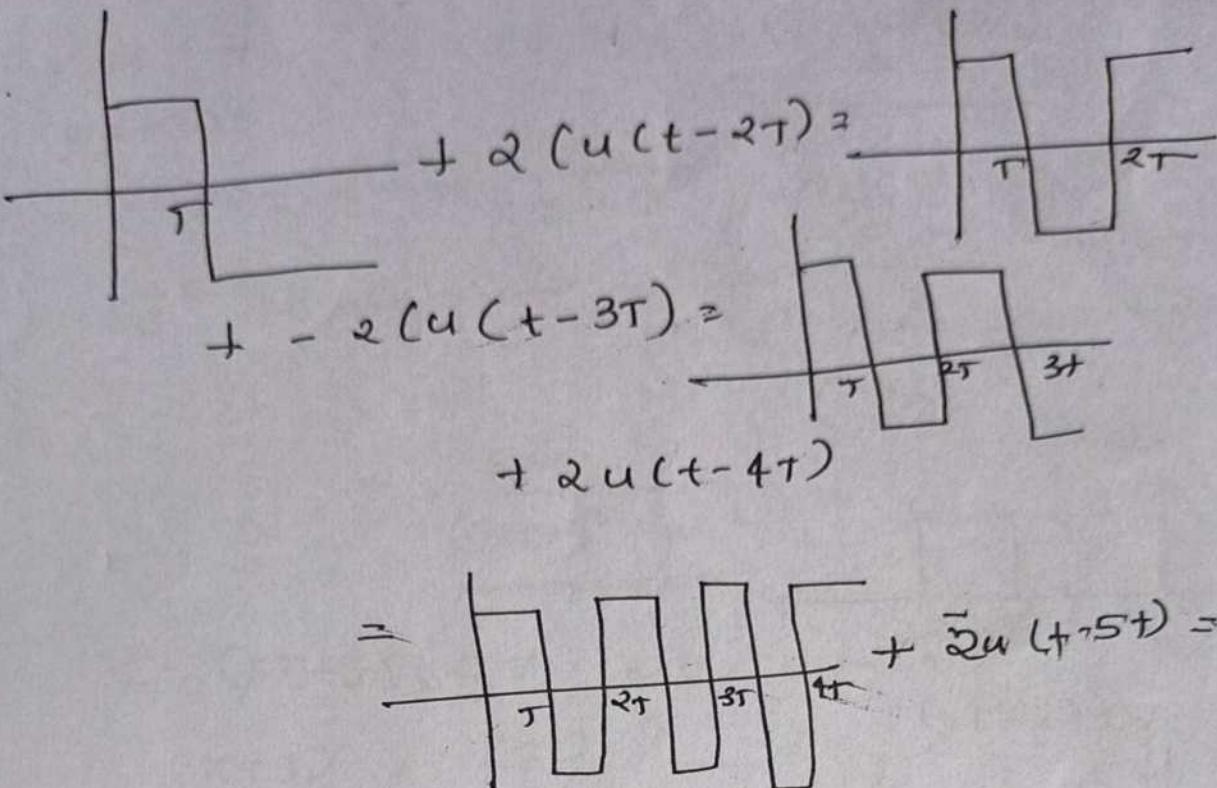
$$X(s) = L\{u(t - T_1)\} - L\{u(t - T_2)\}$$

$$= \frac{e^{-ST_1}}{s} - \frac{e^{-ST_2}}{s} = \frac{e^{-ST_1} - e^{-ST_2}}{s}$$

Q Obtain the Laplace Transform of the square wave given

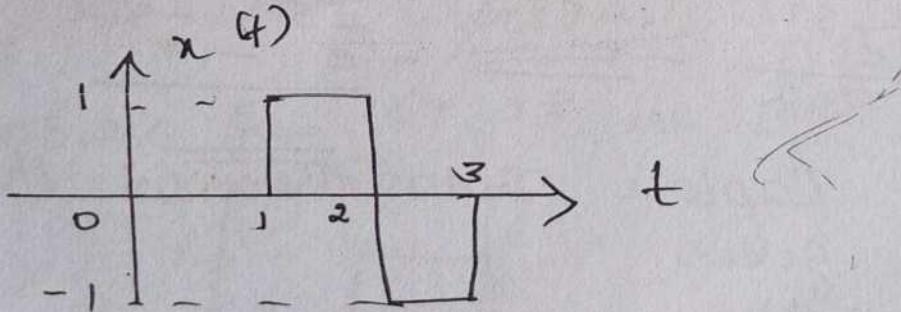


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$$\therefore x(t) = u(t) - 2u(t-T) + 2u(t-2T) - 2u(t-3T) + 2u(t-4T) - 2u(t-5T) + \dots$$

$$X(s) = \frac{1}{s} - \frac{2e^{-st}}{s} + \frac{2e^{-2st}}{s} - \frac{2e^{-3st}}{s} + \frac{2e^{-4st}}{s} + \dots$$



$$x(t) = u(t-1) - 2u(t-2) + u(t-3)$$

$$X(s) = \frac{1 \cdot e^{-s}}{s} - \frac{2e^{-2s}}{s} + \frac{e^{-3s}}{s}$$

Given  $X(s) = \frac{2}{s} - \frac{1}{s+3}$ . Find  $x(\infty)$   
According to final value theorem

$$x(\infty) = \lim_{s \rightarrow 0} sX(s)$$

$$= \lim_{s \rightarrow 0} \left( 2 - \frac{3}{s+3} \right)$$

$$= \underline{\underline{2}}$$

Q: Given  $X(s) = M \begin{bmatrix} (s+\alpha)\sin\theta & +\beta \cos\theta \\ (s+\alpha)^2 + \beta^2 & (s+\alpha)^2 + \beta^2 \end{bmatrix}$

Show that initial value  $x(0)$  is  $M\sin\theta$

According to initial value theorem,

$$x(0) = \lim_{s \rightarrow \infty} sX(s)$$

$$= M \lim_{s \rightarrow \infty} \begin{bmatrix} s(s+\alpha)\sin\theta & +\beta \cos\theta \cdot s \\ (s+\alpha)^2 + \beta^2 & (s+\alpha)^2 + \beta^2 \end{bmatrix}$$

$$= M \lim_{s \rightarrow \infty} \begin{bmatrix} s^2(1 + \frac{\alpha}{s})\sin\theta & + \cancel{s} \frac{\beta \cos\theta \cdot s}{(1 + \alpha/s)^2 + \beta^2} \\ \cancel{s^2} (1 + \alpha/s)^2 + \beta^2 & \cancel{s^2} (1 + \alpha/s)^2 + \beta^2 \end{bmatrix}$$

$$= M \lim_{s \rightarrow \infty} \begin{bmatrix} (1 + \alpha/s)\sin\theta & + \frac{\beta \cos\theta}{s} \\ (1 + \alpha/s)^2 + \beta^2 & (1 + \alpha/s)^2 + \beta^2 \end{bmatrix}$$

$$= M \begin{bmatrix} (1 + 0)\sin\theta & + \frac{\beta \cos\theta}{s} \\ 1 + 0 & (1 + 0)^2 + \beta^2 \end{bmatrix}$$

$$= \underline{\underline{M \sin\theta}}$$

End