

MODULE 05

# DESIGN OF SPRINGS

**Spring** is defined as an elastic machine element(flexible element) that deflects under the action of load and returns to its original shape when load is removed.

## **IMPORTANT FUNCTIONS AND APPLICATIONS OF SPRING**

1. Springs are used to absorb shocks and vibrations

eg: vehicle suspension springs, railway buffers to control energy, buffer springs in elevators and vibration mounts for machinery.

2. Measuring forces :Spring balances, gages

3. Storing of energy in clocks ,toys ,movie cameras, circuit breakers ,starters

4. Springs are used to apply force and control motion.

# TYPES OF SPRINGS

1. Helical coil springs

a) helical compression spring ;b)helical extension spring ;c)helical torsion spring.

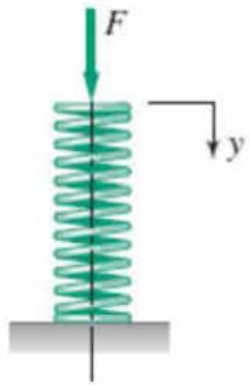
1. Torsion bar springs

2. Leaf spring (beam spring)

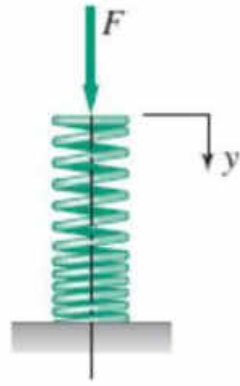
3. Volute springs

4. Pneumatic spring

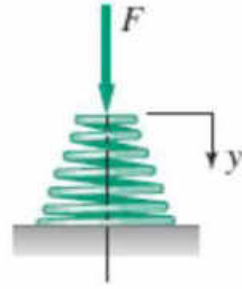
5. Belleville spring(coned disk spring)



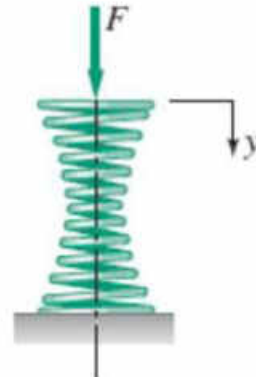
(a) Standard compression; fixed pitch; linear; constant rate; pushes.



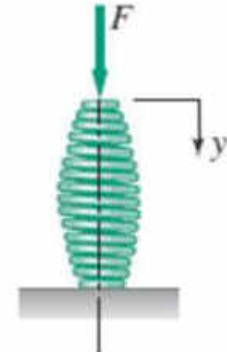
(b) Variable pitch; nonlinear; pushes; resists resonance.



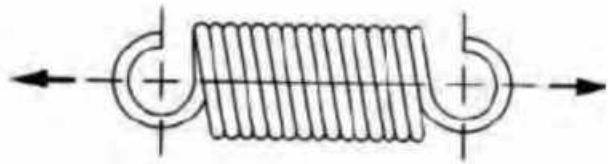
(c) Conical; linear or hardening; pushes; minimum solid height.



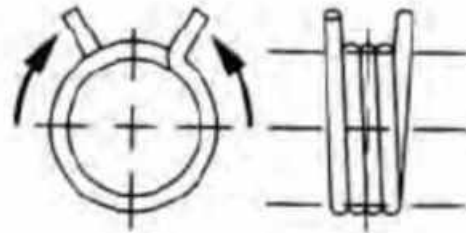
(d) Hourglass; nonlinear; pushes; resists resonance.



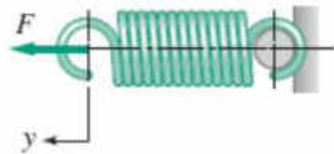
(e) Barrel; nonlinear; pushes; resists resonance.



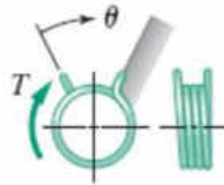
(b) Helical extension spring



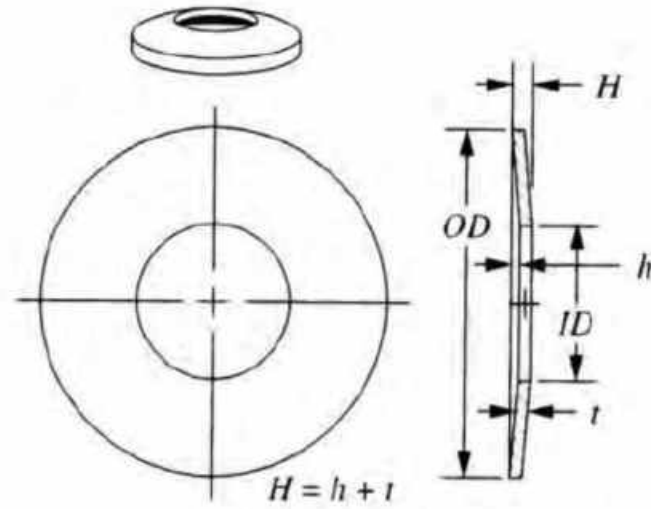
(d) Helical torsion spring



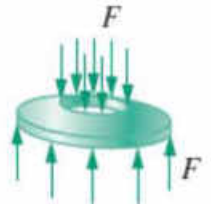
(f) Standard closed-coil extension; linear after coils open; pulls.



(h) Helical torsion; linear; constant rate; twists.



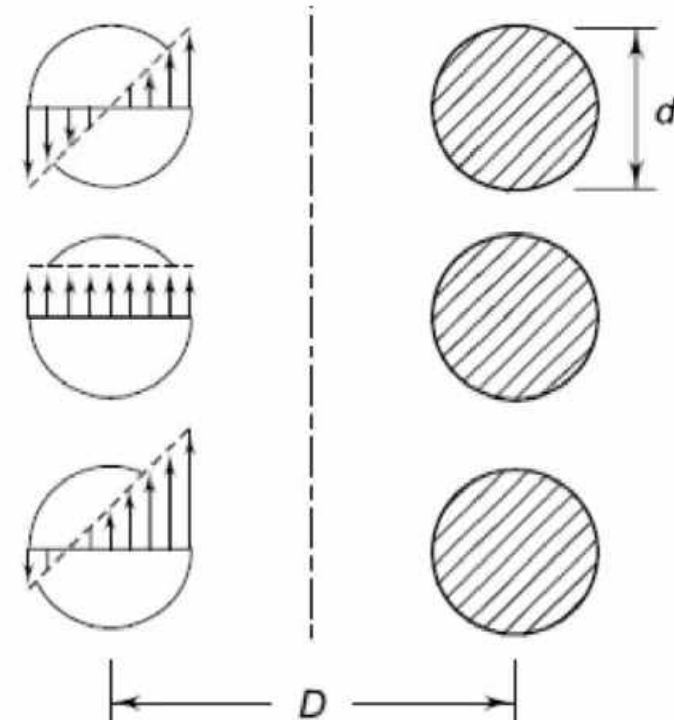
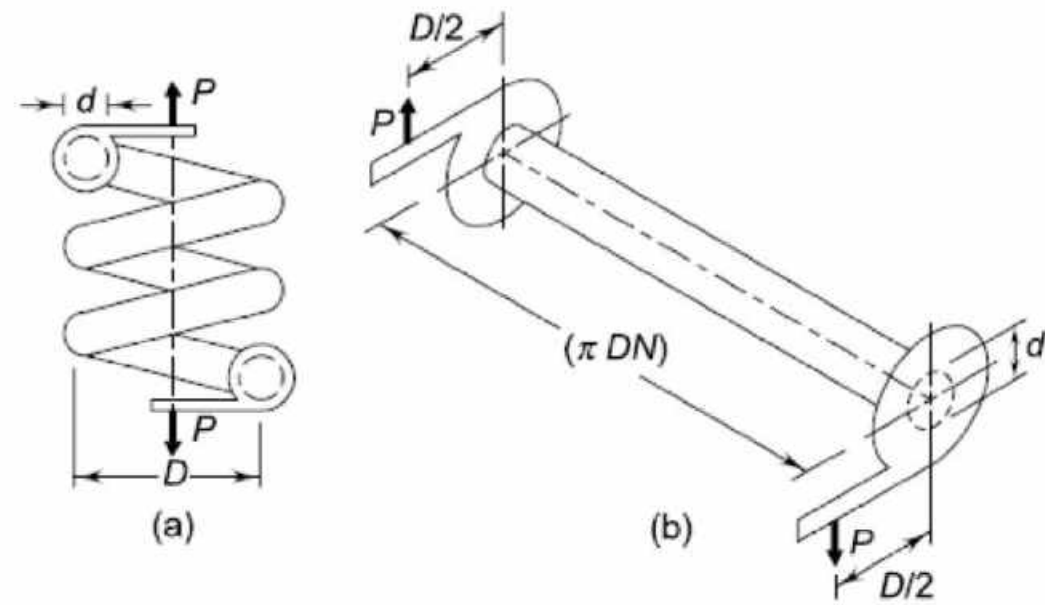
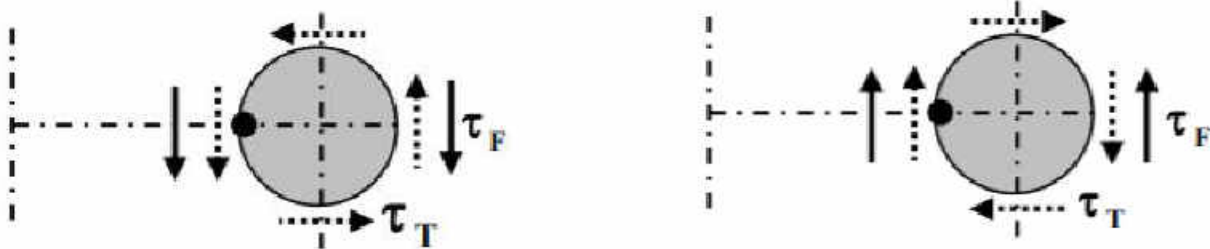
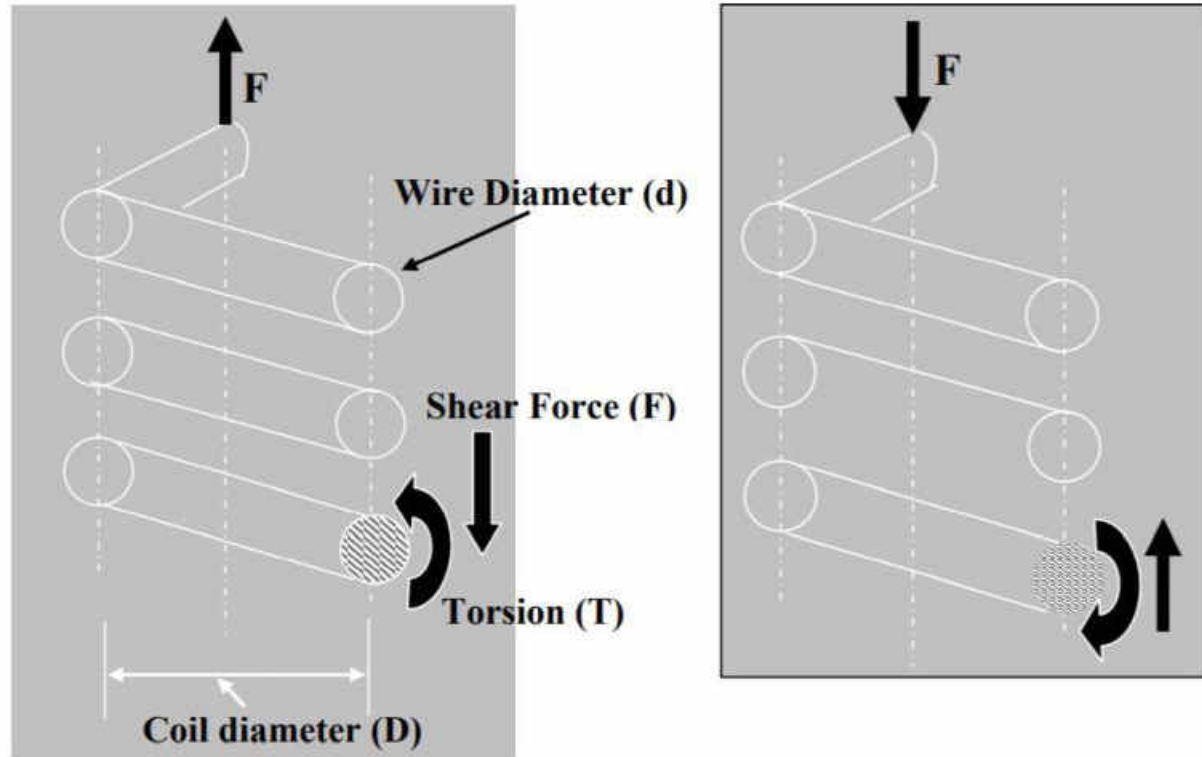
(e) Belleville spring



(f) Bellville washer; high loads; nonlinear; pushes.

# **STRESS ,DEFLECTION AND SPRING RATE OF AXIALLY LOADED HELICAL SPRINGS**

# STRESSES IN HELICAL SPRING





$$T = F \times \frac{D}{2}$$

$$J = \frac{\pi d^4}{32}$$

shear stress in the spring wire due to torsion is

$$\tau_T = \frac{Tr}{J} = \frac{F \times \frac{D}{2} \times \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{8FD}{\pi d^3}$$

Average shear stress in the spring wire due to force F is

$$\tau_F = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

Therefore, maximum shear stress the spring wire is

$$\tau_T + \tau_F = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$\text{or } \tau_{\max} = \frac{8FD}{\pi d^3} \left( 1 + \frac{1}{\frac{2D}{d}} \right)$$

$$\text{or } \tau_{\max} = \frac{8FD}{\pi d^3} \left( 1 + \frac{1}{2C} \right)$$

where,  $C = \frac{D}{d}$ , is called the spring index.

$$\text{finally, } \tau_{\max} = (K_s) \frac{8FD}{\pi d^3} \quad \text{where, } K_s = 1 + \frac{1}{2C}$$

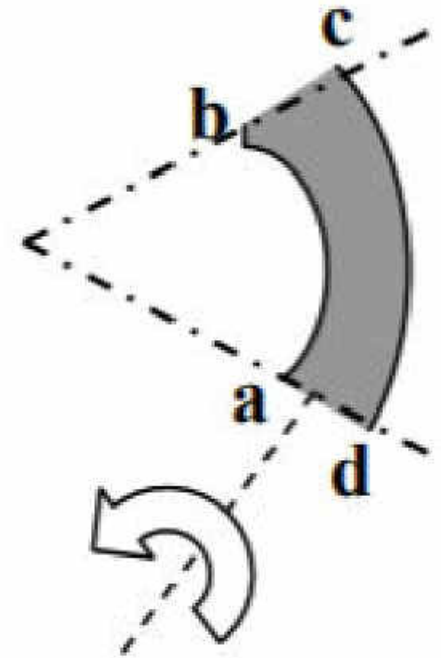
The above equation gives maximum shear stress occurring in a spring.  $K_s$  is the shear stress correction factor.

# CURVATURE EFFECT

For springs where the wire diameter is comparable with the coil diameter, in a given segment of the spring, the inside length of the spring segment is relatively shorter than the outside length.

Hence, for a given magnitude of torsion, shearing strain is more in the inner segment than the outer segment

This unequal shearing strain is called the **curvature effect**.



Curvature effect decreases with the increase in spring index.

So more is the spring index ( $C = D/d$ ) the lesser will be the curvature effect.

For example, the suspensions in the railway carriages use helical springs.

These springs have large wire diameter compared to the diameter of the spring itself.

In this case curvature effect will be predominantly high.



In the design of helical springs, the designer should use good judgement in assuming the value of the spring index  $C$ . The spring index indicates the relative sharpness of the curvature of the coil. A low spring index means high sharpness of curvature. When the spring index is low ( $C < 3$ ), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high ( $C > 15$ ), it results in large variation in the coil diameter. Such a spring is prone to buckling and also tangles easily during handling. A spring index from 4 to 12 is considered best from manufacturing considerations. Therefore, in practical applications, the spring index usually varies from 4 to 12. However, a spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

To take care of the curvature effect, the earlier equation for maximum shear stress in the spring wire is modified as,

$$\tau_{\max} = (K_w) \frac{8FD}{\pi d^3}$$

Where,  $K_w$  is Wahl correction factor, which takes care of both curvature effect and shear stress correction factor and is expressed as,

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

# Derivation of deflection in helical compression spring

The equation of torsion is  $\frac{M_t}{J} = \frac{G\theta}{l}$ .

$\theta$  = angle of twist in radians. For small angle,  $\tan \theta = \theta = \frac{y}{R}$  and  $l$  = length of wire =  $(\pi D) \cdot i$

Substituting,  $\frac{(F \cdot R)}{\left(\frac{\pi d^4}{32}\right)} = \frac{G \cdot (y/R)}{i \pi D}$

$$\frac{F \cdot \frac{D}{2} \times 32}{\pi d^4} = \frac{G \cdot y}{i \pi D \cdot \frac{D}{2}}$$

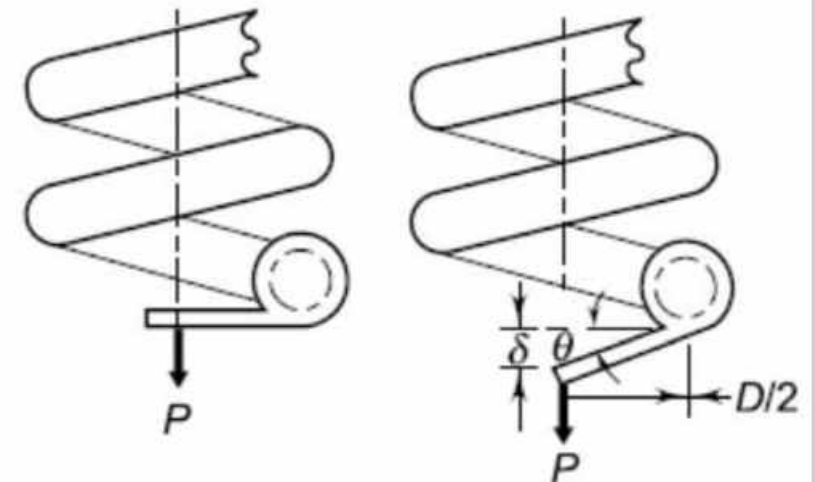
$\therefore$  Deflection  $y = \frac{F \cdot \frac{D}{2} \times 32 \times i \pi D \cdot \frac{D}{2}}{\pi d^4 G}$

$$y = \frac{8FD^3i}{Gd^4} = \frac{8FC^3i}{Gd} \quad \left( \because C = \frac{D}{d} \right)$$

Knowing the deflection for a given load, one can easily estimate the no. of active turns requires using

$$i = \frac{yGd^4}{8FD^3} = \frac{yGd}{8FC^3}$$

$$\begin{aligned} \delta &= \theta \times (\text{length of bracket}) \\ &= \theta \times (D/2) \end{aligned}$$



# Deflection in helical spring

Deflection in a helical spring,  $\delta$

$$y = \frac{8FD^3i}{Gd^4}$$

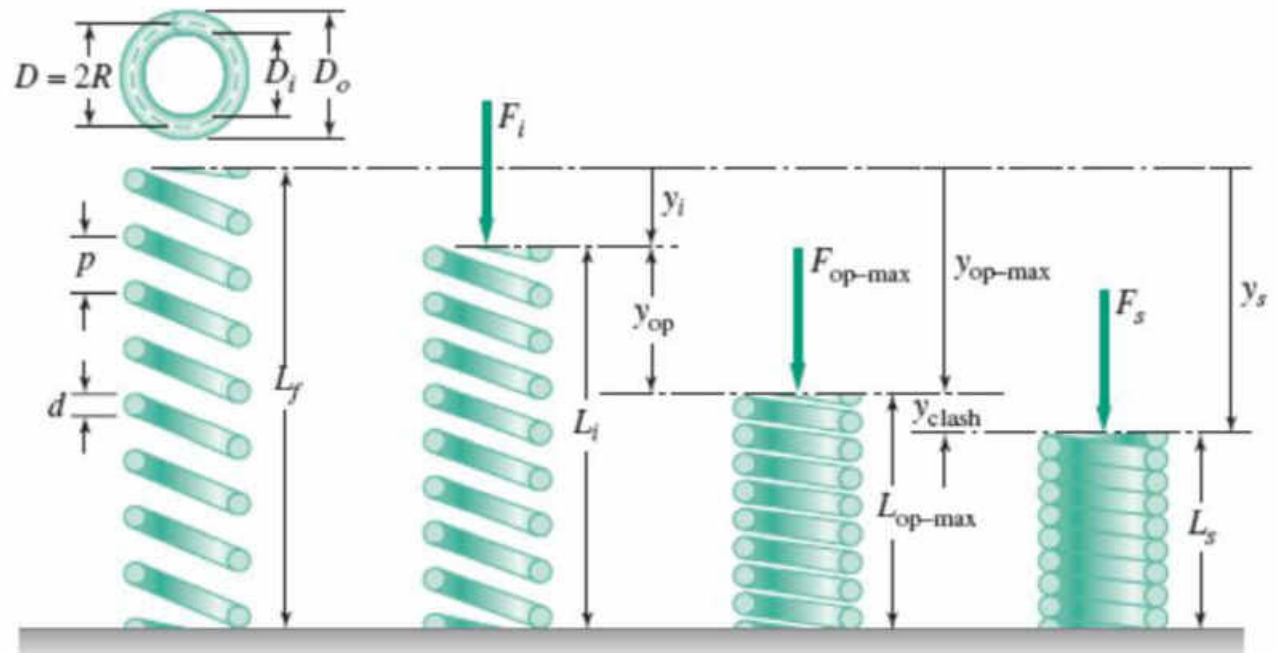
Design parameter (spring rate or stiffness)

$$F_0 = \frac{F}{y} = \frac{Gd^4}{8D^3i}$$

- $i$  : number of active coils
- $F$  : Axial force
- $D$ : Mean diameter of coil
- $G$ : Modulus of rigidity
- $d$ : diameter of spring wire
- $F_0$  : spring stiffness ,spring rate

# TERMINOLOGY OF HELICAL SPRINGS

The design of helical-coil springs involves selection of a material, and determination of the wire diameter,  $d$ , mean coil radius,  $R$ , number of active coils,  $N$ , and other spring parameters so that the desired force-deflection response is obtained, without exceeding the design stress under the most severe operating conditions.



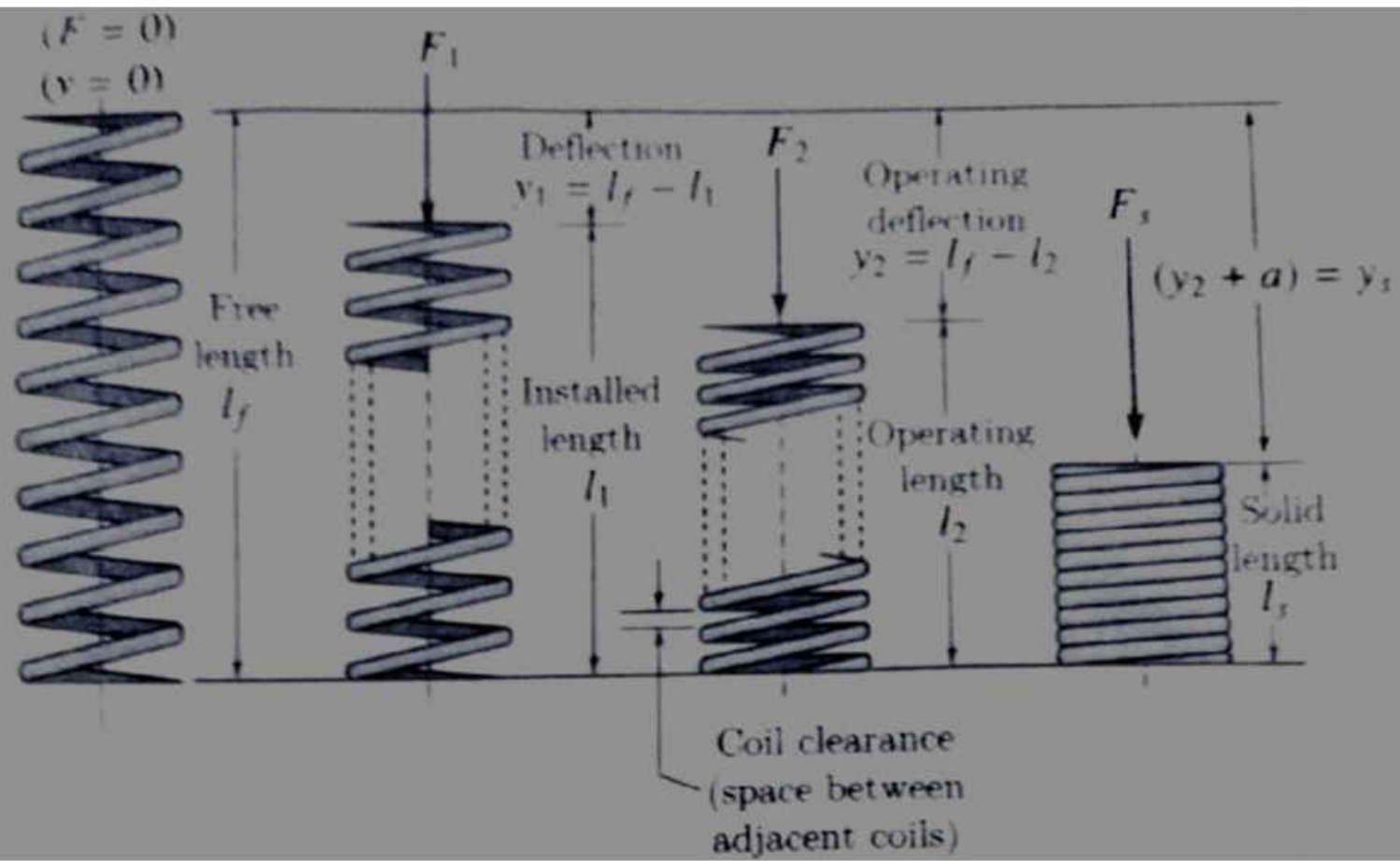
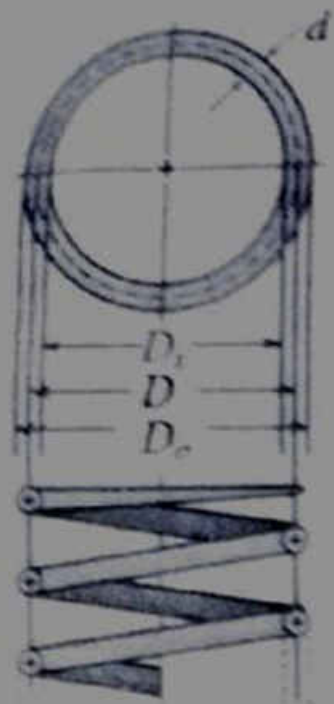
(b) Helical-coil compression spring at free length,  $L_f$ , no load.

(c) Same spring with initial preload,  $F_i$ .

(d) Same spring at maximum operating load.

(e) Same spring at solid height,  $L_s$  (to be avoided).



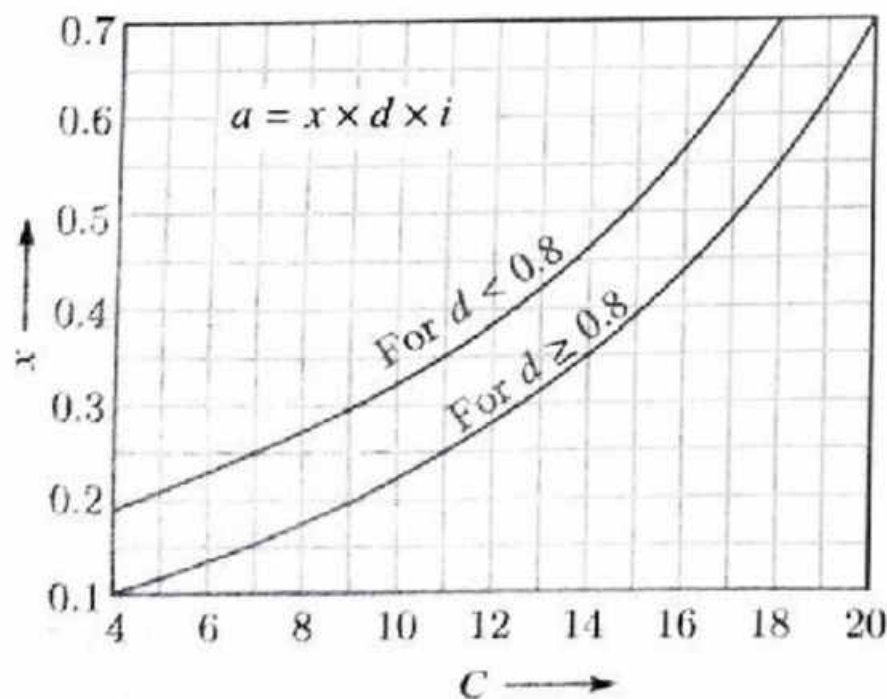


### Minimum free length of the spring: (Table 11.4)

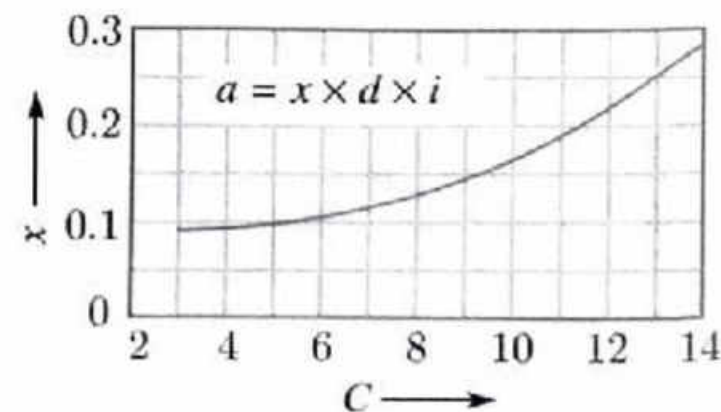
If the ends are bent before grinding	$l_o \geq (i + 2)d + y + a$	11.18(a)
If the ends are either ground or bent	$l_o \geq (i + 1)d + y + a$	11.18(b)
If the ends are neither ground nor bent	$l_o \geq (i)d + y + a$	11.18(c)

where  $a = xdi$ , total clearance between working coils, mm

$i$  is the number of active coils (Table 11.4)



(a) Cold coiled springs



(b) Hot coiled springs

**Fig. 11.6: Value of  $x$  as a function of coil ratio ( $C$ )**

# DESIGN OF SPRINGS

- It should possess sufficient strength to withstand the external load.
- It should have the required load deflection characteristics
- It should not buckle under the external load.

# DESIGN OF HELICAL SPRINGS

## DESIGN OF HELICAL SPRINGS

- (i) For the given application, estimate the maximum spring force ( $F$ ) and the corresponding required deflection ( $y$ ) of the spring. In some cases, maximum spring force ( $F$ ) and stiffness  $F_0$  which is ( $F/y$ ) are specified.
- (ii) Select a suitable spring material and find out ultimate tensile strength ( $S_{ut}$ ) from the data. Calculate the permissible shear stress for the spring wire by following relationship:  $\tau = 0.30 S_{ut}$  or  $0.50 S_{ut}$
- (iii) Assume a suitable value for the spring index ( $C$ ). For industrial applications, the spring index varies from 8 to 10. A spring index of 8 is considered as a good value. The spring index for springs in valves and clutches is 5. The spring index should never be less than 3.
- (iv) Calculate the Wahl factor by the following equation:

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

- (v) Determine wire diameter ( $d$ ) by Eq.  $\tau = K \left( \frac{8FC}{\pi d^3} \right)$

# DESIGN OF HELICAL SPRINGS

- (vi) Determine mean coil diameter ( $D$ ) by the following relationship:  $D = Cd$
- (vii) Determine the number of active coils ( $i$ ) by Eq.  $y = \frac{8FD^3 i}{Gd^4}$  The modulus of rigidity ( $G$ ) for steel wires is  $81\,370\text{ N/mm}^2$ .
- (viii) Decide the style of ends for the spring depending upon the configuration of the application. Determine the number of inactive coils. Adding active and inactive coils, find out the total number of coils  $i_t$
- (ix) Determine the solid length of the spring by the following relationship: Solid length =  $i_t d$
- (x) Determine the actual deflection of the spring by Eq.  $y = \frac{8FD^3 N}{Gd^4}$



**Factor of Safety** The factor of safety in the design of springs is usually 1.5 or less. The use of a relatively low factor of safety is justified on the following grounds:

- (i) In most of the applications, springs operate with well defined deflections. Therefore, the forces acting on the spring and corresponding stresses can be precisely calculated. It is not necessary to take higher factor of safety to account for uncertainty in external forces acting on the spring.
- (ii) In case of helical compression springs, an overload will simply close up the gaps between coils without a dangerous increase in deflection and stresses.
- (iii) In case of helical extension springs, usually overload stops are provided to prevent excessive deflection and stresses.
- (iv) The spring material is carefully controlled at all stages of manufacturing. The thin and uniform wire cross-section permits uniform heat treatment and cold working of the entire spring.

## DESIGN PROCEDURE OF HELICAL SPRINGS

Given load  $F$ , spring index ' $C$ ', deflection ' $y$ ', allowable shear stress ' $\tau$ ' and the rigidity modulus ' $G$ '.

**Step 1.** Design of wire diameter.

$$\text{Shear stress} \quad \tau = \frac{8FCK}{\pi d^3}$$

where  $K = \text{curvature factor} = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

Find wire diameter ' $d$ '. select standard wire diameter. (Nearest higher value is to be adopted if calculated value is not standard one.)

**Step 2.** Mean coil diameter  $D = Cd$

Inside diameter of coil  $D_i = D - d$

Outside diameter of coil  $D_o = D + d$